

Inflation in general covariant Hořava-Lifshitz gravity without projectability

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ABSTRACT: In this paper, we study inflation in the general covariant Hořava-Lifshitz gravity without the projectability condition. We write down explicitly the equations of the linear scalar perturbations of the FRW universe for a single scalar field without specifying to any gauge. Applying these equations to a particular gauge, we are able to obtain a master equation of the perturbations, in contrast to all the other versions of the theory without the projectability condition. This is because in the current version of the theory it has the same degree of freedom of general relativity. To calculate the power spectrum and index, we first define the initial conditions as the ones that minimize the energy of the ground state. Then, we obtain the full solutions of the equation of motion. From these solutions, we calculate the power spectrum and spectrum index of the comoving curvature perturbations and find the corrections due to the high order spatial derivative terms of the theory to those standard results obtained in general relativity. Remarkably, partly of corrections are a direct consequence of the non-projectability condition. It is also shown that the perturbations are still of scale-invariance, and the results obtained in the general covariant Hořava-Lifshitz gravity without the projectability condition are consistent with all current cosmological observations.

KEYWORDS: Cosmology of Theories beyond the SM, Models of Quantum Gravity, Gauge Symmetry

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1 Introduction

Recently, Hořava formulated a theory of quantum gravity, whose scaling at short distances exhibits a strong anisotropy between space and time [1–3],

$$\mathbf{x} \rightarrow b^{-1}\mathbf{x}, \quad t \rightarrow b^{-z}t. \tag{1.1}$$

In order for the theory to be power-counting renormalizable, in $(3 + 1)$ -dimensions the critical exponent z needs to be $z \geq 3$ [1–6]. The gauge symmetry of the theory now is broken from the general covariance, $\delta x^\mu = -\delta x^\mu(t, x)$ ($\mu = 0, 1, 2, 3$), down to the foliation-preserving diffeomorphisms $\text{Diff}(M, \mathcal{F})$,

$$\delta t = -f(t), \quad \delta x^i = -\zeta^i(t, \mathbf{x}). \tag{1.2}$$

Abandoning the general covariance gives rise to a proliferation of independently coupling constants [7–9], which could potentially limit the prediction powers of the theory. To reduce the number of these constants, Hořava imposed two conditions, *the projectability and detailed balance* [1–3]. The former assumes that the lapse function N in the Arnowitt-Deser-Misner (ADM) decompositions [10] is a function of t only,

$$N = N(t), \tag{1.3}$$

while the latter assumes that gravitational potential \mathcal{L}_V can be obtained from a superpotential W_g that is defined on the three-spatial hypersurfaces $t = \text{Constant}$. With these conditions, the number of independently coupling constants reduces to 5 [1–3].

However, with the detailed balance condition, the Newtonian limit does not exist [11], and a scalar field in the UV is not stable [12]. Thus, it is generally believed that this condition should be abandoned [13–19]. But, due to several remarkable features [20], Borzou, Lin and Wang recently studied it in detail, and found that the scalar field can be stabilized in both regimes, UV and IR, if the detailed balance condition is allowed to be broken softly [21].

On the other hand, giving up the detailed balance condition, but still keeping the projectability one, the number of the independent coupling constants can also be significantly reduced. In fact, together with the assumptions of the parity and time-reflection symmetry, it can be reduced from more than 70 to 11 [22, 23] (See also [24]). However, in this model the Minkowski spacetime is not stable [22, 23, 25],¹ although the de Sitter spacetime is [31, 32]. In addition, such a theory also faces the strong coupling problem [32–36].² Note that both of these two problems are closely related to the existence of a spin-0 graviton [26–29], because of the foliation-preserving diffeomorphisms.³ Another problem related to the presence of this spin-0 graviton is the difference of its speed from that of the spin-2 graviton. Since they are not related by any symmetry, it poses a great challenge for any attempt to restore Lorentz symmetry at low energies where it has been well tested experimentally. In particular, one needs a mechanism to ensure that in those energy scales all species of matter and gravity have the same effective speed and light cones.

To overcome the above mentioned problems, recently, together with Shu and Wu, two of the present authors proposed a model without the projectability condition, but assuming that: (a) the detailed balance condition is softly broken; and (b) the symmetry of theory is enlarged to included a local U(1) symmetry [42, 43],⁴

$$U(1) \times \text{Diff}(M, \mathcal{F}). \tag{1.4}$$

¹In the literature, the ghost problem was often mentioned [26–29]. But, by restricting the coupling constant λ to the regions $\lambda \geq 1$ or $\lambda < 1/3$, this problem is solved (at least in the classical level) [1–3, 22, 23, 25, 30]. In addition, when $\lambda \in (1/3, 1)$, the instability problem disappears. Therefore, one of these two problems can be always avoided by properly choosing λ . In this paper, we choose $\lambda \geq 1$, so the ghost problem does not exist.

²It should be noted that strong coupling is not a real problem, but an indication that the linear perturbations involved are broken down, and nonlinear effects are needed to be taken into account. If the theory is consistent with observations, after those nonlinear effects are taken into account, it still represents a viable theory. One example is the massive gravity [37–39]. In the content of the Hořava-Lifshitz theory with the projectability condition, such effects were also studied in both spherically symmetric static spacetimes [29] and cosmological models [32, 40, 41], and found that the spin-0 gravitons decouple after nonlinear effects are taken into account. As a result, the relativistic limits indeed exist.

³Since the foliation-preserving diffeomorphisms (1.2) is also assumed in the version without the projectability condition [7, 8], the spin-0 graviton exists there, too.

⁴The enlarged symmetry was first introduced by Hořava and Melby-Thompson (HMT) in the case with the projectability condition and $\lambda = 1$ [44], and was soon generalized to the case with any λ [47], where λ is a coupling constant, which characterizes the deviation from general relativity (GR) in the IR. In such a setup, the spin-0 gravitons are eliminated, and the degree of the freedom of the gravitational sector is the same as that in GR [44–49]. In this model, cosmology and spherically symmetric spacetimes were also studied in, respectively, [50] and [51–56].

The detailed balance condition considerably reduces the number of independently coupling constants, in addition to the desired features mentioned above, while allowing it to be softly broken yields a healthy IR limit. On the other hand, the presence of the local U(1) symmetry eliminates the existence of the spin-0 graviton, whereby all the problems related to it, including instability, strong coupling, and different speeds, are now all out of questions [42, 43]. The enlarged symmetry (1.4) is realized by intruding a U(1) gauge field A , and a Newtonian prepotential φ , the same as those in the case with the projectability condition [44], where under the U(1) transformations, A and φ transform as,

$$\delta_\alpha A = \dot{\alpha} - N^i \nabla_i \alpha, \quad \delta_\alpha \varphi = -\alpha, \tag{1.5}$$

where α denotes the U(1) generator, $\dot{\alpha} \equiv d\alpha/dt$, N^i is the shift vector in the ADM decompositions, and ∇_i the covariant derivative with respect to the 3-metric g_{ij} , defined on the hypersurfaces $t = \text{Constant}$. A remarkable feature of this model is that the lapse function N depends on both t and x , and the model has the same freedom (only spin-2 gravitons exist) as that in GR.

The Hořava-Lifshitz (HL) theory modifies GR in strong gravitational regimes. Therefore, a natural application of it is the early universe. In this paper, we shall study inflation of a single scalar field in the setup of [42, 43], by paying particular attention on two important issues: (i) the consistency of the theory with cosmological observations; and (ii) its distinguishable signatures from other theories of gravity, including GR. Specifically, the paper is organized as follows. In section 2, we present a brief review of the general covariant Hořava-Lifshitz gravity without the projectability condition, while in section 3, we study the linear scalar perturbations, and discuss their gauge choices and gauge-invariants. After working out explicitly the general equations for the cosmological linear scalar perturbations in a flat FRW universe without specifying to a particular gauge, we derive the master equation (3.37) for the scalar perturbations in the specific gauge (3.23). In section 4, we first define the initial vacuum state of the scalar perturbation as the one that minimizes the energy of the ground state, and then calculate the power spectrum and index of the comoving curvature perturbation. Finally, in section 5 we present our main conclusions.

It should be noted that linear perturbations of the Friedmann-Robertson-Walker (FRW) universe in the non-projectable HL theory without the enlarged symmetry (1.4) has been studied by several authors [57–61]. Due to the presence of the spin-0 graviton modes, they are quite different from the ones presented here. In particular, because of the coupling of this mode to matter fields, a master equation does not exist [62].

In addition, in the current setup, the coupling constant Λ_g , defined explicitly in eq. (2.2) below, has to vanish, although quantum mechanically it is expected to be subjected to radiative corrections. It is still an open question whether $\Lambda_g = 0$ is the low energy fixed point or not, as the corresponding RG flow of the theory has not been worked out, yet.

2 General covariant Hořava-Lifshitz gravity without projectability

In this section, we shall give a very brief introduction to the general covariant HL gravity with the enlarged symmetry (1.4) but without the projectability condition. For detail, we

refer readers to [42, 43]. The total action of the theory can be written as,

$$S = \zeta^2 \int dt d^3x \sqrt{g} N \left(\mathcal{L}_K - \mathcal{L}_V + \mathcal{L}_A + \mathcal{L}_\varphi + \frac{1}{\zeta^2} \mathcal{L}_M \right), \quad (2.1)$$

where $g = \det(g_{ij})$, and

$$\begin{aligned} \mathcal{L}_K &= K_{ij} K^{ij} - \lambda K^2, \\ \mathcal{L}_A &= \frac{A}{N} (2\Lambda_g - R), \\ \mathcal{L}_\varphi &= \varphi \mathcal{G}^{ij} (2K_{ij} + \nabla_i \nabla_j \varphi + a_i \nabla_j \varphi) + (1 - \lambda) \left[(\Delta\varphi + a_i \nabla^i \varphi)^2 + 2(\Delta\varphi + a_i \nabla^i \varphi) K \right] \\ &\quad + \frac{1}{3} \hat{\mathcal{G}}^{ijkl} \left[4(\nabla_i \nabla_j \varphi) a_{(k} \nabla_{l)} \varphi + 5(a_{(i} \nabla_{j)} \varphi) a_{(k} \nabla_{l)} \varphi + 2(\nabla_{(i} \varphi) a_{j)(k} \nabla_{l)} \varphi \right. \\ &\quad \left. + 6K_{ij} a_{(l} \nabla_{k)} \varphi \right]. \end{aligned} \quad (2.2)$$

Here $\Delta \equiv g^{ij} \nabla_i \nabla_j$, and Λ_g is a coupling constant. The Ricci and Riemann tensors R_{ij} and $R^i{}_{jkl}$ all refer to the 3-metric g_{ij} , with $R_{ij} = R^k{}_{ikj}$ and

$$\begin{aligned} K_{ij} &\equiv \frac{1}{2N} (-\dot{g}_{ij} + \nabla_i N_j + \nabla_j N_i), \\ \mathcal{G}_{ij} &\equiv R_{ij} - \frac{1}{2} g_{ij} R + \Lambda_g g_{ij}. \end{aligned} \quad (2.3)$$

\mathcal{L}_M is the Lagrangian of matter fields.

When the projectability condition is abandoned, it gives rise to a proliferation of a large number of independently coupling constant [7–9]. Following Horava, the detailed balance condition is generalized to [42, 43],

$$\mathcal{L}_{(V,D)} = \begin{pmatrix} E_{ij} & A_i \end{pmatrix} \begin{pmatrix} \mathcal{G}^{ijkl} & 0 \\ 0 & -g^{ij} \end{pmatrix} \begin{pmatrix} E_{kl} \\ A_j \end{pmatrix}, \quad (2.4)$$

where \mathcal{G}^{ijkl} denotes the generalized De Witt metric, defined as $\mathcal{G}^{ijkl} = \frac{1}{2}(g^{ik}g^{jl} + g^{il}g^{jk}) - \lambda g^{ij}g^{kl}$, and E_{ij} and A_i are given by

$$E^{ij} = \frac{1}{\sqrt{g}} \frac{\delta W_g}{\delta g_{ij}}, \quad A^i = \frac{1}{\sqrt{g}} \frac{\delta W_a}{\delta a_i}. \quad (2.5)$$

The super-potentials W_g and W_a are constructed as

$$\begin{aligned} W_g &= \frac{1}{w^2} \int_\Sigma \omega_3(\Gamma) + \mu \int d^3x \sqrt{g} (R - 2\Lambda), \\ W_a &= \int d^3x \sqrt{g} \sum_{n=0}^1 \mathcal{B}_n a^i \Delta^n a_i, \end{aligned} \quad (2.6)$$

where $\omega_3(\Gamma)$ denotes the gravitational 3-dimensional Chern-Simons term, w , μ , Λ and \mathcal{B}_n are arbitrary constants. However, to have a healthy infrared limit, the detailed balance

condition is allowed to be broken softly, by adding all the low dimensional operators, so that the potential finally takes the form [42, 43],

$$\begin{aligned} \mathcal{L}_V = & \gamma_0 \zeta^2 - \left(\beta_0 a_i a^i - \gamma_1 R \right) + \frac{1}{\zeta^2} \left(\gamma_2 R^2 + \gamma_3 R_{ij} R^{ij} \right) \\ & + \frac{1}{\zeta^2} \left[\beta_1 (a_i a^i)^2 + \beta_2 (a^i{}_i)^2 + \beta_3 (a_i a^i) a^j{}_j + \beta_4 a^{ij} a_{ij} + \beta_5 (a_i a^i) R + \beta_6 a_i a_j R^{ij} + \beta_7 R a^i{}_i \right] \\ & + \frac{1}{\zeta^4} \left[\gamma_5 C_{ij} C^{ij} + \beta_8 (\Delta a^i)^2 \right], \end{aligned} \quad (2.7)$$

where all the coefficients, β_n and γ_n , are dimensionless and arbitrary, except for the ones of the sixth-order derivative terms, γ_5 and β_8 , which must satisfy the conditions,

$$\gamma_5 > 0, \quad \beta_8 < 0, \quad (2.8)$$

in order for the theory to be unitary in the UV. C_{ij} denotes the Cotton tensor, defined by

$$C^{ij} = \frac{e^{ikl}}{\sqrt{g}} \nabla_k \left(R_l^j - \frac{1}{4} R \delta_l^j \right), \quad (2.9)$$

with $e^{123} = 1$. Using the Bianchi identities, one can show that $C_{ij} C^{ij}$ can be written in terms of the five independent sixth-order derivative terms in the form

$$C_{ij} C^{ij} = \frac{1}{2} R^3 - \frac{5}{2} R R_{ij} R^{ij} + 3 R_j^i R_k^j R_i^k + \frac{3}{8} R \Delta R + (\nabla_i R_{jk}) (\nabla^i R^{jk}) + \nabla_k G^k, \quad (2.10)$$

where

$$G^k = \frac{1}{2} R^{jk} \nabla_j R - R_{ij} \nabla^j R^{ik} - \frac{3}{8} R \nabla^k R. \quad (2.11)$$

To be consistent with observations in the IR, we assume that

$$\zeta^2 = \frac{1}{16\pi G}, \quad \gamma_1 = -1, \quad (2.12)$$

where G denotes the Newtonian constant, and

$$\Lambda \equiv \frac{1}{2} \zeta^2 \gamma_0, \quad (2.13)$$

is the cosmological constant.

It should be noted that it is still not clear whether the detailed balance condition remains after quantum effects are taken into account [1–3, 26–28], despite the fact that it has several desirable features [20, 21], and helps to reduce the number of independent constants significantly. Without such a condition, the total number of independent coupling constants are about 100 [7–9, 42].

Variations of the total action (2.1) with respect to N , N^i , A , φ and g_{ij} yield, respectively, the Hamiltonian, momentum, A -, and φ -constraints, and dynamical equations, which are given explicitly in [43]. From those equations, one can see clearly that the mathematical structure of the theory is completely different from the one without the U(1) symmetry [7, 8].

In addition, assuming the translation symmetry of the action (1.4), one obtains the conservation laws of energy and momentum [43]. However, because of the reduced symmetry, $\delta t = f(t)$, the energy is conserved only globally, in contrast to that of GR.

3 Inflation of a single scalar field

Inflation in the projectable case with the enlarged symmetry (1.4) has been studied recently in [50], and in this section we shall closely follow it, although the two setups have completely different mathematical as well as physical structures, as to be shown below.

A scalar field in the current setup is described by

$$S_\chi = \int dt d^3x \sqrt{g} N \mathcal{L}_M, \tag{3.1}$$

where

$$\begin{aligned} \mathcal{L}_M &= \mathcal{L}_\chi^{(A,\varphi)} + \mathcal{L}_\chi^{(0)}, \\ \mathcal{L}_\chi^{(A,\varphi)} &= \frac{A - \mathcal{A}}{N} [c_1(\chi)\Delta\chi + c_2(\chi)(\nabla\chi)^2] + \frac{f}{2} [(\nabla^k\varphi)(\nabla_k\chi)]^2 \\ &\quad - \frac{f}{N} (\dot{\chi} - N^i\nabla_i\chi)(\nabla^k\varphi)(\nabla_k\chi), \end{aligned} \tag{3.2}$$

$$\mathcal{L}_\chi^{(0)} = \frac{f}{2N^2} (\dot{\chi} - N^i\nabla_i\chi)^2 - \mathcal{V}, \tag{3.3}$$

and

$$\begin{aligned} \mathcal{V} &= V(\chi) + \left(\frac{1}{2} + V_1(\chi)\right) (\nabla\chi)^2 + V_2(\chi)\mathcal{P}_1^2 \\ &\quad + V_3(\chi)\mathcal{P}_1^3 + V_4(\chi)\mathcal{P}_2 + V_5(\chi)(\nabla\chi)^2\mathcal{P}_2 + V_6\mathcal{P}_1\mathcal{P}_2, \end{aligned} \tag{3.4}$$

$$\begin{aligned} \mathcal{A} &\equiv -\dot{\varphi} + N^i\nabla_i\varphi + \frac{1}{2}N(\nabla_i\varphi)(\nabla^i\varphi), \\ \mathcal{P}_n &\equiv \Delta^n\chi, \quad V_6 \equiv -\sigma_3^2, \end{aligned} \tag{3.5}$$

where σ_3 is a constant. The coefficient f in (3.1) is a function of λ only, subjected to the requirements: (i) the scalar field must be ghost-free in all the energy scales, including the UV and IR; (ii) in the IR limit, the scalar field has a well-defined velocity, which should be equal or very closed to its relativistic value; and (iii) the stability condition in the IR requires $f(\lambda) > 0$ [21, 43]. Usually, one chooses $f = 1$. In this paper, we shall leave this possibility open. To study the inflationary model of such a scalar field, we first consider the slow-roll conditions in the FRW background.

3.1 Slow-roll inflation

We consider a general FRW space-time,

$$ds^2 = a^2(\eta)(-d\eta^2 + \gamma_{ij}dx^i dx^j), \tag{3.6}$$

where

$$\gamma_{ij} = \frac{\delta_{ij}}{(1 + kr^2/4)^2}, \tag{3.7}$$

and $k = 0, \pm 1$. Then, it can be shown that the A - and φ - constraints yield [43],

$$\frac{\mathcal{H}}{a} \left(\Lambda_g - \frac{k}{a^2} \right) = -\frac{8\pi G}{3} \hat{J}_\varphi, \quad (3.8)$$

$$\frac{k}{a^2} - \frac{\Lambda_g}{3} = \frac{4\pi G \hat{J}_A}{3}, \quad (3.9)$$

where $\mathcal{H} = (da/d\eta)/a \equiv a'/a$, and

$$\begin{aligned} \hat{J}_\varphi &\equiv -\frac{\delta \mathcal{L}_M}{\delta \varphi} = 0, \\ \hat{J}_A &\equiv 2\frac{\delta(N\mathcal{L}_M)}{\delta A} = 0, \end{aligned} \quad (3.10)$$

as one can see from eqs. (3.2)–(3.5) when only the FRW background is considered. Then, from eqs. (3.8) and (3.9) we find that

$$k = 0 = \Lambda_g. \quad (3.11)$$

That is, the FRW universe is necessarily flat in the current setup for the scalar field χ , given by eq. (3.1). It is interesting to note that in such a setup, the coupling constant Λ_g has to vanish, although quantum mechanically it is expected to be subjected to radiative corrections. It is still an open question whether $\Lambda_g = 0$ is the low energy fixed pointed or not, as the corresponding RG flow of the theory has not been worked out, yet. Since we assume that inflation occurred in the regime $H \ll M_*$ [See the discussions given below], where H is the Hubble constant during the inflation epoch, and M_* the suppression energy scale of the high-order derivative terms [43], in the rest of the paper it is sufficient for us to consider that eq. (3.11) holds classically.

On the other hand, from the dynamical equations, we find that

$$\mathcal{H}^2 = \frac{8\pi \tilde{G} a^2}{3} \left(\frac{1}{2} \hat{\chi}'^2 + \tilde{V}(\hat{\chi}) \right), \quad (3.12)$$

$$2\mathcal{H}' + \mathcal{H}^2 = 8\pi G a^2 \left(\frac{1}{2} \hat{\chi}'^2 - \tilde{V}(\hat{\chi}) \right), \quad (3.13)$$

where $\hat{\chi} = \hat{\chi}(\eta)$ denotes the background scalar field, and

$$\tilde{G} \equiv \frac{2fG}{3\lambda - 1}, \quad \tilde{V}(\hat{\chi}) \equiv \frac{V(\hat{\chi})}{f(\lambda)}. \quad (3.14)$$

Note that in writing the above equations, we had set the cosmological constant $\Lambda = 0$. On the other hand, the Klein-Gordon equation in the flat FRW background reads,

$$\hat{\chi}'' + 2\mathcal{H}\hat{\chi}' + a^2 \tilde{V}_{,\chi} = 0. \quad (3.15)$$

eqs. (3.12), (3.13), and (3.15) are identical to these given in GR [64], if one identifies \tilde{G} and \tilde{V} to the Newtonian constant and scalar potential, respectively. As a result, all the

conditions for inflationary models obtained in GR are equally applicable to the current case, as long as the background is concerned. In particular, the slow-roll conditions,

$$\tilde{\epsilon}_V, |\tilde{\eta}_V| \ll 1, \quad (3.16)$$

need to be imposed here, too, in order to get enough e-fold and solve all the problems encountered in GR, where

$$\begin{aligned} \tilde{\epsilon}_V &\equiv \frac{\tilde{M}_{\text{pl}}^2}{2} \left(\frac{\tilde{V}'}{\tilde{V}} \right)^2 = \frac{3\lambda - 1}{2f} \epsilon_V, \\ \tilde{\eta}_V &\equiv \tilde{M}_{\text{pl}}^2 \left(\frac{\tilde{V}''}{\tilde{V}} \right) = \frac{3\lambda - 1}{2f} \eta_V, \end{aligned} \quad (3.17)$$

with $\tilde{M}_{\text{pl}}^2 \equiv 1/(8\pi\tilde{G})$, and ϵ_V and η_V are the ones defined in GR [65]. However, due to the presence of high-order spatial derivatives, the perturbations will be dramatically different, as to be shown below.

3.2 Gauge freedom and invariants

The linear scalar perturbations are given by

$$\begin{aligned} \delta N &= a\phi, & \delta N_i &= a^2 \partial_i B, & \delta g_{ij} &= -2a^2(\psi \delta_{ij} - \partial_i \partial_j E), \\ A &= \hat{A} + \delta A, & \varphi &= \hat{\varphi} + \delta\varphi, & \chi &= \hat{\chi} + \delta\chi, \end{aligned} \quad (3.18)$$

where \hat{A} and $\hat{\varphi}$ denote the background fields of A and φ , and are functions of η only. Under the gauge transformations (1.2), the scalar perturbations transform as

$$\begin{aligned} \tilde{\phi} &= \phi - \mathcal{H}\xi^0 - \xi^{0'}, & \tilde{\psi} &= \psi + \mathcal{H}\xi^0, \\ \tilde{B} &= B + \xi^0 - \xi', & \tilde{E} &= E - \xi, \\ \tilde{\delta\varphi} &= \delta\varphi - \xi^0 \hat{\varphi}', & \tilde{\delta A} &= \delta A - \xi^0 \hat{A}' - \xi^{0'} \hat{A}, \end{aligned} \quad (3.19)$$

where $f = -\xi^0(\eta)$, $\zeta^i = -\xi^{i'}(\eta, x)$. Under the U(1) gauge transformations, on the other hand, we find that

$$\begin{aligned} \tilde{\phi} &= \phi, & \tilde{E} &= E, & \tilde{\psi} &= \psi, & \tilde{B} &= B - \frac{\epsilon}{a}, \\ \tilde{\delta\varphi} &= \delta\varphi + \epsilon, & \tilde{\delta A} &= \delta A - \epsilon', \end{aligned} \quad (3.20)$$

where $\epsilon = -\alpha(\eta, x)$. Then, the gauge transformations of the whole group $U(1) \times \text{Diff}(M, \mathcal{F})$ will be the linear combination of the above two. Out of the six unknowns, $(\phi, B, \psi, E, \delta A, \delta\varphi)$, one can construct four gauge-invariant quantities,

$$\begin{aligned} \gamma &= \Delta\phi, \\ \Phi &= \phi - \frac{1}{a - \hat{\varphi}'} (a\sigma - \delta\varphi)' - \frac{1}{(a - \hat{\varphi}')^2} (\hat{\varphi}'' - \mathcal{H}\hat{\varphi}') (a\sigma - \delta\varphi), \\ \Psi &= \psi + \frac{\mathcal{H}}{a - \hat{\varphi}'} (a\sigma - \delta\varphi), \\ \Gamma &= \delta A + \left[\frac{a(\delta\varphi - \hat{\varphi}'\sigma) - \hat{A}(a\sigma - \delta\varphi)}{a - \hat{\varphi}'} \right]', \end{aligned} \quad (3.21)$$

which is different from the projectable case, in which only three of gauge-invariants exist [46], because the projectability condition requires $\phi = \phi(\eta)$, which can be eliminated by the gauge transformation $f = -\xi^0(\eta)$. But, now since $\phi = \phi(t, x)$, this becomes impossible. Thus, from above discussions and the gauge transformations (3.19) and (3.20), one can choose the following four different gauges,

$$\begin{aligned}
 (i) \quad & B = 0, \quad \delta\varphi = 0; & (ii) \quad & B = 0, \quad \delta A = 0; \\
 (iii) \quad & E = 0, \quad \delta\varphi = 0; & (iv) \quad & E = 0, \quad \delta A = 0.
 \end{aligned} \tag{3.22}$$

The field equations for the linear scalar perturbations without specifying to any gauge are presented in appendix A.

3.3 Equations of linear perturbations

In this subsection, we consider the cosmological perturbations with the gauge,

$$\hat{\varphi}(\eta) = 0, \quad E = 0 = \delta\varphi. \tag{3.23}$$

Then, eqs. (A.7)–(A.12) can be cast in the following forms,

$$\begin{aligned}
 & \frac{3\lambda - 1}{2} \mathcal{H} [3\psi' + 3\phi\mathcal{H} + \partial^2 B] - \wp\partial^2\psi + \frac{1}{2}\eth\partial^2\phi \\
 & = -4\pi G a^2 \left[\frac{f\hat{\chi}'}{a^2} (\delta\chi' - \hat{\chi}'\phi) + \frac{V_4}{a^4} \partial^4\delta\chi + V'\delta\chi \right],
 \end{aligned} \tag{3.24}$$

$$\partial^2\psi = 4\pi G c_1 \partial^2\delta\chi, \tag{3.25}$$

$$\begin{aligned}
 & \partial^2 \left[(1 - \lambda)(3\psi' + \partial^2 B) + 2\mathcal{H}\psi + (1 - 3\lambda)\mathcal{H}\phi \right] \\
 & = 8\pi G \left[(c_1'\hat{\chi}' + c_1\mathcal{H} - f\hat{\chi}')\partial^2\delta\chi + c_1\partial^2\delta\chi' \right],
 \end{aligned} \tag{3.26}$$

$$\partial^2 \left[(3\lambda - 1)(\psi' + \phi\mathcal{H}) + (\lambda - 1)\partial^2 B \right] = 8\pi G f \hat{\chi}' \partial^2\delta\chi, \tag{3.27}$$

$$-\sigma' - 2\mathcal{H}\sigma - \psi - \alpha_1\partial^2\psi + \wp\phi + \frac{\hat{A}\psi - \delta A}{a} = 0, \tag{3.28}$$

$$\begin{aligned}
 & \psi'' + \mathcal{H}(\phi' + 2\psi') + (\mathcal{H}^2 + 2\mathcal{H}')\phi + \frac{\lambda - 1}{3\lambda - 1} \partial^2 \left[\psi + \alpha_1\partial^2\psi - \wp\phi - \frac{\hat{A}\psi - \delta A}{a} \right] \\
 & = \frac{8\pi G a^2}{3\lambda - 1} \left[\frac{f\hat{\chi}'}{a^2} (\delta\chi' - \hat{\chi}'\phi) - V'\delta\chi \right].
 \end{aligned} \tag{3.29}$$

The Klein-Gordon equation (A.13) now reads

$$\begin{aligned}
 & f\delta\chi'' + 2\mathcal{H}f\delta\chi' - f\hat{\chi}'(\partial^2 B) - 2\mathcal{H}f\hat{\chi}'\phi - f\hat{\chi}''\phi - f\hat{\chi}'\phi' + a^2 V''\delta\chi \\
 & \quad - 2 \left(\frac{1}{2} + V_1 - \frac{V_2 + v_4'}{a^2} \partial^2 - \frac{V_6}{a^4} \partial^4 \right) (\partial^2\delta\chi) \\
 & \quad - 3f\hat{\chi}'\psi' - 2(c_1' - c_2) \frac{\hat{A}}{a} (\partial^2\delta\chi) - c_1\partial^2 \left(\frac{\delta A}{a} \right) + a^2 V'\phi + V_4(\partial^4\phi) = 0.
 \end{aligned} \tag{3.30}$$

On the other hand, the comoving curvature perturbation, defined by

$$\mathcal{R} = \psi + \frac{\mathcal{H}}{\hat{\chi}'}\delta\chi, \tag{3.31}$$

is gauge-invariant even in the current setup. Then, with the help of its definition, and from eqs. (3.24), (3.25), (3.27), and (3.28), we can express ψ , B and δA in terms of $\delta\chi$, and then submit them into eq. (A.1), we obtain the second order action in terms of \mathcal{R} ,

$$S^{(2)} = \frac{1}{2} \int d\eta d^3x a^2 h^2 \left[\mathcal{R}' e_0 \mathcal{R}' + \mathcal{R} e_1 \mathcal{R} \right], \quad (3.32)$$

where

$$\begin{aligned} h &\equiv \left(4\pi G c_1 + \frac{H}{\dot{\chi}} \right)^{-1} = \frac{\delta\chi}{\mathcal{R}}, \\ e_0 &\equiv f + \frac{4\pi G c_1^2}{|c_\psi^2|} - \frac{4\zeta^2 d_1^2}{d_0}, \\ e_1 &\equiv -a^2 V'' + 4\pi G a^2 V' c_1 \left(\frac{c_1'}{f|c_\psi^2|} - \frac{1}{|c_\psi^2|} + 3 \right) + \frac{8\pi G f \hat{\chi}'^2}{\lambda - 1} (f - c_1') - \frac{4\pi G c_1 c_1'' \hat{\chi}'^2}{|c_\psi^2|} \\ &\quad + \left[1 + 2V_1 + 2\frac{\hat{A}}{a} (c_1' - c_2) - 4\pi G c_1^2 \left(1 - \frac{\hat{A}}{a} \right) \right] \partial^2 - 2 \left(\frac{V_2 + V_4'}{a^2} + 2\pi G c_1^2 \alpha_1 \right) \partial^4 - \frac{2V_6}{a^4} \partial^6 \\ &\quad - \frac{4\zeta^2 d_2^2}{d_0} + \frac{8\zeta^2 \mathcal{H} d_1 d_2}{d_0} + 4\zeta^2 \left(\frac{d_1 d_2}{d_0} \right)' - \left(\frac{h''}{h} \right) e_0 - \left(\frac{h'}{h} \right) (e_0' + 2\mathcal{H} e_0), \end{aligned} \quad (3.33)$$

with

$$\begin{aligned} c_\psi^2 &\equiv \frac{\lambda - 1}{1 - 3\lambda}, \\ d_0 &\equiv \left(\frac{3\lambda - 1}{\lambda - 1} \right) \mathcal{H}^2 + 4\pi G f \hat{\chi}'^2 - \frac{1}{2} \delta \partial^2, \\ d_1 &\equiv 4\pi G \left[f \hat{\chi}' - \left(\frac{3\lambda - 1}{\lambda - 1} \right) c_1 \mathcal{H} \right], \\ d_2 &\equiv 4\pi G \left[a^2 V' + \left(\frac{3\lambda - 1}{\lambda - 1} \right) (f \hat{\chi}' - c_1' \hat{\chi}' \mathcal{H}) + \frac{V_4}{a^2} \partial^4 - c_1 \delta \partial^2 \right]. \end{aligned} \quad (3.34)$$

From the above expressions, it can be shown that eq. (3.24) can be cast in the form,

$$d_0 \phi = d_1 \delta\chi' + d_2 \delta\chi. \quad (3.35)$$

Introducing the variables,

$$v \equiv z\mathcal{R}, \quad z^2 \equiv a^2 h^2 e_0, \quad (3.36)$$

from eq. (3.32) one obtains the equation of mode function v_k in the momentum space, which reads

$$v_k'' + \beta^2(\eta, k) v_k = 0, \quad (3.37)$$

where

$$\begin{aligned} \beta^2(\eta, k) &= - \left(\frac{z''}{z} + \frac{e_1}{e_0} \right)_{\partial_i \rightarrow ik_i} \\ &= \omega^2(\eta, k) + m_{\text{eff}}^2, \end{aligned} \quad (3.38)$$

and the exact expressions of $\beta^2(\eta, k)$, $\omega^2(\eta, k)$, and m_{eff} are given in appendix B. It is remarkable to note that with the U(1) symmetry, a master equation (3.37) exists, in contrast to the case without it [62]. This is closely related to the fact that the U(1) symmetry eliminates the spin-0 graviton, and the resulted theory enjoys the same degree of freedom as that of GR.

4 Power spectrum and index

To calculate the power spectrum of the linear scalar perturbations with slow-roll approximations, as one usually does in GR, we should first choose a specific vacuum state of the quantum field at initial time η_i (the initial conditions) and solve the linearized equation of motion. Then, using both the initial conditions the solution is uniquely determined, from which we calculate the power spectrum and index.

However, due to the complexity of the function $\beta^2(\eta, k)$ defined in eq. (3.38), the analysis becomes very much involved mathematically. In the following we follow [66, 67]. First, we notice that, at the beginning of the inflationary period, the physical wavelengths of comoving scales which correspond to present large-scale structure of the Universe are usually much smaller than the corresponding Hubble length. This means that at the early stage of inflation the high order spatial derivative terms of the theory are dominant. In this case, a natural choice of the initial state is the one that minimizes the energy of the field at the initial moment η_i [66, 67],

$$\begin{aligned} v_k(\eta_i) &= \frac{1}{\sqrt{2\beta(\eta_i, k)}}, \\ v'_k(\eta_i) &= \pm i \sqrt{\frac{\beta(\eta_i, k)}{2}}. \end{aligned} \tag{4.1}$$

Then, with the slow roll approximations, one has

$$\beta^2(\eta_i, k) \simeq \gamma^2 \eta^4 k^6, \tag{4.2}$$

where

$$\gamma^2 \equiv -\frac{2(1-4\epsilon_V)V_6 H^4}{f+4\pi G c_1^2/|c_\psi^2|} \simeq (1-4\epsilon_V) \left(\frac{H}{M_\star}\right)^4 c_s^2. \tag{4.3}$$

In the above expression, c_s is defined by (4.11) and $M_\star \leq M_{\text{pl}}$. For the field to be stable in the UV, the condition $V_6 < 0$ has to be satisfied, which in the following will be assumed to be always the case.

Secondly, as can be seen from (B.1), the expression of $\beta^2(\eta, k)$ is too complicated to solve the corresponding equation of motion analytically. Following [66, 67], we divide the momentum space into three regions:

- Region I, in which we have $k_{\text{phy}} > M_\star$, and the k^6 term of $\beta^2(\eta, k)$ dominates;
- Region II, in which we have $M_\star > k_{\text{phy}} > H/c_s$, and the k^2 term in $\beta^2(\eta, k)$ dominates;
- Region III, in which we have $k_{\text{phy}} < H/c_s$, and the m_{eff} term dominates,

In the following, we will solve the equation of motion (3.37) in each of these three regions, separately, and then using the matching conditions on the boundaries to connect the integration constants obtained in each of the three regions. In particular, the coefficients of the two fundamental solutions in Region I are fixed by the initial conditions (4.1). Then, we perform the matching of v_k and v'_k at the transitions between region I and region II, which occurs at time η_1 , and between regions II and III, which occurs at time η_2 , to obtain the coefficients of the two fundamental solutions in region III, from which the power spectrum and index can be calculated.

4.1 Region I

In Region I, the equation of motion for the mode function v_k reduces to

$$v_k'' + \gamma^2 \eta^4 k^6 v_k = 0, \quad (4.4)$$

which has the general solution,

$$v_k^I(\eta) = A_1 |\eta|^{1/2} J_\nu(z) + A_2 |\eta|^{1/2} J_{-\nu}(z), \quad (4.5)$$

where $J_\nu(z)$ denotes the usual Bessel function, with $\nu = 1/6$ and $z = \gamma |\eta|^3 k^3 / 3$. The coefficients A_1 and A_2 are to be determined by the initial conditions (4.1) at η_i , i.e.,

$$\begin{aligned} A_1 J_\nu(z_i) + A_2 J_{-\nu}(z_i) &= v_k(\eta_i) |\eta_i|^{-1/2}, \\ -A_1 J_{\nu-1}(z_i) + A_2 J_{1-\nu}(z_i) &= \frac{v'_k(\eta_i) |\eta_i|^{1/2}}{3z_i}. \end{aligned} \quad (4.6)$$

Using the Wronskian relation $J_{-\nu}(z)J_{\nu-1}(z) + J_{1-\nu}(z)J_\nu(z) = 2 \sin[\nu\pi]/(\pi z)$, we find that

$$\begin{aligned} A_1 &= \pi z_i v_k(\eta_i) |\eta_i|^{-1/2} [J_{1-\nu}(z_i) \mp i J_{-\nu}(z_i)], \\ A_2 &= \pi z_i v_k(\eta_i) |\eta_i|^{-1/2} [J_{\nu-1}(z_i) \pm i J_\nu(z_i)]. \end{aligned} \quad (4.7)$$

Since $z_i \gg 1$, by introducing two new functions,

$$x(\eta) \equiv z(\eta) + \frac{\nu\pi}{2} - \frac{\pi}{4}, \quad y(\eta) \equiv z(\eta) - \frac{\nu\pi}{2} - \frac{\pi}{4}, \quad (4.8)$$

and using the asymptotic forms of the Bessel function for large arguments, we find

$$\begin{aligned} A_1 &\simeq \mp i \sqrt{2\pi} z_i v_k(\eta_i) |\eta_i|^{-1/2} e^{\pm i x_i}, \\ A_2 &\simeq \pm i \sqrt{2\pi} z_i v_k(\eta_i) |\eta_i|^{-1/2} e^{\pm i y_i}. \end{aligned} \quad (4.9)$$

4.2 Region II

In Region II, the dispersion relation is approximatively to be linear, thus the equation of motion reduces to

$$v_k'' + c_s^2 k^2 v_k = 0, \quad (4.10)$$

with

$$c_s^2 \equiv \frac{1 + 2V_1 + 2\bar{A}(c'_1 - c_2) - 4\pi G c_1^2 \left(1 - \bar{A} - \frac{2}{\beta_0}\right)}{f + 4\pi G c_1^2 / |c_\psi^2|}. \quad (4.11)$$

Therefore, the solution can be expressed as

$$v_k^{II}(\eta) = B_1 e^{ic_s k \eta} + B_2 e^{-ic_s k \eta}. \quad (4.12)$$

The coefficients B_1 and B_2 are to be determined by matching of v_k and v'_k at the transitions time η_1 between regions I and II, which yield

$$\begin{aligned} B_1 e^{ic_s k \eta_1} &= \frac{A_1 |\eta_1|^{1/2}}{2} \left[J_\nu(z_1) - \frac{3z_1 |\eta_1|^{-1}}{ic_s k} J_{\nu-1}(z_1) \right] \\ &\quad + \frac{A_2 |\eta_1|^{1/2}}{2} \left[J_{-\nu}(z_1) + \frac{3z_1 |\eta_1|^{-1}}{ic_s k} J_{1-\nu}(z_1) \right], \\ B_2 e^{-ic_s k \eta_1} &= \frac{A_1 |\eta_1|^{1/2}}{2} \left[J_\nu(z_1) + \frac{3z_1 |\eta_1|^{-1}}{ic_s k} J_{\nu-1}(z_1) \right] \\ &\quad + \frac{A_2 |\eta_1|^{1/2}}{2} \left[J_{-\nu}(z_1) - \frac{3z_1 |\eta_1|^{-1}}{ic_s k} J_{1-\nu}(z_1) \right]. \end{aligned} \quad (4.13)$$

For $z_1 \gg 1$, we obtain

$$\begin{aligned} B_1 &\simeq \frac{A_1 |\eta_1|^{1/2} e^{-ic_s k \eta_1}}{\sqrt{2\pi z_1}} \left[\cos y_1 + \frac{3z_1}{ic_s k |\eta_1|} \sin y_1 \right] \\ &\quad + \frac{A_2 |\eta_1|^{1/2} e^{-ic_s k \eta_1}}{\sqrt{2\pi z_1}} \left[\cos x_1 + \frac{3z_1}{ic_s k |\eta_1|} \sin x_1 \right], \\ B_2 &\simeq \frac{A_1 |\eta_1|^{1/2} e^{ic_s k \eta_1}}{\sqrt{2\pi z_1}} \left[\cos y_1 - \frac{3z_1}{ic_s k |\eta_1|} \sin y_1 \right] \\ &\quad + \frac{A_2 |\eta_1|^{1/2} e^{ic_s k \eta_1}}{\sqrt{2\pi z_1}} \left[\cos x_1 - \frac{3z_1}{ic_s k |\eta_1|} \sin x_1 \right], \end{aligned} \quad (4.14)$$

where A_1 and A_2 are given by eq. (4.9) and the transitions time η_1 is given by

$$|\eta_1| = \frac{1 + \epsilon_V M_\star}{H} \frac{1}{k}. \quad (4.15)$$

4.3 Region III and the power spectrum

In Region III, the equation of motion reads

$$v_k'' + m_{\text{eff}}^2 v_k = 0, \quad (4.16)$$

where m_{eff}^2 is the effective mass term and is given by eq. (B.4). Then, the solution can be simply expressed as

$$v_k^{III}(\eta) \simeq C |\eta|^{\frac{1-\sqrt{9-4w}}{2}}, \quad (4.17)$$

where $w = 3\eta_V/f + 9(2|c_\psi^2| - 1)\epsilon_V$, and the coefficient C can be determined by matching v_k and v'_k at $c_s k = aH$, which gives

$$C = \left(B_1 e^{ic_s k \eta_2} + B_2 e^{-ic_s k \eta_2} \right) |\eta_2|^{\frac{-1+\sqrt{9-4w}}{2}}. \quad (4.18)$$

Set Then the power spectrum of comoving curvature perturbation \mathcal{R} can be calculated, and is given by

$$\frac{k^3}{2\pi^2} P_{\mathcal{R}} = \left(\frac{H}{2\pi}\right)^2 \left(\frac{1+2\epsilon_V}{c_s^3 h^2 e_0}\right) |c_s k \eta|^{-2w/3}. \quad (4.19)$$

Setting the slow roll parameters to zero exactly, the power spectrum given above can be put in the simple form,

$$\frac{k^3}{2\pi^2} P_{\mathcal{R}} = \left(\frac{H}{2\pi}\right)^2 \left(\frac{1}{c_s^3 h^2 e_0}\right)_{c_s k \eta \rightarrow 0}, \quad (4.20)$$

here

$$e_0|_{c_s k \eta \rightarrow 0} = f + \frac{4\pi G c_1^2}{|c_\psi^2|} \left\{ 1 - \frac{1}{1/|c_\psi^2| + \frac{1}{2}\beta_0} \right\}. \quad (4.21)$$

We can see from above that the scale-invariance of the spectrum is maintained in our theory. In the relativistic limit ($c_1 = c_2 = \beta_0 = \bar{A} = V_1 = 0$, $\lambda = 1 = f$), the power spectrum reduces to the well known result obtained in GR.

The spectrum index can be calculated as

$$n_{\mathcal{R}} - 1 = 2\eta_V - 6\epsilon_V + \Delta n_{\mathcal{R}1} + \Delta n_{\mathcal{R}2}, \quad (4.22)$$

where the modification of the index is

$$2\eta_V - 6\epsilon_V + \Delta n_{\mathcal{R}1} \equiv -\frac{2}{3}w, \quad (4.23)$$

which comes from the effective mass term m_{eff} , and

$$\Delta n_{\mathcal{R}2} \equiv -\frac{d \ln e_0}{d \ln k}, \quad (4.24)$$

which represent the contributions from higher curvature terms in e_0 . To estimate the effect from higher curvature terms on the index, let us first define

$$\delta \equiv \frac{2|c_\psi^2|}{\beta_0|c_\psi^2|+2}, \quad M_A \equiv \frac{M_{pl}}{|\beta_2 + \beta_4|^{1/2}}, \quad M_B \equiv \frac{M_{pl}}{|\beta_8|^{1/4}}. \quad (4.25)$$

Then, e_0 can be expressed as

$$e_0 = f + \frac{4\pi G c_1^2}{|c_\psi^2|} - \frac{4\pi G c_1^2}{1 + \frac{1}{2}\beta_0|c_\psi^2|} \frac{1}{1 - \delta\epsilon_1|c_s k \eta|^2 - \delta\epsilon_2^2|c_s k \eta|^4}, \quad (4.26)$$

where

$$\epsilon_1 = \frac{H^2}{M_A^2}, \quad \epsilon_2 = \frac{H^2}{M_B^2}. \quad (4.27)$$

Usually, the introduced scales M_A and M_B are much greater than H when inflation just starts, i.e., $\epsilon_{1,2} \ll 1$. For the sake of simplicity, we assume that $c_1 \simeq c_2 \simeq M_\star$ [50]. Then, we have

$$\frac{d \ln e_0}{d \ln k} \simeq -\frac{1}{e_0(1 + \frac{1}{2}\beta_0|c_\psi^2|)} \left(\frac{M_\star}{M_{\text{pl}}}\right)^2 (\delta\epsilon_1 + 2\delta\epsilon_1^2 + 2\delta\epsilon_2^2). \quad (4.28)$$

Two comments are in order. First, it appears that the nonlinear dispersion relation $\beta^2(\eta, k)$ has insignificant contributions to the standard spectrum $k^3 P_{\mathcal{R}}^{\text{GR}}$. This is consistent with the results obtained in refs. [68, 69], when two conditions are satisfied. The first one is the adiabaticity that is satisfied for the mode propagation before horizon crossing, i.e., $\mathcal{C} \equiv |\beta'(\eta, k)/\beta^2(\eta, k)| \ll 1$. As shown in refs. [68, 69], only when this condition is violated, can the nonlinear dispersion relation have significant contributions to the spectrum. In this paper, we assumed that the adiabatic condition $\mathcal{C} \ll 1$ is always satisfied before horizon crossing. The second condition is that $\beta^2(\eta, k)$ is approximately linear in region II, thus when we evaluate the power spectrum at horizon crossing, all the high order derivative terms are neglected. More accurately, one can expand $k^3/\beta^3(\eta, k)$ in order of ϵ_{HL} in the form,

$$\left. \frac{k^3}{\beta^3(\eta, k)} \right|_{\beta(\eta_3, k)=aH} \simeq \frac{1}{c_s^3} (1 + a_1 \epsilon_{\text{HL}} + a_2 \epsilon_{\text{HL}}^2 + \dots), \quad (4.29)$$

where $\epsilon_{\text{HL}} \simeq H^2/M_\star$, and $M_A \simeq M_B \simeq M_\star$. Thus, the power spectrum which contains contributions from $\beta^2(\eta, k)$ can be expressed as

$$\frac{k^3}{2\pi^2} P_{\mathcal{R}} = \left(\frac{H}{2\pi}\right)^2 \left(\frac{1}{c_s^3 h^2 e_0}\right) (1 + a_1 \epsilon_{\text{HL}} + a_2 \epsilon_{\text{HL}}^2 + \dots). \quad (4.30)$$

Second, for the spectrum index acquires corrections from both the effective mass term m_{eff} and $1/e_0$. As shown in (4.28), corrections from $1/e_0$ are of order ϵ_{HL} . From the first term $\mathcal{R}'_{e_0}(\eta, \partial^{2n})\mathcal{R}'$ in the second order action (3.32), one can see that these corrections come from the mixture of spatial derivative and time derivative of the comoving curvature perturbations. And more interesting, such mixture is a consequence of the fact that the lapse function ϕ is a function of both η and x , i.e., without the projectability condition. Thus, such corrections to the spectrum index presents a distinguishable signature of the model from the version with the projectability condition.

5 Conclusions and discussions

In this paper, we have studied inflation driven by a single scalar field in the general covariant Hořava-Lifshitz gravity without projectability condition, formulated in [42, 43]. Because of the enlarged symmetry (1.4), the gauge invariants for cosmological perturbations are different from those given in GR. They are also different from those given in general covariant Hořava-Lifshitz theory with projectability conditions. By using the gauge transformations for perturbations, we have constructed all the four gauge invariants, given by eq. (3.21).

The most general cosmological linear perturbation equations have been derived without specifying any gauge, and presented in appendix A. By using these equations, we have shown that a master equation for the mode function of the scalar perturbations exists, in contrast to the case without the U(1) symmetry [62]. Following the method developed in ref. [66, 67], we have first divided the momentum space into three different regions, and then solved the equation of motion in each of the three regions, separately. With the initial conditions that the vacuum state minimizes the energy of the field, we have determined the coefficients of the solutions by matching the solutions and its time derivative at the initial time and transition times on the boundaries of the three regions.

With these solutions, we have also calculated the power spectra and spectrum index of the scalar perturbations in the slow-roll approximations. Comparing with the standard results obtained in GR, the power spectrum and index acquire tiny corrections from the theory, as the adiabatic condition holds in the case considered here. Remarkably, partly of corrections are of the consequence of the non-projectability condition, i.e., $N = N(t, x)$. In the relativistic limit, the spectrum and index reduce to the standard results given in GR. It is also true that the scale invariance of the comoving curvature perturbation remains almost the same as that given in GR, although the initial conditions chosen here are very different from those in GR. Thus, the general covariant Hořava-Lifshitz gravity without the projectability condition is consistent with all current observations.

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A Cosmological perturbations

In this section, we shall consider the linear perturbations given by eq. (3.18) in a flat FRW universe, $k=0$. Then, it can be shown that the action to second-order of $(\phi, B, \psi, E, \delta A, \delta\varphi)$ takes the form,

$$S^{(2)} = \zeta^2 \int d\eta d^3x \left[\delta_2(\sqrt{g}N\mathcal{L}_K) - \delta_2(\sqrt{g}N\mathcal{L}_V) + \delta_2(\sqrt{g}N\mathcal{L}_A) + \delta_2(\sqrt{g}N\mathcal{L}_\varphi) + \frac{\delta_2(\sqrt{g}N\mathcal{L}_\chi)}{\zeta^2} \right], \quad (\text{A.1})$$

where

$$\begin{aligned} \delta_2(\sqrt{g}N\mathcal{L}_K) = & (1 - 3\lambda)a^2 \left[6\mathcal{H}(\phi + \psi)\psi' + 3\psi'^2 + 3\mathcal{H}^2\phi^2 + \frac{9}{2}\mathcal{H}^2\psi^2 + 9\mathcal{H}^2\phi\psi \right. \\ & - 2\mathcal{H}(\phi + \psi)(\partial^2\sigma) - 2\psi'(\partial^2\sigma) - 2\mathcal{H}\psi'(\partial^2E) - 2\mathcal{H}(\partial^2E)(\partial^2\sigma) \\ & \left. - 2\mathcal{H}\psi(\partial^2B) - 2\mathcal{H}(\partial^2E)(\partial^2B) - 3\mathcal{H}^2(\phi + \psi)(\partial^2E) - \frac{3}{2}\mathcal{H}^2(\partial^2E)^2 \right] \\ & + (1 - \lambda)a^2(\partial^2\sigma)^2, \end{aligned} \quad (\text{A.2})$$

$$\begin{aligned} \delta_2(\sqrt{g}N\mathcal{L}_V) &= a^4\Lambda \left[-6\phi\psi + 2(\phi - \psi)(\partial^2 E) + 3\psi^2 - (\partial^2 E)^2 \right] \\ &\quad + a^2 \left[2\psi(\partial^2\psi) + 2\alpha_1\psi(\partial^4\psi) + \phi(\partial^2\phi) - 4\phi(\wp\partial^2\psi) \right], \end{aligned} \quad (\text{A.3})$$

$$\begin{aligned} \delta_2(\sqrt{g}N\mathcal{L}_A) &= a^4\Lambda_g \left[\frac{2\delta A}{a} (-3\psi + \partial^2 E) + \frac{\hat{A}}{a} (3\psi^2 - 2\psi\partial^2 E - (\partial^2 E)^2) \right] \\ &\quad + 2a^2 \left[\frac{\hat{A}}{a}\psi\partial^2\psi - \frac{2\delta A}{a}\partial^2\psi \right], \end{aligned} \quad (\text{A.4})$$

$$\begin{aligned} \delta_2(\sqrt{g}N\mathcal{L}_\varphi) &= 2a^3\Lambda_g \left[\hat{\varphi} \left[\psi(\partial^2 B) + (\partial^2 E)(\partial^2 B) + \psi(\partial^2\sigma) - 3\psi\psi' + \psi'(\partial^2 E) \right. \right. \\ &\quad \left. \left. + (\partial^2 E)(\partial^2\sigma) - \frac{9}{2}\mathcal{H}\psi^2 + 3\mathcal{H}\psi(\partial^2 E) + \frac{3}{2}\mathcal{H}(\partial^2 E)^2 \right] \right. \\ &\quad \left. + \delta\varphi \left[3\psi' - \partial^2\sigma + 3\mathcal{H}(3\psi - \partial^2 E) \right] \right] + a^2\Lambda_g\delta\varphi(\partial^2\delta\varphi) \\ &\quad + a \left[-2\hat{\varphi}[\mathcal{H}\psi(\partial^2\psi) + 2\psi'(\partial^2\psi)] + 4\mathcal{H}(\psi - \phi)(\partial^2\delta\varphi) \right] \\ &\quad + 2a(1 - \lambda) \left[3\mathcal{H}\phi(\partial^2\delta\varphi) + 3\psi'(\partial^2\delta\varphi) - (\partial^2\delta\varphi)(\partial^2\sigma) \right] \\ &\quad + (1 - \lambda)(\partial^2\delta\varphi)^2, \\ \delta_2(\sqrt{g}N\mathcal{L}_\chi) &= \frac{a^2 f}{2} [\delta\chi'^2 + 2\hat{\chi}'\delta\chi(\partial^2 B) - 4\hat{\chi}'\phi\delta\chi' + 3\hat{\chi}'^2\phi^2] - \frac{1}{2}a^4 V''\delta\chi^2 \\ &\quad + \left(\frac{1}{2} + V_1 \right) a^2\delta\chi(\partial^2\chi) - (V_2 + V_4')\delta\chi(\partial^4\delta\chi) - \frac{V_6}{a^2}\delta\chi(\partial^6\delta\chi) \\ &\quad + a(\hat{A} + \hat{\varphi}')(c_1' - c_2)\delta\chi(\partial^2\delta\chi) + ac_1(\delta A + \delta\varphi')(\partial^2\delta\chi) + af\hat{\chi}'\delta\varphi(\partial^2\chi) \\ &\quad + (\phi - 3\psi + \partial^2 E) \left[a^2 f(\hat{\chi}'\delta\chi' - \phi\hat{\chi}'^2) - a^4 V'\delta\chi \right] - V_4\phi(\partial^4\delta\chi) \\ &\quad + \left(\frac{a^2 f}{2}\hat{\chi}'^2 - a^4 V \right) \left[\frac{3}{2}\psi^2 - \psi(\partial^2 E) - \frac{1}{2}(\partial^2 E)^2 + \phi(-3\psi + \partial^2 E) \right]. \end{aligned} \quad (\text{A.5})$$

In writing the above expressions, we have used the following definitions

$$\begin{aligned} \sigma &\equiv E' - B, \\ \alpha_1 &\equiv \frac{8\gamma_2 + 3\gamma_3}{a^2\zeta^2}, \\ \partial &\equiv \beta_0 + \frac{\beta_2 + \beta_4}{a^2\zeta^2}\partial^2 - \frac{\beta_8}{a^4\zeta^4}\partial^4, \\ \wp &\equiv 1 - \frac{\beta_7}{a^2\zeta^2}\partial^2. \end{aligned} \quad (\text{A.6})$$

Then Variation of the action (A.1) with respect to ϕ , ψ , B , E , δA , and $\delta\varphi$ yield, respectively

$$\begin{aligned} &\frac{3\lambda - 1}{2}\mathcal{H} \left[3\psi' + 3\phi\mathcal{H} - \partial^2\sigma + \frac{1}{a}\partial^2\delta\varphi \right] - \wp\partial^2\psi + \frac{1}{2}\partial\partial^2\phi \\ &= -4\pi G a^2 \left[\frac{f\hat{\chi}'}{a^2}(\delta\chi' - \hat{\chi}'\phi) + \frac{V_4}{a^4}\partial^4\delta\chi + V'\delta\chi \right], \end{aligned} \quad (\text{A.7})$$

$$\partial^2\psi = 4\pi Gc_1\partial^2\delta\chi, \quad (\text{A.8})$$

$$\begin{aligned} \partial^2\left[(1-\lambda)\left(3\psi' - \partial^2\sigma + \frac{\partial^2\delta\varphi}{a}\right) + 2\mathcal{H}\psi + (1-3\lambda)\mathcal{H}\phi\right] \\ = 8\pi G\left[(c_1'\hat{\chi}' + c_1\mathcal{H} - f\hat{\chi}')\partial^2\delta\chi + c_1\partial^2\delta\chi'\right], \end{aligned} \quad (\text{A.9})$$

$$\partial^2\left[(3\lambda-1)(\psi' + \phi\mathcal{H}) - (\lambda-1)\left(\partial^2\sigma - \frac{\partial^2\delta\varphi}{a}\right)\right] = 8\pi Gf\hat{\chi}'\partial^2\delta\chi, \quad (\text{A.10})$$

$$-\sigma' - 2\mathcal{H}\sigma - \psi - \alpha_1\partial^2\psi + \wp\phi + \frac{\hat{A}\psi - \delta A + \hat{\varphi}'\psi + \mathcal{H}\delta\varphi}{a} = 0, \quad (\text{A.11})$$

$$\begin{aligned} \psi'' + \mathcal{H}(\phi' + 2\psi') + (\mathcal{H}^2 + 2\mathcal{H}')\phi \\ + \frac{\lambda-1}{3\lambda-1}\partial^2\left[\psi + \alpha_1\partial^2\psi - \wp\phi - \frac{\hat{A}\psi - \delta A + \hat{\varphi}'\psi + 2\mathcal{H}\delta\varphi + \delta\varphi'}{a}\right] \\ = \frac{8\pi Ga^2}{3\lambda-1}\left[\frac{f\hat{\chi}'}{a^2}(\delta\chi' - \hat{\chi}'\phi) - V'\delta\chi\right], \end{aligned} \quad (\text{A.12})$$

and variation with respect to $\delta\chi$, one can get the linear Klein-Gordon equation

$$\begin{aligned} f\delta\chi'' + 2\mathcal{H}f\delta\chi' + f\hat{\chi}'(\partial^2\sigma) - 2\mathcal{H}f\hat{\chi}'\phi - f\hat{\chi}''\phi - f\hat{\chi}'\phi' + a^2V''\delta\chi \\ - 2\left(\frac{1}{2} + V_1 - \frac{V_2 + v_4'}{a^2}\partial^2 - \frac{V_6}{a^4}\partial^4\right)(\partial^2\delta\chi) - 3f\hat{\chi}'\psi' - 2(c_1' - c_2)\frac{\hat{A} + \hat{\varphi}'}{a}(\partial^2\delta\chi) \\ - c_1\partial^2\left(\frac{\delta A + \delta\varphi'}{a}\right) - f\hat{\chi}'\partial^2\left(\frac{\delta\varphi}{a}\right) + a^2V'\phi + V_4(\partial^4\phi) = 0. \end{aligned} \quad (\text{A.13})$$

B $\beta^2(\eta, k)$, $\omega^2(\eta, k)$, and $m_{\text{eff}}(\eta)$

The functions $\beta^2(\eta, k)$, $\omega^2(\eta, k)$, and m_{eff} , defined in eq. (3.38) are given by,

$$\begin{aligned} \beta^2 = & -\mathcal{H}^2 - \mathcal{H}' - \mathcal{H}\frac{e_0'(k)}{e_0(k)} - \frac{1}{2}\frac{e_0''(k)}{e_0(k)} + \frac{1}{4}\left(\frac{e_0'(k)}{e_0(k)}\right)^2 \\ & - \frac{1}{e_0(k)}\left\{4\pi Ga^2V'c_1\left(\frac{c_1'}{f|c_\psi^2|} - \frac{1}{|c_\psi^2|} + 3\right) - a^2V'' + \frac{8\pi Gf\hat{\chi}'^2}{\lambda-1}(f - c_1) - \frac{4\pi Gc_1c_1'\hat{\chi}'^2}{|c_\psi^2|}\right. \\ & - \left[1 + 2V_1 + 2\left(\frac{\hat{A}}{a}\right)(c_1' - c_2) - 4\pi Gc_1^2\left(1 - \frac{\hat{A}}{a}\right)\right]k^2 \\ & - 2\left(\frac{V_2 + V_4'}{a^2} + 2\pi Gc_1^2\alpha_1\right)k^4 + \frac{2V_6}{a^4}k^6 \\ & \left. - 4\zeta^2\frac{d_2^2(k)}{d_0(k)} + 8\zeta^2\mathcal{H}\frac{d_1d_2(k)}{d_0(k)} + 4\zeta^2\left(\frac{d_1d_2(k)}{d_0(k)}\right)'\right\}, \end{aligned} \quad (\text{B.1})$$

$$m_{\text{eff}}^2 = \beta^2(\eta, 0), \quad (\text{B.2})$$

where

$$\begin{aligned}
 \omega^2(\eta, k) &\equiv \beta^2(\eta, k) - m_{\text{eff}}^2(\eta), \\
 e_0(k) &= f + \frac{4\pi G c_1^2}{|c_\psi^2|} - 4\zeta^2 \frac{d_1^2}{d_0(k)}, \\
 d_0(k) &= \frac{\mathcal{H}^2}{|c_\psi^2|} + 4\pi G f \hat{\chi}^{\prime 2} + \frac{1}{2} \beta_0 k^2 - \frac{\beta_2 + \beta_4}{2a^2 \zeta^2} k^4 - \frac{\beta_8}{2a^4 \zeta^4} k^6 \\
 d_2(k) &= 4\pi G \left[a^2 V' + \frac{(f \hat{\chi}' - c_1' \hat{\chi}') \mathcal{H}}{|c_\psi^2|} + \frac{V_4}{a^2} k^4 + c_1 k^2 + \frac{c_1 \beta_7}{a^2 \zeta^2} k^4 \right]. \tag{B.3}
 \end{aligned}$$

Upto first-order of the slow-roll parameters, the mass m_{eff} can be simplified to

$$m_{\text{eff}}^2 \simeq -\frac{2 - 3\eta_V + 3\epsilon_V}{\eta^2} - \frac{1}{\eta^2} \left[3 \left(1 - \frac{1}{f} \right) \eta_V + \frac{12}{3\lambda - 1} \epsilon_V \right]. \tag{B.4}$$

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