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Flavored orbifold GUT — an $\mathrm{SO}(10) \times S_4$ model

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Abstract: Orbifold grand unified theories (GUTs) solve several problems in GUT model building. Therefore, it is intriguing to investigate similar constructions in the flavor context. In this paper, we propose that a flavor symmetry might emerge due to orbifold compactification of one orbifold and broken by boundary conditions of another orbifold. The combination of the orbifold parities in gauge and flavor space determines the zero modes. We demonstrate the construction in a supersymmetric (SUSY) $\mathrm{SO}(10) \times S_4$ orbifold GUT model, which predicts the tribimaximal mixing at leading order in the lepton sector as well as the Cabibbo angle in the quark sector.

Keywords: Beyond Standard Model, Neutrino Physics, GUT

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1 Introduction

The leptonic mixing matrix is compatible with the tribimaximal mixing matrix [1] and hints towards a flavor symmetry. Especially, non-abelian discrete flavor symmetries have been studied. However, the origin of a potential flavor symmetry is unclear. The embedding in a continuous gauge symmetry seems disfavored because the explanation of the flavor structure requires large representations of flavon fields which cannot couple directly to the three generations of the SM fermions [2, 3]. In the context of string theory, string selection rules may lead to a discrete flavor symmetry [4–6]. Moreover, in magnetized extra dimensional models, a discrete flavor symmetry may arise from the localization behavior of zero modes [7, 8]. In addition, a discrete symmetry might be a remnant of an orbifold compactification [5, 9–13], which we consider in this study. Orbifolds, even several, are used in heterotic string theory model building and can therefore be consistently embedded in a UV complete theory. An orbifold compactification of a GUT can lead to its breaking and nicely solve e.g. the doublet-triplet splitting problem [14]. Recently, a flavor symmetry originating from an orbifold compactification has been studied in a GUT context [15]. A gauge symmetry is broken in an orbifold construction by the non-trivial transformation of the gauge fields under the orbifold parities [16, 17] or Wilson lines [18]. Similarly, it is possible to break a discrete flavor symmetry by a non-trivial transformation of the bulk fermions [19] as well as by Wilson lines [20]. Alternatively, the orbifold compactification can generate the alignment of vacuum expectation values (VEVs) of flavons [21] transmitting the flavor symmetry breaking into the fermion mass matrices.

In this paper, we study the combination of the origin of a flavor symmetry as well as its breaking by the VEV alignment of flavons from an orbifold. We assume two orbifolds, where the flavor symmetry originates from the special geometry of one orbifold and it is broken on another orbifold. We demonstrate it with a simple model in the context of a SUSY SO(10) orbifold GUT with S_4 flavor symmetry [22–44]. The model predicts the tribimaximal mixing at leading order in the lepton sector and the Cabibbo angle in the quark sector by implementing the Gatto-Sartori-Tonin relation [45, 46]. Note that this does not depend on SUSY. We assume that the orbifold parities act on gauge, SUSY as well as flavor space. Hence, the zero modes are determined by the overall parity.

2 Flavor symmetry from orbifolding

We study the $T^2/\mathbf{Z}_2$ orbifold (see figure 1) with radii $R = R_5 = R_6$, which we choose as $2\pi R = 1$ for simplicity. The discussion of the flavor symmetry does not change for $2\pi R \neq 1$. It is defined by

$$T_1 : z \to z+1, \qquad T_2 : z \to z+\gamma, \qquad Z : z \to -z. \tag{2.1}$$

where $z = x_5 + \mathrm{i}\, x_6$ and $\gamma = e^{\mathrm{i}\pi/3}$. The shape of this orbifold is a regular tetrahedron. This choice of equal radii and $\gamma = e^{\mathrm{i}\pi/3}$ is motivated by the possibility to stabilize the shape by Casimir energy as discussed in [47, 48]. It has been shown in [12] that the breaking of Poincaré symmetry from 6d to 4d through compactification on the orbifold leads to a remnant S_4 flavor symmetry. Concretely, the orbifold has four fixed points, $(z_1, z_2, z_3, z_4) = (1/2,\ (1+\gamma)/2,\ \gamma/2,\ 0)$, which are permuted by two translation operations S_i, the rotation T_R, and two parity operations $P^{(\prime)}$

$$S_1 : z \to z + 1/2, \quad S_2 : z \to z + \gamma/2, \quad T_R : z \to \gamma^2 z, \quad P : z \to z^*, \quad P' : z \to -z^*. \tag{2.2}$$

One can also write these operations explicitly in terms of the interchange of the fixed points, $S_1[(14)(23)]$, $S_2[(12)(34)]$, $T_R[(123)(4)]$, $P[(23)(1)(4)]$ and $P'[(23)(1)(4)]$. From these elements we can define two generators of S_4 as $S = S_2 P$ and $T = T_R$ satisfying the generator relation, $S^4 = T^3 = (ST^2)^2 = 1$. The localization of a brane field defines its representation of S_4. Concretely, the generators S, T can be represented by the matrices,

$$S = \begin{pmatrix} 0&0&1&0\\ 1&0&0&0\\ 0&0&0&1\\ 0&1&0&0 \end{pmatrix}, \quad T = \begin{pmatrix} 0&0&1&0\\ 1&0&0&0\\ 0&1&0&0\\ 0&0&0&1 \end{pmatrix}, \tag{2.3}$$

acting on the brane field $\psi(x_\mu) = (\psi_1, \psi_2, \psi_3, \psi_4)^T$, where $\psi_i = \psi(x_\mu, z_i)$ is a field localized at fixed point z_i. We denote this basis as localization basis. The characters of S and T show that the four dimensional representation generated by $\langle S, T\rangle$ can be decomposed in $3_1 \oplus 1_1$. The explicit unitary transformation is

$$S \to U^\dagger S U = \begin{pmatrix} S_3^{\mathrm{fl}} & \\ & 1 \end{pmatrix}, \quad T \to U^\dagger T U = \begin{pmatrix} T_3^{\mathrm{fl}} & \\ & 1 \end{pmatrix} \tag{2.4}$$

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with $U = \Big((\alpha_{ij}) \ (\beta_i) \Big)$ and

$$(\alpha_{ij}) = \begin{pmatrix} -\frac{1}{2\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{2\sqrt{3}} & \frac{\omega}{\sqrt{3}} & \frac{\omega^2}{\sqrt{3}} \\ -\frac{1}{2\sqrt{3}} & \frac{\omega^2}{\sqrt{3}} & \frac{\omega}{\sqrt{3}} \\ \frac{\sqrt{3}}{2} & 0 & 0 \end{pmatrix}, \qquad (\beta_i) = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}, \tag{2.5}$$

where $\omega = e^{2\pi i/3}$ and $S_3^{\mathrm{fl}}, T_3^{\mathrm{fl}}$ are the three dimensional generators of S_4 acting on a triplet 3_1 in flavor basis. The transformation of a field $\psi(x)$ is accordingly related to the flavor basis $\psi^{\mathrm{fl}}(x_\mu) = U^\dagger \psi(x_\mu)$ as well as the Clebsch-Gordan coefficients. The first three components of ψ^{fl} form a triplet 3_1 and the last one a singlet 1_1. It is possible to remove one of the representations from the low-energy spectrum by adding a bulk field transforming as 3_1 (1_1) and oppositely charged to the brane field, such that they acquire a Dirac mass term.

The representations 1_2 and 3_2 are analogously obtained by using the freedom to change the phase of each brane field in a symmetry transformation. Concretely, by replacing S with $-S$, we obtain a representation of S_4 which decomposes as $\langle -S, T\rangle = 3_2 \oplus 1_2$. Similarly, the 2 of S_4 can be obtained from the four dimensional representation $\left\langle S := T_R^2 P T_R,\, T := T_R^2 \right\rangle = 2 \oplus 1_1 \oplus 1_1$. Analogously, the edges are exchanged by $S[(ab)(cf)(de)]$ and $T[(ace)(bdf)]$. S and T form a six dimensional representation which can be expressed in matrix form by

$$S = \begin{pmatrix} 0&1&0&0&0&0 \\ 1&0&0&0&0&0 \\ 0&0&0&0&0&1 \\ 0&0&0&0&1&0 \\ 0&0&0&1&0&0 \\ 0&0&1&0&0&0 \end{pmatrix}, \qquad T = \begin{pmatrix} 0&0&0&0&1&0 \\ 0&0&0&0&0&1 \\ 1&0&0&0&0&0 \\ 0&1&0&0&0&0 \\ 0&0&1&0&0&0 \\ 0&0&0&1&0&0 \end{pmatrix}. \tag{2.6}$$

It can be decomposed in

$$6 = 2 \oplus 2 \oplus 1_1 \oplus 1_2 \tag{2.7}$$

and the explicit unitary transformation which transforms S and T from the localization basis $(S,T) \to (U^\dagger S U, U^\dagger T U)$ into flavor basis is given by

$$U = \begin{pmatrix} 0 & \frac{\omega^2}{\sqrt{3}} & \frac{\omega^2}{\sqrt{3}} & 0 & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \\ \frac{\omega^2}{\sqrt{3}} & 0 & 0 & \frac{\omega^2}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ 0 & \frac{1}{\sqrt{3}} & \frac{\omega}{\sqrt{3}} & 0 & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \\ \frac{\omega}{\sqrt{3}} & 0 & 0 & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ 0 & \frac{\omega}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & 0 & \frac{\omega}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} \end{pmatrix}. \tag{2.8}$$

Indeed, the symmetry generated by $S : z \to z^* + \gamma/2$ and T is a symmetry of the whole

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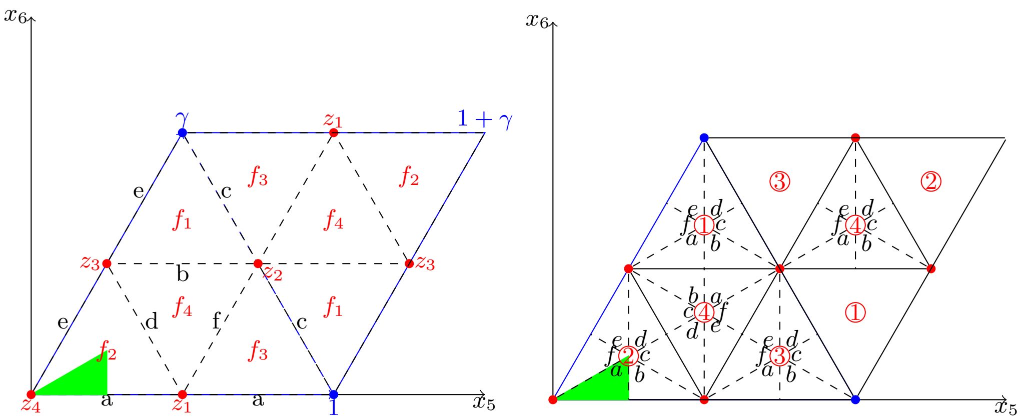

Figure 1. The orbifold $T^2/\mathbf{Z}_2$ with its four fixed points (z_1, z_2, z_3, z_4). The symmetry of interchanging the fixed points forms the discrete group S_4. S_4 is a symmetry of the whole orbifold, where the edges (a, b, c, d, e, f) form a six dimensional representation and the triangles building up the faces a 24 dimensional representation.

orbifold, because S and T fulfill the S_4 generator relations

$$S^4 : z \xrightarrow{S} z^* + \gamma/2 \xrightarrow{S} z + \gamma^*/2 + \gamma/2 \xrightarrow{S^2} z^* + \gamma^*/2 \to z\,, \tag{2.9a}$$

$$T^3 : z \xrightarrow{T} \gamma^2 z \xrightarrow{T^2} \gamma^6 z = z\,, \tag{2.9b}$$

$$(ST^2)^2 : z \xrightarrow{ST^2} \gamma^2 z^* + \gamma/2 \xrightarrow{ST^2} \gamma^2(\gamma^2 z^* + \gamma/2)^* + \gamma/2 = z\,. \tag{2.9c}$$

The orbits under the action of S_4, i.e. the equivalence classes with respect to the group action, can be represented by the points within the green (gray) triangle. Hence, the fundamental domain is reduced to the green (gray) triangle. The 6d Poincaré symmetry is broken to 4d Poincaré symmetry and the discrete symmetry S_4. The generators can be expressed as $S : [(1243)] \otimes [(ab)(cf)(ed)]$ and $T : [(123)] \otimes [(ace)(bdf)]$. Hence, the 24 dimensional representation is constructed by the tensor product of the four dimensional representation of the vertices and the six dimensional representation of the edges. It can be decomposed into irreducible representations as follows

$$\begin{aligned} 4 \otimes 6 &= (1_1 \oplus 3_1) \otimes (1_1 \oplus 1_2 \oplus 2 \oplus 2) \\ &= 1_1 \oplus 3_1 \oplus 1_2 \oplus 3_2 \oplus 2 \oplus 3_1 \oplus 3_2 \oplus 2 \oplus 3_1 \oplus 3_2\,. \end{aligned} \tag{2.10}$$

We note that for the remaining sections we will work in the flavor basis with irreducible representations only and omit $^{\text{fl}}$ for simplicity.

3 Symmetry breaking by boundary conditions

In order to demonstrate how the flavor structure can be obtained and broken appropriately from an orbifold, we implement it in an 8d SUSY SO(10) model with a breaking of gauge symmetry, which nicely leads to a splitting of the doublet and triplet components in the **<u>10</u>**-plet similar to the 6d model in [49, 50]. As the different boundary conditions lift the

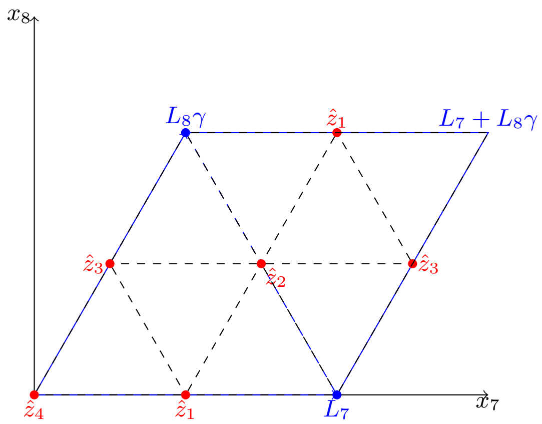

Figure 2. The gauge symmetry SO(10) as well as the flavor symmetry S_4 is broken by boundary conditions on the orbifold $T^2/(\mathbf{Z}_2^I \times \mathbf{Z}_2^{PS} \times \mathbf{Z}_2^{GG})$.

degeneracy of the fixed points, the flavor symmetry has to emerge from a different orbifold than the one where it is broken. (We note that in the first orbifold we can impose only one boundary condition without lifting the degeneracy of the fixed points, however, this is not enough for breaking the gauge symmetry, therefore, the second orbifold is needed.) For simplicity, we assume that the orbifold on which the symmetry is broken is also $T^2/\mathbf{Z}_2$ with two additional boundary conditions at $\hat{z}_1$, $\hat{z}_3$, i.e. $T^2/(\mathbf{Z}_2^I \times \mathbf{Z}_2^{PS} \times \mathbf{Z}_2^{GG})$ (see figure 2) and then we assume that the gauge fields are bulk fields of the two orbifolds, while all other bulk fields of the second orbifold are brane fields of the first orbifold. The $N = 1$ SUSY in 8d leads to $N = 4$ SUSY in 4d [51]. Its gauge vector multiplet can be decomposed in terms of one vector multiplet and three chiral multiplets of the unbroken $N = 1$ SUSY in 4d. In order to break $N = 4$ SUSY down to $N = 1$ SUSY, we use the orbifold parity of the first orbifold combined with one orbifold parity from the second orbifold, such that only the vector muliplet of $N = 1$ SUSY remains as zero mode.

The resulting GUT scale M_{GUT} is determined by the compactification scale. We neglect anomaly cancellation [52, 53] for the purpose of this study. The gauge fields transform under the orbifold parities as

$$\begin{aligned} P_0 V(x_\mu, -z, \hat{z}) P_0^{-1} &= \eta_0 V(x_\mu, z, \hat{z})\,, \\ P_I V(x_\mu, z, -\hat{z}) P_I^{-1} &= \eta_I V(x_\mu, z, \hat{z})\,, \\ P_{PS} V(x_\mu, z, -\hat{z}+\hat{z}_1) P_{PS}^{-1} &= \eta_{PS} V(x_\mu, z, \hat{z}+\hat{z}_1)\,, \\ P_{GG} V(x_\mu, z, -\hat{z}+\hat{z}_3) P_{GG}^{-1} &= \eta_{GG} V(x_\mu, z, \hat{z}+\hat{z}_3)\,, \end{aligned} \tag{3.1}$$

with $\hat{z} = x_7 + \mathrm{i}x_8$ where the matrices,

$$\begin{aligned} P_0 = P_I &= \mathbf{1}\,, \\ P_{PS} &= \mathrm{diag}(-1,-1,-1,1,1) \otimes \sigma^0\,, \\ P_{GG} &= \mathrm{diag}(1,1,1,1,1) \otimes \sigma^2\,, \end{aligned} \tag{3.2}$$

act on the gauge space with σ^0 being the 2×2 unity matrix and σ^2 one of the Pauli matrices. The parities are chosen as $\eta_0 = \eta_I = \eta_{PS} = \eta_{GG} = +1$. The first two parities

$G_{SM'}$	G_{GG}	G_{PS}	$(V_\mu,\, \lambda_1)$		
			Z_2^I	Z_2^{GG}	Z_2^{PS}
$(\underline{\mathbf{8}}, \underline{\mathbf{1}}, 0)_0$	$\underline{\mathbf{24}}_0$	$(\underline{\mathbf{15}}, \underline{\mathbf{1}}, \underline{\mathbf{1}})$	$+$	$+$	$+$
$(\underline{\mathbf{3}}, \underline{\mathbf{2}},\text{-}5)_0$	$\underline{\mathbf{24}}_0$	$(\underline{\mathbf{6}}, \underline{\mathbf{2}}, \underline{\mathbf{2}})$	$+$	$+$	$-$
$(\underline{\bar{\mathbf{3}}}, \underline{\mathbf{2}}, 5)_0$	$\underline{\mathbf{24}}_0$	$(\underline{\mathbf{6}}, \underline{\mathbf{2}}, \underline{\mathbf{2}})$	$+$	$+$	$-$
$(\underline{\mathbf{1}}, \underline{\mathbf{3}}, 0)_0$	$\underline{\mathbf{24}}_0$	$(\underline{\mathbf{1}}, \underline{\mathbf{3}}, \underline{\mathbf{1}})$	$+$	$+$	$+$
$(\underline{\mathbf{1}}, \underline{\mathbf{1}}, 0)_0$	$\underline{\mathbf{24}}_0$	$(\underline{\mathbf{1}}, \underline{\mathbf{1}}, \underline{\mathbf{3}})$	$+$	$+$	$+$
$(\underline{\mathbf{3}}, \underline{\mathbf{2}}, 1)_{+4}$	$\underline{\mathbf{10}}_{+4}$	$(\underline{\mathbf{6}}, \underline{\mathbf{2}}, \underline{\mathbf{2}})$	$+$	$-$	$-$
$(\underline{\bar{\mathbf{3}}}, \underline{\mathbf{1}}, -4)_{+4}$	$\underline{\mathbf{10}}_{+4}$	$(\underline{\mathbf{15}}, \underline{\mathbf{1}}, \underline{\mathbf{1}})$	$+$	$-$	$+$
$(\underline{\mathbf{1}}, \underline{\mathbf{1}}, 6)_{+4}$	$\underline{\mathbf{10}}_{+4}$	$(\underline{\mathbf{1}}, \underline{\mathbf{1}}, \underline{\mathbf{3}})$	$+$	$-$	$+$
$(\underline{\bar{\mathbf{3}}}, \underline{\mathbf{2}}, -1)_{-4}$	$\underline{\overline{\mathbf{10}}}_{-4}$	$(\underline{\mathbf{6}}, \underline{\mathbf{2}}, \underline{\mathbf{2}})$	$+$	$-$	$-$
$(\underline{\mathbf{3}}, \underline{\mathbf{1}}, 4)_{-4}$	$\underline{\overline{\mathbf{10}}}_{-4}$	$(\underline{\mathbf{15}}, \underline{\mathbf{1}}, \underline{\mathbf{1}})$	$+$	$-$	$+$
$(\underline{\mathbf{1}}, \underline{\mathbf{1}}, -6)_{-4}$	$\underline{\overline{\mathbf{10}}}_{-4}$	$(\underline{\bar{\mathbf{1}}}, \underline{\mathbf{1}}, \underline{\mathbf{3}})$	$+$	$-$	$+$
$(\underline{\mathbf{1}}, \underline{\mathbf{1}}, 0)_0$	$\underline{\mathbf{1}}_0$	$(\underline{\mathbf{15}}, \underline{\mathbf{1}}, \underline{\mathbf{1}})$	$+$	$+$	$+$

Table 1. Parity assignments for the components $V_M^A = \frac{1}{2}\mathrm{tr}(T^A V_M)$ of the $\underline{\mathbf{45}}$-plet of SO(10). $G_{SM'} = \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_Y \times \mathrm{U}(1)_X$, $G_{GG} = \mathrm{SU}(5) \times \mathrm{U}(1)$, and $G_{PS} = \mathrm{SU}(4) \times \mathrm{SU}(2) \times \mathrm{SU}(2)$.

$G_{SM'}$	G_{GG}	G_{PS}	H_d		H_u	
			Z_2^{PS}	Z_2^{GG}	Z_2^{PS}	Z_2^{GG}
$(\underline{\mathbf{1}}, \underline{\mathbf{2}}, -1/2)_{-2}$	$\underline{\bar{\mathbf{5}}}_{-2}$	$(\underline{\mathbf{1}}, \underline{\mathbf{2}}, \underline{\mathbf{2}})$	$+$	$+$	$+$	-
$(\underline{\mathbf{1}}, \underline{\mathbf{2}}, +1/2)_{+2}$	$\underline{\mathbf{5}}_{+2}$	$(\underline{\mathbf{1}}, \underline{\mathbf{2}}, \underline{\mathbf{2}})$	$+$	-	$+$	$+$
$(\underline{\bar{\mathbf{3}}}, \underline{\mathbf{1}}, +1/3)_{-2}$	$\underline{\bar{\mathbf{5}}}_{-2}$	$(\underline{\mathbf{6}}, \underline{\mathbf{1}}, \underline{\mathbf{1}})$	-	$+$	-	-
$(\underline{\mathbf{3}}, \underline{\mathbf{1}}, -1/3)_{+2}$	$\underline{\mathbf{5}}_{+2}$	$(\underline{\mathbf{6}}, \underline{\mathbf{1}}, \underline{\mathbf{1}})$	-	-	-	$+$

Table 2. Parity assignments for the components of the $\underline{\mathbf{10}}$ hypermultiplet.

(corresponding to fixed point z_4 and $\hat{z}_4$) are used to break $N = 4$ SUSY to $N = 1$ SUSY and the remaining two parities are used to break the gauge symmetry [54, 55]. The parity assignment of the different components of the bulk fields is given in tables 1, 2 and 3 [53, 55].

The different boundary conditions break the gauge symmetry to different subgroups at the different fixed points, concretely Pati-Salam $\mathrm{SU}(4) \times \mathrm{SU}(2) \times \mathrm{SU}(2)$ at fixed point $\hat{z}_1$,

$G_{SM'}$	G_{GG}	G_{PS}	$\bar{\Delta}_1$		$\bar{\Delta}_2$	
			Z_2^{PS}	Z_2^{GG}	Z_2^{PS}	Z_2^{GG}
$(\overline{\underline{\mathbf{3}}}, \underline{\mathbf{2}}, -1/6)_{+1}$	$\overline{\underline{\mathbf{10}}}_{+1}$	$(\overline{\underline{\mathbf{4}}}, \underline{\mathbf{2}}, \underline{\mathbf{1}})$	-	-	+	-
$(\underline{\mathbf{1}}, \underline{\mathbf{2}}, +1/2)_{-3}$	$\underline{\mathbf{5}}_{-3}$	$(\overline{\underline{\mathbf{4}}}, \underline{\mathbf{2}}, \underline{\mathbf{1}})$	-	+	+	+
$(\underline{\mathbf{3}}, \underline{\mathbf{1}}, +2/3)_{+1}$ $\oplus(\underline{\mathbf{1}}, \underline{\mathbf{1}}, -1)_{+1}$	$\overline{\underline{\mathbf{10}}}_{+1}$	$(\underline{\mathbf{4}}, \underline{\mathbf{1}}, \underline{\mathbf{2}})$	+	-	-	-
$(\underline{\mathbf{3}}, \underline{\mathbf{1}}, -1/3)_{-3}$ $\oplus(\underline{\mathbf{1}}, \underline{\mathbf{1}}, 0)_{+5}$	$\underline{\mathbf{5}}_{-3} \oplus \underline{\mathbf{1}}_{+5}$	$(\underline{\mathbf{4}}, \underline{\mathbf{1}}, \underline{\mathbf{2}})$	+	+	-	+

Table 3. Parity assignments for the components of the $\overline{\underline{\mathbf{16}}}$ hypermultiplet.

$SU(5) \times U(1)_X$ at $\hat{z}_3$ and flipped $SU(5)' \times U(1)'$ at $\hat{z}_2$, while SO(10) is unbroken at $\hat{z}_4$. The remnant gauge symmetry of the orbifold is the intersection of the gauge symmetries at each fixed point, which is $G_{SM'} = SU(3) \times SU(2) \times U(1)_Y \times U(1)_X$. The remaining $U(1)_X$ can be broken by a right-handed neutrino mass term. All flavons, i.e. gauge singlets, in the bulk have positive gauge parities. Hence, the gauge and flavor breaking sectors are separated.

Analogously, by assigning the parities to the flavons living in the bulk of the second orbifold, a zero mode is singled out and consequently the flavor symmetry is broken. As the flavons are the bulk fields of only one orbifold, the flavons inherit $N = 2$ SUSY in 4d, which can be broken by using one orbifold parity. If the zero mode acquires a VEV, the breaking of the symmetry is transmitted to the fermion masses. We assume that flavons transform non-trivially,

$$P_1 \phi(x_\mu, z, -\hat{z}) = \eta_1 \, \phi(x_\mu, z, \hat{z}), \tag{3.3a}$$

$$P_2 \phi(x_\mu, z, -\hat{z} + \hat{z}_1) = \eta_2 \, \phi(x_\mu, z, \hat{z} + \hat{z}_1), \tag{3.3b}$$

$$P_3 \phi(x_\mu, z, -\hat{z} + \hat{z}_3) = \eta_3 \, \phi(x_\mu, z, \hat{z} + \hat{z}_3), \tag{3.3c}$$

where the $P_{1,2,3} = Z$, $T_1 Z$, $T_2 Z$ are formed by the combination of the translation operators $T_{1,2}$ and parity operator Z acting on flavor space. While the first parity is used to break $N = 2$ SUSY to $N = 1$, the remaining two parity operators are used to generate the VEV alignment of the flavons by singling out the appropriate zero modes. We note that, in fact, the flavor symmetry is already broken when the boundary conditions are introduced, however, in order to predict the neutrino mixing, the breaking has to be transmitted by a certain flavon VEV alignment to the fermion masses.

In general, the flavon VEV alignment is determined by a possibly complicated flavon potential. However, in an orbifold context, (part of) the VEV alignment can be obtained from the action of the orbifold parities in flavor space, i.e. the simultaneous eigenvectors of the set of parity operators in flavor space. Therefore, the flavon potential can remain simple. As the square of the parity operators has to be the identity [21], they have eigenvalues ± 1.

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$\mathcal{C}_2$	3_i	$\mathcal{C}_4$	3_1	3_2	2
EV	1	EV	-1	1	1
S^2	$\begin{pmatrix}1 & 1 & 1\end{pmatrix}$	TST	$\begin{pmatrix}-2 & 1 & 1\end{pmatrix}$		$\begin{pmatrix}1 & 1\end{pmatrix}$
TS^2T^2	$\begin{pmatrix}1 & \omega^2 & \omega\end{pmatrix}$	T^2S	$\begin{pmatrix}-2 & \omega^2 & \omega\end{pmatrix}$		$\begin{pmatrix}1 & \omega\end{pmatrix}$
$S^2TS^2T^2$	$\begin{pmatrix}1 & \omega & \omega^2\end{pmatrix}$	ST^2	$\begin{pmatrix}-2 & \omega & \omega^2\end{pmatrix}$		$\begin{pmatrix}1 & \omega^2\end{pmatrix}$
		$TSTS^2$	$\begin{pmatrix}0 & -1 & 1\end{pmatrix}$		$\begin{pmatrix}1 & 1\end{pmatrix}$
		STS^2	$\begin{pmatrix}0 & -\omega^2 & \omega\end{pmatrix}$		$\begin{pmatrix}1 & \omega\end{pmatrix}$
		S^2TS	$\begin{pmatrix}0 & -\omega & \omega^2\end{pmatrix}$		$\begin{pmatrix}1 & \omega^2\end{pmatrix}$

Table 4. Eigenvector structure of the elements of the conjugacy classes $\mathcal{C}_{2,4}$ in representation 2 and 3_i, where S, T are generators of S_4 and $\omega = \mathrm{e}^{2\pi\mathrm{i}/3}$. EV denotes a/the non-degenerate eigenvalue and the vector in the corresponding row and column is the eigenvector of the group element given to the eigenvalue shown at the top of the column.

We restrict ourselves to parity operators which are within our flavor group, because other choices of parity operators, which are not elements of the flavor group, do not preseve the Clebsch-Gordan coefficients and would forbid e.g. the kinetic term of the flavon field. For the VEV alignment, we are only interested in elements which have both $+1$ and -1 as eigenvalues to obtain one single zero mode. VEVs originating from the same set of parity operators are orthogonal. However, parity operators can be chosen differently for different representations of S_4, and therefore VEVs of fields in different representations do not have to be orthogonal. We note that the couplings between the fields with different parity operators are forbidden because they are not invariant under the orbifold parity.

Concretely, in S_4, the elements of order two are in the conjugacy classes $\mathcal{C}_{2,4}$ according to the notation in [56] and its eigenvalues can be inferred from its character. The eigenvectors to the non-degenerate eigenvalues of each element are shown in table 4. With two parity operators, it is possible to obtain every eigenvector shown in table 4 as zero mode as well as the orthogonal complement of any two chosen ones. We give one example, how to obtain the VEV alignment (1, 1, 1), which we use in the model in the following section. In order to obtain the VEV alignment for a triplet $\phi \sim 3_1$, we choose

$$P_1 = \mathbf{1}, \qquad P_2 = TST, \qquad P_3 = TSTS^2 . \tag{3.4}$$

$$\text{i.e.} \qquad P_2 = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & 2 & -1 \\ 2 & -1 & 2 \end{pmatrix}, \qquad P_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

As P_1 is the unit matrix, it does not affect the zero mode. Therefore, the zero mode is entirely determined by $P_{2,3}$. P_2 and P_3 are simultaneously diagonalized by the unitary matrix U

$$U = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{6}} & 0 \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} \end{pmatrix}, \qquad \begin{aligned} U^\dagger P_2 U &= \mathrm{diag}(1,\,-1,\,1)\,, \\ U^\dagger P_3 U &= \mathrm{diag}(1,\,1,\,-1)\,, \end{aligned} \tag{3.5}$$

The eigenvalues and eigenvectors can also be read of from table 4. The eigenvector to the non-degenerate eigenvalue -1 of $P_2 = TST$ is $(-2,\,1,\,1)$ corresponding to the second column of U. The two eigenvectors to the eigenvalue 1 are not explicitly given, but they are orthogonal to $(-2,\,1,\,1)$, which is sufficient for the analysis. Similarly, the eigenvector to the non-degenerate eigenvalue -1 of $P_3 = TSTS^2$ is $(0,\,-1,\,1)$ according to table 4. This eigenvector determines the third column of U. The two eigenvectors to eigenvalue $+1$ are also in the orthogonal complement. Taking the parity assignment of ϕ, $\eta_2 = \eta_3 = +1$, the eigenvectors to the eigenvalue $+1$ of $P_{2,3}$ are chosen and the zero mode lies in the intersection of the two-dimensional subspaces spant by the eigenvectors to the eigenvalue $+1$ of $P_{2,3}$. Hence, we can determine the zero mode by either taking the cross product of $(-2,\,1,\,1)$ with $(0,\,-1,\,1)$ or equivalently the first column of U which corresponds to $P_{2,3}$ having eigenvalue $+1$. Hence, the VEV alignment is

$$\langle\phi\rangle = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}^T / \sqrt{3} \left\langle\tilde{\phi}\right\rangle\,, \tag{3.6}$$

with $\tilde{\phi}$ being the single zero mode of ϕ. In order to demonstrate this, we present a model which is based on the previous discussion.

4 Model

As discussed in the previous sections, the gauge field is a bulk field of the two orbifolds in our setup, while the other fields are brane fields of the first orbifold $T^2/\mathbf{Z}_2$, leading to the flavor symmetry S_4, and can be either a brane or bulk field of the second orbifold $T^2/(\mathbf{Z}_2^I \times \mathbf{Z}_2^{PS} \times \mathbf{Z}_2^{GG})$, which is used to break the GUT and the flavor symmetry. Besides the gauge field, we introduce a $\underline{\mathbf{16}}$ brane field ψ, which is localized on the SO(10) brane and leads to the SM matter, together with the SO(10) singlet fields $S_{N,\nu}$ on the SO(10) brane, the bulk Higgs fields $H_{u,d} \sim \underline{\mathbf{10}}$ generating fermion masses and breaking electroweak symmetry as well as $\Phi \sim \underline{\mathbf{45}}$ and the bulk Higgs fields $\bar{\Delta}_i \sim \underline{\overline{\mathbf{16}}}$ leading to the mass terms of neutrinos. Furthermore, there are the flavons $\phi_{1,1,1}$, ξ and $\chi_{1,0,0}$ in the bulk as well as $\chi_{0,1,0}$ and $\chi_{0,0,1}$ on the SO(10) brane. The index of each flavon field denotes its VEV alignment, i.e. $\chi_{a,b,c} \sim (a,\,b,\,c)\tilde{\chi}_{a,b,c}$. The $\mathrm{U}(1)_R$ helps to implement the driving field mechanism for the flavon potential and the additional $\mathbf{Z}_N$ charge is introduced to separate the particles in the different sectors. We note that the Yukawa coupling related to a bulk zero mode have to be rescaled by the rescaling factor $1/\sqrt{\Lambda^2 V} < 1$, where Λ is the cut-off of the bulk theory and $1/\sqrt{V}$ is the volume normalization factor of the zero mode as can be seen in appendix B. In our case, $V = \frac{1}{2} L_7 L_8 \sin\theta$ with θ being the angle between the basis vectors spanning the second orbifold and $L_{7,8}$ being their respective length. In principle, they are free parameters, but for concreteness, we set $\theta = \pi/3$ and $L_7 = L_8 = 2\pi R$. In the following, we will only write the zero mode of each bulk field, since we are interested in

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Field	SO(10)	S_4	$\mathbf{Z}_N$	$\mathrm{U}(1)_R$	$\eta_I\eta_1$	$\eta_{PS}\eta_2$	$\eta_{GG}\eta_3$
ψ	$\underline{\mathbf{16}}$	3_2	0	1			
H_u	$\underline{\mathbf{10}}$	1_1	-2	0	$+$	$+$	$-$
H_d	$\underline{\mathbf{10}}$	1_1	-2	0	$+$	$+$	$+$
Φ	$\underline{\mathbf{45}}$	1_1	4	0	$+$	$+$	$+$
$\chi_{1,0,0}$	$\underline{\mathbf{1}}$	3_1	-3	0	$+$	$-$	$-$
$\chi_{0,0,1}$	$\underline{\mathbf{1}}$	3_2	-1	0			
$\chi_{0,1,0}$	$\underline{\mathbf{1}}$	3_2	1	0			
$\bar{\Delta}_1$	$\overline{\underline{\mathbf{16}}}$	1_1	-1	0	$+$	$-$	$-$
$\bar{\Delta}_2$	$\overline{\underline{\mathbf{16}}}$	1_1	3	0	$+$	$+$	$-$
S_N	$\underline{\mathbf{1}}$	3_2	1	1			
S_ν	$\underline{\mathbf{1}}$	3_2	-3	1			
$\phi_{1,1,1}$	$\underline{\mathbf{1}}$	3_1	6	0	$+$	$+$	$+$
ξ	$\underline{\mathbf{1}}$	1_1	6	0	$+$	$+$	$+$

Table 5. Particle content of the model. Bulk fields are classified by three orbifold parities in addition to the symmetry groups. The lower index of the flavon fields denote their zero modes.

the low-energy spectrum. The particle content and all charges are given in table 5. The orbifold parities of each Higgs field determines its respective zero mode and also which component obtains a VEV. In particular the VEVs of the $\underline{\mathbf{10}}$-plets are chosen to be at the electroweak scale. The singlet component of $\bar{\Delta}_1$ acquires a VEV of the order of the GUT scale, whereas the VEV of the doublet component of $\bar{\Delta}_2$ is assumed to be of the order of the electroweak scale. This additional Higgs doublet component and its fermionic superpartner modify the running of the electroweak gauge couplings, which might destroy gauge coupling unification. There are several possible solutions. For instance, corrections due to Kaluza-Klein threshold effects [57, 58] might restore the unification of gauge couplings. Otherwise, it is possible to introduce additional fields either to obtain complete GUT representations or magic combinations, which do not spoil gauge coupling unification [59]. This, however, leads to additional effects at the low energy scale, e.g. through new colored states. As we are concentrating on the flavor structure, we are assuming in the following that a successful gauge coupling unification can be obtained. The parities of $\Phi \sim \underline{\mathbf{45}}$ are chosen such that the two total SM singlets remain as zero modes, as it can be seen in table 1. In the following, we assume, that only the component in $B-L$ direction obtains a VEV, i.e. $\langle\Phi\rangle \sim B-L$ which leads to the Georgi-Jarlskog factor [60] at the GUT scale.

Small neutrino masses can be generated by the seesaw mechanism starting from

$$\mathbf{W}_\nu = \frac{y_s}{\sqrt{\Lambda^2 V}} \psi \bar{\Delta}_2 S_\nu + \frac{1}{\sqrt{\Lambda^2 V}} \left(y_\phi^\nu \phi_{1,1,1} + y_\xi^\nu \xi \right) S_\nu S_\nu \,. \tag{4.1}$$

After $\phi_{1,1,1}$ and ξ obtain VEVs, S_ν becomes massive

$$M_{\mathrm{SS}} = \frac{1}{\sqrt{\Lambda^2 V}} \left(y_\phi^\nu \langle \phi_{1,1,1} \rangle + y_\xi^\nu \langle \xi \rangle \right) \tag{4.2}$$

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and leads to neutrino masses

$$M_\nu = -\frac{1}{\Lambda^2 V}(y_s \langle\bar{\Delta}_2\rangle)(M_{\rm SS}^{-1})(y_s \langle\bar{\Delta}_2\rangle)^T = -m_0 U_{\rm tbm}^* \begin{pmatrix} \frac{1}{3a+b} & 0 & 0 \\ \cdot & \frac{1}{b} & 0 \\ \cdot & \cdot & \frac{1}{3a-b} \end{pmatrix} U_{\rm tbm}^\dagger \,, \tag{4.3}$$

with $m_0 = y_s^2 \langle\bar{\Delta}_2\rangle^2 / \langle\xi\rangle / \sqrt{\Lambda^2 V}$ as well as $a = y_\phi^\nu \langle\phi\rangle / \langle\xi\rangle$ and $b = y_\xi^\nu$. As $M_{\rm SS}$ is diagonalized by the tribimaximal mixing matrix and the structure of the Yukawa couplings y_s does not affect the tribimaximal mixing, the neutrino mixing matrix is of the tribimaximal form and the neutrino masses are given by

$$(m_1,\, m_2,\, m_3) = m_0 \left(\frac{1}{3a+b}, \frac{1}{b}, \frac{1}{3a-b}\right) . \tag{4.4}$$

In order to suppress the usual type-I seesaw contribution to the neutrino mass matrix, we introduce an additional SO(10) singlet S_N which combines with the SM singlet component of ψ

$$\mathbf{W}_N = \frac{y_N}{\sqrt{\Lambda^2 V}} \psi \bar{\Delta}_1 S_N \,, \tag{4.5}$$

to form a pseudo-Dirac particle and there is only a small mixing with the light neutrinos of the order of $\langle H_u\rangle / \langle\bar{\Delta}_1\rangle$ with $\langle\bar{\Delta}_1\rangle$ being of the order of the GUT scale. Therefore the SM singlet component essentially decouples from the low-energy theory. Furthermore, this term breaks the remnant $\mathrm{U}(1)_X$.

For clarity, we present the quark sector of the model in terms of effective higher-dimensional operators and demonstrate in appendix A, how the operators can be obtained by integrating out vector-like brane fields. The third generation Yukawa couplings are described by

$$\mathbf{W}_3 \supset \frac{1}{\sqrt{\Lambda^2 V}} \frac{y_3^{u,d}}{M_3^2} (\psi \chi_{0,1,0})_{1_1} (\psi \chi_{0,1,0})_{1_1} H_{u,d} \,, \tag{4.6}$$

where $(\psi \chi_{0,1,0})_{1_1}$ denotes that the fields within the brackets are contracted to the trivial singlet 1_1 and M_3 is the mass of the decoupled vector-like field. After the flavon $\chi_{0,1,0}$ obtains a VEV, a mass term

$$\mathbf{W}_3 \supset \frac{y_3^{u,d} \langle\chi_{0,1,0}\rangle^2 H_{u,d}}{\sqrt{\Lambda^2 V} M_3^2} \psi_3^2 \tag{4.7}$$

of the third generation is generated [44, 61]. The masses of the top and bottom quark are

$$m_{t,b} = \frac{y_3^{u,d} \langle\chi_{0,1,0}\rangle^2}{\sqrt{\Lambda^2 V} M_3^2} \langle H_{u,d}\rangle \,, \tag{4.8}$$

which requires a large top Yukawa coupling, y_3^u, to overcome the suppression from the extra-dimensional volume and the mass scale M_3. The VEV $\langle H_{u,d}\rangle$ is of the order of the electroweak scale and $y_3^{u,d} \langle\chi_{0,1,0}\rangle^2 / (\sqrt{\Lambda^2 V} M_3^2)$ is of order one. The factor m_b/m_t can be written in term of $\tan\beta = \langle H_u\rangle / \langle H_d\rangle$ as $m_b/m_t = y_3^d/(y_3^u \tan\beta)$.

Similarly, the Yukawa couplings for the first and second generation can be obtained from

$$\mathbf{W}_{12} \supset \frac{1}{(\sqrt{\Lambda^2 V})^2}\left(\frac{y_2^{u,d}}{M_2^3}(\psi\chi_{0,0,1})_{1_1}(\psi\chi_{0,0,1})_{1_1}\Phi H_{u,d}+\frac{y_{12}^{u,d}}{\sqrt{\Lambda^2 V}M_{12}^3}(\psi\chi_{1,0,0})_{1_1}(\psi\chi_{0,0,1})_{1_1}\xi H_{u,d}\right). \tag{4.9}$$

The scale, at which the operators are generated, is denoted by M_2 and M_{12}. As discussed at the beginning of the section, $\Phi \sim \underline{\mathbf{45}}$ obtains a VEV in $B-L$ direction leading to the Georgi-Jarlskog factor [60] at the GUT scale. This contributes to the mass matrices subdominantly and leads to the mass of the first generations as well as the Cabibbo angle. The charged fermion mass matrices are given by

$$M_{u,d} = m_{t,b}\begin{pmatrix} 0 & a^{u,d} & 0 \\ a^{u,d} & b^{u,d} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad M_l = m_b\begin{pmatrix} 0 & a^d & 0 \\ a^d & 3\,b^d & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{4.10}$$

where

$$a^{u,d} = \frac{1}{(\sqrt{\Lambda^2 V})^2}\frac{y_{12}^{u,d}}{y_3^{u,d}}\frac{M_3^2\,\langle\chi_{1,0,0}\rangle\,\langle\chi_{0,0,1}\rangle\,\langle\xi\rangle}{M_{12}^3\,\langle\chi_{0,1,0}\rangle^2} \quad \text{and} \quad b^{u,d} = \frac{1}{\sqrt{\Lambda^2 V}}\frac{y_2^{u,d}}{y_3^{u,d}}\frac{M_3^2\,\langle\chi_{0,0,1}\rangle^2\,\langle\Phi\rangle}{M_2^3\,\langle\chi_{0,1,0}\rangle^2} \tag{4.11}$$

with $a^{u,d}$ being naturally smaller than $b^{u,d}$ through the additional suppression by the volume factor. The Cabibbo angle θ_c is approximately given by $\sin\theta_c = a^d/b^d - a^u/b^u$. Hence, the dominant contribution to the Cabibbo angle is from the down-type quark mass matrix

$$0.04 - 0.05 \simeq \sqrt{\frac{m_u}{m_c}} = \frac{a^u}{b^u} \ll \frac{a^d}{b^d} = \sqrt{\frac{m_d}{m_s}} \simeq 0.23\,. \tag{4.12}$$

We note that the other quark mixing angles can be obtained from higher order corrections. As the structure of the charged lepton mass matrix in eq. (4.10) is connected to the down-type mass matrix M_d and the CKM mixing is mainly generated by M_d, there is a correction to the tribimaximal mixing matrix [62–64] which results in a small deviation from the tribimaximal mixing

$$s_{12}^2 = \frac{1}{3} - \frac{2\theta_c}{9} + \frac{\theta_c^2}{54}, \qquad s_{13}^2 = \frac{\theta_c^2}{18}, \qquad s_{23}^2 = \frac{1}{2} - \frac{\theta_c^2}{36} \tag{4.13}$$

with $s_{ij} = \sin\theta_{ij}$. Note that the solar mixing angle θ_{12} is corrected towards smaller values. Depending on the absolute mass scale of neutrino masses, the angles are further corrected by the renormalization group evolution, which we can neglect in case of a hierarchical spectrum [65].

Finally, we discuss the flavon VEV alignment and show that we obtain the required one. The VEV alignment of the bulk flavon fields is readily obtained from the boundary conditions, which project out one single zero mode. However, we still have to show that the zero mode develops a VEV and the VEV alignment of the brane flavon fields is achieved. The alignment of $\phi_{1,1,1}$ has been demonstrated in the previous section. In order

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Field	SO(10)	S_4	$\mathbf{Z}_N$	$\mathrm{U}(1)_R$	$\eta_I\eta_1$	$\eta_{PS}\eta_2$	$\eta_{GG}\eta_3$
$\bar{\xi}$	$\underline{\mathbf{1}}$	1	-6	0	$+$	$+$	$+$
$\delta_0^{(i)}$	$\underline{\mathbf{1}}$	1_1	0	2			
$\delta_2^{(i)}$	$\underline{\mathbf{1}}$	1_1	2	2			
δ_{-2}	$\underline{\mathbf{1}}$	1_1	-2	2			
δ_4	$\underline{\mathbf{1}}$	1_1	4	2			
δ_{-12}	$\underline{\mathbf{1}}$	1_1	-12	2			

Table 6. The additional flavon field $\bar{\xi}$ as well as driving fields $\delta_n^{(i)}$ on the SO(10) brane which are needed to obtain the relevant VEV alignment. The lower index of δ denotes its $\mathbf{Z}_N$ charge and a possible upper index labels different driving fields with the same $\mathbf{Z}_N$ charge.

to obtain the VEV alignment for the triplet $\chi_{1,0,0} \sim 3_2$, we choose the parity operators, in flavor space as

$$P_1 = \mathbf{1}, \qquad P_2 = STS^2 \qquad P_3 = TSTS^2 . \tag{4.14}$$

which leads to

$$\langle \chi_{1,0,0} \rangle = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}^T \langle \tilde{\chi}_{1,0,0} \rangle \tag{4.15}$$

with $\tilde{\chi}_{1,0,0}$ denoting the single zero mode of flavon $\chi_{1,0,0}$ and its parities are given in table 5.

The remaining VEV alignment can be obtained from the flavon potential by using the driving field mechanism

$$\begin{aligned} \mathbf{W}_{\text{fl}} = & \sum_i \delta_0^{(i)} \left(\lambda_{ij} X_j - M_i^2 \right) \\ & + \frac{\delta_{-12}}{(\sqrt{\Lambda^2 V})^2} (\alpha_1 \xi^2 - \alpha_2 \phi_{1,1,1}^2) + \frac{\alpha_3 \delta_4}{\sqrt{\Lambda^2 V}} \chi_{1,0,0} \chi_{0,0,1} + \alpha_4 \delta_{-2} \chi_{0,1,0}^2 + \delta_2^{(i)} \kappa_{ij} Y_j \end{aligned} \tag{4.16}$$

with

$$X = \left(\frac{\bar{\xi}\xi}{(\sqrt{\Lambda^2 V})^2} \, , \, \frac{\chi_{1,0,0}^2}{(\sqrt{\Lambda^2 V})^2} \, , \, \chi_{0,0,1}\chi_{0,1,0} \right)^T \quad \text{and} \quad Y = \left(\frac{1}{\sqrt{\Lambda^2 V}} \chi_{1,0,0}\chi_{0,1,0} \, , \, \chi_{0,0,1}^2 \right)^T . \tag{4.17}$$

The matrices (λ_{ij}) and (κ_{ij}) are non-singular as well as the couplings $\alpha_i \neq 0$ and $M_i \neq 0$. We introduced one additional flavon field $\bar{\xi}$ to form a singlet $\bar{\xi}\xi$ and several driving fields $\delta_n^{(i)}$. The index of each driving field denotes its $\mathbf{Z}_N$ charge. They are listed in table 6. In the SUSY limit, the minima of the flavon potential can be obtained from the condition of vanishing F-terms

$$F_\delta = \frac{\partial \mathbf{W}_{\text{fl}}}{\partial \delta} \stackrel{!}{=} 0 \tag{4.18}$$

with δ being one of the driving (or flavon) fields. The F term conditions of the flavon fields are readily satisfied by the vanishing of all driving field VEVs. The F term conditions of the driving fields determine the flavon VEVs. All components of X obtain a VEV due to the non-singularity of $(\lambda)_{ij}$. The F term condition $F_{\delta_4} = 0$ forces the VEV in the first component of $\langle\chi_{0,0,1}\rangle$ to vanish. Similarly, the non-singularity of (κ_{ij}) leads to a vanishing of the first component of the VEV of $\langle\chi_{0,1,0}\rangle$ as well as the product $\langle\chi_{0,0,1}\rangle_2\langle\chi_{0,0,1}\rangle_3$. Therefore, either the third or second component of $\langle\chi_{0,0,1}\rangle$ vanishes. We choose the second component. Furthermore, $F_{\delta_{-2}}=0$ together with the non-vanishing of $\langle\chi_{0,0,1}\chi_{0,1,0}\rangle$ leads to the vanishing of the third component of $\langle\chi_{0,1,0}\rangle$. The new fields do not introduce additional couplings up to a certain order depending on the $\mathbf{Z}_N$ charge.

5 Conclusions

In this paper, we investigated a model, where an S_4 flavor symmetry arises from an orbifold compactification and it is broken by the boundary conditions together with the SO(10) gauge symmetry on another orbifold. All possible VEV alignments by only using elements of S_4 as parity operators have been summarized in table 4. Finally, we gave an example in the context of SO(10) $\times$ S_4 which leads to a phenomenologically viable neutrino mass matrix as well as enables to fit the masses of quarks and charged leptons. The model predicts the tribimaximal mixing at leading order in the lepton sector and the Gatto-Sartori-Tonin relation [45, 46] in the quark sector, which leads to a quantitatively correct Cabibbo mixing angle. The solar mixing angle θ_{12} receives a small correction from the charged leptons towards a smaller value and a small θ_{13} is generated. The VEV alignment obtained from the orbifold is an essential ingredient to obtain the required VEV alignment. We state that the VEV alignment mechanism can also be used for flavor symmetries arising from different orbifolds [13] as well as for models with flavored Higgs fields.

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A Generation of effective operators

In this section, we demonstrate how to obtain the effective operators, which have been used to generate the charged fermion mass matrices in section 4. The vector-like brane fields, which are integrated out, are summarized in table 7. The effective term which leads to the third generation masses in eq. (4.6) can be obtained from

$$\mathbf{W}_{\mathrm{LO}3}=h_3(\psi\chi_{0,1,0})\bar{\Psi}_3+\frac{h_{u,d}}{\sqrt{\Lambda^2V}}\Psi_3H_{u,d}\Psi_3+M_{\Psi3}\bar{\Psi}_3\Psi_3\,. \tag{A.1}$$

The scale M_3^2 is therefore given by $M_{\Psi3}^2$. The first term in eq. (4.9), which generates the mass of the second generation is obtained by integrating out two vector-like fields

$$\mathbf{W}_{\mathrm{LO}2}=h_2(\psi\chi_{0,0,1})\bar{\Psi}_2+\frac{h_{45}}{\sqrt{\Lambda^2V}}\Psi_2\Phi\bar{\Psi}_2'+\frac{h_{u,d}'}{\sqrt{\Lambda^2V}}\Psi_2H_{u,d}\Psi_2'+M_{\Psi2}\bar{\Psi}_2\Psi_2+M_{\Psi2'}\bar{\Psi}_2'\Psi_2'\,. \tag{A.2}$$

Field	$\mathrm{SO}(10)_{\mathbf{Z}_N}$	S_4	$\mathrm{U}(1)_R$
$\Psi_3 \oplus \bar{\Psi}_3$	$\underline{\mathbf{16}}_1 \oplus \underline{\overline{\mathbf{16}}}_{-1}$	1_1	1
$\Psi_2 \oplus \bar{\Psi}_2$	$\underline{\mathbf{16}}_{-1} \oplus \underline{\overline{\mathbf{16}}}_{1}$	1_1	1
$\Psi'_2 \oplus \bar{\Psi}'_2$	$\underline{\mathbf{16}}_3 \oplus \underline{\overline{\mathbf{16}}}_{-3}$	1_1	1
$\Psi_{12} \oplus \bar{\Psi}_{12}$	$\underline{\mathbf{16}}_{-3} \oplus \underline{\overline{\mathbf{16}}}_{3}$	1_1	1
$\Psi'_{12} \oplus \bar{\Psi}'_{12}$	$\underline{\mathbf{16}}_5 \oplus \underline{\overline{\mathbf{16}}}_{-5}$	1_1	1

Table 7. Vector-like brane fields which generate the required effective operators.

After integrating out, $\Psi'_2 + \bar{\Psi}'_2$, we obtain the effective dimension 4 operator

$$\mathbf{W}_{d=4} \supset \frac{h'_{u,d} h_{45}}{(\sqrt{\Lambda^2 V})^2 M_{\Psi 2'}} \Psi_2 \Phi H_{u,d} \Psi_2 \,, \tag{A.3}$$

which leads to eq. (4.9) after integrating out $\Psi_2 + \bar{\Psi}_2$. Hence, the scale of the operator in eq. (4.9) M_2^3 equals $M_{\Psi 2}^2 M_{\Psi 2'}$. Finally, the Cabibbo mixing angle can be generated from

$$\mathbf{W}_{\mathrm{LO12}} = \frac{h_{12}}{\sqrt{\Lambda^2 V}} (\psi \chi_{1,0,0}) \bar{\Psi}_{12} + \frac{h''_{u,d}}{\sqrt{\Lambda^2 V}} \Psi_{12} H_{u,d} \Psi'_{12} + \frac{h_\xi}{\sqrt{\Lambda^2 V}} \bar{\Psi}'_{12} \Psi_2 \xi + M_{\Psi 12'} \bar{\Psi}'_{12} \Psi'_{12} \tag{A.4}$$

in combination with eq. (A.2). The second term in eq. (4.9) is then generated in two steps. After integrating out $\Psi'_{12} + \bar{\Psi}'_{12}$, the effective dimension 4 operator

$$\mathbf{W}_{d=4} \supset \frac{h''_{u,d} h_\xi}{(\sqrt{\Lambda^2 V})^2 M'_{\Psi 12}} \Psi_{12} H_{u,d} \Psi_2 \xi \tag{A.5}$$

is generated and a subsequent decoupling of $\Psi_{12} + \bar{\Psi}_{12}$ and $\Psi_2 + \bar{\Psi}_2$ leads to the required term with $M_{12}^3 = M_{\Psi 2} M_{\Psi 12} M_{\Psi 12'}$.

B Mode expansion

The possible boundary conditions of functions on the orbifold $T^2/(Z_2^I \times Z_2^{PS} \times Z_2^{GG})$ are characterized by three parities ($a, b = +, -$),

$$\begin{aligned} \phi_{\pm ab}(x, z, -\hat{z}) &= \pm \phi_{\pm ab}(x, z, \hat{z}) \,, \\ \phi_{a \pm b}(x, z, -\hat{z} + \hat{z}_1) &= \pm \phi_{a \pm b}(x, z, \hat{z} + \hat{z}_1) \,, \\ \phi_{ab \pm}(x, z, -\hat{z} + \hat{z}_3) &= \pm \phi_{ab \pm}(x, z, \hat{z} + \hat{z}_3) \,. \end{aligned} \tag{B.1}$$

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The mode expansion of functions with the boundary conditions reads

$$\phi_{+++}(x,z,\hat{z}) = \frac{2}{\sqrt{2^{\delta_{n,0}\delta_{m,0}} L_7 L_8 \sin\theta}} \left[\delta_{0,m}\sum_{n=0}^{\infty} + \sum_{m=1}^{\infty}\sum_{n=-\infty}^{\infty}\right] \phi_{+++}^{(2m,2n)}(x,z) \times \cos\left(\frac{2m\pi}{L_7}(x_7 - \frac{x_8}{\tan\theta}) + \frac{2n\pi}{L_8\sin\theta}x_8\right), \tag{B.2a}$$

$$\phi_{++-}(x,z,\hat{z}) = \frac{2}{\sqrt{L_7 L_8 \sin\theta}} \left[\delta_{0,m}\sum_{n=0}^{\infty} + \sum_{m=1}^{\infty}\sum_{n=-\infty}^{\infty}\right] \phi_{++-}^{(2m,2n+1)}(x,z) \times \cos\left(\frac{2m\pi}{L_7}(x_7 - \frac{x_8}{\tan\theta}) + \frac{(2n+1)\pi}{L_8\sin\theta}x_8\right), \tag{B.2b}$$

$$\phi_{+-+}(x,z,\hat{z}) = \frac{2}{\sqrt{L_7 L_8 \sin\theta}} \left[\sum_{m=0}^{\infty}\sum_{n=-\infty}^{\infty}\right] \phi_{+-+}^{(2m+1,2n)}(x,z) \times \cos\left(\frac{(2m+1)\pi}{L_7}(x_7 - \frac{x_8}{\tan\theta}) + \frac{2n\pi}{L_8\sin\theta}x_8\right), \tag{B.2c}$$

$$\phi_{+--}(x,z,\hat{z}) = \frac{2}{\sqrt{L_7 L_8 \sin\theta}} \left[\sum_{m=0}^{\infty}\sum_{n=-\infty}^{\infty}\right] \phi_{+--}^{(2m+1,2n+1)}(x,z) \times \cos\left(\frac{(2m+1)\pi}{L_7}(x_7 - \frac{x_8}{\tan\theta}) + \frac{(2n+1)\pi}{L_8\sin\theta}x_8\right), \tag{B.2d}$$

$$\phi_{-++}(x,z,\hat{z}) = \frac{2}{\sqrt{L_7 L_8 \sin\theta}} \left[\sum_{m=0}^{\infty}\sum_{n=-\infty}^{\infty}\right] \phi_{-++}^{(2m+1,2n+1)}(x,z) \times \sin\left(\frac{(2m+1)\pi}{L_7}(x_7 - \frac{x_8}{\tan\theta}) + \frac{(2n+1)\pi}{L_8\sin\theta}x_8\right), \tag{B.2e}$$

$$\phi_{-+-}(x,z,\hat{z}) = \frac{2}{\sqrt{L_7 L_8 \sin\theta}} \left[\sum_{m=0}^{\infty}\sum_{n=-\infty}^{\infty}\right] \phi_{-+-}^{(2m+1,2n)}(x,z) \times \sin\left(\frac{(2m+1)\pi}{L_7}(x_7 - \frac{x_8}{\tan\theta}) + \frac{2n\pi}{L_8\sin\theta}x_8\right), \tag{B.2f}$$

$$\phi_{--+}(x,z,\hat{z}) = \frac{2}{\sqrt{L_7 L_8 \sin\theta}} \left[\delta_{0,m}\sum_{n=0}^{\infty} + \sum_{m=1}^{\infty}\sum_{n=-\infty}^{\infty}\right] \phi_{++-}^{(2m,2n+1)}(x,z) \times \sin\left(\frac{2m\pi}{L_7}(x_7 - \frac{x_8}{\tan\theta}) + \frac{(2n+1)\pi}{L_8\sin\theta}x_8\right), \tag{B.2g}$$

$$\phi_{---}(x,z,\hat{z}) = \frac{2}{\sqrt{L_7 L_8 \sin\theta}} \left[\delta_{0,m}\sum_{n=0}^{\infty} + \sum_{m=1}^{\infty}\sum_{n=-\infty}^{\infty}\right] \phi_{---}^{(2m,2n)}(x,z) \times \sin\left(\frac{2m\pi}{L_7}(x_7 - \frac{x_8}{\tan\theta}) + \frac{2n\pi}{L_8\sin\theta}x_8\right), \tag{B.2h}$$

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with θ being the angle between the basis vectors spanning the orbifold $T^2/(Z_2^I \times Z_2^{PS} \times Z_2^{GG})$ and $L_{7,8}$ being their respective length. The wave functions are normalized by the volume of the fundamental region.

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