

Spatially modulated instabilities of magnetic black branes

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ABSTRACT: We investigate spatially modulated instabilities of magnetically charged $\text{AdS}_2 \times \mathbb{R}^2$, $\text{AdS}_3 \times \mathbb{R}^2$ and $\text{AdS}_2 \times \mathbb{R}^3$ backgrounds in a broad class of theories, including those arising from KK reductions of ten and eleven dimensional supergravity. We show that magnetically charged black brane solutions in $D = 4, 5$ spacetime dimensions, whose zero temperature near horizon limit approach these backgrounds, can have instabilities that are dual to phases with current density waves that spontaneously break translation symmetry. Our examples include spatially modulated instabilities for a new class of magnetic black brane solutions of $D = 5$ $\text{SO}(6)$ gauged supergravity, that we construct in closed form, which are dual to new phases of $N = 4$ SYM theory.

KEYWORDS: Holography and condensed matter physics (AdS/CMT), Black Holes in String Theory

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1 Introduction

The AdS/CFT correspondence can be used to analyse the dynamics of strongly coupled gauge theories at finite temperature by constructing and studying black hole solutions of $D = 10$ and $D = 11$ supergravity. The stability properties of the black hole solutions play an important role as they are related to the thermodynamical stability properties of the gauge theories. Typically, at the onset of an instability new black hole solutions appear which are dual to new phases of the dual gauge theory, which may have applications to condensed matter systems.

Several new classes of black hole solutions have been found in this way, principally in the context of *electrically* charged black brane solutions of Einstein-Maxwell theory. Recall that these are simply AdS-Reissner-Nordström (AdS-RN) black branes and are dual to field theories at finite temperature and charge density. A prominent class of instabilities appear after embedding the solutions into theories of gravity with additional charged fields. The resulting instabilities lead to black brane solutions with charged hair that spontaneously break a global abelian symmetry and hence are dual to superfluid phases. Such superfluid black branes were first analysed in phenomenological theories of gravity [1–3] and then in consistent truncations of $D = 10, 11$ supergravity [4–7]. They were first studied using D-brane probes in [8].

The electrically charged AdS-RN black branes can also have spatially modulated instabilities leading to interesting new classes of black brane solutions that are dual to phases which spontaneously break translation invariance. Such instabilities have been investigated for a class of $D = 5$ gravity theories with a single gauge-field and a Chern-Simons coupling in [9, 10] (for earlier related work see [11]). It has also been shown [12] that these instabilities are present in the Sakai-Sugimoto model. A study of the effect of some higher derivative corrections was made in [13].

Similar instabilities are also present in $D = 4$ [14]. In this case they are associated with “striped” black brane solutions which are dual to phases with both current density waves and charged density waves. The $D = 4$ theories studied in [14] have a neutral pseudo-scalar field φ coupled to one or two vector fields and are of a form that is very natural in the context of $N = 2$ gauged supergravity. The key couplings in the Lagrangian that drive the instabilities are either $\varphi F \wedge F$ or $\varphi F \wedge G$, where F, G are the field strengths of the two vector-fields. Indeed these terms give rise to a mixing of the linearised modes via a linear dependence on the spatial momentum on the black brane. It was also shown in [14] how some of the $D = 4$ models which exhibit the spatially modulated instabilities can be embedded into $D = 10, 11$ supergravity. Holographic striped instabilities were also studied in the context of probe-branes in [15] utilising, essentially, the same mechanism.

In this paper we show that *magnetically* charged black brane solutions can also have spatially modulated instabilities. We will analyse several different models, including some top-down examples. We will first analyse black brane solutions of Einstein-Maxwell theory. Recall that in $D = 4$ the magnetically charged black brane solutions are again the standard AdS-RN black brane solutions, while in $D = 5$ they have only been constructed numerically [16]. In both cases, at zero temperature, the solutions interpolate between AdS_D in the UV and $\text{AdS}_{D-2} \times \mathbb{R}^2$ in the IR. It is natural to view the AdS_{D-2} region as describing the strongly coupled dynamics of the lowest Landau-level excitations of the $D - 1$ dimensional dual gauge-theory.

We will show that spatially modulated instabilities of the magnetic black branes of Einstein-Maxwell theory can appear after embedding them in a class of D -dimensional theories of gravity that involve two vector fields and a single scalar field ϕ . In these models the key coupling in the Lagrangian that drives the instability is now $\phi * F \wedge G$. The class of D -dimensional theories that we consider naturally arise in $N = 2$ gauged-supergravity, and we discuss some specific embeddings into $D = 10, 11$ supergravity. The simplest way to test for instabilities of the finite temperature black brane solutions is to look for linearised perturbations of the $\text{AdS}_{D-2} \times \mathbb{R}^2$ limiting solution that violate the AdS_{D-2} BF bound. We investigate this in section 3 and find that spatially modulated instabilities arise very naturally within the class of models that we consider. We have not yet been able to find examples of these instabilities within any consistent truncations of $D = 10, 11$ supergravity, but we expect that they can be found.

For $D = 4$, where the magnetic black brane solutions are known analytically, we go beyond the near horizon $\text{AdS}_{D-2} \times \mathbb{R}^2$ region in section 4, and analyse linearised instabilities in the full geometry. More specifically we look for spatially modulated zero modes that

appear just prior to the appearance of an instability. For some representative models that have a spatially modulated instability in the $\text{AdS}_{D-2} \times \mathbb{R}^2$ region we determine the critical temperatures at which the zero modes appear. We will explain how these instabilities are associated with phases that have current density waves without having charge density waves (in contrast to the $D = 4$ electrically charged striped black branes of [14]).

For models that do not have a spatially modulated instability in the $\text{AdS}_{D-2} \times \mathbb{R}^2$ region, it is possible that there still can be spatially modulated instabilities in the full black brane geometry. We have analysed a model arising in the $\text{SU}(3)$ invariant sector of $D = 4$ $\text{SO}(8)$ gauged supergravity, but our numerical results only allow us to conclude that if the black brane has such an instability it will be at a very low temperature.

In section 5 we discuss instabilities of a different class of magnetic black brane solutions. They arise in theories of gravity coupled to a scalar field and a single gauge field but cannot be truncated to Einstein-Maxwell theory. We focus on models that have $\text{AdS}_{D-2} \times \mathbb{R}^2$ solutions and discuss the spatially modulated instabilities.

An interesting class of $\text{AdS}_3 \times \mathbb{R}^2$ and $\text{AdS}_2 \times \mathbb{R}^3$ solutions of $D = 5$ $\text{SO}(6)$ gauge supergravity, and hence type IIB supergravity, have recently been discussed in [17, 18], which carry magnetic charges with respect to $\text{U}(1)^3 \subset \text{SO}(6)$. These include both supersymmetric and non-supersymmetric examples. For the former class, an investigation of some instabilities, including those of the type discussed in [19], was made. In section 6 we will first present a new magnetic black brane solution in closed form that at zero temperature and in the near horizon limit approaches the $\text{AdS}_2 \times \mathbb{R}^3$ solution of [17]. By considering perturbations about the $\text{AdS}_2 \times \mathbb{R}^3$ solution we find spatially modulated modes that violate the AdS_2 BF bound. This example thus provides a top-down setting in which magnetic black branes exhibit spatially modulated instabilities. Interestingly it corresponds to a phase of $N = 4$ SYM with both current density waves and charged density waves.

2 Models extending Einstein-Maxwell

We start with Einstein-Maxwell theory in D spacetime dimensions with Lagrangian given by

$$\mathcal{L} = \left[\frac{1}{2}R + \lambda^2 (D - 3) \right] * 1 - \frac{1}{2} * F \wedge F, \tag{2.1}$$

where $F \equiv dA$. The negative cosmological constant has been written in terms of the constant λ for convenience. We are interested in asymptotically AdS_D black brane solutions of this theory that are supported by magnetic flux in a single \mathbb{R}^2 -plane. When $D = 4$ these solutions are the standard magnetically charged AdS-Reissner-Nordström black brane solutions. When $D = 5$ the solutions have been constructed numerically in [16]. The solutions¹ have not yet been constructed for $D \geq 6$.

¹When $D \geq 6$ one can also have magnetic fields switched on in additional planes. The special case when the skew eigenvalues of the two-form field strength are all equal was discussed in [16] and the solutions are similar to the $D = 4, 5$ cases depending on whether D is even or odd, respectively.

In both the $D = 4$ and $D = 5$ cases, at zero temperature, the near horizon limit of these black brane solutions approach the magnetically charged $\text{AdS}_{D-2} \times \mathbb{R}^2$ solutions of (2.1) given by

$$\begin{aligned} ds^2 &= L^2 ds^2(\text{AdS}_{D-2}) + dx_1^2 + dx_2^2, & L^2 &= \frac{D-3}{2\lambda^2} \\ F &= \sqrt{2}\lambda dx_1 \wedge dx_2, \end{aligned} \quad (2.2)$$

where $ds^2(\text{AdS}_{D-2})$ is the metric on a unit radius AdS_{D-2} space. We expect that this will similarly be true in $D \geq 6$.

We can look for instabilities of the magnetically charged black brane solutions by analysing linearised perturbations about the $\text{AdS}_{D-2} \times \mathbb{R}^2$ solutions. If the perturbations have mass squared that violate the AdS_{D-2} BF bound, given by

$$L^2 M^2 \geq -\frac{(D-3)^2}{4}, \quad (2.3)$$

then we can conclude that the black brane solution is also unstable. In order to establish the precise temperature at which the instability appears one needs to analyse the full finite temperature black brane solution which, as we mentioned above, is only known in $D = 4, 5$. We shall return to this point in section 4.

We now embed these solutions in a theory of gravity that has an additional scalar field, ϕ , and a massive vector field, B , with Lagrangian

$$\begin{aligned} \mathcal{L} &= \frac{1}{2}R * 1 - V(\phi) * 1 - \frac{1}{2} * d\phi \wedge d\phi - \frac{1}{2}t(\phi) * F \wedge F \\ &\quad - \frac{1}{2}v(\phi) * G \wedge G - \frac{1}{2}m_v^2 * B \wedge B - u(\phi) * F \wedge G \end{aligned} \quad (2.4)$$

where $G \equiv dB$ and m_v^2 is a constant. The corresponding equations of motion are given by

$$\begin{aligned} R_{\mu\nu} &= \frac{2}{D-2}V g_{\mu\nu} + m_v^2 B_\mu B_\nu + \partial_\mu \phi \partial_\nu \phi + t \left(F_{\mu\rho} F_\nu{}^\rho - \frac{1}{2(D-2)} g_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma} \right) \\ &\quad + v \left(G_{\mu\rho} G_\nu{}^\rho - \frac{1}{2(D-2)} g_{\mu\nu} G_{\rho\sigma} G^{\rho\sigma} \right) + 2u \left(G_{(\mu}{}^\rho F_{\nu)\rho} - \frac{1}{2(D-2)} g_{\mu\nu} G_{\rho\sigma} F^{\rho\sigma} \right), \\ &\quad d * (tF + uG) = 0, \\ &\quad d * (vG + uF) - (-1)^D m_v^2 * B = 0, \\ (-1)^D d * d\phi + V' * 1 + \frac{1}{2}t' * F \wedge F + \frac{1}{2}v' * G \wedge G + u' * F \wedge G &= 0. \end{aligned} \quad (2.5)$$

We will assume that the functions V, t, u and v have the following expansion

$$\begin{aligned} V(\phi) &= -\lambda^2 (D-3) + \frac{1}{2} m_s^2 \phi^2 + \dots, \\ t(\phi) &= 1 - n \phi^2 + \dots, \\ u(\phi) &= s \phi + \dots, \\ v(\phi) &= 1 + \dots. \end{aligned} \quad (2.6)$$

where λ, m_s, n and s are constant. It is then consistent to set $\phi = B = 0$ in the equations of motion to recover the equations of motion of the Einstein-Maxwell theory (2.1). In particular, both the black brane solutions (for $D = 4, 5$) and the $\text{AdS}_{D-2} \times \mathbb{R}^2$ solutions of Einstein-Maxwell theory are solutions of this more general class of theories.

The Lagrangian (2.4) has been chosen to provide a simple setting to display spatially modulated instabilities. There are certainly many ways in which additional fields and interactions can be incorporated which will lead to generalisations of the instabilities that we describe. Note that we will only be considering configurations for which $F \wedge F = F \wedge G = G \wedge G = 0$ and hence various terms one might consider, such as Chern-Simons terms in odd dimensions and coupling to pseudo-scalars in even dimensions, will not play a role.

The form of (2.4) for $D = 4$ is also rather natural from the point of $N = 2$ gauged supergravity. Indeed we can make contact with a top-down construction this way. Recall that $D = 4$ $\text{SO}(8)$ gauge-supergravity is a consistent truncation of $D = 11$ supergravity. We then consider the further consistent truncation to the $\text{SU}(3) \subset \text{SO}(8)$ invariant sector which is described by an $N = 2$ $D = 4$ supergravity theory coupled to a vector multiplet and a hypermultiplet [20, 21]. Restricting to configurations in which $F \wedge F = F \wedge G = G \wedge G = 0$, where F and G are the field strengths of the two-vector fields, it is consistent to set to zero the four scalar fields in the hypermultiplet as well as the imaginary part of the complex scalar field in the vector multiplet. This leads² to a theory with two gauge fields and a real scalar field and we obtain equations of motion as in (2.5), (2.6) with $m_s^2 = -4$, $m_v^2 = 0$, $\lambda^2 = 6$, $n = -1$ and $s = -\sqrt{2}$. This can also be obtained from the semi-consistent $\text{U}(1)^4 \subset \text{SO}(8)$ truncation of [23] and one should be aware that the details will affect the higher order terms in (2.6). For example, the sub-truncation of [23] considered in section 2.2.2 of [22] has a \mathbb{Z}_2 symmetry which flips the sign of both ϕ and B but this symmetry is absent in the $\text{SU}(3) \subset \text{SO}(8)$ invariant case.

In $D = 5$ the model (2.4) also arises in Roman's $N = 4^+$ $\text{SU}(2) \times \text{U}(1)$ gauged supergravity theory [24], which is a consistent truncation of type IIB [25] and $D = 11$ supergravity [26]. We will use the action for Roman's theory given in eq. (2.14) of [26]. We set $m = 1$, $C = F^1 = F^2 = 0$ and restrict to configurations in which the Chern-Simons terms can be set to zero. We then define $X = e^{\frac{1}{\sqrt{3}}\phi}$ and also $G \rightarrow \sqrt{\frac{2}{3}}(F - \sqrt{2}G)$, $F^3 \rightarrow \sqrt{\frac{2}{3}}(\sqrt{2}F + G)$ to obtain equations of motion as in (2.5), (2.6) with $n = -4/3$, $m_s^2 = -4$, $m_v^2 = 0$, $\lambda = \sqrt{3}$ and $s = -2\sqrt{\frac{2}{3}}$. It is interesting to point out that starting with the consistent truncation of $\text{SO}(6)$ gauged supergravity given in [27], we obtain a model with the same parameters. Indeed, starting with the action in eq (2.7) of [27], which is only valid for configurations in which the Chern-Simons terms play no role, one can set $\varphi_i = 0$ and either $\beta = 0$, $A^1 = A^2$ or $\beta = 3\alpha$, $A^1 = A^3$.

²E.g. set $\rho = \chi = 0$ in eq. (2.22) of [22].

3 Instabilities of magnetic AdS_{D-2} × ℝ²

In the models extending Einstein-Maxwell theory with Lagrangian (2.4), we consider the following simple perturbation about the AdS_{D-2} × ℝ² solution (2.2):

$$\phi = \delta\phi(x^\alpha) \cos(kx_1), \quad B = \delta B(x^\alpha) \sin(kx_1) dx_2, \quad (3.1)$$

where x^α are coordinates on AdS_{D-2} and k is a constant.³ Introducing the vector $\mathbf{v} = (\delta\phi, \delta B)$ we find that, at linearised order, the equations of motion (2.5) expanded around the solution (2.2) yield

$$\square_{D-2}\mathbf{v} - L^2M^2\mathbf{v} = 0, \quad (3.2)$$

where \square_{D-2} is the Laplacian of the unit radius AdS_{D-2} and the mass matrix is given by

$$M^2 = \begin{pmatrix} \tilde{m}_s^2 + k^2 & \sqrt{2}\lambda s k \\ \sqrt{2}\lambda s k & m_v^2 + k^2 \end{pmatrix}, \quad (3.3)$$

with $\tilde{m}_s^2 \equiv m_s^2 - 2\lambda^2 n$. Let us first consider no spatial modulation i.e. $k = 0$. We see that just as in the $D = 4$, purely electric case studied in [14], if n is positive and large enough, the BF bound (2.3) can be violated leading to an instability. We have not been able to find any top-down examples where this occurs.

We now consider $k \neq 0$. The eigenvalues of the mass matrix (3.3) are given by

$$m_\pm^2 = k^2 + \frac{1}{2} (\tilde{m}_s^2 + m_v^2) \pm \frac{1}{2} \sqrt{(\tilde{m}_s^2 - m_v^2)^2 + 8k^2s^2\lambda^2}. \quad (3.4)$$

We deduce that if

$$2s^2\lambda^2 > |\tilde{m}_s^2 - m_v^2|, \quad (3.5)$$

the branch m_-^2 develops a minimum at

$$k_{\min} = \frac{1}{2\sqrt{2}s\lambda} \sqrt{4s^4\lambda^4 - (\tilde{m}_s^2 - m_v^2)^2}, \quad (3.6)$$

with

$$m_{\min}^2 = \frac{1}{2} (\tilde{m}_s^2 + m_v^2) - \frac{1}{8s^2\lambda^2} (\tilde{m}_s^2 - m_v^2)^2 - \frac{1}{2}s^2\lambda^2, \quad (3.7)$$

and it is possible for this to violate the AdS_{D-2} BF bound (2.3).

We now investigate whether these instabilities of AdS_{D-2} × ℝ² are present in the top-down models we discussed above. First consider the SU(3) invariant sector of $D = 4$ SO(8) gauged supergravity which has $m_s^2 = -4$, $m_v^2 = 0$, $\lambda^2 = 6$, $n = -1$ and $s = -\sqrt{2}$. This gives rise to $L^2m_{\min}^2 = -2/9$ (at $k \neq 0$) which is very close to but does not violate the AdS₂ BF bound of $-1/4$. For the $D = 5$ Romans theory we have $n = -4/3$, $m_s^2 = -4$, $m_v^2 = 0$, $\lambda = \sqrt{3}$ and $s = -2\sqrt{2/3}$. We find $L^2m_{\min}^2 = -3/4$ (at $k \neq 0$) which again does not violate the AdS₃ BF bound of -1 . In the appendix we will discuss some other top down constructions which are similar to (2.4) but involve a second scalar field; we find that they also do not lead to a violation of the AdS_{D-2} BF bound (2.3).

³Note that the background has $F \wedge F = 0$ and that the perturbation satisfies $F \wedge G = G \wedge G = 0$.

4 Instabilities of $D = 4$ magnetic AdS-RN black branes

In this section we analyse spatially modulated instabilities of the full magnetic black brane solutions in the models of section 2. More precisely, we look for zero modes that appear at the onset of instabilities. Since the $D = 5$ magnetic black brane solutions are only known numerically, we restrict our considerations to $D = 4$.

Fixing the cosmological constant by setting $\lambda^2 = 6$ the $D = 4$ magnetic black brane solution of Einstein-Maxwell theory is the AdS-RN solution given by

$$\begin{aligned} ds^2 &= -f dt^2 + \frac{dr^2}{f} + r^2 (dx^2 + dy^2) , \\ F &= r_+ dx \wedge dy , \end{aligned} \tag{4.1}$$

where

$$f = 2r^2 - \left(2r_+^2 + \frac{1}{2}\right) \frac{r_+}{r} + \frac{1}{2} \frac{r_+^2}{r^2} . \tag{4.2}$$

The zero temperature limit of this black hole is obtained when $r_+ = 1/\sqrt{12}$ and we note that (2.2) can be recovered as the near horizon limit after rescaling the spatial coordinates by r_+ .

4.1 First order perturbations

For simplicity, we now assume that $m_s^2 = -4$ and $m_v^2 = 0$ which covers the case of $\mathcal{N} = 8$ gauged supergravity. To construct the zero modes we consider the perturbation

$$\phi = \delta\phi(r) \cos(kx) , \quad B = \delta B(r) \sin(kx) dy . \tag{4.3}$$

The equations of motion (2.5) then yield

$$\begin{aligned} \frac{1}{r^2} \partial_r (r^2 f \partial_r \delta\phi) + \left[4 - \frac{k^2}{r^2} + \frac{nr_+^2}{r^4}\right] \delta\phi - \frac{skr_+}{r^4} \delta B &= 0 , \\ \partial_r (f \partial_r \delta B) - \frac{k^2}{r^2} \delta B - \frac{skr_+}{r^2} \delta\phi &= 0 . \end{aligned} \tag{4.4}$$

At the horizon $r = r_+$ we impose the boundary conditions

$$\delta\phi = \phi_0 + \mathcal{O}(r - r_+) , \quad \delta B = b_0 + \mathcal{O}(r - r_+) . \tag{4.5}$$

Since we are dealing with a linear and homogeneous system of equations, we can use a scaling symmetry to set $\phi_0 = 1$. At infinity, on the other hand, we have the asymptotic expansion

$$\delta\phi = \frac{\phi_1}{r} + \frac{\phi_2}{r^2} + \dots , \quad \delta B = b_0 + \frac{b_1}{r} + \dots . \tag{4.6}$$

We are only interested in zero modes that spontaneously break translation invariance so we shall demand $b_0 = 0$. Note that since we have chosen $m_v^2 = 0$, the massless gauge-field is

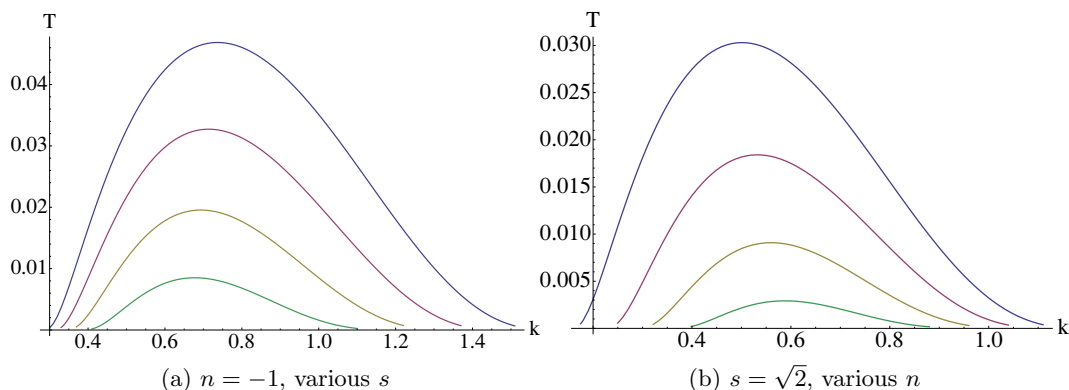


Figure 1: Plots of critical temperatures T versus k for the existence of normalisable zero modes about the $D = 4$ magnetically charged AdS-RN black brane solutions in the models of section 2. All cases have $m_s^2 = -4$ and $m_v^2 = 0$. The left frame has $n = -1$ and, from the top going down, $s = 2, 1.9, 1.8, 1.7$. The right frame has $s = \sqrt{2}$ and, from the top going down, $n = -0.25, -0.375, -0.5, -0.675$. Note that $\mathcal{N} = 8$ gauged supergravity has $n = -1$ and $s = \sqrt{2}$.

dual to a conserved current j^B in the dual field theory. For definiteness, we will assume that ϕ is dual to an operator \mathcal{O}_ϕ with dimension $\Delta = 1$, as it is in SO(8) gauged supergravity when we quantise with maximal supersymmetry, and thus we will also demand that $\phi_2 = 0$.

We have solved numerically the differential equations with these boundary conditions for two particular models, and determined the critical temperature as a function of k at which the spatially modulated zero modes appear. We have displayed some of our results in figure 1. The first frame has $n = -1$ and various values of $s = 2, 1.9, 1.8, 1.7$ and the second frame has $s = \sqrt{2}$ and various values of $n = -2/8, -3/8, -4/8, -5/8$. All of these cases have a violation of the AdS₂ BF bound in the IR at finite values of k . Note that the values $n = -1$ and $s = -\sqrt{2}$ are relevant for the magnetic black brane embedded in the SU(3) invariant sector of $D = 4$ $\mathcal{N} = 8$ gauged supergravity. Unfortunately we have not been able to stabilise our numerics for these values. All that we can conclude is that if there is an instability for these values of n, s it will be at very low temperatures indeed, as indicated by the figures.

For a given model, at the highest critical temperature T_c at which a static normalisable zero mode appears (the maxima of the curves in figure 1), a new branch of black brane solutions appear. This new branch will have a spatial modulation fixed by k_c , where k_c is the critical wave-number corresponding to T_c . These black branes, assuming that they are thermodynamically preferred, describe a spatially modulated phase in the dual field theory in which, near T_c ,

$$\langle \mathcal{O}_\phi \rangle \sim \cos k_c x, \quad \langle j_y^B \rangle \sim \sin k_c x, \quad (4.7)$$

where \mathcal{O}_ϕ is the operator dual to ϕ and j^B is the current dual to B . In particular, there is a current density wave. In [14], spatially modulated instabilities of electrically charged

black-branes were studied. At first order, current density waves were observed and then at second order, charged density waves. We now show that this does not occur for the magnetically charged black branes being considered here.

4.2 Second order perturbations

We now consider the second order perturbations. We find that a consistent set of equations is obtained if we take

$$\begin{aligned}
 \phi &= \epsilon [\delta\phi(r) \cos(kx)] + \epsilon^2 [\phi^{(0)}(r) + \phi^{(1)}(r) \cos(2kx)] , \\
 B_y &= \epsilon [\delta B(r) \sin(kx)] + \epsilon^2 [b_y^{(1)}(r) \sin(2kx)] , \\
 \delta g_{tt} &= \epsilon^2 [h_{tt}^{(0)}(r) + h_{tt}^{(1)}(r) \cos(2kx)] , \\
 \delta g_{xx} &= \epsilon^2 [h_{xx}^{(0)}(r) + h_{xx}^{(1)}(r) \cos(2kx)] , \\
 \delta g_{yy} &= \epsilon^2 [h_{yy}^{(0)}(r) + h_{yy}^{(1)}(r) \cos(2kx)] , \\
 \delta A_y &= \epsilon^2 [a_y^{(1)}(r) \sin(2kx)] ,
 \end{aligned} \tag{4.8}$$

where ϵ is a small parameter that can be taken to be $\epsilon^2 = (T - T_c)/T_c$. Expanding the equations of motion (2.5) to $O(\epsilon^2)$ we obtain a set of ordinary differential equations. The functions $\phi^{(\alpha)}$, $b_y^{(\alpha)}$, $h_{xx}^{(\alpha)}$, $h_{yy}^{(\alpha)}$ and $a_y^{(\alpha)}$ satisfy a system of inhomogeneous second order equations, the function $h_{tt}^{(0)}$ satisfies a first order inhomogeneous equation while an algebraic equation completely determines $h_{tt}^{(1)}$.

Thus, at second order, we see that in the dual field theory the stress tensor is becoming spatially modulated, as expected. Furthermore, the current, j^A , dual to the gauge-field A , is also becoming spatially modulated. For the branch appearing at T_c with modulation fixed by k_c we have

$$\langle j_y^A \rangle \sim \sin 2k_c x , \tag{4.9}$$

and hence the current density wave for j^A has half the period of that for j^B . The absence of A_t and B_t terms in (4.8) implies that there are no CDWs as commented above.

4.3 The non-linear ansatz

We can also investigate the structure of the spatially modulated black brane solutions by finding a consistent ansatz that contain the solutions. Concretely, we consider

$$\begin{aligned}
 ds^2 &= -e^{2\alpha(r,x)} dt^2 + dr^2 + e^{2\beta_1(r,x)} dx^2 + e^{2\beta_2(r,x)} dy^2 , \\
 A &= a(r,x) dy, \quad B = b(r,x) dy, \quad \phi = \phi(r,x) .
 \end{aligned} \tag{4.10}$$

This includes the magnetic black brane background (4.1) as well as the perturbations considered in (4.8) after a simple redefinition of the coordinate r . Note, in particular, that this ansatz is not associated with CDWs.

To see that it is a well defined ansatz, we proceed as follows. We first find that the equations of motion for the matter fields leads to three pde's, second order in r and x , for

the three functions $a(r, x), b(r, x), \phi(r, x)$. We next observe that if we write the Einstein's equations in the form $E_{\mu\nu} = 0$, then there are five non-trivial components. The equations $E_{tt} = E_{xx} = E_{yy} = 0$ can also be written as three pdes, second order in r and x , for the three functions $\alpha(r, x), \beta_1(r, x), \beta_2(r, x)$. This leaves two more equations $E_{rr} = E_{rx} = 0$. However, the Bianchi identities give two relations

$$\begin{aligned}
 e^{-\beta_1} \partial_r \left(e^{\alpha+\beta_1+\beta_2} E_{rx} \right) &= -\partial_x \left(e^{\alpha-2\beta_1+\beta_2} E_{xx} \right) + \frac{1}{2} e^{\alpha+\beta_2} \left(E_{tt} \partial_x e^{-2\alpha} - E_{yy} \partial_x e^{-2\beta_2} \right), \\
 \partial_r \left(e^{\alpha+\beta_1+\beta_2} E_{rr} \right) &= -\partial_x \left(e^{\alpha-\beta_1+\beta_2} E_{rx} \right) \\
 &\quad + \frac{1}{2} e^{\alpha+\beta_1+\beta_2} \left(E_{tt} \partial_r e^{-2\alpha} - E_{yy} \partial_r e^{-2\beta_2} - E_{xx} \partial_r e^{-2\beta_1} \right). \quad (4.11)
 \end{aligned}$$

From these we observe that if the three equations $E_{tt} = E_{xx} = E_{yy} = 0$ are satisfied everywhere, then we also have $E_{rr} = E_{rx} = 0$ everywhere provided that we just demand that $E_{rr} = E_{rx} = 0$ on a Cauchy surface $r = r_0$. Thus, the ansatz leads to a well defined set of equations, and in particular, will lead to solutions with spatially modulated current density waves without charge density waves.

5 Instabilities of $\text{AdS}_{D-2} \times \mathbb{R}^2$ in other models

We now briefly discuss instabilities of magnetic solutions in models that involve a scalar field coupled to just a single gauge-field which, unlike the models considered above, cannot be truncated to Einstein-Maxwell theory. We consider the Lagrangian

$$\mathcal{L} = \frac{1}{2} R * 1 - V(\phi) * 1 - \frac{1}{2} * d\phi \wedge d\phi - \frac{1}{2} t(\phi) * F \wedge F \quad (5.1)$$

and note that the equations of motion can be obtained from (2.4), (2.5). We now consider more general V, t than the expansions given in (2.6) and we note, in particular, that if $V'(0)$ or $t'(0)$ are non-zero then we cannot consistently set $\phi = 0$ in the equations of motion. Particular examples of these models have been considered in the context of AdS/CMT from a different point of view in [28–30].

What is of interest here is that for certain V, t these models can have magnetically charged asymptotically AdS_D black branes that approach $\text{AdS}_{D-2} \times \mathbb{R}^2$ in the IR at zero temperature. It would go beyond the scope of this paper to make a detailed analysis of these magnetic black brane solutions. Instead we will focus on the $\text{AdS}_{D-2} \times \mathbb{R}^2$ solutions and see how spatially modulated instabilities can show up as perturbations that have imaginary AdS_{D-2} scaling dimensions.

We first note that the equations of motion to (5.1) admit the magnetically⁴ charged $\text{AdS}_{D-2} \times \mathbb{R}^2$ solution with constant scalar field, $\phi = \phi_0$, and

$$\begin{aligned}
 ds^2 &= L^2 ds^2(\text{AdS}_{D-2}) + dx_1^2 + dx_2^2, & L^2 &= -\frac{(D-3)t'(\phi_0)}{2V'(\phi_0)t(\phi_0)}, \\
 F &= \left(-\frac{2V'(\phi_0)}{t'(\phi_0)} \right)^{1/2} dx_1 \wedge dx_2, & & (5.2)
 \end{aligned}$$

⁴An analogous analysis can be carried out for electrically charged $\text{AdS}_2 \times \mathbb{R}^{D-2}$ solutions.

provided that

$$\begin{aligned} t'(\phi_0) V(\phi_0) &= (D-3)V'(\phi_0)t(\phi_0), \\ t'(\phi_0) V'(\phi_0) &< 0, \quad t(\phi_0) > 0. \end{aligned} \tag{5.3}$$

We have not yet found a top-down embedding of such solutions, but we expect that they can be found.

We next consider spatially modulated instabilities of this solution. Let us write the metric on the unit radius AdS_{D-2} space as

$$ds^2(\text{AdS}_{D-2}) = \rho^2(-dt^2 + dy_a dy_a) + \frac{d\rho^2}{\rho^2} \tag{5.4}$$

Restricting to $D = 4$ for simplicity, we consider the time independent linear perturbation given by

$$\begin{aligned} \delta g_{tt} &= \rho^2 h_{tt}(\rho) \cos(kx_1), & \delta g_{x_i x_i} &= h_{ii}(\rho) \cos(kx_1), \\ \delta \phi &= w(\rho) \cos(kx_1), & \delta A &= a(\rho) \sin(kx_1) dx_2. \end{aligned} \tag{5.5}$$

After substituting into the equations of motion for (5.1), we find that we can solve an algebraic equation to obtain h_{tt} . This then leads to a second order differential equation for the 4-vector $\mathbf{v} \equiv (h_{11}, h_{22}, w, a)$ whose coefficients involve the data $\{t(\phi_0), V(\phi_0), t'(\phi_0), V'(\phi_0), t''(\phi_0), V''(\phi_0)\}$ and k . These equations admit modes of the form $\mathbf{v} = \mathbf{v}_0 \rho^\delta$ where \mathbf{v}_0 is a constant vector and δ is the scaling dimension. By suitable choice of the data it is simple to obtain complex values for δ , with $k \neq 0$, which corresponds to a violation of the AdS_{D-2} BF bound.

6 An instability in type IIB supergravity

We now consider instabilities of magnetic black brane solutions within SO(6) $D = 5$ gauged supergravity and hence within type IIB supergravity. In fact the solutions will lie within a consistent truncation of SO(6) gauged supergravity that has two scalar fields ϕ_1, ϕ_2 and $U(1)^3 \subset \text{SO}(6)$ gauge fields with Lagrangian [31]

$$\mathcal{L} = (R - V) * 1 - \frac{1}{2} \sum_{a=1}^2 *d\phi_a \wedge d\phi_a - \frac{1}{2} \sum_{i=1}^3 X_i^{-2} * F^i \wedge F^i + F^1 \wedge F^2 \wedge A^3 \tag{6.1}$$

where $F^i = dA^i$ and

$$\begin{aligned} V &= -4 \sum_{i=1}^3 X_i^{-1} \\ X_1 &= e^{-\frac{1}{\sqrt{6}}\phi_1 - \frac{1}{\sqrt{2}}\phi_2}, \quad X_2 = e^{-\frac{1}{\sqrt{6}}\phi_1 + \frac{1}{\sqrt{2}}\phi_2}, \quad X_3 = e^{\frac{2}{\sqrt{6}}\phi_1} \end{aligned} \tag{6.2}$$

Our first result is that this theory admits the following magnetically charged black hole solutions

$$\begin{aligned} ds_5^2 &= -f dt^2 + \frac{dr^2}{f} + r^2 (dx_1^2 + dx_2^2 + dx_3^2) \\ F^1 &= \epsilon_1 B dx_2 \wedge dx_3, \quad F^2 = \epsilon_2 B dx_3 \wedge dx_1, \quad F^3 = \epsilon_3 B dx_1 \wedge dx_2 \end{aligned} \tag{6.3}$$

where

$$f = r^2 - \frac{r_+^4}{r^2} + \frac{B^2}{2r^2} \log \frac{r_+}{r} \quad (6.4)$$

and $\epsilon_i = \pm 1$. We choose $0 \leq B \leq 2\sqrt{2}r_+^2$ so that the outer event horizon is located at $r = r_+$ and the temperature is $T = \frac{8r_+^4 - B^2}{8\pi r_+^3}$. At zero temperature, when $B = 2\sqrt{2}r_+^2$, the near horizon limit of (6.3) approaches the magnetic $\text{AdS}_2 \times \mathbb{R}^3$ solution constructed in [17]

$$ds_5^2 = \frac{1}{8} ds^2(\text{AdS}_2) + dx_1^2 + dx_2^2 + dx_3^2$$

$$F^1 = \epsilon_1 2\sqrt{2} dx_2 \wedge dx_3, \quad F^2 = \epsilon_2 2\sqrt{2} dx_3 \wedge dx_1, \quad F^3 = \epsilon_3 2\sqrt{2} dx_1 \wedge dx_2 \quad (6.5)$$

(after scaling $x_i \rightarrow x_i/r_+$). Note that we can change the signs of two of the spatial coordinates without changing the $D = 5$ orientation and this would change the signs of two of the three F^i . Thus, there are two independent solutions, with fixed orientation, depending on whether $\epsilon_1 \epsilon_2 \epsilon_3 = \pm 1$. In fact both solutions have the same spatially modulated instability as we now show.

We first consider perturbations about the $\text{AdS}_2 \times \mathbb{R}^3$ solution (6.5). Specifically, using the coordinates for AdS_2 as in (5.4), we take

$$\begin{aligned} \delta g_{x_1 x_1} &= -\delta g_{x_2 x_2} = h(\rho) \cos(k x_3) \\ \delta A^1 &= \epsilon_1 a(\rho) \sin(k x_3) dx_2, & \delta A^2 &= \epsilon_2 a(\rho) \sin(k x_3) dx_1, \\ \delta A^3 &= \epsilon_1 \epsilon_2 \rho u(\rho) \sin(k x_3) dt, & \delta \phi_2 &= w(\rho) \cos(k x_3) \end{aligned} \quad (6.6)$$

for which the equations of motion yield

$$\begin{aligned} 8 (\rho^2 w')' - (12 + k^2) w - 8\sqrt{2} h + 8k a &= 0 \\ 8 (\rho^2 a')' - k^2 a + 4k w + 2\sqrt{2} k h - 16\sqrt{2} (\rho u)' &= 0 \\ 8 (\rho^2 u')' - k^2 u - 4\sqrt{2} \rho a' &= 0 \\ 8 (\rho^2 h')' - (k^2 + 8) h - 8\sqrt{2} w + 4\sqrt{2} k a &= 0 \end{aligned} \quad (6.7)$$

Notice that we take into account the mixing of the metric and the scalar even for $k = 0$ (in contrast to the analysis of [17]). We now look for solutions of the form $(w, a, u, h) = \mathbf{v} \rho^\delta$ with \mathbf{v} a constant vector. The system of equations (6.7) then takes the form $\mathbf{M} \mathbf{v} = 0$ where \mathbf{M} is a 4×4 matrix that depends on k only. Demanding that non-trivial values of \mathbf{v} exist implies that $\det \mathbf{M} = 0$ and this equation specifies the possible values of δ as functions of k . In [17], where only modes with $k = 0$ were considered, it was argued that the $\text{AdS}_2 \times \mathbb{R}^3$ background is stable. Here, even after properly taking into account the mixing with the metric, we still find that for $k = 0$ the system is stable. However, for general k , a numerical analysis shows that there is a range of $k \neq 0$ for which δ has a non-zero imaginary part signalling an unstable background.

We now turn our attention to spatially modulated perturbations about the full magnetic black brane solution (6.1). Specifically we consider

$$\begin{aligned}
 \delta g_{x_1 x_1} &= -\delta g_{x_2 x_2} = r^2 h(r) \cos(k x_3) \\
 \delta A^1 &= \epsilon_1 a(r) \sin(k x_3) dx_2, & \delta A^2 &= \epsilon_2 a(r) \sin(k x_3) dx_1, \\
 \delta A^3 &= \epsilon_1 \epsilon_2 u(r) \sin(k x_3) dt, & \delta \phi_2 &= w(r) \cos(k x_3)
 \end{aligned} \tag{6.8}$$

which leads to the linear system of equations

$$\begin{aligned}
 r (r^3 f w')' - (2B^2 + k^2 r^2 - 4r^4) w + 2\sqrt{2} B k a - \sqrt{2} B^2 h &= 0 \\
 r (r f a')' - k^2 a + \sqrt{2} B k w + B k h - r B u' &= 0 \\
 (r^3 u')' - k^2 r f^{-1} u - 2B a' &= 0 \\
 r (r^3 f h')' - r^2 (k^2 + 8r^2 - 4f - 2r f') h + 2B k a - \sqrt{2} B^2 w &= 0
 \end{aligned} \tag{6.9}$$

At the black hole horizon we impose the following boundary conditions

$$\begin{aligned}
 w &= w^{(0)} + w^{(1)} (r - r_+) + \dots \\
 a &= a^{(0)} + a^{(1)} (r - r_+) + \dots \\
 u &= u^{(1)} (r - r_+) + \dots \\
 h &= h^0 + h^{(1)} (r - r_+) + \dots
 \end{aligned} \tag{6.10}$$

As usual we are only interested in spatially modulated zero modes that correspond to spontaneously breaking of translation invariance. Thus, asymptotically as $r \rightarrow \infty$ we impose the boundary conditions

$$\begin{aligned}
 w &= \frac{v_1}{r^2} + \dots \\
 a &= \frac{v_2}{r^2} + \dots \\
 u &= \frac{v_3}{r^2} + \dots \\
 h &= \frac{v_4}{r^4} + \dots
 \end{aligned} \tag{6.11}$$

with the v_i fixing the expectation values of the corresponding operators in the dual $N = 4$ SYM theory.

Our analysis of the instabilities of the $\text{AdS}_2 \times \mathbb{R}^3$ solution implies that there will be solutions of the ODEs (6.9) with these boundary conditions at a specific temperature for a given value of k . Unfortunately the temperatures are very low and so we have not been able to stabilise the numerics. The highest critical temperature T_c will occur for a critical wave number k_c . At T_c a new branch of spatially modulated black branes will exist with, at leading order,

$$\begin{aligned}
 \langle \mathcal{O}_{\phi_2} \rangle &\sim \sin k_c x_3, \\
 \langle j_{x_2}^1 \rangle &\sim \sin k_c x_3, & \langle j_{x_1}^2 \rangle &\sim \sin k_c x_3 & \langle j_t^3 \rangle &\sim \sin k_c x_3
 \end{aligned} \tag{6.12}$$

where \mathcal{O}_{ϕ_2} is the operator dual to ϕ_2 and j^i are the three U(1) currents dual to A^i in $N = 4$ SYM theory. Observe that j^1, j^2 exhibit current density waves, while j^3 exhibits charge density waves and that they are in phase with each other. It would be interesting to explore what happens at next order in perturbation theory.

7 Final comments

We have shown that magnetically charged black branes can exhibit spatially modulated instabilities. In the dual field theory, these correspond to the spontaneous breaking of translation invariance via current density waves, and in some cases also charge density waves, when the field theory is placed in a magnetic field. It would be very interesting to go beyond our perturbative analysis and construct the fully back reacted black brane solutions as well as determining the zero temperature ground states.

For the class of black brane solutions which have an $\text{AdS}_{D-2} \times \mathbb{R}^2$ region in the IR at zero temperature, we have not yet been able to find any top-down model that exhibits an instability either in the models extending Einstein-Maxwell theory or those considered in section 5. However, we think it is likely that they can be found. On the other hand for a new class of black brane solutions of $D = 5$ SO(6) gauge supergravity with an $\text{AdS}_2 \times \mathbb{R}^3$ region at zero temperature that we constructed in section 6 we did find spatially modulated instabilities. It would be interesting to explore this example further and determine, for example, whether or not the instability we found is the dominant instability within type IIB supergravity.

Another direction is to extend our analysis to dyonic black brane solutions, carrying both electric and magnetic charges. For the $D = 4$ case the dyonic black brane solutions of Einstein-Maxwell theory are again of the AdS-RN form. For the $D = 5$ case, dyonic black brane solutions have been constructed numerically for a class of gravity theories, including minimal gauged supergravity, in [32–34]. There are a variety of ways in which these theories can be coupled to additional fields and it is clear that, again, there is a rich spectrum of spatially modulated black brane solutions.

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A Top down perturbations of magnetic $\text{AdS}_{D-2} \times \mathbb{R}^2$

Here we consider perturbations of magnetic $\text{AdS}_{D-2} \times \mathbb{R}^2$ solutions that appear in some top-down models extending Einstein-Maxwell theory that are similar to (2.4), but involve a second scalar field. In all cases we find that the models do not have spatially modulated instabilities. As in section 4, it is still possible that the magnetic brane solutions do have them, but we will not investigate this here.

We first consider the consistent truncation of $D = 11$ supergravity on a SE_7 manifold to a $D = 4$ $N = 2$ gauged supergravity [35]. It is convenient to use the Lagrangian given in eq (2.6) of [6] and we then set $h = \chi = a = 0$ to obtain

$$\mathcal{L} = \frac{R}{2} - 12\partial U^2 - \frac{3}{4}\partial V^2 - 3\partial U\partial V - \frac{1}{8}e^{6U+3V} F_{\mu\nu}F^{\mu\nu} - \frac{3}{8}e^{-V-2U} H_{\mu\nu}H^{\mu\nu} - 9e^{-12U} (B - \epsilon A)^2 + 24e^{-8U-V} - 3e^{-10U+V} - 9e^{-18U-3V}, \quad (\text{A.1})$$

with $F = dA$, $H = dB$ and $\epsilon = \pm 1$. Note that this truncation is only valid for configurations which satisfy $F \wedge H = 0$. The basic AdS_4 vacuum uplifts to an $\text{AdS}_4 \times SE_7$ solution of $D = 11$ supergravity: when $\epsilon = 1$ it is the supersymmetric solution and when $\epsilon = -1$ it is the skew-whiffed solution. We now redefine $\epsilon A \rightarrow \frac{1}{2\sqrt{2}}A - \frac{\sqrt{3}}{2\sqrt{2}}B$, $B \rightarrow \frac{1}{2\sqrt{2}}A + \frac{1}{2\sqrt{6}}B$ and $U \rightarrow \frac{1}{6}\sqrt{1 - \frac{1}{\sqrt{13}}}u - \frac{1}{6}\sqrt{1 + \frac{1}{\sqrt{13}}}v$, $V \rightarrow \frac{1}{3}\sqrt{4 - \frac{10}{\sqrt{13}}}u + \frac{1}{3}\sqrt{4 + \frac{10}{\sqrt{13}}}v$. We can then expand around the $\text{AdS}_2 \times \mathbb{R}^2$ solution of the model and find that u and v each combine with B as in (3.1) to give a mass matrix as in (3.3) with

- $\tilde{m}_u^2 = 8(7 + \sqrt{13})$, $m_{vec}^2 = 48$, $s = \sqrt{1 + \frac{1}{\sqrt{13}}}$, $\lambda = \sqrt{12}$.
- $\tilde{m}_v^2 = 8(7 - \sqrt{13})$, $m_{vec}^2 = 48$, $s = \sqrt{1 - \frac{1}{\sqrt{13}}}$, $\lambda = \sqrt{12}$.

In both cases we find that the minimum mass-squared eigenvalue of (3.3) is at $k = 0$ and does not violate the AdS_2 BF bound.

We next consider the consistent truncation of $D = 11$ supergravity on $S^4 \times H^3$ to a $D = 4$ $N = 2$ gauged supergravity that was derived in [36]. Starting with the action given in eq. (4.14) of [36] we set $l = -1$, $\chi = \theta = \xi = a = 0$, $T = \delta$. Restricting to configurations in which $\tilde{H} \wedge \tilde{H} = \tilde{H} \wedge F = 0$ we can also set $\beta = 0$. We next redefine the scalars via: $\phi_0 \rightarrow -\frac{1}{2}\ln 2 + \frac{1}{\sqrt{15}}(\sqrt{5 + \sqrt{5}}\phi_0 - \sqrt{5 - \sqrt{5}}\phi_1)$, $\phi_1 \rightarrow 2\left(\frac{1}{\sqrt{5 + \sqrt{5}}}\phi_0 + \frac{1}{\sqrt{5 - \sqrt{5}}}\phi_1\right)$ and the vectors via: $A \rightarrow 2^{1/4}(A - \sqrt{3}B)$, $\tilde{B} \rightarrow 2^{-3/4}\left(A + \frac{1}{\sqrt{3}}B\right)$. Focussing on perturbations about the $\text{AdS}_2 \times \mathbb{R}^2$ solution we find

- $\tilde{m}_{\phi_0}^2 = 3\sqrt{2} + \sqrt{10}$, $m_v^2 = 2\sqrt{2}$, $s = -\sqrt{1 + \frac{1}{\sqrt{5}}}$, $\lambda = \frac{3^{1/2}}{2^{1/4}}$.
- $\tilde{m}_{\phi_1}^2 = 3\sqrt{2} - \sqrt{10}$, $m_v^2 = 2\sqrt{2}$, $s = \sqrt{1 - \frac{1}{\sqrt{5}}}$, $\lambda = \frac{3^{1/2}}{2^{1/4}}$.

The minimum mass-squared eigenvalue of (3.3) for ϕ_0 is at $k \neq 0$, while for ϕ_1 it is at $k = 0$ and neither violates the AdS_2 BF bound.

Finally we consider the consistent truncation of type IIB supergravity on an arbitrary SE_5 space derived in [37]. We start with the action given in eq. (4.21) of [37]. We then redefine the scalars via: $u \rightarrow -\frac{1}{5\sqrt{5}}(u - 2\sqrt{6}v)$, $v \rightarrow \frac{1}{5}\sqrt{\frac{2}{15}}(2\sqrt{6}u + v)$ and the vectors via $\mathcal{A} \rightarrow \sqrt{\frac{2}{3}}(A - \sqrt{2}B)$, $\mathbf{A} \rightarrow \sqrt{3}B$. Expanding about the $\text{AdS}_3 \times \mathbb{R}^2$ solution we find

- $\tilde{m}_u^2 = 36$, $m_{vec}^2 = 24$, $s = -\frac{2\sqrt{2}}{5}$, $\lambda = \sqrt{3}$.
- $\tilde{m}_v^2 = 16$, $m_{vec}^2 = 24$, $s = \frac{4}{\sqrt{15}}$, $\lambda = \sqrt{3}$.

The minimum mass-squared eigenvalue of (3.3) for u is at $k \neq 0$, while for v it is at $k = 0$ and neither violates the AdS_3 BF bound.

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