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Reduction schemes in cutoff regularization and Higgs decay into two photons

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Abstract: We present a new systematic method to evaluate one-loop tensor integrals in conventional ultraviolet cutoff regularization. By deriving a new recursive relation that describes the momentum translation variance of ultraviolet integrals, we implement this relation in the Passarino-Veltman reduction method. With this method, we recalculate the Higgs boson decay into two photons at one-loop level in the Standard Model. We reanalyze this process carefully and clarify some issues arisen recently in cutoff regularization.

Keywords: Higgs Physics, Electromagnetic Processes and Properties, Standard Model

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Contents

1 Introduction

Recently, the ATLAS [1] and CMS [2] collaborations have renewed efforts to search for the Higgs boson at the CERN LHC with data integrated up to $\mathcal{O}(fb^{-1})$. The excluded mass region for the Standard Model(SM) Higgs boson has been extended to most of the region between 145 and 466 GeV. In the low mass region of the Higgs boson, the two-photon mode of Higgs decay plays a crucial role in experimental studies.

R. Gastmans et al. recently recalculated $H \to \gamma\gamma$ via W-boson loop [3, 4], which yielded a result in contradiction with the old ones in the literature [5–8]. Their computation was carried out in four-momentum cutoff regularization rather than dimensional regularization (DREG). To reduce the number of Feynman diagrams, Gastmans et al. chose unitary gauge. In their treatment, the new result, which satisfies the decoupling theorem [9], was favored by the authors. Later, several authors [10–13] have pointed out that the old results are still correct and the decoupling theorem is violated by the $H\phi^+\phi^-$ coupling in this case. However, we are still unsatisfactory with the explanations about the problems with the calculations of R. Gastmans et al., since there have never been doubts about the correctness of their algebra. In order to clarify this problem, we develop a new method to do one-loop calculation in cutoff regularization.

Although DREG has proven its superiority and achieved the most widely usage in phenomenological applications, cutoff regularization, the oldest regularization, still has some advantages compared with DREG theoretically. For instance, in DREG, one is unable to obtain the correct divergent terms higher than logarithmic divergences, which means that quadratic divergent terms of SM Higgs self-energy diagrams disappeared in DREG. Pauli-Villars regularization is flawed because it violates chiral symmetry, while the symmetry is preserved in cutoff regularization. Moreover, from Wilsonian effective field theory viewpoint, cutoff regularization scheme is also a more intuitive and straightforward scheme. Therefore, the introduction of an explicit cutoff is sometimes advantageous.

However, there are still many drawbacks in this four-momentum regularization that should be mentioned.Considering the truncation in momentum modes, this regularization is flawed because it violates gauge invariance and translation invariance regarding the loop momentum. The latter condition signifies that the results may ambiguously depend on the manner how the propagators are written. Hence, in the present paper a new recursive relation for loop momentum translation is derived first. Then the Passarino-Veltman reduction method[1][14, 15] is modified to reduce the tensor integrals in this regularization. One can follow Dyson's prescription [16, 17] to obtain a gauge invariant result, just as shown in our calculations of $H \to \gamma\gamma$.

As an example, we reconsider the process $H \to \gamma\gamma$ in this four-dimensional momentum cutoff regularization with our proposed approach. Given that the process of $H \to \gamma\gamma$ is free from infrared and mass singularities, only the ultraviolet cutoff is considered here. Readers who need to handle infrared or mass singularities should turn to the mass regularization scheme demonstrated in the literature e.g. [18].

The present paper is organized as follows. In section 2, a new recursive relation for the loop momentum translation in cutoff regularization is demonstrated. Then, it is implemented in the Passarino-Veltman reduction schemes in section 3. With this approach, calculations and analysis of $H \to \gamma\gamma$ are performed in section 4. Our conclusion is present in section 5. In Appendex A, the expressions for $J^N_{\mu_1\ldots\mu_s}$ used in section 2 is derived. Finally, some scalar integrals can be found in appendix B.

2 A new recursive relation

In this section, we will show how to calculate

$$I^{\Delta}_{\mu_1\ldots\mu_s}(b,a^2) \equiv \int \mathrm{d}^4k \frac{k_{\mu_1}\ldots k_{\mu_s}}{(-(k-b)^2+a^2)^n} - \int \mathrm{d}^4k \frac{(k+b)_{\mu_1}\ldots(k+b)_{\mu_s}}{(-k^2+a^2)^n}. \tag{2.1}$$

This integration has a superficial divergence degree $\Delta \equiv s+4-2n$.

A negative Δ evidently simplifies the calculation, because the limits of the integrals in eq. (2.1) can be set to infinity and the translation shift $k \to k+b$ does not change these limits. Therefore, $I^{\Delta}_{\mu_1\ldots\mu_s}$ completely vanishes when $\Delta < 0$. However, results may vary when the integrals in eq. (2.1) are ultraviolet divergent because there is an artificial four-momentum cutoff scale $\mathbf{\Lambda}$ in these integrals. These conditions are then considered in the following.

$I^{\Delta}_{\mu_1\ldots\mu_s}$ can be rewritten as

$$I^{\Delta}_{\mu_1\ldots\mu_s}(b,a^2) = \left(\int \mathrm{d}^4k \frac{k_{\mu_1}\ldots k_{\mu_s}}{(-(k-b)^2+a^2)^n} - \int \mathrm{d}^4k \frac{k_{\mu_1}\ldots k_{\mu_s}}{(-k^2+a^2)^n}\right) - \int \mathrm{d}^4k \frac{f^{\mathrm{rem}}(b)}{(-k^2+a^2)^n}, \tag{2.2}$$

[1] Note that, integration by parts (IBP) reduction methods are not valid in this case due to nonvanishing surface terms.

with the remainder $f^{\rm rem}(b) \equiv (k+b)_{\mu_1}\dots(k+b)_{\mu_s} - k_{\mu_1}\dots k_{\mu_s}$. Using the identity

$$\frac{1}{A^n} - \frac{1}{B^n} = \int_0^1 \mathrm{d}x \frac{n(B-A)}{(xA+(1-x)B)^{n+1}}, \tag{2.3}$$

one arrived at

$$\begin{aligned} I^{\Delta}_{\mu_1\dots\mu_s}(b,a^2) &= \int \mathrm{d}^4k \int_0^1 \mathrm{d}x \frac{n(-2b\cdot k + b^2)k_{\mu_1}\dots k_{\mu_s}}{(-(k-c)^2+d^2)^{n+1}} \\ &\quad - \int \mathrm{d}^4k \frac{f^{\rm rem}(b)}{(-k^2+a^2)^n}, \end{aligned} \tag{2.4}$$

where $c \equiv x\, b, d^2 \equiv a^2 - b^2x(1-x)$. The integral momentum k to $k+c$ in the first integral of the previous equation is shifted, so that

$$\begin{aligned} I^{\Delta}_{\mu_1\dots\mu_s}(b,a^2) &= -2n\; b^{\mu_0} \int_0^1 \mathrm{d}x I^{\Delta-1}_{\mu_0\mu_1\dots\mu_s}(c,d^2) + n\; b^2 \int_0^1 \mathrm{d}x I^{\Delta-2}_{\mu_1\dots\mu_s}(c,d^2) \\ &\quad + \int \mathrm{d}^4k \int_0^1 \mathrm{d}x \frac{n(-2b\cdot k + b^2 - 2b\cdot c)k_{\mu_1}\dots k_{\mu_s}}{(-k^2+d^2)^{n+1}} \\ &\quad + \int \mathrm{d}^4k \int_0^1 \mathrm{d}x \frac{n(-2b\cdot k + b^2 - 2b\cdot c)f^{\rm rem}(c)}{(-k^2+d^2)^{n+1}} \\ &\quad - \int \mathrm{d}^4k \frac{f^{\rm rem}(b)}{(-k^2+a^2)^n} \\ &= -2n\; b^{\mu_0} \int_0^1 \mathrm{d}x I^{\Delta-1}_{\mu_0\mu_1\dots\mu_s}(c,d^2) + n\; b^2 \int_0^1 \mathrm{d}x I^{\Delta-2}_{\mu_1\dots\mu_s}(c,d^2) \\ &\quad -2n\; b^{\mu_0} \int \mathrm{d}^4k \int_0^1 \mathrm{d}x \frac{k_{\mu_0}k_{\mu_1}\dots k_{\mu_s}}{(-k^2+d^2)^{n+1}} \\ &\quad -2n \int \mathrm{d}^4k \int_0^1 \mathrm{d}x \frac{b\cdot k f^{\rm rem}(c)}{(-k^2+d^2)^{n+1}} \\ &\quad - \int \mathrm{d}^4k \int_0^1 \mathrm{d}x \frac{\frac{\partial f^{\rm rem}(c)}{\partial x}}{(-k^2+d^2)^n}. \end{aligned} \tag{2.5}$$

In the sixth line of eq. (2.5), a spurious part that is proportional to $(1-2x)$ in the numerator of the integrand is removed. Integration by parts is performed at the end of eq. (2.5).

Aside from the terms expressed in $I^{\Delta-1}$ and $I^{\Delta-2}$, eq. (2.5) can be simplified further using the formulae $J^N_{\mu_1\dots\mu_s}(a^2) \equiv \int \mathrm{d}^4k \frac{k_{\mu_1}k_{\mu_2}\dots k_{\mu_s}}{(-k^2+a^2)^N}$ given in appendix A. After expanding the terms proportional to x^j in the integrands and implementing the expressions for J^N, a lot of terms are canceled. Thus, the final result is

$$\begin{aligned} I^{\Delta}_{\mu_1\dots\mu_s}(b,a^2) &= -2n\; b^{\mu_0} \int_0^1 \mathrm{d}x I^{\Delta-1}_{\mu_0\mu_1\dots\mu_s}(c,d^2) + n\; b^2 \int_0^1 \mathrm{d}x I^{\Delta-2}_{\mu_1\dots\mu_s}(c,d^2) \\ &\quad - \sum_{t=\max(0,4-2n),\ \text{even}}^{2\lfloor\frac{s+1}{2}\rfloor-2n+2} \{g^{s+t-\Delta}b^{\Delta-t}\}_{\mu_1\dots\mu_s} \frac{i\pi^2(-2)^{-\frac{s+t-\Delta}{2}}}{\Gamma(\frac{s+t-\Delta}{2}+3)} h(t,n,\Delta), \end{aligned} \tag{2.6}$$

where the notation $\{g^{s+t-\Delta}b^{\Delta-t}\}_{\mu_1\ldots\mu_s}$ defined in appendix A, $n = \frac{s+4-\Delta}{2}$, $\lfloor y \rfloor$ is a Gaussian function (the greatest integer that is not larger than y), and the function is

$$h(t,n,\Delta) \equiv \sum_{l=0}^{\frac{t}{2}} C_{n-1+l}^{l} \sum_{n_2+n_1+n_0=l}^{n_2,n_1,n_0\geq 0} \frac{(-1)^{n_2+n_0}(\Delta-t)}{2n_2+n_1+\Delta-t} \frac{l!}{n_2!n_1!n_0!} (b^2)^{n_2+n_1}(a^2)^{n_0}\Lambda^{t-2l}. \tag{2.7}$$

We should make some remarks about above equation before going forward. In the recursive relation eq. (2.6), there are two integrals left. However, since all of the $I^{\Delta}_{\mu_1\ldots\mu_s}(b,a^2)$ are only polynomials of b and a^2, i.e., they are only polynomials of integral variable x, the explicit expressions for this recursive relation can be easily obtained with the help of computers. Especially, the explicit expressions for $I^{\Delta}_{\mu_1\ldots\mu_s}(\Delta = 0,1,2,3)$ are

$$\begin{aligned}
I^{0}_{\mu_1\ldots\mu_s}(b,a^2) &= 0,\\
I^{1}_{\mu_1\ldots\mu_s}(b,a^2) &= -\{g^{s-1}b^1\}_{\mu_1\ldots\mu_s}\frac{i\pi^2(-2)^{-\frac{s-1}{2}}}{\Gamma(\frac{s+5}{2})},\\
I^{2}_{\mu_1\ldots\mu_s}(b,a^2) &= \left(n\ b^2\{g^s\}_{\mu_1\ldots\mu_s} + (4n+2)\theta(s-2)\{g^{s-2}b^2\}_{\mu_1\ldots\mu_s}\right)\\
&\qquad \frac{i\pi^2(-2)^{1-n}}{\Gamma(n+2)}, \qquad \text{with} \quad n = \frac{s+2}{2},\\
I^{3}_{\mu_1\ldots\mu_s}(b,a^2) &= -\theta(s-3)\{g^{s-3}b^3\}_{\mu_1\ldots\mu_s}\frac{i\pi^2(-2)^{2-n}(3n^2+6n+2)}{\Gamma(n+3)}\\
&\qquad -\{g^{s-1}b^1\}_{\mu_1\ldots\mu_s}\frac{i\pi^2(-2)^{1-n}}{\Gamma(n+2)}\\
&\qquad \left(\Lambda^2 - n\ a^2 - \frac{n(n+1)}{n+2}b^2\right), \quad \text{with} \quad n = \frac{s+1}{2}.
\end{aligned} \tag{2.8}$$

3 Modified Passarino-Veltman reduction schemes

It is known that the one-loop tensor integrals can be reduced to a linear combination of up to four-point scalar integrals [14]. In this section, a generic one-loop integral

$$T^{N}_{\mu_1\ldots\mu_s} \equiv \int \mathrm{d}^4k \frac{k_{\mu_1}\ldots k_{\mu_s}}{D_0D_1\ldots D_{N-1}}, \tag{3.1}$$

with propagators $D_i \equiv (k+p_i)^2 - m_i^2 + i\varepsilon$ and $p_0 = 0$ is considered. As mentioned in the previous sections, the integrals $T^{N}_{\mu_1\ldots\mu_s}$ may not be translation invariant because of the finite integral limits when they are ultraviolet divergent. Therefore, the expressions for

$$\Delta L^{N}_{\mu_1\ldots\mu_s} \equiv \int \mathrm{d}^4k \frac{k_{\mu_1}\ldots k_{\mu_s}}{D_1\ldots D_N} - \int \mathrm{d}^4k \frac{(k-p_1)_{\mu_1}\ldots(k-p_1)_{\mu_s}}{\tilde{D}_1\ldots\tilde{D}_N}, \tag{3.2}$$

where propagators $D_i \equiv (k+p_i)^2 - m_i^2 + i\varepsilon$ but $p_1 \neq 0$ and $\tilde{D}_i \equiv (k+p_i-p_1)^2 - m_i^2 + i\varepsilon$ should be calculated. After the conventional Feynman parameterization, $\Delta L^N_{\mu_1\ldots\mu_s}$ can be reexpressed as

$$\Delta L^N_{\mu_1\ldots\mu_s} \equiv (-)^N \Gamma(N) \left[\int_{\text{simplex}} \prod_{i=1}^N \mathrm{d}u_i \; I^{s+4-2N}_{\mu_1\ldots\mu_s}(\tilde{b}-p_1, \tilde{a}^2) - \int_{\text{simplex}} \prod_{i=1}^N \mathrm{d}u_i \; \sum_{i=0}^s (-)^{s-i} \{p_1^{s-i} I^{i+4-2N,\ i}(\tilde{b}, \tilde{a}^2)\}_{\mu_1\ldots\mu_s} \right], \quad (3.3)$$

where $\tilde{b} \equiv -\sum_{i=1}^N u_i p_i + p_1$, $\tilde{a}^2 \equiv -\sum_{i=1}^N u_i(p_i^2 - m_i^2) + \sum_{i,j=1}^N u_i u_j p_i \cdot p_j$, and the notation $\{p_1^{s-i} I^{i+4-2N,i}(\tilde{b}, \tilde{a}^2)\}_{\mu_1\ldots\mu_s}$ is defined in appendix A with the divergence degree of $I^{i+4-2N,\ i}$ defined in section 2 is $i+4-2N$. The polynomial dependence of b, a^2 in $I^\Delta(b,a^2)_{\mu_1\ldots\mu_s}$ makes the simplex integration in eq. (3.3) straightforward using

$$\int_{\text{simplex}} \prod_{i=1}^N \mathrm{d}u_i \prod_{i=1}^N u_i^{r_i-1} = \frac{\prod_{i=1}^N \Gamma(r_i)}{\Gamma(\sum_{i=1}^N r_i)}. \quad (3.4)$$

Next, several notations similar to that given in ref. [15] are reintroduced here

$$\begin{aligned}
\Delta L^N_{\mu_1\ldots\mu_s} &\equiv \sum_{2n_0+n_1+\ldots+n_N=s}^{n_0,n_1,\ldots,n_N\geq 0} \{g^{2n_0} p_1^{n_1} \ldots p_N^{n_N}\}_{\mu_1\ldots\mu_s} \Delta L^N_{\underbrace{0\ldots 0}_{2n_0}\ldots\underbrace{N\ldots N}_{n_N}}, \\
T^N_{\mu_1\ldots\mu_s} &\equiv \sum_{2n_0+n_1+\ldots+n_{N-1}=s}^{n_0,n_1,\ldots,n_{N-1}\geq 0} \{g^{2n_0} p_1^{n_1} \ldots p_{N-1}^{n_{N-1}}\}_{\mu_1\ldots\mu_s} T^N_{\underbrace{0\ldots 0}_{2n_0}\ldots\underbrace{(N-1)\ldots(N-1)}_{n_{N-1}}}, \\
T^N_{\mu_1\ldots\mu_s}(0) &\equiv \int \mathrm{d}^4k \frac{k_{\mu_1}\ldots k_{\mu_s}}{D_1\ldots D_N}, \\
T^N_{\mu_1\ldots\mu_s}(k) &\equiv \int \mathrm{d}^4k \frac{k_{\mu_1}\ldots k_{\mu_s}}{D_0\ldots \hat{D}_k \ldots D_N}, \\
\tilde{T}^N_{\mu_1\ldots\mu_s}(0) &\equiv \int \mathrm{d}^4k \frac{k_{\mu_1}\ldots k_{\mu_s}}{\tilde{D}_1\ldots \tilde{D}_N}, \\
T^N_{\mu_1\ldots\mu_s}(0) &\equiv \sum_{2n_0+n_1+\ldots+n_N=s}^{n_0,n_1,\ldots,n_N\geq 0} \{g^{2n_0} p_1^{n_1} \ldots p_N^{n_N}\}_{\mu_1\ldots\mu_s} T^N_{\underbrace{0\ldots 0}_{2n_0}\ldots\underbrace{N\ldots N}_{n_N}}(0), \\
\tilde{T}^N_{\mu_1\ldots\mu_s}(0) &\equiv \sum_{2n_0+n_1+\ldots+n_{N-1}=s}^{n_0,n_1,\ldots,n_{N-1}\geq 0} \{g^{2n_0} (p_2-p_1)^{n_1} \ldots (p_N-p_1)^{n_{N-1}}\}_{\mu_1\ldots\mu_s} \\
&\quad \tilde{T}^N_{\underbrace{0\ldots 0}_{2n_0}\ldots\underbrace{(N-1)\ldots(N-1)}_{n_{N-1}}}(0),
\end{aligned} \quad (3.5)$$

where $D_0 \ldots \hat{D}_k \ldots D_N \equiv D_0 \ldots D_{k-1} D_{k+1} \ldots D_N$ and the caret "^" is employed to indicate the indices omitted.

Thus in this cutoff regularization, eq. (2.9) in ref. [15] should be replaced by

$$T^N_{\underbrace{0\ldots0}_{2n}\underbrace{1\ldots1}_{k}i_{2n+k+1}\ldots i_s}(0) = (-)^k \sum_{l=0}^{k} C^l_k \sum_{i_1,\ldots,i_l=1}^{N-1} \tilde{T}^N_{\underbrace{0,\ldots,0}_{2n},i_1,\ldots,i_l,\; i_{2n+k+1}-1,\ldots,\; i_s-1}(0)$$
$$+\Delta L^N_{\underbrace{0\ldots0}_{2n}\underbrace{1\ldots1}_{k}i_{2n+k+1}\ldots i_s}, \quad i_{2n+k+1},\ldots,i_s > 1. \tag{3.6}$$

When the determinant of Gram matrix

$$Z^{(N)} = \begin{pmatrix} 2p_1\cdot p_1 & \ldots & 2p_1\cdot p_N \\ \vdots & \ddots & \vdots \\ 2p_N\cdot p_1 & \ldots & 2p_N\cdot p_N \end{pmatrix} \tag{3.7}$$

for $(N+1)$-point functions is non-vanishing, the reduction can be continued as

$$T^N_{00i_3\ldots i_s} = \frac{1}{2(3+s-N)}\Bigg[T^{N-1}_{i_3\ldots i_s}(0) + 2m_0^2 T^N_{i_3\ldots i_s} + \sum_{j=1}^{N-1} f_j T^N_{ji_3\ldots i_s}$$
$$+\Delta L^{N-1}_{i_3\ldots i_s} + \sum_{j=1}^{N-1}\Delta L^{N-1}_{ji_3\ldots i_s}\Bigg],$$
$$T^N_{i_1\ldots i_s} = \sum_{j=1}^{N-1}(Z^{(N-1)})^{-1}_{i_1 j}\left(S^s_{ji_2\ldots i_s} - 2\sum_{r=2}^{s}\delta_{ji_r}T^N_{00i_2\ldots\hat{i}_r\ldots i_s}\right), \quad i_1\neq 0, \tag{3.8}$$

where some notations are defined in ref. [15]

$$\begin{aligned} f_k &\equiv p_k^2 - m_k^2 + m_0^2, \\ \bar{\delta}_{ij} &\equiv 1-\delta_{ij}, \\ (i_r)_k &\equiv \begin{cases} i_r & , k > i_r \\ i_r - 1 \,, & k < i_r \end{cases}, \\ S^s_{ki_2\ldots i_s} &\equiv T^{N-1}_{(i_2)_k\ldots(i_s)_k}(k)\bar{\delta}_{ki_2}\ldots\bar{\delta}_{ki_s} - T^{N-1}_{i_2\ldots i_s}(0) - f_k T^N_{i_2\ldots i_s}. \end{aligned} \tag{3.9}$$

Otherwise, when the Gram determinant is zero, there is at least one non-vanishing element $\tilde{Z}^{(N)}_{kl}$ in the adjoint matrix of $Z^{(N)}$

$$\tilde{Z}^{(N)}_{kl} \equiv (-)^{k+l}\begin{vmatrix} 2p_1p_1 & \ldots & 2p_1p_{l-1} & 2p_1p_{l+1} & \ldots & 2p_1p_N \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 2p_{k-1}p_1 & \ldots & 2p_{k-1}p_{l-1} & 2p_{k-1}p_{l+1} & \ldots & 2p_{k-1}p_N \\ 2p_{k+1}p_1 & \ldots & 2p_{k+1}p_{l-1} & 2p_{k+1}p_{l+1} & \ldots & 2p_{k+1}p_N \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 2p_Np_1 & \ldots & 2p_Np_{l-1} & 2p_Np_{l+1} & \ldots & 2p_Np_N \end{vmatrix} \tag{3.10}$$

and one non-zero element $\tilde{X}_{0j}^{(N)}$ in the adjoint matrix of the following matrix

$$X^{(N)} \equiv \begin{pmatrix} 2m_0^2 & f_1 & \dots & f_N \\ f_1 & 2p_1p_1 & \dots & 2p_1p_N \\ \vdots & \vdots & \ddots & \vdots \\ f_N & 2p_Np_1 & \dots & 2p_Np_N \end{pmatrix}. \tag{3.11}$$

One-loop reduction can be applied using the following equations

$$\begin{aligned} T^N_{i_1\dots i_s} &= -\frac{1}{\tilde{X}_{0j}^{(N-1)}}\sum_{n=1}^{N-1}\tilde{Z}_{jn}^{(N-1)}\left(\hat{S}^{s+1}_{ni_1\dots i_s} - 2\sum_{r=1}^{s}\delta_{ni_r}T^N_{00i_1\dots\hat{i}_r\dots i_s}\right), \\ T^N_{00i_1\dots i_s} &= \frac{1}{2(6+s-N+\sum_{r=1}^{s}\bar{\delta}_{i_r0})\tilde{Z}_{kl}^{(N-1)}}\Bigg[\tilde{Z}_{kl}^{(N-1)}S^{s+2}_{00i_1\dots i_s} \\ &\quad + \sum_{n=1}^{N-1}\left(\tilde{Z}_{nl}^{(N-1)}\hat{S}^{s+2}_{nki_1\dots i_s} - \tilde{Z}_{kl}^{(N-1)}\hat{S}^{s+2}_{nni_1\dots i_s}\right) \\ &\quad - \sum_{n,m=1}^{N-1}\tilde{\tilde{Z}}_{(kn)(lm)}^{(N-1)}\Bigg(f_n\hat{S}^{s+1}_{mi_1\dots i_s} + 2\sum_{r=1}^{s}\delta_{ni_r}\hat{S}^{s+2}_{m00i_1\dots\hat{i}_r\dots i_s} \\ &\qquad\qquad - f_nf_mT^N_{i_1\dots i_s} - 2\sum_{r=1}^{s}\left(f_n\delta_{mi_r}+f_m\delta_{ni_r}\right)T^N_{00i_1\dots\hat{i}_r\dots i_s} \\ &\qquad\qquad - 4\sum_{r,t=1,r\neq t}^{s}\delta_{ni_r}\delta_{mi_t}T^N_{0000i_1\dots\hat{i}_r\dots\hat{i}_t\dots i_s}\Bigg)\Bigg]. \end{aligned} \tag{3.12}$$

Some notations in eq. (3.12) should be recalled, i.e.

$$\begin{aligned} \tilde{\tilde{Z}}_{(ik)(jl)}^{(N)} &\equiv (-)^{i+j+k+l}\mathrm{sgn}(i-k)\mathrm{sgn}(l-j) \\ &\quad \begin{vmatrix} 2p_1p_1 & \dots & 2p_1p_{j-1} & 2p_1p_{j+1} & \dots & 2p_1p_{l-1} & 2p_1p_{l+1} & \dots & 2p_1p_N \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 2p_{i-1}p_1 & \dots & 2p_{i-1}p_{j-1} & 2p_{i-1}p_{j+1} & \dots & 2p_{i-1}p_{l-1} & 2p_{i-1}p_{l+1} & \dots & 2p_{i-1}p_N \\ 2p_{i+1}p_1 & \dots & 2p_{i+1}p_{j-1} & 2p_{i+1}p_{j+1} & \dots & 2p_{i+1}p_{l-1} & 2p_{i+1}p_{l+1} & \dots & 2p_{i+1}p_N \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 2p_{k-1}p_1 & \dots & 2p_{k-1}p_{j-1} & 2p_{k-1}p_{j+1} & \dots & 2p_{k-1}p_{l-1} & 2p_{k-1}p_{l+1} & \dots & 2p_{k-1}p_N \\ 2p_{k+1}p_1 & \dots & 2p_{k+1}p_{j-1} & 2p_{k+1}p_{j+1} & \dots & 2p_{k+1}p_{l-1} & 2p_{k+1}p_{l+1} & \dots & 2p_{k+1}p_N \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 2p_Np_1 & \dots & 2p_Np_{j-1} & 2p_Np_{j+1} & \dots & 2p_Np_{l-1} & 2p_Np_{l+1} & \dots & 2p_Np_N \end{vmatrix}, \\ &\quad N > 2, \\ \tilde{\tilde{Z}}_{(ik)(jl)}^{(2)} &\equiv \delta_{il}\delta_{kj} - \delta_{ij}\delta_{kl}, \\ \hat{S}^s_{ki_2\dots i_s} &\equiv T^{N-1}_{(i_2)_k\dots(i_s)_k}(k)\bar{\delta}_{ki_2}\dots\bar{\delta}_{ki_s} - T^{N-1}_{i_2\dots i_s}(0). \end{aligned} \tag{3.13}$$

For $\det(Z^{(N-1)}) = 0$,$\det(X^{(N-1)}) = 0$ and all $\tilde{X}_{0k}^{(N-1)} = 0$ but $\tilde{Z}_{kl}^{(N-1)} \neq 0$ and $\tilde{X}_{ij}^{N-1} \neq 0$, following equations

$$\begin{aligned} T^N_{\underbrace{0\ldots0}_{r}\underbrace{l\ldots l}_{n}i_1\ldots i_m} &= \frac{1}{2(n+1)\tilde{Z}_{kl}^{(N-1)}}\left[-2\sum_{j=1}^{m}\tilde{Z}_{ki_j}^{(N-1)}T^N_{\underbrace{0\ldots0}_{r}\underbrace{l\ldots l}_{n+1}i_1\ldots\hat{i}_j\ldots i_m}\right.\\ &\quad\left.+\sum_{j=1}^{N-1}\tilde{Z}_{kj}^{(N-1)}\hat{S}^{r+n+m}_{j\underbrace{0\ldots0}_{r-2}\underbrace{l\ldots l}_{n+1}i_1\ldots i_m}\right], i_1,\ldots,i_m\neq 0,l,\\ T^N_{i_1\ldots i_s} &= \frac{1}{\tilde{X}_{ij}^{(N-1)}}\left[\tilde{Z}_{ij}^{(N-1)}\left(2(5+s-N)T^N_{00i_1\ldots i_s} - T^{N-1}_{i_1\ldots i_s}(0)\right.\right.\\ &\quad\left. -\Delta L^{N-1}_{i_1\ldots i_s} - \sum_{n=1}^{N-1}\Delta L^{N-1}_{ni_1\ldots i_s}\right)\\ &\quad\left.+\sum_{m,n=1}^{N-1}\tilde{\tilde{Z}}^{(N-1)}_{(in)(jm)}f_n\left(\hat{S}^{s+1}_{mi_1\ldots i_s} - 2\sum_{r=1}^{s}\delta_{mi_r}T^N_{00i_1\ldots\hat{i}_r\ldots i_s}\right)\right] \end{aligned} \tag{3.14}$$

can be used. Other details in the derivation of these equations can be found in [15].

4 Higgs decay into two photons

In this section, one-loop reduction which is illustrated in the previous section is applied to the process $H \to \gamma\gamma$.

In unitary gauge, the three diagrams via the W-boson loop that contribute to this process with a specific loop momentum configuration are shown in figure (1). A direct calculation of amplitude (dropping the polarization vectors of external photons) yields

$$\begin{aligned} \mathcal{M}^{\mu\nu}_{\text{unitary}} &= -\frac{3e^3 m_W}{8\pi^4 m_H^2 s_w}\left[2k_2^\mu k_1^\nu\left(i\pi^2 - (m_H^2 - 2m_W^2)C_0(0,0,m_H^2,m_W^2,m_W^2,m_W^2)\right)\right.\\ &\quad\left.-k_1\cdot k_2 g^{\mu\nu}\left(i\pi^2 - 2(m_H^2-2m_W^2)C_0(0,0,m_H^2,m_W^2,m_W^2,m_W^2)\right)\right]\\ &= \frac{3ie^3 m_W}{8\pi^2 m_H^4 s_w}\left[-k_2^\mu k_1^\nu\left(2m_H^2 + 4(m_H^2-2m_W^2)\mathbf{f}(\frac{m_H^2}{4m_W^2})\right)\right.\\ &\quad\left.+k_1\cdot k_2 g^{\mu\nu}\left(m_H^2 + 4(m_H^2-2m_W^2)\mathbf{f}(\frac{m_H^2}{4m_W^2})\right)\right], \end{aligned} \tag{4.1}$$

with

$$\mathbf{f}(x) \equiv \begin{cases} \arcsin(\sqrt{x})^2 & , x \leq 1 \\ -\frac{1}{4}\left[\ln\left(\frac{1+\sqrt{1-x^{-1}}}{1-\sqrt{1-x^{-1}}}\right) - i\pi\right]^2 & , x > 1 \end{cases}, \tag{4.2}$$

and the scalar integral C_0 is given in appendix B. However, gauge invariance is spoiled in this four-momentum cutoff regularization. Therefore, a term should be subtracted from the above expressions to recover gauge invariance. In this gauge, a requirement of $\mathcal{M}^{\mu\nu}(k_1 = k_2 = 0) = 0$ should be made. However,

$$\mathcal{M}^{\mu\nu}_{\text{unitary}}(k_1 = k_2 = 0) = \frac{-3ie^3 m_W}{16\pi^2 s_w} g^{\mu\nu} \neq 0. \tag{4.3}$$

Following Dyson's prescription [16, 17], gauge invariance is recovered after making subtraction from eq. (4.3), and the final result is

$$\mathcal{M}^{\mu\nu}_{\text{unitary}} = -\frac{3ie^3}{16\pi^2 m_W s_w}\left(k_2^\mu k_1^\nu - g^{\mu\nu} k_1\cdot k_2\right)\left(\tau^{-1} + (2\tau^{-1} - \tau^{-2})\mathbf{f}(\tau)\right) \tag{4.4}$$

with $\tau = \frac{m_H^2}{4m_W^2}$ following the notations of refs. [3, 4]. Eq. (4.4) is the same as those in refs. [3, 4] up to a factor of $-2i$ from the symmetry factor of loops and different conventions of Feynman rules. However, in this gauge, there are high degrees of ultraviolet divergence in each diagram. The expressions for amplitude may be different under different choices of loop momentum. One may suspect that the discrepancy between eq. (4.4) and the result given in DREG

$$\mathcal{M}^{\mu\nu}_{\text{DREG}} = -\frac{ie^3}{16\pi^2 m_W s_w}\left(k_2^\mu k_1^\nu - g^{\mu\nu} k_1\cdot k_2\right)\left(2 + 3\tau^{-1} + 3(2\tau^{-1} - \tau^{-2})\mathbf{f}(\tau)\right) \tag{4.5}$$

is originated from the bad loop momentum choices in eq. (4.4). However, in our calculation we find that the terms

$$\begin{aligned}\Delta\mathcal{M}^{\mu\nu}(p) = -\frac{ie^3}{96\pi^2 m_W^3 s_w}&\Big[\left(k_2^\mu p^\nu - p^\mu k_1^\nu\right)\left(-3\Lambda^2 - 2m_H^2 - 6m_W^2 + (k_1+k_2)\cdot p - p^2\right)\\ &+2g^{\mu\nu}(k_1-k_2)\cdot p\left(-3\Lambda^2 - 2m_H^2 + 3m_W^2 + (k_1+k_2)\cdot p - p^2\right)\Big]\end{aligned} \tag{4.6}$$

should be added to eq. (4.4) if loop momentum k is shifted to $k+p$. From the symmetric consideration of k_1, k_2, a proper choice of p is $\frac{k_1+k_2}{2}$ which is the same as that presented in refs. [3, 4]. Since $\Delta\mathcal{M}^{\mu\nu}(\frac{k_1+k_2}{2}) = 0$ in eq. (4.6), the result in eq. (4.4) remains unchanged. From eq. (4.6), it seems hopeless that the difference between eq. (4.4) and eq. (4.5) can be eliminated through shifting the integral momentum k.

In 't Hooft-Feynman gauge ($\xi = 1$), the amplitude with one-loop diagrams shown in figure (2) is

$$\begin{aligned}\mathcal{M}^{\mu\nu}_{\xi=1} &= \frac{e^3}{16\pi^4 m_H^2 m_W s_w}\Big\{2k_2^\mu k_1^\nu\Big[-i\pi^2(m_H^2+6m_W^2)\\ &\quad+6m_W^2(m_H^2-2m_W^2)C_0(0,0,m_H^2,m_W^2,m_W^2,m_W^2)\Big] + k_1\cdot k_2 g^{\mu\nu}\Big[i\pi^2(m_H^2+6m_W^2)\\ &\quad-12m_W^2\left(m_H^2-2m_W^2\right)C_0(0,0,m_H^2,m_W^2,m_W^2,m_W^2)\Big]\Big\}\\ &= \frac{ie^3}{16\pi^2 m_H^2 m_W s_w}\Big[-2k_1^\nu k_2^\mu\left(m_H^4 + 6m_H^2 m_W^2 + 12m_W^2\left(m_H^2-2m_W^2\right)\mathbf{f}(\tau)\right)\\ &\quad+k_1\cdot k_2 g^{\mu\nu}\left(m_H^4 + 6m_H^2 m_W^2 + 24m_W^2(m_H^2-2m_W^2)\mathbf{f}(\tau)\right)\Big].\end{aligned} \tag{4.7}$$

Following a similar procedure to obtain a gauge invariant result, the amplitude at $k_1 = k_2 = 0$ is calculated as

$$\mathcal{M}^{\mu\nu}_{\xi=1}(k_1=k_2=0) = -\frac{3ie^3 m_W}{16\pi^2 s_w}g^{\mu\nu}, \tag{4.8}$$

which is the same as eq. (4.4). However, the gauge invariant amplitude is non-vanishing at $k_1 = k_2 = 0$ because of the contributions of diagrams (g) and (h) in figure (2) in this gauge. These contributions are

$$\mathcal{M}^{\mu\nu}_{\xi=1,(g,h)}(k_1 = k_2 = 0) = \frac{ie^3 m_H^2}{32\pi^2 m_W s_w} g^{\mu\nu}. \tag{4.9}$$

Therefore, the subtracted terms should be $\mathcal{M}^{\mu\nu}_{\xi=1}(k_1 = k_2 = 0) - \mathcal{M}^{\mu\nu}_{\xi=1,(g,h)}(k_1 = k_2 = 0)$ instead of $\mathcal{M}^{\mu\nu}_{\xi=1}(k_1 = k_2 = 0)$. The final result is

$$\begin{aligned}\mathcal{M}^{\mu\nu}_{\xi=1} &= -\frac{ie^3}{16\pi^2 m_W s_w}\left(k_2^\mu k_1^\nu - g^{\mu\nu} k_1\cdot k_2\right)\left(2 + 3\tau^{-1} + 3(2\tau^{-1} - \tau^{-2})\mathbf{f}(\tau)\right)\\ &= \mathcal{M}^{\mu\nu}_{\text{DREG}}.\end{aligned} \tag{4.10}$$

The term generated by the contributions of the Goldstone triangle diagrams (d, e) in figure (2) spoils the decoupling theorem, as pointed out by Shifman et al. recently [10]. Given that there are only logarithmic divergences under this covariant gauge, the result in eq. (4.10) is unique with a different loop momentum chosen. To best of our knowledge, it is the first derivation in the 't Hooft-Feynman gauge in cutoff regularization.

It seems there are some problems with unitary gauge in this cutoff regularization. Given that the top quark loop (figure (3)) does not suffer from any ambiguities in the gauge or loop momentum choices, the diagrammatic expressions are expected to be the same in DREG and in this cutoff regularization. These conditions have been verified following the same procedures. The result is as follows

$$\mathcal{M}^{\mu\nu}_{\text{top}} = \frac{ie^3 N_c}{18\pi^2 m_W s_w}\left(k_2^\mu k_1^\nu - k_1\cdot k_2 g^{\mu\nu}\right)\left(\chi^{-1} + (\chi^{-1} - \chi^{-2})\mathbf{f}(\chi)\right), \tag{4.11}$$

where $\chi = \frac{m_H^2}{4m_t^2}$.

The authors of refs. [11, 12] have also calculated this Higgs decay process in Pauli-Villars regularization and dimensional regularization respectively, and obtained the same result as the old ones [5–8]. Their statement about this issue is that the integral (in Euclidean space)

$$I_{\mu\nu} \equiv \int \frac{k^2 g_{\mu\nu} - 4k_\mu k_\nu}{(k^2 + m^2)^3} \tag{4.12}$$

is vanishing in cutoff regularization, while it is nonzero in DREG, which is also pointed out by R. Gastmans et al. [3, 4]. They also argued that $I_{\mu\nu}$ contained the difference of two logarithmic divergencies and should be regulated. Therefore, the integral that violates electromagnetic gauge invariance may suffer from some ambiguities. Actually, this issue was first discussed by R. Jackiw [21] in a more general case. However, we think that the vanishing of $I_{\mu\nu}$ in 4 dimensions is just a result of the fact that the integral intervals are symmetric about the origin even when there is a cutoff $\mathbf{\Lambda}$, and the replacement of $k_\mu k_\nu \to \frac{g_{\mu\nu}k^2}{4}$ in the integrand is also proper.

Moreover, very recently R. Jackiw also pointed out that by combining the two terms in the integrand of $I_{\mu\nu}$ one can avoid infinities but the difference of the integrals in these two regularization schemes is still the same, thus both evaluations are mathematically defensible [22]. So, what are the physical reasons for these ambiguities? In the following, we will try to clarify this issue.

Considering that the diagrammatic expressions in unitary gauge are not well-defined in the four-momentum cutoff regularization, how to recover the correct result under this condition may be still an open question. We investigate the Lagrangian for the Standard Model in unitary gauge, similar to the treatments in ref. [19]. The covariant terms for scalars are

$$\begin{aligned} \mathfrak{L}_{\text{scalar}} &\equiv (\mathrm{D}_\mu\Phi)^\dagger(\mathrm{D}^\mu\Phi) - \mathrm{V}(\Phi), \\ \text{with}\ \ \mathrm{V}(\Phi) &\equiv -\mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2, \mathrm{D}_\mu\Phi \equiv \left(\partial_\mu - \frac{i}{2}\mathrm{g}\tau^i\mathrm{W}^i_\mu - \frac{i}{2}\mathrm{g}'\mathrm{B}_\mu\right)\Phi. \end{aligned} \tag{4.13}$$

The scalar doublet produces the vacuum expectation value through the Higgs mechanism as

$$\langle\Phi\rangle_0 = \begin{pmatrix} 0 \\ \frac{\mathrm{v}}{\sqrt{2}} \end{pmatrix}, \mathrm{v} = \left(\frac{\mu^2}{\lambda}\right)^{1/2}. \tag{4.14}$$

Therefore, the scalar fields can be redefined as

$$\Phi = \begin{pmatrix} \phi^+ \\ \frac{\mathrm{v}}{\sqrt{2}} + \mathrm{h} + i\phi_3 \end{pmatrix}. \tag{4.15}$$

There are terms like

$$\begin{aligned} &m_W^2\mathrm{W}^+_\mu\mathrm{W}^{-,\mu} + im_W\left(\mathrm{W}^-_\mu\partial^\mu\phi^+ - \mathrm{W}^+_\mu\partial^\mu\phi^-\right) \\ &\qquad = m_W^2\left(\mathrm{W}^+_\mu + \frac{i}{m_W}\partial_\mu\phi^+\right)\left(\mathrm{W}^{-,\mu} - \frac{i}{m_W}\partial^\mu\phi^-\right) - \partial_\mu\phi^+\partial^\mu\phi^- \end{aligned} \tag{4.16}$$

after expanding the Lagrangian given in eq. (4.13), where $W^\pm_\mu \equiv \frac{W^1_\mu \mp iW^2_\mu}{\sqrt{2}}$. By following the prescription in ref. [19], the W-boson fields in unitary gauge can be redefined as

$$\begin{aligned} \tilde{\mathrm{W}}^+_\mu &\equiv \mathrm{W}^+_\mu + \frac{i}{m_W}\partial_\mu\phi^+, \\ \tilde{\mathrm{W}}^-_\mu &\equiv \mathrm{W}^-_\mu - \frac{i}{m_W}\partial_\mu\phi^-. \end{aligned} \tag{4.17}$$

In this gauge there are no kinetic term $\partial_\mu\phi^+\partial^\mu\phi^-$ and mass term for the Goldstone ϕ^+, ϕ^- because of the cancelation between the last term in eq. (4.16) and the original kinetic term of the W-boson's Goldstone in $\mathfrak{L}_{\text{scalar}}$. However, terms such as $h\phi^+\phi^-$ still exist in the original Lagrangian. In DREG, $\phi^+ = \phi^- = 0$ can be set safely, similar to a previous work by Grosse-Knetter [20], because all the momentum modes can be included in this regularization.[2] Hence, the conventional Lagrangian in unitary gauge only with physical fields is obtained. However, the results are in contrast to those of the four-momentum cutoff regularization, because an artificial scale $\mathbf{\Lambda}$ is introduced in the Lagrangian. The absence of a kinetic term for ϕ^+, ϕ^- does not mean that these Goldstone fields are vanishing intuitively,

[2]Note that the limits of loop integrals are taken to be infinity

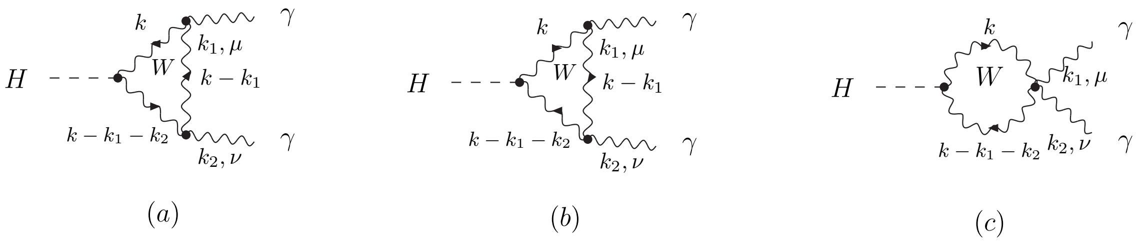

Figure 1. Feynman diagrams via W-boson loop in unitary gauge for $H \to \gamma\gamma$.

but because the theory does not provide any information above $\mathbf{\Lambda}$ in this regularization. If the mass of ϕ^+, ϕ^- is assumed to be $\mathcal{O}(\mathbf{\Lambda})$, there are still finite contributions from the Goldstone triangle diagrams when $\mathbf{\Lambda} \to \infty$. From this viewpoint, the cutoff regularization in unitary gauge is problematic. The violation of the property in gauge invariance can also be attributed to the absence of large momentum modes. Therefore, the old results in the literature for $H \to \gamma\gamma$ are still valid.

In fact, dimensional and Pauli-Villars regularization schemes are free of missing large momentum modes, and can maintain gauge invariance. Therefore, these results are correct also in unitary gauge. From the evaluations of R. Jackiw, the integral $I_{\mu\nu}$ can be dealt without any infinities, and the only difference is from surface terms (i.e., the large momentum region), which also verifies our conclusion.

5 Summary

A method for systematical evaluations of one-loop tensor integrals in cutoff regularization is proposed by deriving a new recursive relation eq. (2.6) and implementing it in the Passarino-Veltman reduction method. The result has been expressed in a form that can be directly translated into computer codes. Similar to the methods presented in ref. [15], our results are also numerical stable for up to four-point integrals. Surely, our method can be extended to deal with high-point integrals straightforwardly.

With this approach, we have calculated the amplitudes for Higgs decay into two photons via the W-boson loop and the top-quark loop. The correctness of the method has been confirmed by evaluating these processes and checking other programs, and it is certainly useful in both theoretical and phenomenological aspects. Moreover, we also reanalyze the Higgs decay process and make our efforts to find the physical reasons for some puzzles appeared in the calculations of this process.

Acknowledgments

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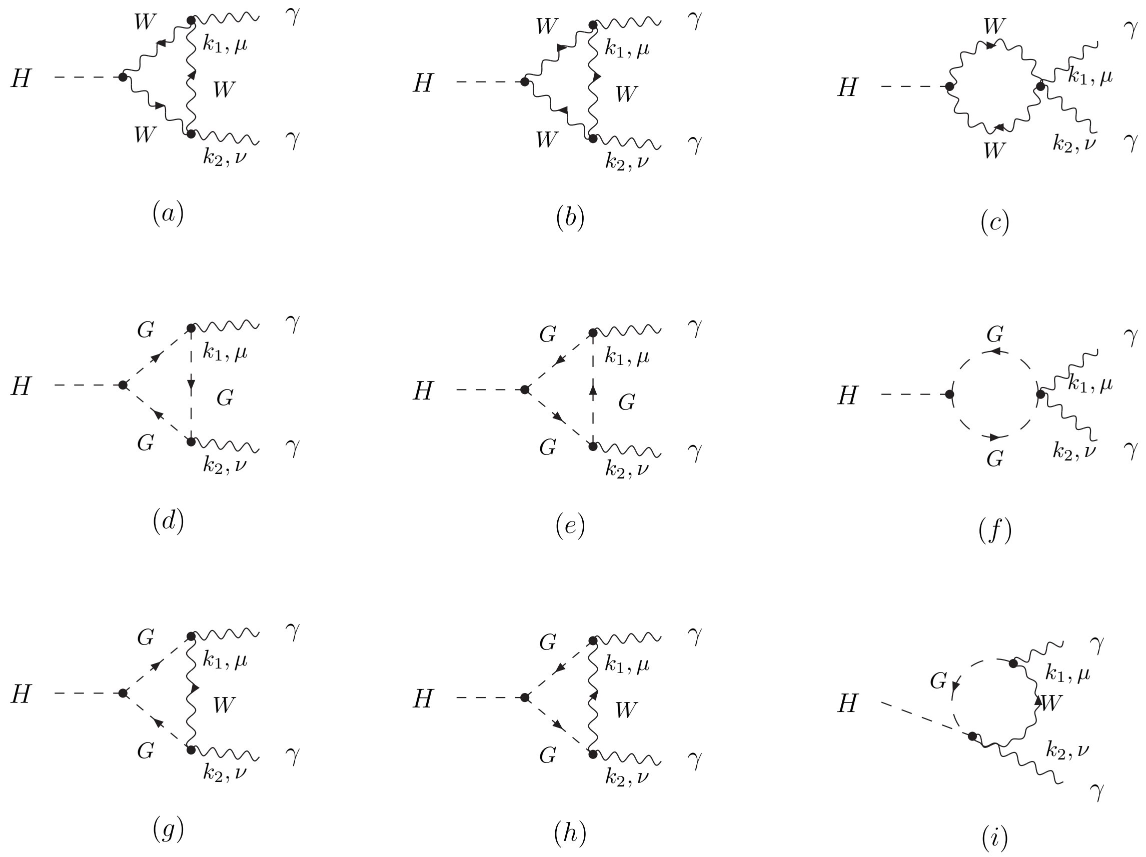

Figure 2. Some representative Feynman diagrams via W-boson loop in 't Hooft-Feynman gauge for $H \to \gamma\gamma$.

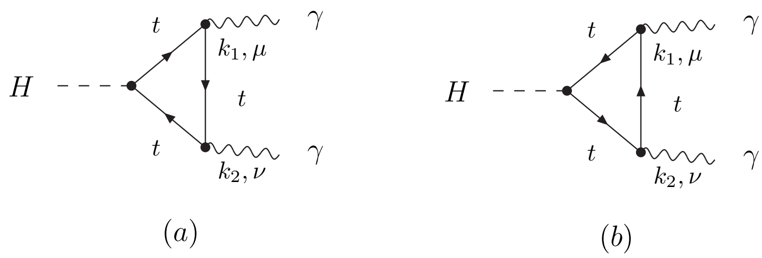

Figure 3. Feynman diagrams via top-quark loop for $H \to \gamma\gamma$.

A Derivation of expressions for $J^N_{\mu_1\mu_2\ldots\mu_s}$

In this appendix, the general formulae for $J^N_{\mu_1\mu_2\ldots\mu_s} \equiv \int \mathrm{d}^4k \frac{k_{\mu_1}k_{\mu_2}\ldots k_{\mu_s}}{(-k^2+a^2)^N}$ used in section 2 are derived first. Obviously, $J^N_{\mu_1\mu_2\ldots\mu_s}$ is vanishing unless $\mathbf{s}$ is even. Therefore, $\mathbf{s}$ should be an even and non-negative integer and N should be positive in the following context.

A notation (similar to but a little different from that in ref. [15]) is introduced first in order to write down the tensor decomposition in a concise way. We use curly braces to denote symmetrization with respect to Lorentz indices, where all non-equivalent permutations of the Lorentz indices on metric tensor **g** and momenta p are implicitly understood. A generic notation $\{g^{2n_0}p_1^{n_1}\dots p_k^{n_k}\}_{\mu_1\dots\mu_t}$ with $t=\sum_{l=1}^k n_l+2n_0$ means a sum that the $\mathbf{2n_0}$ of Lorentz indices $\mu_1,\dots,\mu_t$ are distributed to $\mathbf{n_0}$ metric tensors **g** while $\mathbf{n_l}$ of them are distributed to $\mathbf{n_l}$ momenta $\mathbf{p_l}$ with equal weights. For instance,

$$\begin{aligned}\{g^4\}_{\mu\nu\rho\sigma} &\equiv g_{\mu\nu}g_{\rho\sigma}+g_{\mu\rho}g_{\nu\sigma}+g_{\mu\sigma}g_{\nu\rho},\\ \{g^2p^1\}_{\mu\nu\rho} &\equiv g_{\mu\nu}p_\rho+g_{\nu\rho}p_\mu+g_{\rho\mu}p_\nu,\\ \{p_1^2p_2^1\}_{\mu\nu\rho} &\equiv p_{1\mu}p_{1\nu}p_{2\rho}+p_{1\mu}p_{1\rho}p_{2\nu}+p_{1\nu}p_{1\rho}p_{2\mu}.\end{aligned} \tag{A.1}$$

The Lorentz covariance ensures us to make the following replacement

$$k_{\mu_1}k_{\mu_2}\dots k_{\mu_s}\longrightarrow \{g^s\}_{\mu_1\mu_2\dots\mu_s}\frac{(k^2)^{\frac{s}{2}}}{\Gamma(\frac{s}{2}+2)2^{\frac{s}{2}}} \tag{A.2}$$

in the integral $J^N_{\mu_1\dots\mu_s}$, which can be proven by the induction of the integer **s**. After this replacement and subsequent Wick rotation, spherical coordinate system transformation and some trivial variable substitutions, one arrived

$$J^N_{\mu_1\dots\mu_s}=\{g^s\}_{\mu_1\dots\mu_s}\frac{i\pi^2(-2)^{-\frac{s}{2}}}{\Gamma(\frac{s}{2}+2)}\int_0^{\Lambda^2}\mathrm{d}K\frac{K^{\frac{s+2}{2}}}{(K+a^2)^N}, \tag{A.3}$$

where $\mathbf{\Lambda}$ was denoted as the ultraviolet cutoff scale.

Eq. (A.3) can be solved directly when $a^2=0$, i.e.

$$J^N_{\mu_1\dots\mu_s}=\{g^s\}_{\mu_1\dots\mu_s}\frac{i\pi^2(-2)^{-\frac{s}{2}}}{\Gamma(\frac{s}{2}+2)}\frac{2\Lambda^\Delta}{\Delta} \tag{A.4}$$

when superficial degree of ultraviolet divergence $\Delta\equiv s-2N+4>0$. In the case of $\Delta\le 0$, eq. (A.3) encounters infrared divergence, which is not considered in this article. When $a^2\neq 0$, result becomes a little more complicated than the previous one. However, this problem can be resolved after implementing the tricks of using integration by parts and following integral formulae

$$\begin{aligned}\int \mathrm{d}x\; x^n\ln(x+a) = \frac{1}{n+1}\Bigg(&x^{n+1}\ln(x+a)-\sum_{k=0}^{n}\frac{(-a)^k x^{n+1-k}}{n+1-k}\\ &-(-a)^{n+1}\ln(a+x)\Bigg), \qquad n\in\mathbf{N}\end{aligned} \tag{A.5}$$

into eq. (A.3). The explicit expressions for $J^N_{\mu_1\dots\mu_s}$ can also be obtained, i.e. $a^2\neq 0$ and superficial degree of divergence $\Delta\equiv s-2N+4\ge 0$ yields

$$\begin{aligned}J^N_{\mu_1\dots\mu_s} =& \{g^s\}_{\mu_1\dots\mu_s}\,(-2)^{-\frac{s}{2}}\frac{i\pi^2}{\Gamma(N)}\\ &\Bigg[-\sum_{k=1}^{N-1}\frac{\Gamma(N-k)}{\Gamma(\frac{s}{2}+3-k)}\Bigg(\sum_{l=0}^{\frac{\Delta}{2}}C^l_{N-k+l-1}(-a^2)^l\Lambda^{\Delta-2l}\Bigg)\\ &+\frac{\Gamma(1)}{\Gamma(\frac{\Delta}{2}+1)}\Bigg(\sum_{k=0}^{\frac{\Delta}{2}-1}(-a^2)^k\frac{2\Lambda^{\Delta-2k}}{\Delta-2k}+(-a^2)^{\frac{\Delta}{2}}\ln(\frac{\Lambda^2}{a^2})\Bigg)\Bigg],\end{aligned} \tag{A.6}$$

while $a^2 \neq 0$ but $\Delta < 0$ returns

$$J^N_{\mu_1\ldots\mu_s} = \{g^s\}_{\mu_1\ldots\mu_s} \, (-2)^{-\frac{s}{2}} \frac{i\pi^2\Gamma(-\frac{\Delta}{2})}{\Gamma(N)} a^\Delta. \tag{A.7}$$

B Some scalar integrals

After the reduction of one-loop integrals using the modified Passarino-Veltman reduction formulas given in the section 2, every tensor integral can be expressed as a linear combination of up to four-point scalar integrals. In this appendix, the analytical expressions for some scalar integrals are listed below. Some of them may be used in the one-loop calculations of the process Higgs decay to two photons.

First of all, the conventions for the scalar integrals used in this article are fixed as follows

$$T^N_0 \equiv \int \mathrm{d}^4k \frac{1}{D_0 D_1 \ldots D_{N-1}}. \tag{B.1}$$

For one-point functions,

$$\begin{aligned}
A_0(0) &= -i\pi^2 \, \Lambda^2, \\
A_0(m_0^2) &= i\pi^2 m_0^2 \left(\ln(\frac{\Lambda^2}{m_0^2}) - \frac{\Lambda^2}{m_0^2} \right), \\
A_{\underbrace{0\ldots0}_{2n}}(m_0^2) &= \frac{(-)^{n+1} i\pi^2}{\Gamma(n+2)2^n} \left(\sum_{i=1}^{n+1} \frac{(-)^{n+1-i}}{i} \Lambda^{2i} m_0^{2n+2-2i} + (-)^{n+1} m_0^{2n+2} \ln(\frac{\Lambda^2}{m_0^2}) \right).
\end{aligned} \tag{B.2}$$

Two-point functions can be easily verified as

$$\begin{aligned}
B_0(p_1^2, m_0^2, m_1^2) &= i\pi^2 \left(\ln(\frac{\Lambda^2}{p_1^2}) + 1 + \sum_{i=1}^{2} [\gamma_i \ln(\frac{\gamma_i - 1}{\gamma_i}) - \ln(\gamma_i - 1)] \right), \\
\text{with} \quad \gamma_{1,2} &= \frac{p_1^2 - m_1^2 + m_0^2 \pm \sqrt{\left(p_1^2 - m_1^2 + m_0^2\right)^2 - 4p_1^2 m_0^2}}{2p_1^2}, \\
B_0(0, 0, m^2) &= i\pi^2 \ln(\frac{\Lambda^2}{m^2}), \\
B_0(p^2, 0, 0) &= i\pi^2 \left(\ln(\frac{\Lambda^2}{p^2}) + 1 \right), \\
B_0(p^2, 0, m^2) &= i\pi^2 \left(\ln(\frac{\Lambda^2}{m^2}) + 1 + \frac{m^2 - p^2}{p^2} \ln(\frac{m^2 - p^2}{m^2}) \right), \\
B_0(0, m_0^2, m_1^2) &= i\pi^2 \frac{m_0^2 \ln(\frac{\Lambda^2}{m_0^2}) - m_1^2 \ln(\frac{\Lambda^2}{m_1^2})}{m_0^2 - m_1^2}, \\
B_0(0, m^2, m^2) &= i\pi^2 \left(\ln(\frac{\Lambda^2}{m^2}) - 1 \right).
\end{aligned} \tag{B.3}$$

Moreover, two special finite three-point functions are

$$
\begin{aligned}
C_0(0,0,p^2,m^2,m^2,m^2) &= \frac{i\pi^2}{2p^2}\left(\ln^2\left(\frac{1-\sqrt{1-\frac{4m^2}{p^2}}}{1+\sqrt{1-\frac{4m^2}{p^2}}}\right)-\pi^2\right), \\
C_0(0,0,0,m^2,m^2,m^2) &= -\frac{i\pi^2}{2m^2}.
\end{aligned}
\tag{B.4}
$$

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