

## Plasma conductivity at finite coupling

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**ABSTRACT:** By taking into account the full  $\mathcal{O}(\alpha^3)$  type IIB string theory corrections to the supergravity action, we compute the leading finite 't Hooft coupling  $\mathcal{O}(\lambda^{-3/2})$  corrections to the conductivity of strongly-coupled  $SU(N)$   $\mathcal{N} = 4$  supersymmetric Yang-Mills plasma in the large  $N$  limit. We find that the conductivity is *enhanced* by the corrections, in agreement with the trend expected from previous perturbative weak-coupling computations.

**KEYWORDS:** Holography and quark-gluon plasmas, Gauge-gravity correspondence, AdS-CFT Correspondence

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**1 Introduction**

The gauge/string duality [1–3] has become an essential framework for the study of strongly-coupled systems, in which perturbative quantum field theory techniques cannot be successfully applied. Particularly, during the last decade, and with remarkable results, gauge/string duality techniques have been applied to study properties of strongly coupled SYM plasmas. In a certain temperature range, these idealized systems can perhaps provide a qualitative guide to the quark-gluon plasma (QGP) which is produced as a result of heavy ion collisions at RHIC and the LHC. Some references discussing the phenomenology of the QGP are [4–10]. For a summary of the results of the Pb-Pb ion collisions carried out recently at the LHC, see [11], and for an interesting discussion of the predictions of heavy ion collisions at the LHC we refer the reader to [12].

While the work involving the gauge/string duality in the top-down sense mainly deals with highly symmetric field theories, the real world at low energies is governed by QCD. One of the most challenging aspects of studying the experimentally-produced QGP is how to deform the gravity backgrounds available in string theory in order to break some of the symmetries of the dual field theories, thus bringing them closer to QCD. Another approach is to improve the AdS/CFT results for highly symmetric theories, in order to understand those theories better despite their differences with QCD, in the hope that one can extract universal properties, or make statements that apply to a wide class of strongly-coupled field theories. In this work, we adopt this approach, focussing our study on the hydrodynamics, and in particular charge transport, in  $\mathcal{N} = 4$  SYM theory at finite yet strong 't Hooft coupling.

The hydrodynamic regime of the  $\mathcal{N} = 4$  SYM plasma has been studied extensively over the past decade. Using the rules developed in references [13, 14], the two-point correlators for the energy-momentum tensor  $T_{\mu\nu}$ , and the  $R$ -charge vector currents  $J_\mu$  in the low-momentum (hydrodynamic) regime have been computed. The transport coefficients of both energy-momentum and charge were extracted from the two-point functions, yielding

the viscosity [15–17] and the  $R$ -charge conductivity [14, 16, 18–20]. The work done in these references pertained to the large  $N$  limit (where  $N$  is the rank of the gauge group, i.e. for an infinite number of colour degrees of freedom), and infinite 't Hooft coupling ( $\lambda$ ). Therefore, the gravitational dual in which the work was carried out was given by zeroth order solutions of type IIB supergravity, i.e. the gravity backgrounds were computed to zeroth order in  $\alpha'$ , consistent with the assertion that the  $\lambda \rightarrow \infty$  limit in the field theory corresponds to the  $\alpha' \rightarrow 0$  limit on the gravity side. Precisely, the solutions obtained from the minimal quadratic type IIB supergravity action, correct to  $\mathcal{O}(\alpha'^0)$ , are perturbed in the directions which source the relevant boundary operator, obtaining an  $\mathcal{O}(\alpha'^0)$  action for the supergravity perturbation. The equations of motion derived from this action are solved, thus obtaining the correlators of operators of  $\mathcal{N} = 4$  SYM theory at infinite 't Hooft coupling. In the case of  $\mathcal{N} = 4$  SYM plasma, the  $\mathcal{O}(\alpha'^0)$  ten-dimensional supergravity background is the product of an AdS<sub>5</sub>-Schwarzschild black hole and a five-sphere, with a constant Ramond-Ramond five-form field strength and a constant dilaton. All  $\mathcal{N} = 4$  SYM correlators obtained by perturbing this background are correct at infinite 't Hooft coupling.

A natural question to then ask is the following: what are leading order finite-coupling corrections to the hydrodynamic transport coefficients in  $\mathcal{N} = 4$  SYM plasma? These turn out to arise at order  $\lambda^{-3/2}$ , and to compute them from supergravity one must include  $\mathcal{O}(\alpha'^3)$  corrections to the minimal supergravity action. This was carried out for the shear viscosity and the mass-density diffusion constant in [21–27]. As mentioned above, momentum-transport is governed by correlators of the energy-momentum tensor  $T_{\mu\nu}$ , the dual field of which is  $h_{\mu\nu}$ , which is the graviton of the AdS theory, where  $\mu, \nu$  are both in the AdS factor of the bulk geometry.

In this article, we wish to compute the  $\mathcal{O}(\lambda^{-3/2})$  corrections to the electrical conductivity of the plasma, obtained from the correlators of  $J_\mu$ , the dual field to which is  $A_\mu$ , one of the SO(6) gauge fields in the five-dimensional AdS theory. The ten-dimensional parents of these gauge fields are  $h_{\mu a}$ , where  $\mu$  is in the AdS factor, and  $a$  is in the  $S^5$ . To obtain the corrections to the correlators of  $J_\mu$ , we must therefore compute the equations of motion of  $h_{\mu a}$  starting from the ten-dimensional type IIB supergravity action *plus* the  $\mathcal{O}(\alpha'^3)$  string theory corrections. The schematic form of the  $\mathcal{O}(\alpha'^3)$  corrections is  $C^4 + C^3\mathcal{T} + C^2\mathcal{T}^2 + C\mathcal{T}^3 + \mathcal{T}^4$ , where  $C$  is the ten-dimensional Weyl tensor, while the rank-6 tensor  $\mathcal{T}$  is defined in terms of the Ramond-Ramond five-form field strength and its covariant derivative. The big difference between this computation and the equivalent one for the shear viscosity is that, as shown in [21–27], the only operator that affects the tensor fluctuations  $h_{\mu\nu}$  (and hence the shear viscosity) is the operator  $C^4$ , which only involves the metric in ten dimensions. For the fluctuations  $h_{\mu a}$  which govern charge-transport, on the other hand, we must include the full set of  $\mathcal{O}(\alpha'^3)$  operators listed above, which makes the problem much more challenging. In our recent work [28], we considered this problem from a general viewpoint, without performing the dimensional reduction explicitly.<sup>1,2</sup> In

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<sup>1</sup>Previously we also have studied vector fluctuations on the  $\mathcal{O}(\alpha'^3)$ -corrected metric for short distances compared to the inverse of the plasma equilibrium temperature [29].

<sup>2</sup>Corrections to the holographic conductivity from certain higher-dimensional operators were considered in [26, 30].

the work we present in this article, we compute the exact and full correction to the electrical conductivity at order  $\mathcal{O}(\lambda^{-3/2})$ . We find that the conductivity is enhanced by the corrections, giving:

$$\sigma(\lambda) = \sigma_\infty \left( 1 + \frac{\zeta(3)}{8} \frac{14993}{9} \lambda^{-3/2} \right), \quad (1.1)$$

where  $\sigma_\infty$  is the conductivity at infinite 't Hooft coupling, and  $\zeta(3) \sim 1.202$  is the Riemann Zeta function. In the next section, we briefly discuss the interpretation of this quantity as the electrical conductivity of the plasma, before explaining in detail how we obtained this result.

## 2 Plasma conductivity

We would like to compute the electrical conductivity of the  $SU(N)$   $\mathcal{N} = 4$  SYM plasma in the large  $N$  limit. We mean the following [20]: the  $\mathcal{N} = 4$  SYM theory has a global  $SU(4)$   $R$ -symmetry group. Focus on a  $U(1)$  subgroup of this  $SU(4)$  symmetry, which, being global, does not come with any gauge fields. Then, let us gauge this  $U(1)$  group, and couple the newly-introduced  $U(1)$  gauge-field minimally to the SYM Lagrangian with coupling  $e$ , in the usual way. We also make the  $U(1)$  gauge-field dynamical by adding its kinetic term to the action. We call this theory SYM-EM, following the notation of [20]. The current which couples to the  $U(1)$  gauge field is  $J_\mu^{em}$ , and to leading order in the coupling  $e$ , is given by  $J_\mu^3$  (where the superscript 3 simply signifies that this is the  $R$ -symmetry current in the un-gauged  $U(1)$  direction) *plus* some terms which are subleading in  $e$ . Therefore, to leading order in  $e$  and to full non-perturbative order in  $\lambda$  (the 't Hooft coupling of the  $\mathcal{N} = 4$  SYM theory), the two-point function of  $J_\mu^{em}$  is given by the two-point function of the  $R$ -symmetry currents  $J_\mu^3$  calculated entirely in the  $\mathcal{N} = 4$  SYM theory. The problem thus simplifies to computing the two-point function of the  $R$ -symmetry currents  $J_\mu$ .

The quantity we are interested in is the retarded correlator of  $R$ -symmetry currents at non-zero frequency  $\omega$  and vanishing three-momentum  $\vec{q}$ , defined by

$$R_{\mu\nu}(\omega, \vec{q} = 0) = -i \int d^4x e^{i\omega t} \Theta(t) \langle [J_\mu(x), J_\nu(0)] \rangle, \quad (2.1)$$

where  $\Theta(t)$  is the usual Heaviside function, and  $J_\mu(x)$  is the conserved current associated with the relevant  $U(1)$  subgroup of the  $R$ -symmetry group. The brackets denote the expectation value considered as a thermal average over the statistical ensemble of an  $\mathcal{N} = 4$  SYM plasma at equilibrium temperature  $T$ . Setting the electromagnetic coupling to be  $e$ , the electrical conductivity  $\sigma$  of the plasma is given by

$$\sigma = - \lim_{\omega \rightarrow 0} \text{Im} \frac{e^2}{\omega} R_{xx}(\omega, \vec{q} = 0). \quad (2.2)$$

Our aim in this work is to derive the conductivity  $\sigma$ , working in the holographic dual model, and including the full set of  $\mathcal{O}(\alpha'^3)$  corrections to type IIB supergravity. In the next section, we define the ten-dimensional corrected background and describe in detail the field which is dual to the current  $J_\mu$ , the vector perturbation  $A_\mu$ . We then derive the equations of motion for the  $A_x$  component of the  $U(1)$  gauge field, and use their solution to obtain the plasma conductivity corrected to  $\mathcal{O}(\lambda^{-3/2})$ .

### 3 Setting the holographic dual background

The type IIB supergravity action corrected to  $\mathcal{O}(\alpha'^3)$  is given by:

$$S_{IIB} = S_{IIB}^0 + S_{IIB}^{\alpha'}, \quad (3.1)$$

where  $S_{IIB}^0$  denotes the minimal (two-derivative) type IIB supergravity action, and  $S_{IIB}^{\alpha'}$  encodes the corrections. The minimal action  $S_{IIB}^0$  contains the Einstein-Hilbert action coupled to the dilaton and the Ramond-Ramond five-form field strength

$$S_{IIB}^0 = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G} \left[ R_{10} - \frac{1}{2} (\partial\phi)^2 - \frac{1}{4.5!} (F_5)^2 \right]. \quad (3.2)$$

The leading 't Hooft coupling corrections, on the other hand, are accounted for by the following schematic action [24, 31]

$$S_{IIB}^{\alpha'} = \frac{R^6}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G} \left[ \gamma e^{-\frac{3}{2}\phi} (C^4 + C^3\mathcal{T} + C^2\mathcal{T}^2 + C\mathcal{T}^3 + \mathcal{T}^4) \right], \quad (3.3)$$

where  $\gamma$  provides the dependence on the 't Hooft coupling  $\lambda$  through the definition  $\gamma \equiv \frac{1}{8} \zeta(3) (\alpha'/R^2)^3$ , with  $R^4 = 4\pi g_s N \alpha'^2$ . Setting  $\lambda = g_{YM}^2 N \equiv 4\pi g_s N$ , we get  $\gamma = \frac{1}{8} \zeta(3) \frac{1}{\lambda^{3/2}}$ .

The  $C^4$  term is a dimension-eight operator, defined by the following contractions

$$C^4 = C^{hmnk} C_{pmnq} C_h^{rsp} C_{rsk}^q + \frac{1}{2} C^{hkmn} C_{pqmn} C_h^{rsp} C_{rsk}^q, \quad (3.4)$$

where  $C_{rsk}^q$  is the Weyl tensor. We define the tensor  $\mathcal{T}$  as:

$$\mathcal{T}_{abcdef} = i\nabla_a F_{bcdef}^+ + \frac{1}{16} \left( F_{abcmn}^+ F_{def}^{+mn} - 3F_{abfmn}^+ F_{dec}^{+mn} \right), \quad (3.5)$$

where the r.h.s. must be antisymmetrized in  $[a, b, c]$  and  $[d, e, f]$  and symmetrized with respect to interchange of  $abc \leftrightarrow def$  [31], and in addition we have

$$F^+ = \frac{1}{2}(1 + *)F_5. \quad (3.6)$$

The dual to  $\mathcal{N} = 4$   $SU(N)$  SYM plasma at infinite 't Hooft coupling and infinite  $N$  is the following maximally-symmetric solution of the equations of motion of the minimal action  $S_{IIB}^0$ : an  $AdS_5$ -Schwarzschild black hole multiplied by a five-sphere. The five-form field strength is the volume form on the sphere  $\epsilon$ , with  $N$  units of flux through the sphere. This is a solution of the equations of motion arising from the action  $S_{IIB}^0$  of eq. (3.2). The current operator of the SYM theory  $J_\mu(x)$  is dual to the  $s$ -wave mode of the vectorial fluctuation on this background, and we shall denote it by  $A_\mu$ . In order to obtain the Lagrangian for the vectorial perturbation in this background, we have to construct a consistent perturbed *Ansatz* for both the metric and the Ramond-Ramond five-form field strength, such that a proper  $U(1)$  subgroup of the  $R$ -symmetry group is obtained [32, 33]. As shown in the previous references, the consistent perturbation *Ansatz* yields the minimal  $U(1)$  gauge field kinetic term in the  $AdS_5$ -Schwarzschild black-hole geometry. Therefore, by studying the bulk solutions of the Maxwell equations in the  $AdS_5$ -Schwarzschild black hole with certain

boundary conditions, we can obtain the retarded correlation functions [13, 14, 20] of the operator  $J_\mu(x)$ . We wish to carry out this program using the fully-corrected action  $S_{IIB}$ . As pointed out in the introduction, this program has already been carried out for the tensor fluctuations  $h_{\mu\nu}$  dual to the energy momentum tensor  $T_{\mu\nu}$  [21, 23, 34], yielding the viscosity and mass-diffusion constant of the SYM plasma.

To begin the computation of the  $\mathcal{O}(\alpha'^3)$  equations of motion of the gauge field  $A_\mu$  dual to the current  $J_\mu$ , we must first describe the effects of  $S_{IIB}^{\alpha'}$  on the geometry of the gravitational dual to the SYM plasma. Firstly, we note that the higher curvature corrections do not modify the metric at zero temperature [35]. This is simply the statement that the SYM theory is fully conformal at all orders at zero temperature, and so the gravitational dual must necessarily reflect this preserved conformality. The situation at finite temperature is rather different. In references [36, 37], the effect of the string theory leading corrections to the metric were investigated, focussing upon the study of their corrections to thermodynamic quantities of the five-dimensional AdS-Schwarzschild black hole. The corrections to the metric were revisited in references [38–40]. Remarkably, Myers, Paulos and Sinha [24] have shown that the metric itself is only corrected by  $C^4$ , a consequence of the fact that the tensor  $\mathcal{T}$  vanishes on the uncorrected supergravity solution. The corrected metric is given by [36, 37, 39]

$$ds^2 = \left(\frac{r_0}{R}\right)^2 \frac{1}{u} (-f(u) K^2(u) dt^2 + d\vec{x}^2) + \frac{R^2}{4u^2 f(u)} P^2(u) du^2 + R^2 L^2(u) d\Omega_5^2, \quad (3.7)$$

where  $f(u) = 1 - u^2$  and  $R$  is the radius of the AdS<sub>5</sub> and the five-sphere. The AdS-boundary is at  $u = 0$  and the black hole horizon is at  $u = 1$ . For the AdS<sub>5</sub> coordinates we use indices  $m$ , where  $m = \{\mu = 0, 1, 2, 3, 5\}$ . We have

$$K(u) = \exp[\gamma(a(u) + 4b(u))], \quad P(u) = \exp[\gamma b(u)], \quad L(u) = \exp[\gamma c(u)], \quad (3.8)$$

where the exponents are given by:

$$\begin{aligned} a(u) &= -\frac{1625}{8} u^2 - 175 u^4 + \frac{10005}{16} u^6, \\ b(u) &= \frac{325}{8} u^2 + \frac{1075}{32} u^4 - \frac{4835}{32} u^6, \\ c(u) &= \frac{15}{32} (1 + u^2) u^4. \end{aligned} \quad (3.9)$$

Finally, we have the following expression for the extremality parameter  $r_0$ :

$$r_0 = \frac{\pi T R^2}{(1 + \frac{265}{16} \gamma)}, \quad (3.10)$$

where  $T$  is identified as the *physical* equilibrium temperature of the plasma. Having obtained the corrected metric, the next step is to deduce the appropriate perturbation *Ansätze* for the vectorial fluctuations  $A_\mu$  of the corrected supergravity background. Notice that the vector perturbation enters the perturbed metric *in addition* to the perturbed  $F_5$  solution, which means that all the operators inside  $S_{IIB}^{\alpha'}$  can influence the calculation. The plan

is to formulate the perturbation *Ansätze*, correct to linear order in  $\gamma$  and plug them into  $S_{IIB}$ . Due to the fact that we are working to linear order in  $\gamma$ , we must insert the fully corrected *Ansätze* into the  $S_{IIB}^0$  piece, but it is sufficient to insert the  $\mathcal{O}(\gamma^0)$  *Ansätze* into  $S_{IIB}^{\alpha'}$  because the latter part of the action carries an explicit factor of  $\gamma$ . In the remainder of this section, we will describe the perturbation *Ansätze* and display the result of inserting them into  $S_{IIB}^0$ . The insertion of the perturbation *Ansätze* into  $S_{IIB}^{\alpha'}$  will be described in the next section.

The metric *Ansatz* reads as follows

$$ds^2 = \left[ g_{mn} + \frac{4}{3} R^2 L(u)^2 A_m A_n \right] dx^m dx^n + R^2 L(u)^2 d\Omega_5^2 + \frac{4}{\sqrt{3}} R^2 L(u)^2 \times (\sin^2 y_1 dy_3 + \cos^2 y_1 \sin^2 y_2 dy_4 + \cos^2 y_1 \cos^2 y_2 dy_5) A_m dx^m, \quad (3.11)$$

where  $d\Omega_5^2$  is metric of the unit five-sphere given by

$$d\Omega_5^2 = dy_1^2 + \cos^2 y_1 dy_2^2 + \sin^2 y_1 dy_3^2 + \cos^2 y_1 \sin^2 y_2 dy_4^2 + \cos^2 y_1 \cos^2 y_2 dy_5^2.$$

Since we are only interested in the terms which are quadratic in the gauge-field perturbation we can write the  $F_5$  *Ansatz* as follows

$$F_5 = -\frac{4}{R} \bar{\epsilon} + \frac{R^3 L(u)^3}{\sqrt{3}} \left( \sum_{i=1}^3 d\mu_i^2 \wedge d\phi_i \right) \wedge \bar{*} F_2, \quad (3.12)$$

where  $F_2 = dA$  is the Abelian field strength and  $\bar{\epsilon}$  is a deformation of the volume form of the metric of the AdS<sub>5</sub>-Schwarzschild black hole. We stress that we are not interested in the part of  $F_5$  which does not contain the vector perturbations, as we are only concerned with the quadratic action of  $A_\mu$ . The Hodge dual  $*$  is taken with respect to the ten-dimensional metric, while  $\bar{*}$  denotes the Hodge dual with respect to the five-dimensional metric piece of the black hole. In addition, we have the usual definitions for the coordinates on the  $S^5$

$$\begin{aligned} \mu_1 &= \sin y_1, & \mu_2 &= \cos y_1 \sin y_2, & \mu_3 &= \cos y_1 \cos y_2, \\ \phi_1 &= y_3, & \phi_2 &= y_4, & \phi_3 &= y_5. \end{aligned} \quad (3.13)$$

Inserting these *Ansätze* into eq. (3.2), and discarding all the higher (massive) Kaluza-Klein harmonics of the five-sphere, we get the following action for the zero-mode Abelian gauge field  $A_m$

$$S_{IIB}^{SUGRA} = -\frac{\tilde{N}^2}{64\pi^2 R} \int d^4x du \sqrt{-g} L^7(u) g^{mp} g^{nq} F_{mn} F_{pq}. \quad (3.14)$$

In the previous equation, the Abelian field strength is defined as  $F_{mn} = \partial_m A_n - \partial_n A_m$ , the partial derivatives are  $\partial_m = \partial/\partial x^m$ , while  $x^m = (t, \vec{x}, u)$ , where  $t$  and  $\vec{x} = (x_1, x_2, x_3)$  are the Minkowski four-dimensional spacetime coordinates, and  $g \equiv \det(g_{mn})$ , where  $g_{mn}$  is the metric of AdS<sub>5</sub>-Schwarzschild black hole. The factor of  $L(u)^7$  arises straightforwardly from the dimensional reduction [16], and the volume of the five-sphere has been included in  $\tilde{N}$ .

The next step is to obtain the effect of the eight-derivative corrections of eq. (3.3). As in [28], it is sufficient to use the uncorrected *Ansätze* at this point, i.e. eqs. (3.11) and (3.12) in the limit  $\gamma \rightarrow 0$ , so taking  $L(u), K(u), P(u) \rightarrow 1$  and  $\bar{\epsilon} \rightarrow \epsilon$ . We carry this out in the next section.

#### 4 't Hooft corrections to the $R$ -charge conductivity

We here insert the *Ansätze* in eqs. (3.11) and (3.12) (with  $\gamma \rightarrow 0$ ) into  $S'_{IIB}$ . We begin by explicitly writing all the various operators comprising  $S'_{IIB}$ . For this purpose it is convenient to use the definitions given by Paulos [31] which can be explicitly written from eq. (3.3) as

$$\begin{aligned} S'_{IIB} &= \frac{R^6}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G} [\gamma e^{-\frac{3}{2}\phi} (C^4 + C^3\mathcal{T} + C^2\mathcal{T}^2 + C\mathcal{T}^3 + \mathcal{T}^4)] \\ &\equiv \frac{1}{86016} \frac{R^6}{2\kappa_{10}^2} \sum_i n_i \int d^{10}x \sqrt{-G} [\gamma e^{-\frac{3}{2}\phi} M_i], \end{aligned} \quad (4.1)$$

where the coefficients  $n_i$  are found in [31]. The first important point to keep in mind is that we are only interested in terms quadratic in the gauge field  $A_\mu$ . We therefore expand the tensors  $C$  and  $\mathcal{T}$  as follows:  $C = C_0 + C_1 + C_2$ , and  $\mathcal{T} = \mathcal{T}_0 + \mathcal{T}_1 + \mathcal{T}_2$ , where the subindex labels the number of times that the Abelian gauge field occurs. The second important point to note is that the tensor  $\mathcal{T}_0$  vanishes for the background considered here (and for any direct-product background which contains a five-dimensional Einstein manifold as the internal space), as proved by [24]. Therefore, we may write  $\mathcal{T} = \mathcal{T}_1 + \mathcal{T}_2$ . This immediately means that the terms  $C\mathcal{T}^3$  and  $\mathcal{T}^4$  cannot contribute to the quadratic action for the gauge field  $A_\mu$  and so we discard them in what follows.

For  $C^4$  there are two contributions which can be written as:

$$C^4 = -\frac{43008}{86016} C_{abcd} C_{abef} C_{cegh} C_{dgfh} + C_{abcd} C_{aecf} C_{bgeh} C_{dgfh}, \quad (4.2)$$

where repeated indices mean usual Lorentz contractions. The contributions from this term can be schematically written as  $C_0^3 C_2$  and  $C_0^2 C_1^2$ .

Next we consider terms of the form  $C^3 \mathcal{T}$

$$C^3 \mathcal{T} = \frac{3}{2} C_{abcd} C_{aefg} C_{bfhi} \mathcal{T}_{cdeghi}. \quad (4.3)$$

The possible contributions from these terms are of the form  $C_0^2 C_1 \mathcal{T}_1$  and  $C_0^3 \mathcal{T}_2$ . We have explicitly checked that the  $C_0^3 \mathcal{T}_2$  term is zero, so that the only contribution here is  $C_0^2 C_1 \mathcal{T}_1$ .

For the operators  $C^2 \mathcal{T}^2$ , there are a few contractions:

$$\begin{aligned} C^2 \mathcal{T}^2 &= \frac{1}{86016} \times (30240 C_{abcd} C_{abce} \mathcal{T}_{dfghij} \mathcal{T}_{efhgij} + 7392 C_{abcd} C_{abef} \mathcal{T}_{cdghij} \mathcal{T}_{efghij} \\ &\quad - 4032 C_{abcd} C_{aecf} \mathcal{T}_{bceghij} \mathcal{T}_{dfghij} - 4032 C_{abcd} C_{aecf} \mathcal{T}_{bghdij} \mathcal{T}_{eghfi} \\ &\quad - 118272 C_{abcd} C_{aefg} \mathcal{T}_{bcehij} \mathcal{T}_{dfhgij} - 26880 C_{abcd} C_{aefg} \mathcal{T}_{bcehij} \mathcal{T}_{dhifgj} \\ &\quad + 112896 C_{abcd} C_{aefg} \mathcal{T}_{bcfhij} \mathcal{T}_{dehgij} - 96768 C_{abcd} C_{aefg} \mathcal{T}_{bcehij} \mathcal{T}_{dfhgij}). \end{aligned} \quad (4.4)$$

The contributions here are of the form  $C_0^2 \mathcal{T}_1^2$ .

What we must now do is clear: compute the ten-dimensional Weyl tensor  $C$  to quadratic order in the gauge field  $A_x$ , and compute  $\mathcal{T}_1$ . We separate the latter into  $\mathcal{T}_1 = \nabla F_5 + \bar{\mathcal{T}}$ , where

$$(\nabla F_5)_{abcdef} = i \nabla_a F_{bcdef}^+, \quad (4.5)$$



and a second piece which does not contain covariant derivatives

$$\bar{\mathcal{T}}_{abcdef} = \frac{1}{16} \left( F_{abcmn}^+ F_{def}^{+ mn} - 3F_{abfmn}^+ F_{dec}^{+ mn} \right). \quad (4.6)$$

We can recast the definition of  $F^+$  above as a sum of an electric and magnetic components  $F^+ = F_{(e)} + F_{(m)}$ . For the electric part we have

$$F_{(e)} = -\frac{4}{R} \epsilon + \frac{R^3}{\sqrt{3}} \left( \sum_{i=1}^3 d\mu_i^2 \wedge d\phi_i \right) \wedge \bar{*}F_2, \quad (4.7)$$

where the  $\bar{*}$  indicates the Hodge dual with respect to the AdS<sub>5</sub>-Schwarzschild black hole metric. It is convenient to split the electric part into a background piece plus a fluctuation piece:  $F_{(e)} = F_{(e)}^{(0)} + F_{(e)}^{(f)}$ , and similarly for the magnetic terms. Then, in components, we can write:

$$(F_{(e)}^{(0)})_{\mu\nu\rho\sigma} = -\frac{4}{R} \sqrt{-g} \epsilon_{\mu\nu\rho\sigma}, \quad (4.8)$$

where as before  $g$  is the determinant of the AdS piece of the metric. The Hodge dual of the previous equation gives

$$(F_{(m)}^{(0)})_{abcde} = -\frac{4}{R} R^5 \sqrt{\det S^5} \epsilon_{abcde}. \quad (4.9)$$

For the conductivity, we are free to restrict ourselves to the  $A_x(u)$  component of the Abelian field. Thus,  $F_2$  has only one component, namely  $F_{ux}(u)$ . We write  $F = dA = \frac{1}{2!} F_{\mu\nu} / \sqrt{3} dx^\mu \wedge dx^\nu$ , obtaining for the fluctuation of the electric part:

$$(F_{(e)}^{(f)})_{y_i y_j t y z} = -\frac{R^3}{\sqrt{3}} \frac{b_{ij}}{2} \sqrt{g} (2F_{ux} G^{xx} G^{uu}) \epsilon_{y_i y_j t y z}, \quad (4.10)$$

where the pairs  $(ij)$  are (13), (14), (15), (24) and (25). The  $b_{ij}$  functions are:

$$\begin{aligned} b_{13} &= 2 \sin y_1 \cos y_1, & b_{14} &= -2 \sin^2 y_2 \sin y_1 \cos y_1, & b_{15} &= -2 \cos^2 y_2 \sin y_1 \cos y_1, \\ b_{24} &= 2 \cos^2 y_1 \sin y_2 \cos y_2, & b_{25} &= -2 \cos^2 y_1 \sin y_2 \cos y_2. \end{aligned} \quad (4.11)$$

For the fluctuations of the magnetic part we have

$$\begin{aligned} F_{(m)}^{(f)} &= \sqrt{-G_{10}} \tilde{F}(u) G^{tt} G^{yy} G^{zz} \times \\ &\quad (m_{13} \epsilon_{uxy_2 y_4 y_5} + m_{14} \epsilon_{uxy_2 y_3 y_5} + m_{15} \epsilon_{uxy_2 y_3 y_4} + m_{24} \epsilon_{uxy_1 y_3 y_5} + m_{25} \epsilon_{uxy_1 y_3 y_4}), \end{aligned} \quad (4.12)$$

where  $G_{10}$  is the determinant of the full ten-dimensional metric and for conciseness we have defined

$$\tilde{F}(u) = -\frac{R^3}{\sqrt{3}} \frac{1}{2} \sqrt{-g} (2F_{ux} G^{xx} G^{uu}). \quad (4.13)$$

The functions  $m_{ij}$  are given by

$$\begin{aligned} m_{13} &= -\frac{4}{R^4} \sin(2y_2) \cos^4 y_1, & m_{14} &= -\frac{8}{R^4} \sin^2 y_1 \cos^2 y_1 \sin y_2 \cos y_2, \\ m_{15} &= \frac{8}{R^4} \sin^2 y_1 \cos^2 y_1 \sin y_2 \cos y_2, \\ m_{24} &= -\frac{8}{R^4} \cos^2 y_2 \sin y_1 \cos y_1, & m_{25} &= -\frac{8}{R^4} \cos y_1 \sin y_1 \sin^2 y_2. \end{aligned} \quad (4.14)$$

We are now in a position to put all the ingredients together. We plug the *Ansatz* into  $S_{IIB}^{\alpha'}$ , multiply by the determinant of the metric, then integrate out the coordinates of the five-sphere. Setting  $f(u) = 1 - u^2$  as above, the result for the covariant derivative piece (the  $C^2(\nabla F_5)^2$  arising from  $C^2\mathcal{T}^2$  operator) is:

$$L_{C^2(\nabla F_+)^2} = -\frac{u^4}{9} [(11839 - 30773u^2 + 25278u^4) A_x'^2 - 2u f(u) (9401u^2 - 6229) A_x' A_x'' + 3773 u^2 f(u)^2 (A_x'')^2]. \quad (4.15)$$

On the other hand, for the terms from  $C^3\mathcal{T}$ , we get

$$L_{C^3\mathcal{T}_1} = -\frac{112u^4}{3} f(u) (A_x')^2. \quad (4.16)$$

Similarly for the terms from  $C^2\mathcal{T}^2$  arising from the  $\bar{\mathcal{T}}$  piece of the tensor, we obtain

$$L_{C^2F_+^2} = \frac{830u^4}{3} f(u) (A_x')^2. \quad (4.17)$$

Finally, for  $C^4$  we obtain

$$L_{C^4} = -\frac{u^4}{3} (4(14 - 67u^2 + 78u^4) A_x'^2 + 4u f(u) (28 - 53u^2) A_x' A_x'' + 33 u^2 f(u)^2 (A_x'')^2). \quad (4.18)$$

We can therefore write the total Lagrangian coming directly from the dimension-eight operators in  $S_{IIB}^{\alpha'}$  as

$$L_{(C^4+\mathcal{T})^4} = c_1 L_{C^2(\nabla F_+)^2} + c_2 L_{C^2F_+^2} + c_3 L_{C^3\mathcal{T}_1} + c_4 L_{C^4}, \quad (4.19)$$

where all the coefficients  $c_i = 1$  and we include them for the purpose of keeping track of the effects of every term in the final expression for the electrical conductivity. The Lagrangian in eq. (4.19) must be augmented by the terms coming from  $S_{IIB}^0$ , that is, the kinetic term of eq. (3.14). Once that term is added, we are left with the following Lagrangian, whose equations of motion must be derived and solved:

$$S_{\text{total}} = -\frac{\tilde{N}^2 r_0^2}{16\pi^2 R^4} \int \frac{d^4 k}{(2\pi)^4} \int_0^1 du [(B_1 + \gamma B_W) A_k' A_{-k}' + \gamma E_W A_k'' A_{-k}'' + \gamma F_W A_k'' A_{-k}'] , \quad (4.20)$$

where we have introduced the following Fourier transform of the field  $A_x$

$$A_x(t, \vec{x}, u) = \int \frac{d^4 k}{(2\pi)^4} e^{-i\omega t + iqz} A_k(u). \quad (4.21)$$

There are also a number of boundary terms that must be included for this higher-derivative Lagrangian to make sense, and this is discussed in detail in [21, 28, 29]. In the Lagrangian of eq. (4.20), the coefficient  $B_1$  arises directly from the kinetic term  $F^2$ . The subscript  $W$

indicates that the particular coefficient comes directly from the eight-derivative corrections. We have

$$\begin{aligned}
 B_1 &= \frac{K(u)f(u)L^7(u)}{P(u)}, \\
 B_W &= -\frac{u^4}{9} (12c_4 [14 - 67u^2 + 78u^4] \\
 &\quad + c_1 [11839 - 30773u^2 + 25278u^4] + [336c_3 - 2490c_2] f(u)) , \\
 E_W &= -\frac{11}{9} [9c_4 + 343c_1] u^6 f(u)^2 , \\
 F_W &= \frac{2}{9} u^5 f(u) (6c_4 [53u^2 - 28] + c_1 [9401u^2 - 6229]) .
 \end{aligned} \tag{4.22}$$

We note that the equation of motion arising from (a more general version of) this Lagrangian was solved in [28] following [21], so we will be very brief in what follows. We also stress that the behaviour of the solution at the black-hole horizon is unchanged by the finite coupling corrections (once it is expressed in terms of the physical temperature and the physical momentum), consistently with the findings of [28]. The solution therefore has the following form:

$$A_k(u) = A_0(u) + \gamma A_1(u) = [1 - u]^{-\delta} (\phi_0(u) + \gamma \phi_1(u)) , \tag{4.23}$$

where  $\delta = i\omega/(4\pi T)$ . Using this *Ansatz* and following the work of [28], the full solution of the equations of motion to linear order in  $\gamma$  and  $\delta$  is:

$$A_k(u) = [1 - u]^{-\delta} \left( \bar{C} + \delta \left\{ \bar{D} + \bar{C} \left( 1 + \gamma \left[ \frac{185}{4} + 2\beta \right] \right) u \right\} \right) , \tag{4.24}$$

where  $\bar{C}, \bar{D}$  drop out of the final result, and we shall reveal  $\beta$  shortly. We must obtain the on-shell action for this solution. The form of the functions  $B_W, E_W, F_W$  means that the on-shell action reduces to [28]

$$S_{\text{on-shell}} = \frac{\tilde{N}^2 r_0^2}{16\pi^2 R^4} \int \frac{d^4 k}{(2\pi)^4} [B_1 A'_k A_{-k}] \Big|_{u=0} . \tag{4.25}$$

Evaluating the previous equation, and differentiating twice with respect to the boundary value of the gauge field, we obtain that the conductivity of the large  $N$  limit of strongly-coupled  $SU(N)$   $\mathcal{N} = 4$  SYM plasma is corrected by the following factor:

$$1 + \gamma(\beta - 10) ,$$

where

$$\beta = \frac{12797}{9} c_1 + \frac{2490}{9} c_2 - \frac{336}{9} c_3 + \frac{44}{3} c_4 . \tag{4.26}$$

Setting the coefficient  $c_i$  to their actual numerical value ( $= 1$ ), we obtain the following final expression for the conductivity

$$\sigma(\lambda) = \sigma_\infty \left( 1 + \frac{\zeta(3)}{8} C \lambda^{-3/2} \right) , \tag{4.27}$$

where the conductivity at infinite 't Hooft coupling is

$$\sigma_\infty = e^2 \frac{N^2 T}{16\pi}, \tag{4.28}$$

where  $e$  is the electric charge, and  $C$  is given by

$$C = \frac{14993}{9} \approx 1665.89, \tag{4.29}$$

For  $\lambda = 100$  the correction  $\frac{\zeta(3)}{8} \frac{14993}{9} \lambda^{-3/2}$  gives 0.25031, which is a 25 percent enhancement of the electrical conductivity compared with its value at infinite 't Hooft coupling. For  $\lambda = 1000$ , the enhancement reduces to just under one percent.

Also notice that for lower values of  $\lambda$ , for example 5, the correction gives a very large number 22.39, which does not make any sense since these values of the coupling are well below the limit where we can trust our results which are limited to consider  $\mathcal{O}(\alpha'^3)$  corrections only. In order to study small values of the coupling, one should also consider  $\mathcal{O}(\alpha'^5)$  (and probably higher order) corrections, and finite  $1/N$  corrections which go like the string coupling  $g_s$ . These corrections are not known even for the AdS<sub>5</sub>-Schwarzschild black hole  $\times S^5$  metric, and they would certainly make the present calculation prohibitively complicated, at least within present-day available gauge/string duality techniques.

## 5 Discussion and concluding remarks

In this article, we computed the finite 't Hooft coupling corrections to the conductivity of the large  $N$  limit of the strongly-coupled SU( $N$ )  $\mathcal{N} = 4$  SYM plasma. The corrections start at  $\mathcal{O}(\lambda^{-3/2})$  and enhance the conductivity from its value at infinite strong coupling. In our previous work [28], we analysed the effect of the  $\mathcal{O}(\alpha'^3)$  dimension-eight operators on the vector fluctuations of the supergravity metric, through an examination of the possible gauge-field operators that can arise in the AdS theory, after integrating out the internal compact space. We found that the most general initial set of 720 five-dimensional gauge-invariant operators containing two powers of the gauge-field  $A_\mu$  can be reduced by symmetry to only 26 operators.<sup>3</sup> The operators are of the schematic form  $\tilde{C}^2 F_2^2$  and  $\tilde{C}^2 (\nabla F_2)^2$ , where  $\tilde{C}$  stands for the AdS<sub>5</sub> Weyl tensor and  $F_2$  for the gauge field strength (containing  $A_\mu$  above). One can easily show that the Lagrangian obtained above in eq. (4.20) is entirely consistent with the operators obtained in [28]. Furthermore, in [28] we showed that the functional behaviour of the solution of  $A_\mu$  at the black-hole horizon is unchanged, provided the physical parameters of the  $\mathcal{N} = 4$  SYM theory are used. To be more explicit, the solution of  $A_\mu$  always takes the form

$$A_x(u) = [1 - u]^{-\delta} (\phi_0(u) + \gamma \phi_1(u)),$$

where  $\phi_{0,1}$  are regular at the horizon, and the index  $\delta$  is *always* given by  $\delta = i\omega/(4\pi T)$ , where  $\omega$  and  $T$  are the physical frequency of the perturbation, and  $T$  the physical temperature of the plasma. Again, our results in this work are entirely consistent with this

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<sup>3</sup>Part of the computation presented in reference [28], concerning the reduction to the set of independent operators was performed using the programme Cadabra [41, 42].

observation. We view the agreement between our present work and that of [28] as a good check on the correctness of our result for the conductivity.

Before moving on we should also compare the result of our calculation with the trend one expects from weak-coupling computations of charge-transport coefficients. The electrical conductivity of a weakly coupled  $\mathcal{N} = 4$  SYM plasma was calculated in [20], using the techniques used of [43]. The result of reference [20] is

$$\sigma = 1.28349 \frac{e^2(N^2 - 1)T}{\lambda^2[-\frac{1}{2} \ln \lambda + a]}, \tag{5.1}$$

where  $a$  is an  $\mathcal{O}(1)$  constant which was not explicitly evaluated in [20]. For perturbative values of the 't Hooft coupling, this expression implies that decreasing the 't Hooft coupling should increase the conductivity. This trend is confirmed by our result in eq. (4.27). The physical reason here is that at weak-coupling the mean free path for particle collisions in the plasma increases, leading to more efficient charge-transport.

We now turn to a brief comparison between our results and lattice QCD calculations. Firstly, the calculation above obviously has its own value, without any reference to phenomenology, but as we mentioned in the introduction, our ultimate aim is to make contact with the real world, and make tentative statements about the QCD plasma produced at RHIC and the LHC. Any such statement in the context of our work must of course be taken with a set of caveats and qualifiers, owing to the disparities between  $\mathcal{N} = 4$  SYM theory and QCD. Indeed, these theories are very different at zero temperature and weak coupling. However, for a temperature  $T$  above the QCD phase-transition temperature  $T_c$  but not significantly higher, both the QCD plasma and the  $\mathcal{N} = 4$  SYM plasma behave like strongly-coupled ideal fluids. Moreover, there are some results from lattice QCD implying that the thermodynamical properties of QCD are reasonably well approximated by conformal dynamics in a range of temperatures from about  $2T_c$  up to some high temperature (for a discussion on numerical results from lattice QCD in comparison with  $\mathcal{N} = 4$  SYM theory, see [44, 45] and references therein). The closeness of the shear viscosity to entropy density ratio observed in RHIC, and that computed from the gauge/gravity duality, also lends support to the idea that there is a parametric region where one can learn about the hydrodynamical properties of QCD by studying the hydrodynamics of  $\mathcal{N} = 4$  SYM.

With this tentative philosophy in mind, it is possible to make contact with QCD lattice calculations to some extent. We must take into account that in those calculations  $N = 3$  and there are other differences with respect to the large  $N$  limit of  $\mathcal{N} = 4$  SYM plasma. There is a recent estimation of the conductivity given by Aarts et al [46]. That paper finds  $\sigma \sim 0.4 e^2 T$ , above the deconfinement transition temperature  $T_c$  of quenched lattice QCD. Assuming that the conductivity scales with  $N^2$ , and setting  $N = 3$ , a naive insertion of  $\sigma \sim 0.4 e^2 T$  in our formula eq. (4.27) gives  $\lambda = 34.52$ . A more recent lattice calculation by Ding et al [47] obtained  $1/3 e^2 T \leq \sigma \leq e^2 T$  from the vector current correlation function for light valence quarks in the deconfined phase of quenched lattice QCD at  $T = 1.45T_c$ . Using these values in our formula eq. (4.27) yields  $14.39 \leq \lambda \leq 43.86$ . It is worth noting that from lattice computations at temperatures about 1.5 to 2  $T_c$ , the values of  $\alpha_s$  are

thought to be around 0.3 to 0.4, based on heavy quark potentials.<sup>4</sup> These values of  $\alpha_s$  give a value  $\lambda$  about 15. While this is of course a naive comparison which must be taken in the context of the many caveats outlined above, it is nonetheless pleasing that our value for the conductivity falls close to those obtained from lattice QCD.

The results we present here have many interesting extensions. Firstly, observe that the electrical conductivity is an extensive quantity that depends on the number of degrees of freedom  $N$  in the theory, so it is not the best quantity for comparing charge-transport in different theories. A more useful quantity for this purpose is the charge-diffusion constant  $\mathcal{D}$ , which is found to be  $1/(2\pi T)$  in the large  $N$  limit of  $\mathcal{N} = 4$  SYM plasma at infinite 't Hooft coupling [14]. This quantity is obtained from the first pole of the  $R_{zz}$  correlator.

Another very interesting quantity that can be obtained from the current two-point function  $R_{xx}$  is the photoemission rate of the plasma [20]. Computing the finite-coupling corrections to this quantity would involve obtaining and solving the equations of  $A_x$  for the whole range of light-like momenta ( $\omega = |\vec{q}|$ ).

A final observation concerns the behaviour of the holographic conductivity obtained here as we change the internal compactification space of the ten-dimensional dual. We remind the reader that the work of [24, 25] proved that the corrections to the shear viscosity to entropy density ratio were independent of the internal compactification space for a large class of holographic duals. Here we computed the corrections to the conductivity using the simplest compactification space  $S^5$ . It would be very interesting to investigate the impact of the compactification space on the corrections computed here, and whether the universality of momentum transport posited in [24, 25] is operative for charge transport too.

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