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# Thermodynamics on the maximally symmetric holographic screen and entropy from conical singularities

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**ABSTRACT:** For a general maximally symmetric (spherically, plane or hyperbola symmetric) holographic screen, we rewrite the equations of motion of general Lovelock gravity into the form of some generalized first law of thermodynamics, under certain ansatz. With this observation together with other two independent ways, exactly the same temperature and entropy on the screen are obtained. So it is argued that the thermodynamic interpretation of gravity is physically meaningful not only on the horizon, but also on a general maximally symmetric screen. Moreover, the formula of entropy is further checked in the (maximally symmetric) general static case and dynamical case. The entropy formula also holds for those cases. Finally, the method of conical singularity is used to calculate the entropy on such screen, and the result again confirms the entropy formula.

**KEYWORDS:** Models of Quantum Gravity, Black Holes

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## 1 Introduction

It is well known that black-hole thermodynamics [1–4] reveals a deep and elegant relation between gravity and thermodynamics. This relation is also an important clue for searching a quantum theory of gravity. In 1995, Jacobson found that the Einstein equations can be obtained from the first law of thermodynamics if assuming the area law of entropy for all local acceleration horizons [5]. This idea has been extended to non-Einstein gravity theories (for a review, see [6]). A highlight on this problem recently is the idea of entropy force proposed by Verlinde [7]. Using the equipartition rule of energy and the holographic principle [8–10], he found that gravity could be understood as a kind of entropy force. This is a very attractive idea both in physics and mathematics. On physical side, it gives a new viewpoint of gravity in terms of holographic principle. On mathematical side, this idea is closely related with initial boundary value problem of Einstein equations. At almost the same time, Padmanabhan reinterpreted the relation  $E = 2TS$  [11] between the Komar energy, temperature and entropy as the equipartition rule of energy [12]. Many research works have been done in this area (see [13–57, 94] for an incomplete list of them).

All these recent works imply that geometric quantities on a general holographic screen also have thermodynamic interpretation as on a black-hole horizon. As pointed out by Verlinde, for static case, an Unruh-Verlinde temperature [7] can be defined on a general holographic screen. The relation between equipartition rule of energy and gravitational field equation strongly supports that such a temperature is a physical temperature. In [58, 59], Wald suggested a general method to get the horizon entropy and first law of black hole for a general diffeomorphism invariant theory. Padmanabhan generalized this method to the off-horizon case in a certain class of gravitational theories and also got one quarter of the screen's area in Einstein's gravity [49]. Many other authors have also suggested  $S = A/4$

for some general holographic screen from different aspects [50–52], for example Fursaev’s result in the case of minimal surfaces [94]. (See, however, [53] for a different entropy formula, as will be discussed later.) In Verlinde’s derivation, he related the Komar mass of the screen to the thermal energy of the screen, then got the Einstein equations. A natural question is whether we can use other types of quasi-local energy on the screen besides the Komar mass. In ordinary thermodynamics, there are many types of energy, such as inner energy, free energy, Gibbs free energy,  $\dots$ , which correspond to different thermal processes. So, it is reasonable to guess that different quasi-local energy corresponds to different kind of thermal energy, and all the hints of thermodynamics on an off-horizon screen should fit into a whole picture. Chen et al have made an attempt to this direction in four dimensional Einstein’s gravity and obtained a generalized first law of thermodynamics for the spherically symmetric screen [54], but the energy appearing there is the Arnowitt-Deser-Misner (ADM) mass instead of some quasi-local energy associated to the screen. Even earlier, Cai et al have also considered the spherically symmetric case in Einstein’s gravity and obtained a relation similar to the generalized first law [55] (see e.g. [60] for the on-horizon case), but that is a dynamical process, while in the usual thermodynamic sense the generalized first law should describe the quasi-static processes. Although most of the present works support some thermodynamic relations on a general screen with  $S = A/4$  in Einstein’s gravity, this entropy formula seems too simple to be falsified. Therefore, it is necessary to investigate more general theories of gravity, where expressions of the entropy and other quantities are complicated enough, and to collect more, different evidence for supporting that conclusion.

The aim of this paper is to consider more general gravity theory in order to find more evidence to support the thermal interpretation of gravity on an off-horizon screen. We consider a general spherical screen in the Lovelock gravity in arbitrary dimensions [61] which is a natural generalization of Einstein’s gravity. Three independent tests support the thermal interpretation in this case. First, we find that we can recast the equations of motion into the form of the first thermodynamical law. From this result, we can read out the quasi-local energy, entropy, temperature associated to the screen directly. We find the quasi-local energy is just the famous Misner-Sharp-like energy. Second, the analysis in [54] is generalized to this case, which for the Reissner-Nordström solution in Einstein’s gravity involves the Tolman-Komar energy inside the screen after a Legendre transformation. Finally, we also find that the entropy obtained previously just agrees with Padmanabhan’s general definition of entropy on the screen, which satisfies some equipartition-like rule. In all these aspects, exactly the same entropy and Unruh-Verlinde temperature arise, so it is convincing that the quantities and thermodynamic interpretations on the screen are physically meaningful.

Moreover, the formula of entropy is further confirmed in the general static spherically symmetric case and dynamical spherically symmetric case, as well as the corresponding plane symmetric and hyperbola symmetric case in parallel. In those cases, all the methods available also get exactly the same result of entropy, but the forms of temperature are slightly different, on which we will give some discussions.

Besides of those results, we also use another independent method to check the entropy formula. It is well known that the entropy of black hole horizon can be calculated by

the method of conical singularity [84]. Such method have also been used to calculate the holographic entanglement entropy [85–90]. We apply this method to some general screen<sup>1</sup> and find that the entropy obtained by this method agrees with Padmanabhan’s general definition of off-horizon entropy (and also what we obtain from previous methods for the maximally symmetric cases). This can be viewed as independent evidence which supports the entropy formula.

This paper is organized as follows. In section II, we illustrate the three methods in the Einstein case for static space-times with spherical symmetry and the metric ansatz  $g_{tt} = g_{rr}^{-1}$ . In section III, we generalize those results into the general Lovelock gravity. The general static case is discussed in section IV. In this section, we also discuss the non-stationary spherical solutions and find that our results still hold in those cases. We also find that our results hold if the spacetime is plane symmetric or hyperbola symmetric. These cases is included in section V. The method of conical singularity is discussed in section VI. In the last section, we give some remarks and discussions on our results.

## 2 Einstein’s gravity

Take Einstein’s gravity in  $n$  space-time dimensions as the simplest example to illustrate our basic strategy. The action functional is

$$I = \int d^n x \left( \frac{\sqrt{-g}}{16\pi} R + \mathcal{L}_{\text{matt}} \right), \tag{2.1}$$

which leads to the equations of motion

$$R_{ab} - \frac{1}{2} R g_{ab} = 8\pi T_{ab} \tag{2.2}$$

with  $T^{ab} = \frac{2}{\sqrt{-g}} \frac{\delta}{\delta g_{ab}} \int \mathcal{L}_{\text{matt}} d^n x$  the stress-energy tensor of matter. The most general static, spherically symmetric metric can be written as

$$ds^2 = -h(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega_{n-2}^2 \tag{2.3}$$

with  $d\Omega_{n-2}^2$  the metric on the unit  $(n-2)$ -sphere. We will consider this general case, as well as the dynamical case, in section 4. Here we assume the ansatz

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega_{n-2}^2 \tag{2.4}$$

for the metric, which means that the Lagrangian density  $\mathcal{L}_{\text{matt}}$  of matter cannot be too arbitrary, while still containing many cases of physical interest, such as electromagnetic fields, the cosmological constant and homogeneous ideal fluids [68], etc. In fact, the above ansatz essentially requires the relation  $T_t^t = T_r^r$  between the components of the stress-energy tensor of matter. In this space-time, the Unruh-Verlinde temperature on the spherical screen of radius  $r$  is easily obtained as

$$T = \frac{-\partial_r g_{tt}}{4\pi \sqrt{-g_{tt} g_{rr}}} = \frac{f'}{4\pi}, \tag{2.5}$$

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<sup>1</sup>We mean even not necessarily a maximally symmetric screen. See the (Euclidean) metric ansatz (6.6).

which is purely geometric and so independent of the gravitational dynamics. Here a prime means differentiation with respect to  $r$ .

Upon substitution of the ansatz (2.4) into the equations of motion (2.2), the nontrivial part of them is [69]

$$r f' - (n - 3)(1 - f) = \frac{16\pi P}{n - 2} r^2 \quad (2.6)$$

with  $P = T_r^r = T_t^t$  the radial pressure. Now we focus on a spherical screen with fixed  $f$  [54] in different static, spherically symmetric solutions of (2.2). In order to do so, we just need to compare two such configurations of infinitesimal difference. In fact, multiplying both sides of (2.6) by the factor

$$\frac{n - 2}{16\pi} \Omega_{n-2} r^{n-4} dr, \quad (2.7)$$

we have after some simple algebra (assuming  $f$  fixed)

$$\frac{f'}{4\pi} d\left(\frac{\Omega_{n-2} r^{n-2}}{4}\right) - d\left(\frac{n - 2}{16\pi} \Omega_{n-2} (1 - f) r^{n-3}\right) = P d\left(\frac{\Omega_{n-2} r^{n-1}}{n - 1}\right). \quad (2.8)$$

The above equation is immediately recognized as the generalized first law

$$T dS - dE = P dV \quad (2.9)$$

with  $T$  the Unruh-Verlinde temperature (2.5) on the screen,

$$S = \frac{\Omega_{n-2} r^{n-2}}{4}, \quad (2.10)$$

$$E = \frac{n - 2}{16\pi} \Omega_{n-2} (1 - f) r^{n-3} \quad (2.11)$$

and  $V = \frac{\Omega_{n-2} r^{n-1}}{n - 1}$  the volume of the (standard) unit  $(n-1)$ -ball. Here  $E$  is just the standard form of the Misner-Sharp energy inside the screen in spherically symmetric space-times [70], which is also identical to the Hawking-Israel energy in this case. More explicitly, solving  $f$  from (2.11) gives

$$f = 1 - \frac{16\pi E}{(n - 2)\Omega_{n-2} r^{n-3}}, \quad (2.12)$$

which is the Schwarzschild solution in  $n$  dimensions for constant  $E$  as its ADM mass, and some general spherically symmetric solution for certain mass function  $E(r)$ .

Some remarks are in order. First, the generalized first law (2.9) is of the same form as that in [69] for the horizon of spherically symmetric black holes, but is valid for general spherical screen with fixed  $f$ , which includes the horizon as the special case  $f = 0$ . Second, the entropy (2.10) in the generalized first law is actually  $S = A/4$ , i.e. one quarter of the area, for a general spherical screen, the same as the result obtained in [54] by the generalized Smarr's approach for the four dimensional case. (Similar results appear in [55] and [49], as mentioned previously.)

In fact, the generalized Smarr's approach can be used in the higher dimensional case without any difficulty. The Reissner-Nordström solution in  $n$  dimensions is

$$f = 1 - \frac{2\mu}{r^{n-3}} + \frac{q^2}{r^{2n-6}}, \quad (2.13)$$

where the mass parameter  $\mu$  is related to the ADM mass  $M$  by  $\mu = \frac{8\pi M}{(n-2)\Omega_{n-2}}$ . For fixed  $f$ , in order to obtain a generalized first law of the form [54]

$$dM = TdS + \phi dq \quad (2.14)$$

with  $T$  the Unruh-Verlinde temperature (2.5) on the screen, one just needs to notice that

$$df(r, M, q) = \frac{\partial f}{\partial r} dr + \frac{\partial f}{\partial M} dM + \frac{\partial f}{\partial q} dq = 0 \quad (2.15)$$

gives

$$dM = -\frac{f'}{4\pi} \left( \frac{\partial f}{\partial M} \right)^{-1} \frac{4\pi dr}{dS} dS - \left( \frac{\partial f}{\partial M} \right)^{-1} \frac{\partial f}{\partial q} dq. \quad (2.16)$$

Comparing (2.14) and (2.16), we see that

$$S = -4\pi \int \left( \frac{\partial f}{\partial M} \right)^{-1} dr = \frac{\Omega_{n-2} r^{n-2}}{4}, \quad (2.17)$$

which is the same as (2.10), and

$$\phi = \left( \frac{\partial f}{\partial M} \right)^{-1} \frac{\partial f}{\partial q} = \frac{(n-2)\Omega_{n-2}q}{8\pi r^{n-3}} \quad (2.18)$$

proportional to the electrostatic potential on the screen.

Furthermore, by straightforwardly working out the Tolman-Komar energy  $K = M - \phi q$  inside the screen, which is just a Legendre transformation of  $M$ , we can obtain another generalized first law

$$dK = TdS - qd\phi, \quad (2.19)$$

which seems even more relevant to the holographic picture, since now all the quantities are closely related to the screen, and the Tolman-Komar energy  $K$  satisfies the equipartition rule [7, 12]. In this case the Misner-Sharp energy  $E = M - \frac{\phi q}{2}$  is different from either the ADM mass  $M$  or the Tolman-Komar energy  $K$ , which is a general fact except for the vacuum case. Anyway, exactly the same temperature  $T$  and entropy  $S$  appear in different kinds of generalized first laws<sup>2</sup> and other places such as [55] and [49], which is strong evidence that the Unruh-Verlinde temperature (2.5) and the entropy (2.10) should make sense in physics. This argument will be further confirmed in more general cases below.

### 3 The Lovelock gravity

Now we consider the general Lovelock gravity. The action functional is

$$I = \int d^n x \left( \frac{\sqrt{-g}}{16\pi} \sum_{k=0}^m \alpha_k L_k + \mathcal{L}_{\text{matt}} \right) \quad (3.1)$$

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<sup>2</sup>There is no obvious relation between (2.19) [or (2.14)] with (2.9), as can be seen more clearly in the discussion around (3.12) for the general Lovelock gravity.

with  $\alpha_k$  the coupling constants and

$$L_k = 2^{-k} \delta_{c_1 d_1 \dots c_k d_k}^{a_1 b_1 \dots a_k b_k} R_{a_1 b_1}^{c_1 d_1} \dots R_{a_k b_k}^{c_k d_k}, \quad (3.2)$$

where  $\delta_{ef\dots gh}^{ab\dots cd}$  is the generalized delta symbol which is totally antisymmetric in both sets of indices. Note that  $\alpha_0$  is proportional to the cosmological constant and  $L_1 = R$ . This theory has the nice feature that it is free of ghosts, and some special cases of it arise naturally as the low-energy effective theories of string models [71, 72].

By the ansatz (2.4) again and extending the approach in [73] for the vacuum case to include  $\mathcal{L}_{\text{matt}}$ , the nontrivial part of the equations of motion is

$$\sum_k \tilde{\alpha}_k \left( \frac{1-f}{r^2} \right)^{k-1} [kr f' - (n-2k-1)(1-f)] = \frac{16\pi P}{n-2} r^2, \quad (3.3)$$

where

$$\tilde{\alpha}_0 = \frac{\alpha_0}{(n-1)(n-2)}, \quad \tilde{\alpha}_1 = \alpha_1, \quad \tilde{\alpha}_{k>1} = \alpha_k \prod_{j=3}^{2k} (n-j). \quad (3.4)$$

Now follow the strategy illustrated in the Einstein case. Multiplying both sides of the above equation by the factor

$$\frac{n-2}{16\pi} \Omega_{n-2} r^{n-4} dr, \quad (3.5)$$

we have after some simple algebra (assuming  $f$  fixed)

$$\begin{aligned} & \frac{f'}{4\pi} d \left( \frac{n-2}{4} \Omega_{n-2} r^{n-2} \sum_k \frac{\tilde{\alpha}_k k}{n-2k} \left( \frac{1-f}{r^2} \right)^{k-1} \right) \\ & - d \left( \frac{n-2}{16\pi} \Omega_{n-2} r^{n-1} \sum_k \tilde{\alpha}_k \left( \frac{1-f}{r^2} \right)^k \right) = Pd \left( \frac{\Omega_{n-2} r^{n-1}}{n-1} \right). \end{aligned} \quad (3.6)$$

Recalling that the Unruh-Verlinde temperature (2.5) is independent of the gravitational dynamics, we again recognize the above equation as the generalized first law (2.9) with

$$S = \frac{n-2}{4} \Omega_{n-2} r^{n-2} \sum_k \frac{\tilde{\alpha}_k k}{n-2k} \left( \frac{1-f}{r^2} \right)^{k-1}, \quad (3.7)$$

$$E = \frac{n-2}{16\pi} \Omega_{n-2} r^{n-1} \sum_k \tilde{\alpha}_k \left( \frac{1-f}{r^2} \right)^k. \quad (3.8)$$

Here  $E$  can be interpreted as some generalization of the Misner-Sharp (or Hawking-Israel) energy to the Lovelock gravity (for certain special case, see [74] for the discussion of the Misner-Sharp energy). In fact, when  $E = M$  is a constant, (3.8) is just the algebraic equation (of arbitrary degree) that  $f$  satisfies for the vacuum case [73], with  $M$  the ADM mass. And when

$$E(r) = M - \frac{\phi(r)q}{2} \quad (3.9)$$

with  $\phi(r)$  given by (2.18), (3.8) gives the Reissner-Nordström-like solution with charge  $q$ , while the Born-Infeld-like case corresponds to more complicated mass function  $E(r)$  [75, 76].

The entropy (3.7) should be discussed further. On the horizon, we have  $f = 0$ , so (3.7) just becomes the well-known entropy of the Lovelock black hole [77]. On a general spherical screen with fixed  $f$ , the generalized Smarr's approach can be applied without knowing the explicit form of  $f$  (which is impossible in the general Lovelock gravity) and still gives the generalized first law (2.14) for the Reissner-Nordström-like solution (3.9), with exactly the same temperature (2.5) and entropy (3.7). In fact, from (3.8) and (3.9) we have

$$M = \frac{n-2}{16\pi} \Omega_{n-2} \left( r^{n-1} \sum_k \tilde{\alpha}_k \left( \frac{1-f}{r^2} \right)^k + \frac{q^2}{r^{n-3}} \right). \quad (3.10)$$

For this case, (2.16) can be rewritten as

$$dM = -\frac{f'}{4\pi} \frac{\partial M}{\partial f} \frac{4\pi dr}{dS} dS - \frac{\partial M}{\partial q} dq. \quad (3.11)$$

Noticing that the entropy (3.7) and energy (3.8) satisfy

$$\frac{\partial S}{\partial r} = -4\pi \frac{\partial E}{\partial f} = -4\pi \frac{\partial M}{\partial f}, \quad (3.12)$$

which is independent of the previous interpretation of (3.6) as the generalized first law (2.9), we see that (2.14) really holds with  $T$  in (2.5),  $S$  in (3.7) and  $\phi$  in (2.18). Actually, the first equality of (3.12) just means that (3.5) is the integrating factor of the left hand side of (3.3), and this integrating factor just renders the right hand side of (3.3) to have the form  $PdV$ , which is an interesting feature of the general Lovelock gravity.

However, if we define the ‘‘Tolman-Komar energy’’  $\tilde{K} = M - \phi q$  and write down a generalized first law (2.19) mimicking the Einstein case, it is not clear whether  $\tilde{K}$  has the meaning of some quasi-local energy inside the screen. Instead, there is a more acceptable Tolman-Komar energy [49], defined as

$$K = \int_{\mathcal{V}} d^{n-1}x \frac{\sqrt{h}}{16\pi} \mathcal{R}_b^a \xi^b n_a \quad (3.13)$$

for a region  $\mathcal{V}$  with induced metric  $h_{ab}$  and normal vector  $n^a$ , where  $\xi^b$  is the Killing vector and

$$\mathcal{R}_b^a = 16\pi P^{acde} R_{bcde} \quad (3.14)$$

with

$$P^{abcd} = \frac{\partial L}{\partial R_{abcd}}. \quad (3.15)$$

Here  $\mathcal{R}_b^a$  can be viewed as the generalization of Ricci tensor to the Lovelock gravity. Note that even for the vacuum case the equations of motion do not imply  $\mathcal{R}_{ab} = 0$ , so  $K$  does not vanish, unlike in Einstein's gravity. In our case, we take  $\xi = \partial_t$  and  $\mathcal{V}$  to be the outside of the screen. If the space-time is asymptotically flat, it is natural to identify

$$K = M - \int_{\mathcal{V}} d^{n-1}x \frac{\sqrt{h}}{16\pi} \mathcal{R}_b^a \xi^b n_a \quad (3.16)$$



as the Tolman-Komar energy inside the screen, where  $M$  is the ADM mass of the space-time. So, unlike in Einstein's gravity, this energy is screen-dependent even in the vacuum case. For a spherically symmetric screen in the RN-like solution (3.9), the Tolman-Komar energy (3.16) can be explicitly computed to be

$$K = (n-2)^2 \Omega_{n-2} \sum_k \frac{\tilde{\alpha}_k k}{n-2k} \left( \frac{1-f}{r^2} \right)^{k-1} \frac{r^{2n-4} \sum_k \tilde{\alpha}_k (n-2k-1) \left( \frac{1-f}{r^2} \right)^k - (n-3)q^2}{16\pi(n-3)r^{n-3} \sum_k \tilde{\alpha}_k k \left( \frac{1-f}{r^2} \right)^{k-1}}. \quad (3.17)$$

It is obvious that  $K \neq M - \phi q$  in generic case, so it is not clear whether a generalized first law concerning this Tolman-Komar energy exists.

Furthermore, Padmanabhan has proposed another definition of entropy off the horizon [49] in a certain class of theories including the Lovelock gravity, generalizing the definition of entropy on the horizon by Wald et al [58, 59]. For a general screen  $\mathcal{S}$ , the associated entropy is suggested to be

$$S = \int_{\mathcal{S}} 8\pi P_{cd}^{ab} \epsilon_{ab} \epsilon^{cd} \sqrt{\sigma} d^{n-2}x, \quad (3.18)$$

$\epsilon_{ab}$  the binormal to  $\mathcal{S}$  and  $\sigma_{ab}$  the metric on  $\mathcal{S}$ , where  $L = \frac{1}{16\pi} \sum_k \alpha_k L_k$  for the Lovelock gravity. This entropy is shown to satisfy the equipartition-like rule with the Unruh-Verlinde temperature (2.5) and some generalized Tolman-Komar energy [49]. In our case, the only non-vanishing components of the binormal are  $\epsilon_{tr} = 1/2 = -\epsilon_{rt}$ , so the only relevant component of (3.15) in (3.18) is

$$P_{tr}^{tr} = \frac{1}{16\pi} \sum_k \alpha_k k 2^{-k} \delta_{trc_2d_2 \dots c_k d_k}^{tra_2b_2 \dots a_k b_k} R_{a_2b_2}^{c_2d_2} \dots R_{a_k b_k}^{c_k d_k} \quad (3.19)$$

with the indices  $a_i, b_i, c_i, d_i$  ( $i = 2, \dots, k$ ) running only among the angular directions. By explicitly working out

$$R_{cd}^{ab} = \frac{1-f}{r^2} \delta_{cd}^{ab} \quad (3.20)$$

for the metric (2.4) and substituting it into (3.19), one can see that the general definition (3.18) of entropy eventually gives (3.7). In fact, substitution of (3.20) into (3.19) gives

$$\begin{aligned} P_{tr}^{tr} &= \frac{1}{16\pi} \sum_k \alpha_k k 2^{-k} \delta_{trc_2d_2 \dots c_k d_k}^{tra_2b_2 \dots a_k b_k} \delta_{a_2b_2}^{c_2d_2} \dots \delta_{a_k b_k}^{c_k d_k} \left( \frac{1-f}{r^2} \right)^{k-1} \\ &= \frac{1}{32\pi} \sum_k \alpha_k k \delta_{tra_2b_2 \dots a_k b_k}^{tra_2b_2 \dots a_k b_k} \left( \frac{1-f}{r^2} \right)^{k-1} \\ &= \frac{1}{32\pi} \sum_k \alpha_k k (n-2)(n-3) \dots (n-2k+1) \left( \frac{1-f}{r^2} \right)^{k-1} \\ &= \frac{n-2}{32\pi} \sum_k \frac{\tilde{\alpha}_k k}{n-2k} \left( \frac{1-f}{r^2} \right)^{k-1} \end{aligned} \quad (3.21)$$

Substituting the above expression into (3.18), we then obtain the entropy (3.7) exactly. Moreover, the entropy (3.7), the Unruh-Verlinde temperature (2.5) and the Tolman-Komar

energy (3.17) satisfy

$$K = \frac{n-2}{n-3}TS, \tag{3.22}$$

as Padmanabhan proved.

#### 4 The general static case and dynamical case

For the general static case in the Lovelock gravity, it is convenient to let  $h(r) = e^{-2c(r)}f(r)$  in the metric (2.3). In this case, the  $tt$  and  $rr$  components

$$\sum_k \tilde{\alpha}_k \left(\frac{1-f}{r^2}\right)^{k-1} [krf' - (n-2k-1)(1-f)] = \frac{16\pi T_t^t}{n-2}r^2, \tag{4.1}$$

$$\sum_k \tilde{\alpha}_k \left(\frac{1-f}{r^2}\right)^{k-1} [kr(f' - 2fc') - (n-2k-1)(1-f)] = \frac{16\pi T_r^r}{n-2}r^2 \tag{4.2}$$

of the equations of motion are independent of each other. Focusing on a screen with fixed  $f$ , taking a linear combination of these two equations and multiplying both sides of the equation by the factor (3.5), we have again the generalized first law (2.9) with exactly the same entropy (3.7) and Misner-Sharp-like energy (3.8), but with a slightly different temperature

$$T = \frac{f' - 2lfc'}{4\pi} = \frac{\partial_r[(g^{rr})^{1-l}(-g_{tt})^l]}{4\pi(-g_{tt}g_{rr})^l} \tag{4.3}$$

from (2.5) and

$$P = (1-l)T_t^t + lT_r^r. \tag{4.4}$$

Now, two special cases should be noted. The first one is the choice  $l = 1/2$ , where  $P = (T_t^t + T_r^r)/2$  and it can be shown that

$$T = \frac{e^c(e^{-c}f)'}{4\pi} = \frac{D_a D^a r}{4\pi} \quad (a = t, r) \tag{4.5}$$

just coincides with Hayward's definition [78, 79] generalized to the off-horizon case [55] in Einstein's gravity.<sup>3</sup> The second one is the choice  $l = 1$ , which is of particular interest, since in this case  $P = T_r^r$  is just the standard expression of the radial pressure and

$$T = \frac{\partial_r g_{tt}}{4\pi g_{tt}g_{rr}} \tag{4.6}$$

differs from the standard Unruh-Verlinde temperature only by a  $\sqrt{-g_{tt}g_{rr}}$  factor. Similar phenomena of non-unique temperatures are extensively observed in the on-horizon case [55, 80, 81].

For the general definition (3.18) of entropy, since (3.20) still holds in this case and it is easily seen that

$$\epsilon_{tr}\epsilon^{tr} = \frac{1}{4} \tag{4.7}$$

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<sup>3</sup>The work density  $w$  there is just  $-P$  here.

for the metric (2.3), the calculation in the previous section leads to exactly the same result (3.7), which is consistent with the above analysis that leads to the generalized first law.

The dynamical case is formally the same. In this case, the metric can be written as

$$ds^2 = -e^{-2c(t,r)} f(t,r) dt^2 + f(t,r)^{-1} dr^2 + r^2 d\Omega_{n-2}^2, \quad (4.8)$$

with  $t$  the so-called Kodama time [82]. It turns out that the above discussion extends to this case straightforwardly, with  $f$ ,  $c$  and other quantities in the same equations (4.1)–(4.6) understood as functions of both  $t$  and  $r$ . However, the thermodynamic meaning of the resulting “first law”

$$TdS - dE = PdV \quad (4.9)$$

is obscure. In fact, we know in the standard thermodynamics that the expression  $dQ = TdS$  holds only in reversible processes, and a reversible process is necessarily quasi-static. Thus, for such a dynamical evolving case, (4.9) cannot be regarded as the generalized first law in the usual thermodynamic sense. Nevertheless, the first-law-like description (4.9) of the Lovelock gravitational dynamics is interesting in its own right. And needless to say, exactly the same, unique expression (3.7) of entropy on the screen involved here further confirms its physical significance. Furthermore, the general definition (3.18) of entropy still gives (3.7) even in this case, since (3.20) and (4.7) still hold for the metric (4.8).

## 5 The plane symmetric and hyperbola symmetric cases

The above discussions can be generalized to the plane symmetric and hyperbola symmetric cases. Generally, the static metric can be written as

$$ds^2 = -e^{-2c(r)} f(r) dt^2 + f(r)^{-1} dr^2 + r^2 d\Omega_{\varepsilon,n-2}^2 \quad (5.1)$$

with  $d\Omega_{\varepsilon,n-2}^2$  the metric on the “unit”  $(n-2)$ -sphere, plane or hyperbola for  $\varepsilon$  equal to 1, 0 or  $-1$ , respectively. In these cases, (4.1) and (4.2) become

$$\sum_k \tilde{\alpha}_k \left( \frac{\varepsilon - f}{r^2} \right)^{k-1} [krf' - (n - 2k - 1)(\varepsilon - f)] = \frac{16\pi T_t^t}{n - 2} r^2, \quad (5.2)$$

$$\sum_k \tilde{\alpha}_k \left( \frac{\varepsilon - f}{r^2} \right)^{k-1} [kr(f' - 2fc') - (n - 2k - 1)(\varepsilon - f)] = \frac{16\pi T_r^r}{n - 2} r^2. \quad (5.3)$$

Similar discussions as in the previous section lead to the generalized first law (2.9) with the entropy and Misner-Sharp energy

$$S = \frac{n - 2}{4} \Omega_{n-2} r^{n-2} \sum_k \frac{\tilde{\alpha}_k k}{n - 2k} \left( \frac{\varepsilon - f}{r^2} \right)^{k-1}, \quad (5.4)$$

$$E = \frac{n - 2}{16\pi} \Omega_{n-2} r^{n-1} \sum_k \tilde{\alpha}_k \left( \frac{\varepsilon - f}{r^2} \right)^k, \quad (5.5)$$

while the temperature (4.3) and pressure (4.4) have the same forms in all these cases.

On the horizon ( $f = 0$ ), these results just become the known ones for the plane symmetric and hyperbola symmetric black holes in the general Lovelock gravity. Especially, in the plane symmetric case ( $\varepsilon = 0$ ), the entropy on the horizon is just  $A/4$  and the corresponding Misner-Sharp energy is simply

$$\frac{n-2}{16\pi} \tilde{\alpha}_0 \Omega_{n-2} r^{n-1} \tag{5.6}$$

even in the general Lovelock gravity [67], while it is obvious that this statement no longer holds off the horizon.

For the dynamical case, it is easy to check that the above discussion applies straightforwardly, with  $f$ ,  $c$  and other quantities in the same equations (5.1)–(5.5) understood as functions of both  $t$  and  $r$ . Furthermore, the definition (3.18) of entropy again gives (5.4) in the general static case and dynamical case, since (4.7) still holds and (3.20) now becomes

$$R_{cd}^{ab} = \frac{\varepsilon - f}{r^2} \delta_{cd}^{ab} \tag{5.7}$$

for the metric (5.1), even with  $f$  and  $c$  understood as functions of both  $t$  and  $r$ .

## 6 The entropy from conical singularities

In the Euclidean approach to the thermodynamics of black holes, the horizon temperature for the metric (2.4) is determined by requiring that its Euclidean counterpart

$$ds^2 = f(r)d\tau^2 + f(r)^{-1}dr^2 + r^2d\Omega_{n-2}^2 \tag{6.1}$$

should have no conical singularity at the origin  $f(r) = 0$  of the  $\tau$ - $\rho$  plane, where  $\rho$  is defined by the differential equation

$$d\rho = f(r)^{-1/2}dr \tag{6.2}$$

with the boundary condition  $\rho = 0$  when  $f(r) = 0$ . Note that this origin just corresponds to the horizon of the black hole. That analysis results in a periodicity

$$\tau \sim \tau + \frac{4\pi}{f'(r_0)} \quad (f(r_0) = 0) \tag{6.3}$$

of the imaginary time  $\tau$  at the origin, which implies a temperature

$$T = \frac{f'(r_0)}{4\pi} \tag{6.4}$$

of the horizon.

On the other hand, if we allow a conical singularity at the origin with an angle  $\beta$ , then the Euclidean action  $I_E$  (upon properly renormalized) will be  $\beta$ -dependent and, correspondingly, there is an entropy

$$S = \lim_{\beta \rightarrow 2\pi} \left( \beta \frac{\partial}{\partial \beta} - 1 \right) I_E \tag{6.5}$$

canonically conjugate to the deficit angle  $\delta = 2\pi - \beta$ , which is shown to agree with the black-hole entropy in the general Lovelock gravity [83]. Note that this viewpoint on the entropy is off shell, i.e. does not rely on the validity of the equations of motion, in contrast to the on-shell ones by the usual black-hole thermodynamics or by the method of Wald et al [58, 59]. An analysis of Riemann manifolds with conical singularities, using some regularization techniques, has been made in [84], under certain requirements on the asymptotic behavior of the metric approaching the singularity. However, we will use another approach [88–90] in the following to analyze the conical singularity and then compute the entropy (6.5), which seems more convenient and relevant to the off-horizon case that we are really interested in throughout this paper.

In fact, we take a more general form

$$ds^2 = g_{\tau\tau}(\tau, r)d\tau^2 + 2g_{\tau r}(\tau, r)d\tau dr + g_{rr}(\tau, r)dr^2 + g_{ij}(\tau, r, x)dx^i dx^j \quad (6.6)$$

of metric, which is assumed regular everywhere and can be (globally) recast as

$$ds^2 = 2g_{w\bar{w}}(w, \bar{w})dw d\bar{w} + g_{ij}(w, \bar{w}, x)dx^i dx^j \quad (6.7)$$

under some conformally flat complex coordinates  $(w, \bar{w})$  on the  $(\tau, r)$  plane. Here  $i, j$  run over indices excluding  $\tau$  and  $r$ . The Euclidean spherically symmetric, plane symmetric and hyperbola symmetric space-times are all special cases of the metric (6.6). For the on-horizon, static case discussed previously the origin  $w = 0 = \bar{w}$  of the complex plane is just at  $f(r) = 0$  (or  $\rho = 0$ ), while for the off-horizon, static case the origin of the complex plane is at  $(\tau, r)$  with  $r$  the position of the screen and  $\tau$  arbitrary. The conically singular, or multi-sheeted, structure is realized by the coordinate transformation

$$w = z^m, \quad (6.8)$$

which is singular at  $z = 0 = \bar{z}$  for  $m \neq 1$ . Precisely, our original Euclidean space-time has topology  $R^2 \times \mathcal{S}$ . To obtain the multi-sheeted Euclidean space-time,  $m$  copy of the  $R^2$  factor is glued together in the standard way as Riemann surfaces [88–90], while the  $\mathcal{S}$  factor remains untouched. The metric (6.7) then becomes

$$ds^2 = 2m^2(z\bar{z})^{m-1}g_{w\bar{w}}dz d\bar{z} + g_{ij}dx^i dx^j. \quad (6.9)$$

Note that the conical angle  $\beta = 2\pi m$ . Since  $\beta \rightarrow 2\pi$  in (6.5), we extend  $m$  from integers to reals and let  $m = 1 + \epsilon$  with  $\epsilon$  an infinitesimal parameter. To the linear order of  $\epsilon$ , the above metric is

$$ds^2 = 2[1 + 2\epsilon + \epsilon \ln(z\bar{z})]g_{w\bar{w}}dz d\bar{z} + g_{ij}dx^i dx^j. \quad (6.10)$$

Straightforward calculations give the Riemann curvature

$${}^\epsilon R_{cd}^{ab} = R_{cd}^{ab} - 8\pi\epsilon g^{z\bar{z}}\epsilon^{ab}\epsilon_{cd}\delta^2(z, \bar{z}) \quad (6.11)$$

with  $R_{cd}^{ab}$  the tensorial part of the Riemann curvature,  $\epsilon^{ab}$  the binormal to the surface  $z = 0 = \bar{z}$  and the second term on the right hand side the non-tensorial part due to the conical singularity, which is similar to the result in [84].

Now suppose that the Euclidean action has the form

$$-I_E = \int F(g_{ef}, R_{cd}^{ab}, \psi) \sqrt{g} dw d\bar{w} d^{n-2}x, \quad (6.12)$$

where  $F(g_{ef}, R_{cd}^{ab}, \psi)$  is some scalar function of the metric, the Riemann curvature (and its contractions) and some matter fields (and their covariant derivatives) collectively denoted by  $\psi$ . Under the transformation (6.8), substitution of (6.11) into (6.12) gives the  $\beta$ -dependent Euclidean action

$$\begin{aligned} {}^\epsilon I_E &= \epsilon \int 8\pi \frac{\partial F}{\partial R_{cd}^{ab}} \epsilon^{ab} \epsilon_{cd} \delta^2(z, \bar{z}) g^{z\bar{z}} \sqrt{g} dz d\bar{z} d^{n-2}x - \int F \sqrt{g} dz d\bar{z} d^{n-2}x \\ &= \epsilon \int 8\pi \frac{\partial F}{\partial R_{cd}^{ab}} \epsilon^{ab} \epsilon_{cd} \sqrt{\sigma} d^{n-2}x - \int F \sqrt{g} dz d\bar{z} d^{n-2}x, \end{aligned} \quad (6.13)$$

where  $\sigma$  is the determinant of the induced metric  $\sigma_{ij}$  on the surface  $z = 0 = \bar{z}$  of codimension 2. Note that the precise meaning of the bulk integral  $\int \sqrt{g} dz d\bar{z} d^{n-2}x$  in (6.13) is to evaluate it for integral  $m$  and extend the result to real  $m \rightarrow 1$  [91, 92], so this integral is by construction proportional to  $\beta$ . Substituting (6.13) into (6.5), we then have

$$S = \int 8\pi \frac{\partial F}{\partial R_{cd}^{ab}} \epsilon^{ab} \epsilon_{cd} \sqrt{\sigma} d^{n-2}x, \quad (6.14)$$

which agrees with (3.18) because we know from (6.12) that  $F$  is just the Euclidean version of the Lagrangian  $L$ . Similar argument has early been made in [93] that the horizon entropy from conical singularities is equivalent to the definition by Wald et al, but introducing and treating the singularities by techniques similar to [84].

## 7 Concluding remarks

In this work, we use three independent ways to check the thermodynamical interpretation of some geometric quantities on a maximally symmetric (spherically, plane or hyperbola symmetric) holographic screen in the general Lovelock theory, first under certain metric ansatz in the static case. All these methods give the same Unruh-Verlinde temperature, Misner-Sharp energy and entropy formula on the screen. This agreement supports the thermal interpretation of these geometric quantities to be physically meaningful. In the general static spherical case and dynamical spherical case, then, exactly the same form of entropy appears, but the definition of temperature is of some ambiguity and the physical meaning of the “generalized first law” in the dynamical case is obscure, which have been clarified in our paper. In fact, we have obtained a series of “generalized first law”, which include that of Hayward as a special case. Similar to the on-horizon case that the entropy is canonically conjugate to the deficit angle of the conical singularity at the “horizon” of the Euclidean space-time, the off-horizon entropy has been shown to be canonically conjugate to the deficit angle of the conical singularity at the “screen” from the Euclidean point of view. This result can be viewed as additional independent evidence of the thermal meaning of geometric quantities on a general screen.

Nevertheless, there are many open questions and/or unclear points in this framework, of which an important one will be described as follows, simply in Einstein's gravity. Although the relation  $S = A/4$  for a general spherically symmetric screen seems rather universal and is supported by many recent works, there is an alternative expression  $S = 2\pi RE$  obtained in [53], which seems also substantial. To get the former form of entropy, we have to focus on a screen with fixed  $f$ , while to get the latter form, we have to focus on a screen with fixed  $r$ . In fact, the former form of entropy just saturates the holographic entropy bound [8, 9], while the latter form just saturates the Bekenstein entropy bound [95]. Furthermore, for the former form of entropy it is easy to write down some generalized first law of thermodynamics as discussed above, but it is not clear how to realize Verlinde's entropy variation formula and then the gravity as an entropic force, while for the latter form there exist the entropy variation formula and the entropic force expression [53] but without a satisfactory generalized first law. How to reconcile these two forms of entropy is a significant open question.

Although in the method of conical singularity, Padmanabhan's general definition of off-horizon entropy has been confirmed in a much larger class of metrics than the maximally symmetric ones, it seems that it is difficult to generalize all the other approaches of investigating the off-horizon entropy, and moreover, the off-horizon thermodynamics to a general (non-maximally-symmetric) holographic screen. This problem should be left for future works.

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