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On the massive gluon propagator, the PT-BFM scheme and the low-momentum behaviour of decoupling and scaling DSE solutions

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Abstract: We study the low-momentum behaviour of Yang-Mills propagators obtained from Landau-gauge Dyson-Schwinger equations (DSE) in the PT-BFM scheme. We compare the ghost propagator numerical results with the analytical ones obtained by analyzing the low-momentum behaviour of the ghost propagator DSE in Landau gauge, assuming for the truncation a constant ghost-gluon vertex and a simple model for a massive gluon propagator. The asymptotic expression obtained for the regular or decoupling ghost dressing function up to the order $\mathcal{O}(q^2)$ is proven to fit pretty well the numerical PT-BFM results. Furthermore, when the size of the coupling renormalized at some scale approaches some critical value, the numerical PT-BFM propagators tend to behave as the scaling ones. We also show that the scaling solution, implying a diverging ghost dressing function, cannot be a DSE solution in the PT-BFM scheme but an unattainable limiting case.

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Contents

1 Introduction

The low-momentum behaviour of the Yang-Mills propagators derived either from the tower of Dyson-Schwinger equations (DSE) or from Lattice simulations in Landau gauge has been a very interesting and hot topic for the last few years. It seems by now well established that, if we assume in the vanishing momentum limit a ghost dressing function behaving as $F(q^2) \sim (q^2)^{\alpha_F}$ and a gluon propagator as $\Delta(q^2) \sim (q^2)^{\alpha_G - 1}$ (or, by following a notation commonly used, a gluon dressing function as $G(q^2) = q^2\Delta(q^2) \sim (q^2)^{\alpha_G}$), two classes of solutions may emerge (see, for instance, the discussion of refs. [1, 2]) from the DSE: (i) those, dubbed *"decoupling"*, where $\alpha_F = 0$ and the suppression of the ghost contribution to the gluon propagator DSE results in a massive gluon propagator (see [3–6] and references therein); and (ii) those, dubbed *"scaling"*, where $\alpha_F \neq 0$ and the low-momentum behaviour of both gluon and ghost propagators are related by the coupled system of DSE through the condition $2\alpha_F + \alpha_G = 0$ implying that $F^2(q^2)G(q^2)$ goes to a non-vanishing constant when $q^2 \to 0$ (see [7–13] and references therein). As a matter of fact, $F^2(q^2)G(q^2)$, which gives the perturbative running for the coupling constant renormalized in Taylor-scheme [14, 15], is a good quantity in discriminating the kind of solutions we deal with. It is worth to remember that, despite the widely accepted nomeclature for the classes of solutions, neither a scale invariance nor a decoupling of the IR dynamics for the theory can be inferred from the low-momentum behaviour of such a Taylor coupling.

How both types of IR solutions for Landau gauge DSE emerge and how the transition between them occurs, being governed by the size of the coupling taken as an integration boundary condition at the renormalization momentum, was initially discussed in ref. [1]

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through the analysis of a ghost propagator DSE combined with a gluon propagator taken from lattice computations. It should be remembered that one needs to know the QCD mass scale to predict the QCD coupling at any momentum. This mass scale should be of course supplied to get a particular solution from DSE and can be univocally related to the boundary condition needed, after applying a trunctation scheme, to solve the equation. The existence of a *critical* value for the coupling at any renormalization momentum was suggested by that partial analysis. No solution was proved to exist for any coupling bigger than the critical one and the unique scaling solution[1] appeared to emerge when the coupling took that critical value. Later, the authors of ref. [13] confirmed, by the analysis of the tower of DSE truncated within two different schemes and also in the framework of the functional renormalization group, that the boundary condition for the DSE integration determined whether a decoupling or the scaling solution occurs. A similar analysis have been recently done in the Coulomb gauge [16, 17] leading to the same pattern of ref. [1], although the authors interpreted the boundary condition in terms of the gauge-fixing ambiguity (see also [18, 19]). Furthermore, an analytic study based on the pinch technique in ref. [20] shows that, within some approximations, there is a lower limit for the gluon mass, which the IR singularities of QCD still persist below, that can be also interpreted as an upper limit to the coupling. Very recently also, a next-to-leading low-momentum asymptotic formula for the decoupling ghost dressing function solutions was obtained by studying the ghost propagator DSE with the assumption, for the truncation, of a constant ghost-gluon vertex and of a simple model for a massive gluon propagator [21]. In this asymptotic formula, the ghost-propagator low-momentum behaviour appeared to be regulated by the zero-momentum effective charge in Taylor scheme [22] and by the Landau-gauge gluon mass scale.

In the present note, the work of ref. [1] will be extended by the analysis of the results [23] obtained by solving the coupled system of Landau gauge ghost and gluon propagators DSE within the framework of the pinching technique in the background field method [24–26] (PT-BFM). Our main goal is investigating whether the same pattern for regular (decoupling) and critical (scaling) solutions in ref. [1] is also found for the PT-BFM solutions. In the PT-BFM scheme, the zero-momentum ghost dressing function will be seen to diverge when the coupling approaches some critical value, as it should be expected for the scaling solution ($\alpha_F \neq 0$). This seems to support the suggestion of a transition from one to another solutions controled by the size of the coupling approaching a critical value [1]. The authors of ref. [13] obtained similar results but they applied the zero-momentum ghost propagator as the boundary condition for the DSEs integration and missed its connection with the value of the coupling at the renormalization momentum (i.e. the particular value of $\Lambda_{\rm QCD}$ one applies to build the solutions) or the critical coupling the scaling behaviour requires to emerge.[2] This connection is an important ingredient because it provides us with a manner,

[1]The authors of [27, 28] proved there, once the scaling behaviour is assumed, the uniqueness for Yang-Mills infrared solutions

[2]These authors furthermore invoked the renormalization group invariance to claim that such a critical coupling does not appear. Nevertheless, the renormalization group invariance only requires for the critical values of the coupling at any two fixed renormalized momenta to be connected by the appropriate renor-

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through a comparison with the physical strong coupling, to discuss whether the scaling critical DSE solution could be allowed by the data. We will also compare the low-momentum analytic results of ref. [21] with the PT-BFM results. In particular, the next-to-leading asymptotic formula for the ghost dressing function with $\alpha_F = 0$ will be shown to nicely describe the low-momentum PT-BFM results for different values of the coupling taken at the renormalization point, as a boundary condition for the DSE integration. It should be emphasized that the ansatz for the massive gluon propagator applied to derive the analytical results in ref. [21] has nothing to do either with the numerical analysis based on lattice results in [1] or with the one based on PT-BFM results in this paper, both leading to obtain a critical coupling. These analytical low momentum results are applied in this paper only for comparative purposes. However, as a result of the comparison, this boundary condition will be put in direct relation to the zero-momentum effective charge in Taylor scheme [22] and to the Landau gauge gluon mass. Finally, we will also argue that a scaling solution cannot exist as a solution of the coupled system of PT-BFM DSE, although the PT-BFM solutions tend to it when the coupling aproaches the critical value. More generally, a diverging ghost dressing function cannot be obtained when the gluon propagator is massive ($\alpha_G = 1$). Of course, this last claim is not a new result: it is well-known that the scaling solution only can emerge if the ghost dominance in the gluon propagator DSE, after assuming $\alpha_F < 0$, implies the gluon propagator to vanish at zero momentum [7–13]. However, it is not worthless to emphasize that, for the gluon propagator, being massive implies not to observe the relation $2\alpha_F + \alpha_G = 0$ and is therefore a sufficient condition for the scaling solution not to appear. It should be also pointed out that LQCD results (see [29–39] and references therein), pinching technique results (see, for instance, [20, 24–26, 40, 41]), refined Gribov-Zwanziger [3] formalism (see [48, 49]) or other approaches like the infrared mapping of $\lambda\phi^4$ and Yang-Mills theories in ref. [50] or the massive extension of the Fadeev-Popov action in ref. [51] appear to support a massive gluon propagator.

We organized this note as follows: first, we briefly review the low-momentum behaviour for the propagator solutions of the ghost propagator DSE in section 2; we then compare the PT-BFM results with the low-momentum analytical expression and discuss their dependence with the size of the coupling at the renormalization point, taken as a boundary condition for DSE integration, in section 3; and we finally conclude in section 4.

2 The two kinds of solutions of the ghost propagator Dyson-Schwinger equation

As was explained in detail in refs. [2, 21], the low-momentum behavior for the solutions of the Dyson-Schwinger equation for the ghost propagator (GPDSE), which can be written

malization group running (the same, of course, for any decoupling solution). Indeed, the renormalization flow for the coupling can be defined by $\alpha(q^2) = \alpha(\mu^2)F^2(q^2)G(q^2)$ to satisfy this and it is found to agree, in the perturbative domain, with the perturbative running given by the β-function in Taylor scheme [14, 15].

[3] In addition, K-I. Kondo triggered very recently an interesting discussion about the Gribov horizon condition and its implications on the Landau-gauge Yang-Mills infrared solutions [42–47].

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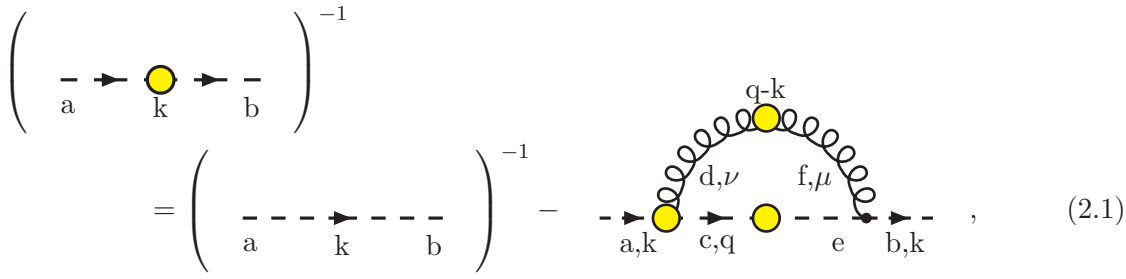
diagrammatically as



(2.1)

can be obtained in Landau gauge by procceeding as follows: we consider eq. (2.1) for two different (although parallel) external ghost momenta, k and p, such that $p^2 - k^2 = \delta^2 k^2$ (δ being an extra parameter that, for the sake of simplicity, will be taken to be small enough as to expand on it around 0) and subtract them, as a regularization prescription for not to have to deal with any UV cut-off. Then, the ghost dressing function, $F(k^2)$, and the gluon propagator form factor, $\Delta(k^2)$, are renormalized by applying the MOM prescription,

$$F_R(\mu^2) \;=\; \mu^2 \Delta_R(\mu^2) \;=\; 1\,, \tag{2.2}$$

where μ^2 is the subtraction point; and, as explained in [2, 21], we choose for the ghost-gluon vertex,

$$\widetilde{\Gamma}^{abc}_{\nu}(-q,k;q-k) \;=\; ig_0 f^{abc}\,(\; q_\nu H_1(q,k) + (q-k)_\nu H_2(q,k)\;)\;, \tag{2.3}$$

to apply the MOM prescription in Taylor kinematics (i.e. with a vanishing incoming ghost momentum) and assume the non-renormalizable bare ghost-gluon form factor, $H_1(q,k) = H_1$, to be constant (at least in the low-momentum regime for the incoming ghost). Thus, after being cast into a renormalized form and the above-mentioned subtraction, the GPDSE reads

$$\frac{1}{F_R(k^2)} - \frac{1}{F_R(p^2)} \;=\; N_C\; g_R^2(\mu^2)\; H_1\; I(k^2)\,, \tag{2.4}$$

where

$$I(k^2) \;=\; \int \frac{d^4q}{(2\pi)^4} \left(\frac{F_R(q^2)}{q^2} \left(\frac{(k\cdot q)^2}{k^2} - q^2 \right) \left[\frac{\Delta_R\left((q-k)^2\right)}{(q-k)^2} - \frac{\Delta_R\left((q-p)^2\right)}{(q-p)^2} \right] \right) \tag{2.5}$$

and where the ghost dressing function and gluon propagator should be understood as renormalized at the subtraction momentum, μ^2, and $g_R(\mu^2)$ is the gauge coupling renormalized at a subtraction point with Taylor kinematics (Taylor MOM scheme). Then, we cut integral domain of $I(k^2)$ in eq. (2.5) into two pieces by introducing some new momentum scale, q_0^2, below which both ghost and gluon are assumed to be well described by the following ansatze

$$\Delta_R(q^2) = \frac{B(\mu^2)}{q^2 + M^2} \;\simeq\; \frac{B(\mu^2)}{M^2}\left(1 - \frac{q^2}{M^2} + \cdots\right)\,, \tag{2.6}$$

$$F_R(q^2) = A(\mu^2)\left(\frac{q^2}{M^2}\right)^{\alpha_F}\left(1 + \cdots\right)\,. \tag{2.7}$$

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Thus, we shall look for the ghost dressing function, $F_{\rm IR}$, its leading behaviour being parameterized through a general power law behaviour where $\alpha_F > -2$ to keep the integral $I(k^2)$ infrared convergent, and for a massive [4] gluon propagator, that implies of course a power law with $\alpha_G = 1$, as the current lattice data seems to point to. It is well-known that the low-momentum behaviour of the integral $I(k^2)$ in eq. (2.5) is dominated by the result of the integration over the IR domain, over $q^2 < q_0^2$, and it can be expanded on $\delta^2 = p^2/k^2 - 1$ around zero with the subtraction momentum μ^2 kept fixed, as explained in ref. [2, 21], to give

$$I(k^2) \simeq I_{\rm IR}(k^2) \simeq -\frac{\delta^2}{M^{2+2\alpha_F}} \frac{2A(\mu^2)B(\mu^2)}{(2\pi)^3} \times \sum_{i=0}^{\infty} (4k^2)^i C_i \int_0^{q_0} q^{3+2i+2\alpha_F} dq \; K_i(q^2;k^2,M^2) + \mathcal{O}(\delta^4) \quad (2.8)$$

where

$$K_i(q^2;k^2,M^2) = \frac{i}{(q^2+k^2+M^2)^{2i+1}} - \frac{i}{(q^2+k^2)^{2i+1}} + k^2 \left(\frac{2i+1}{(q^2+k^2)^{2i+2}} - \frac{2i+1}{(q^2+k^2+M^2)^{2i+2}} \right) \quad (2.9)$$

and

$$C_i = \frac{12\pi^2 4^i}{\Gamma(-3/2-i)\Gamma(1/2-i)\Gamma(5+2i)} \, . \quad (2.10)$$

Now, the two possible cases for the low-momentum behaviour of the GPDSE solutions will be separately analyzed in the following by applying eq. (2.8) to eq. (2.4).

2.1 Critical case: scaling solution

When $\alpha_F < 0$, one can perform the change $t = q^2/k^2$ and integrate over t, the integration in eq. (2.8) for any term of eq. (2.9) being (one by one) convergent as the limit $q_0^2/k^2 \to \infty$ is considered for the upper bound. Thus, the small-momentum asymptotical behaviour of the r.h.s of eq. (2.4) is given by

$$I_{\rm IR}(k^2) \simeq -\delta^2 \left(\frac{k^2}{M^2}\right)^{1+\alpha_F} \frac{2A(\mu^2)B(\mu^2)}{(2\pi)^3} \sum_{i=0}^{\infty} 4^i \frac{C_i}{2} \times \left(-i \int_0^{\infty} dt \frac{t^{1+i+\alpha_F}}{(1+t)^{2i+1}} + (2i+1) \int_0^{\infty} dt \frac{t^{1+i+\alpha_F}}{(1+t)^{2i+2}} \right) + \mathcal{O}(\delta^4) \, , \quad (2.11)$$

while its l.h.s. behaves as:

$$\frac{1}{F_R(k^2)} - \frac{1}{F_R(p^2)} \simeq \delta^2 \frac{\alpha_F}{A(\mu)} \left(\frac{k^2}{M^2}\right)^{-\alpha_F} + \mathcal{O}(\delta^4) \quad (2.12)$$

Then, we should conclude both

[4] This is the massive gluon propagator where the gluon running mass [52, 53], $M(q^2)$, appears to be approximated by its frozen value at vanishing momentum, $M(0)$. It should be also noted that, provided that the gluon propagator is to be multiplicatively renormalized, the mass scale, $M = M(0)$, does not depend on renormalization scale, μ^2.

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(i) that $-\alpha_F = 1 + \alpha_F \;\; \Rightarrow \;\; \alpha_F = -1/2$,

(ii) and, given that

$$\sum_{i=0}^{\infty} 4^i \, C_i \left(-i \int_0^{\infty} dt \frac{t^{\frac{1}{2}+i}}{(1+t)^{2i+1}} + (2i+1) \int_0^{\infty} dt \frac{t^{\frac{1}{2}+i}}{(1+t)^{2i+2}} \right) = \frac{2\pi}{5}, \quad (2.13)$$

one obtains

$$N_C g_R^2(\mu^2) H_1 A^2(\mu^2) B(\mu^2) \simeq 10\pi^2 \; . \quad (2.14)$$

Thus, we know the asymptotic behaviour for the ghost dressing function in this case to be:

$$F_R(q^2) \simeq \frac{\pi}{g_R(\mu^2)} \left(\frac{10}{N_C H_1 \Delta_R(0)} \right)^{1/2} \left(\frac{1}{q^2} \right)^{1/2} \quad (2.15)$$

This is a so-called scaling solution where, in particular, the low-momentum behavior of the massive gluon propagator forces the ghost dressing function to diverge at low-momentum through the requirement [5] for the power exponent, α_F, in (i).

If $\alpha_F > 0$ is assumed, one would be also left with the contradictory scaling condition that $\alpha_F = -1$ [2, 31] and should conclude that no solution exists for that case.

2.2 Regular case: decoupling solution

When $\alpha_F = 0$, the previous ressumation cannot be done and the integral $I(k^2)$ in eq. (2.5) should be consistently expanded in powers of k^2. This case has been deeply studied in ref [21], where it is found that

$$I_{\rm IR}(k^2) \; \simeq \; \delta^2 \, \frac{A(\mu^2)B(\mu^2)}{64\pi^2} \, \frac{k^2}{M^2} \left[\ln \frac{k^2}{M^2} - \frac{5}{6} + \mathcal{O}\left(\frac{M^2}{q_0^2} \right) \right] + \mathcal{O}\left(\frac{k^4}{M^4}, \delta^4 \right) \; . \quad (2.16)$$

Then, the ghost dressing function, including its first correction to the leading constant term, should behave as[6]

$$F_R(q^2) = F_R(0) \left(1 + \frac{N_C H_1}{16\pi} \, \overline{\alpha}_T(0) \, \frac{q^2}{M^2} \left[\ln \frac{q^2}{M^2} - \frac{11}{6} \right] + \mathcal{O}\left(\frac{q^4}{M^4} \right) \right) \quad (2.17)$$

where

$$\overline{\alpha}_T(0) = \lim_{q \to 0} \left(q^2 + M^2 \right) \frac{\alpha_T(q^2)}{q^2} \; = \; M^2 \frac{g_R^2(\mu^2)}{4\pi} F_R^2(0) \Delta_R(0), \quad (2.18)$$

such that the eq. (2.4) could be satisfied. It should again understood that the subtraction momentum for all the renormalization quantities is μ^2. In eq. (2.18), $\alpha_T = g_T^2/(4\pi)$ is

[5] This is of course a particular case, with $\alpha_G = 1$, of the more general scaling condition: $2\alpha_F + \alpha_G = 0$.

[6] It should be also noted that eq. (2.16) imples to take $M^2/q_0^2 \ll 1$. However, any correction to that approximation will not play at the order of the coefficient eqs. (2.18), that will keep the same value disregarding that of M^2/q_0^2, but at the order of the gluon mass, M^2, inside the logarithm, presummably like the UV part of the integral $I(k^2)$ that should be proportional to k^2 and vanish at least like $1/\log(q_0^2/\Lambda_{\rm QCD}^2))$ when $q_0^2/\Lambda_{\rm QCD}^2 \gg 1$.

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the perturbative strong coupling defined in this Taylor scheme [14, 15], while $\overline{\alpha}_T$ is the extension of the non-perturbative effective charge definition from the gluon propagator [54] to the Taylor ghost-gluon coupling [22]. As a consequence of the appropriate *amputation* of a massive gluon propagator, where the gluon mass scale is the same RI-invariant mass scale appearing in eq. (2.6), this Taylor effective charge is frozen at low-momentum and gives a non-vanishing zero-momentum value in terms of which the ghost-dressing-function subleading correction can be expressed.

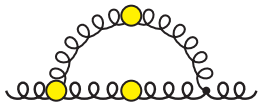
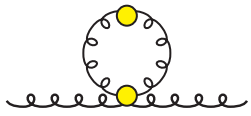
3 Comparison with numerical results from coupled PT-BFM DSE's

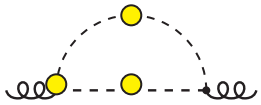
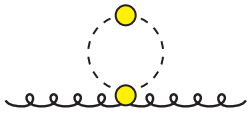
We shall now compare the formulas given by eqs. (2.6), (2.17) with some numerical results for the gluon propagator and ghost dressing function. The aim of the comparison is twofold: testing the asympotical solution we obtained in the previous section, but also checking the consistency of a massive-gluon solution and determining the gluon mass as the best-fit parameter in the comparison. In particular, we will consider the solutions of the coupled system of gluon and ghost DS equations obtained by applying the pinching technique in the background field method (PT-BFM) [24–26] (see also [55] and references therein) to compare with. This PT-BFM framework leaves us with an attractive model for gluon and ghost propagators providing quantitative description of lattice data [6, 56] and giving well account of their main qualitative features: finite gluon propagator and finite ghost dressing function at zero-momentum. Futhermore, the coupled DSE system can be solved with different boundary conditions (see below), the solutions compared with the analytical formula and how their behaviour depends on these boundary conditions can be thus properly studied. This last is the main purpose of this note.

The main feature in the PT-BFM scheme is that the transversality of the gluon self-energy is guaranteed order-by-order in the dressed-loop expansion, this leading to a gauge-invariant truncation of the gluon DSE [24–26]. In this PT-BFM scheme for the coupled DSE system, the ghost propagator DSE is the same as given by eqs. (2.1), (2.4), where the bare ghost-gluon vertex is approximated by $H_1 = 1$. The gluon DSE is given by

$$\frac{(1+G(q^2))^2}{\Delta(q^2)}\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right) = q^2 g_{\mu\nu} - q_\mu q_\nu + i\sum_{i=1}^{4}(a_i)_{\mu\nu} \tag{3.1}$$

where

$a_1 =$  , $a_2 =$ 

$a_3 =$  , $a_4 =$  . (3.2)

In the diagrams of (3.2) for the gluon DSE, eq. (3.1), the external gluons are treated, from the point of view of Feynman rules, as background fields (these diagrams should be also properly regularized, as explained in [55]). The last justifies the four field coupling of two background gluons and two ghosts leading to the contribution a_4. The function $1+G$

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defined in ref. [57, 58] can be, in virtue of the ghost propagator DSE, connected to the ghost propagator [22]. The coupled system is to be solved, by numerical integration, with the two following boundary conditions as the only required inputs: the zero-momentum value of the gluon propagator and that of the coupling at a given perturbative momentum, $\mu = 10\,\mathrm{GeV}$, that will be used as the renormalization point. The latter can be done by fixing different values for the boundary conditions, this providing us with a family of gluon and ghost propagators solutions so determined. In particular, solutions obtained by keeping the zero-momentum value of the gluon propagator fixed (see lefthand plots of figure 1) while $\alpha(\mu^2 = 100\ \mathrm{GeV}^2)$ is ranging from 0.15 to 0.1817 are available [23] and can be confronted to the asymptotical expressions derived in the previous section.

3.1 Decoupling solutions in the PT-BFM scheme

Then, as the gluon propagator solutions in the PT-BFM scheme result to behave as massive ones, the eqs. (2.6), (2.17) must account for the low-momentum behaviour of both gluon propagator and ghost dressing function with $H_1 = 1$ and

$$\overline{\alpha}_T(0) = \alpha_T(\mu^2) F_R^2(0) B(\mu^2) \;=\; \alpha_T(\mu^2) F_R^2(0) M^2 \Delta_R(0)\,, \tag{3.3}$$

$\alpha_T(\mu^2) = g_R^2(\mu^2)/(4\pi)$ being fixed, as a boundary condition, at the moment of the numerical integration of the coupled DSE for each particular solution of the family. $B(\mu^2)$ and M being determined by the best fit of the eq. (2.6) to the numerical solution for the gluon propagator, we shall be left with one only free parameter, $F_R(0)$, to account with eq. (2.17) for the numerical solution for the ghost propagator. Furthermore, the zero-momentum values of the ghost dressing function, $F_R(0)$, can be also taken from the numerical integration of the DSE (for any value of the $\alpha(\mu = 10\mathrm{GeV})$); and these altoghether with the zero-momentum values of the gluon propagator, $\Delta_R(0)$, and the gluon masses, obtained by the fit of eq. (2.6) to the numerical DSE gluon propatator solutions, provide us with all the ingredients to evaluate, with no unknown parameter, eq. (2.17). The gluon masses obtained from the best fits of eq. (2.6) to the numerical data (see the left plots in figure 1) and that of zero-momentum Taylor effective charge, $\overline{\alpha}_T(0)$, computed by applying eq. (3.3), with the zero-momentum ghost dressing function taken from numerical data, can be found in table 1.

Indeed, the expression given by eq. (2.17) can be succesfully applied to describe the solutions all over the range of coupling values, $\alpha(\mu)$, at $\mu = 10\,\mathrm{GeV}$. This can be seen, for instance, for $\alpha = 0.15, 0.16, 0.17$, in the right plots of figure 1, where the ghost dressing functions obtained from the numerical integration of the DSE's appear plotted with black-solid lines and the evaluation of eq. (2.17), as we explained above, with red-dotted ones. Thus, *with no parameter to be fitted ($F_R(0)$ is taken from the numerical integration), we nicely reproduce the low-momentum behaviour of the ghost dressing function obtained through numerical integration.*

In the right plots of figure 1, black-dotted curves obtained by only retaining the logarithmic leading order in eq. (2.17) appear also drawn. The big discrepancy they show with respect to the numerical integration clearly implies the necessity of the next-to-

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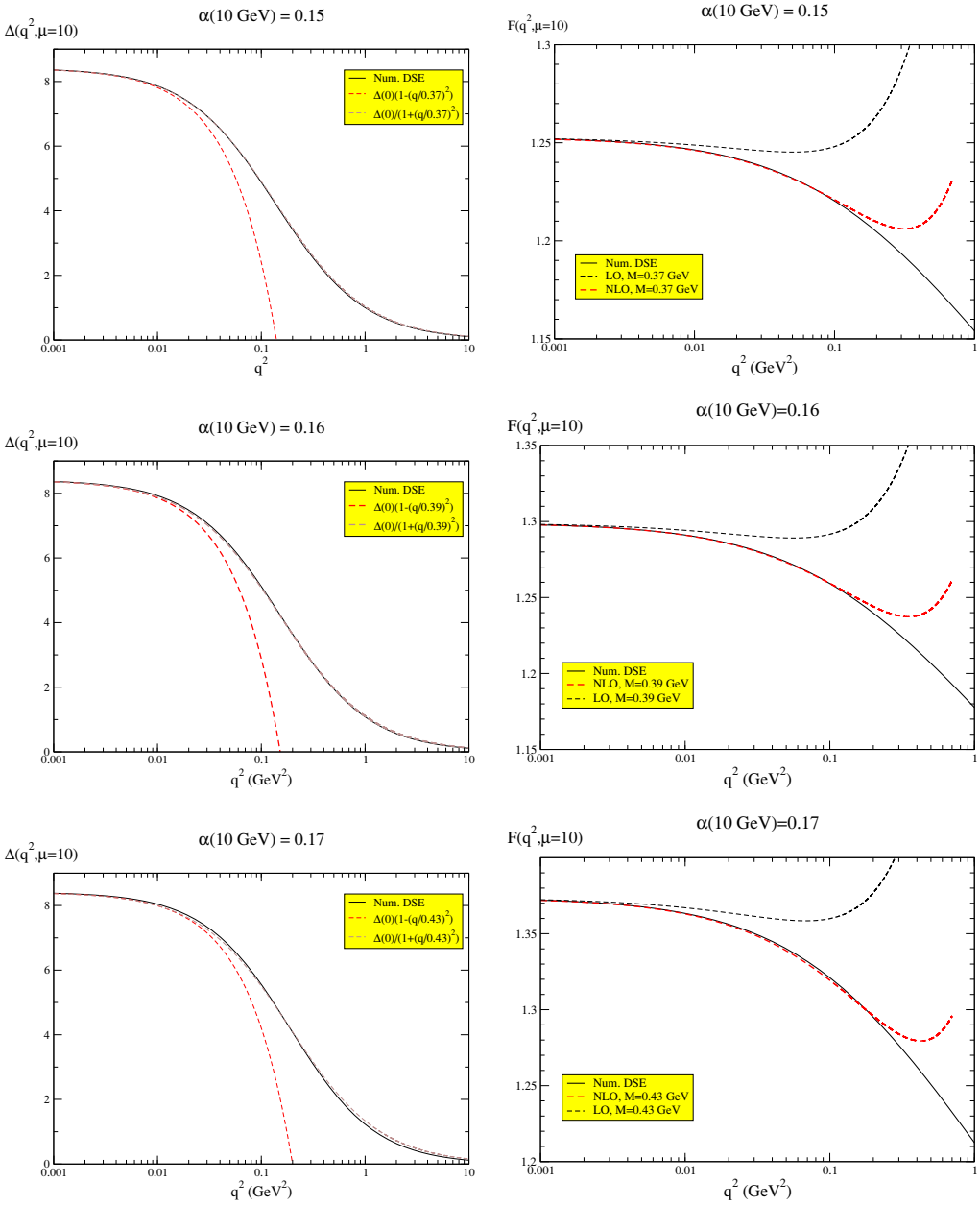

Figure 1. Gluon propagators (left) and ghost dressing functions (right) after the numerical integration of the coupled DSE system for $\alpha(\mu = 10\text{GeV}) = 0.15, 0.16, 0.17$ taken from [23] . The curves for the best fit of eq. (2.6) to the gluon propagator data appear as dotted lines in the lefthand plots. In the righthand plots, the red dotted lines correspond to apply eq. (2.17) with the gluon mass obtained from the gluon fits and with $R = \overline{\alpha}_T(0)/M^2$ determined by the zero-momentum values of gluon propagator and ghost dressing function coming from the numerical integration of the DSE system; the same for the black dotted lines but retaining only the logarithmic leading term in eq. (2.17) by dropping the $-11/6$ away.

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$\alpha(\mu)$	$\overline{\alpha}_T(0)$	M (GeV) [gluon]
0.15	0.24	0.37
0.16	0.30	0.39
0.17	0.41	0.43

Table 1. Gluon masses and the zero-momentum non-perturbative effective charges, obtained as explained in the text, which are applied to describe the gluon propagator and the ghost dressing function numerical data with eqs. (2.6), (2.17).

leading correction in eq. (2.17) when assessing the gluon mass from the low-momentum ghost propagator.

3.2 The "critical" limit in the PT-BFM scheme

There appears to be a *critical* value of the coupling, $\alpha_{\rm crit} = \alpha(\mu^2) \simeq 0.182$ with $\mu = 10$ Gev, above which the coupled DSE system does not converge any longer to a solution [23]. As a matter of the fact, we know from eq. (2.14) that the scaling solution implies for the coupling

$$g^2_{\rm crit} \;=\; g^2_R(\mu^2) \simeq \frac{10\pi^2}{3A^2(\mu^2)B(\mu^2)}\,, \tag{3.4}$$

where $B(\mu^2)$ is determined by the gluon propagator solution that is supposed to behave as eq. (2.6), and $A(\mu^2)$ by the ghost propagator that should behave as

$$F_R(q^2) \;=\; A(\mu^2)\left(\frac{M^2}{q^2}\right)^{1/2}\,, \tag{3.5}$$

where again μ^2 is the momentum at the subtraction point. This is shown in ref. [1], where only the ghost propagator DSE is solved there after extracting a gluon propagator from the lattice data and applying it to build the kernel of the integral, eq. (2.5), appearing in eq. (2.4). In the analysis of ref. [1], a ghost dressing function solution diverging at vanishing momentum appears to exist and verifies eqs. (3.4), (3.5), while regular or decoupling solutions exist for any $\alpha < \alpha_{\rm crit}$.

Now, we can perform a more complete analysis by studying again the dressing function computed by solving eq. (3.1) for the different values of the coupling, $\alpha = \alpha(\mu^2)$, at $\mu^2 = 100$ GeV^2 [23]. A ghost dressing function at vanishing momentum, $F(0,\mu^2)$, diverging as $\alpha \to \alpha_{\rm crit}$ had to be expected, one could try the following power behaviour,

$$F(0) \;\sim\; (\alpha_{\rm crit} - \alpha(\mu^2))^{-\kappa(\mu^2)}\,, \tag{3.6}$$

to describe the vanishing-momentum ghost dressing function in terms of the coupling, $\alpha(\mu^2)$. The coefficient $\kappa(\mu^2)$ should be the positive critical exponent (depending presummably on the renormalization point, μ^2) governing the transition from decoupling ($\alpha < \alpha_{\rm crit}$) to the scaling ($\alpha = \alpha_{\rm crit}$) solutions.

Our strategy will be to let $\alpha_{\rm crit}$ be a free parameter to be fitted by requiring the best linear correlation for $\log[F(0)]$ in terms of $\log[\alpha_{\rm crit} - \alpha]$. In doing so, the best correlation

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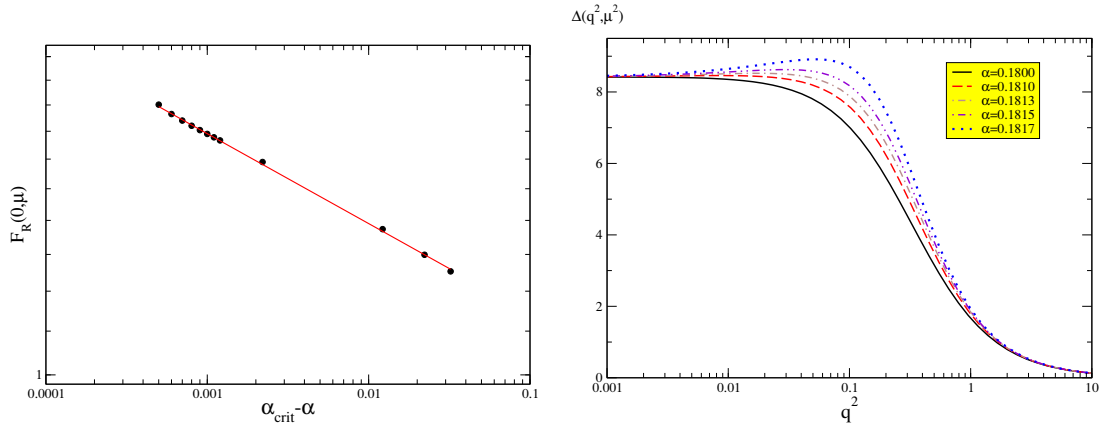

(a) (b)

Figure 2. (a) Log-log plot of the zero-momentum values of the ghost dressing function, obtained by the numerical integration of the coupled DSE system in the PT-BFM scheme, in terms of $\alpha_{\rm crit} - \alpha$. $\alpha = \alpha(\mu = 10\text{GeV})$, the value of the coupling at the renormalization momentum, is an initial condition for the integration; while $\alpha_{\rm crit}$ is fixed to be 0.1822, as explained in the text, by requiring the best linear correlation. (b) Gluon propagator solutions in terms of q^2 for the same coupled DSE system for different values of $\alpha(\mu = 10\text{GeV})$, all very close to the critical value, ranging from 0.18 to 0.1817 .

coefficient is 0.9997 for $\alpha_{\rm crit} = 0.1822$, which is pretty close to the critical value of the coupling above which the coupled DSE system does not converge any more, and then we obtain

$$\kappa(\mu^2) = 0.0854(6) \ . \tag{3.7}$$

In figure 2.(a), the log-log plot of $F_R(0)$ in terms of $\alpha_{\rm crit} - \alpha$ is shown for $\alpha_{\rm crit} = 0.1822$, where the linear behaviour corresponding to the best correlation coefficient can be strikingly seen and the negative slope indicates a zero-momentum ghost propagator diverging as $\alpha \to \alpha_{\rm crit}$. However, *no critical or scaling solution of the coupled DSE system seems to appear with a massive gluon propagator as solution of the coupled DSE system in the PT-BFM, although the decoupling solutions obtained for any* $\alpha < \alpha_{\rm crit} = 0.1822$ *seem to approach the behaviour of a scaling one when* $\alpha \to \alpha_{\rm crit}$. The absence of the scaling solution can be well understood by analysing eq. (3.1). As explained in [6], after the appropriate regularization and renormalization, the contribution of a_3 to the inverse of the gluon propagator, its momentum vanishing, will be dominated by [7]

$$a_3 \to \int_0^{q_0} d(q^2) \left(F^2(q^2) - 1\right) \ , \tag{3.8}$$

where q_0 is again some UV cut-off above which the ghost dressing function can be taken to be perturbative. Provided that one deals with a decoupling solution, the ghost dressing

[7] The regularisation procedure in [6] implies the subtraction of the perturbative part, as well as we evaluate eq. (2.4) for two different scales and subtract them in order not to have to deal with any UV cut-off in eq. (2.4).

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function reaching some constant as $q^2 \to 0$, this contribution is finite and negligible (the same happens for a_4). On the other hand, had we considered the scaling solution, the ghost dressing function would behave as $1/q$ and would lead to a divergent contribution and to a vanishing gluon propagator at vanishing momentum. Then, a massive gluon propagator, as lattice solutions points to and as required in ref. [6, 23], cannot appear as a scaling solution. The same has been already proven in literature by applying different truncation schemes and also on general grounds, and indeed agrees with the very well-known infrared behaviour obtained from the coupled DSE system in refs. [7–13], where an unique scaling solution with $\alpha_F \simeq -0.595$ and $\alpha_G = -2\alpha_F \simeq 1.190$ emerges, the gluon propagator vanishing thus at zero-momentum, as the inverse of the zero-momentum ghost dressing is assumed to vanish. However, for the sake of completeness, a very general argumentation addressing this issue is presented in appendix A. We should recall at this point that the numerical analysis of the ghost propagator DSE in ref. [1] left us with a divergent ghost dressing function for the critical value of the coupling, even after assuming a finite gluon propagator at zero momentum. However, this resulted from a partial analysis where we dealt only with the ghost propagator DSE and not with the gluon propagator one. Thus, we did not solve the gluon DSE and taking a massive gluon propagator from the lattice to build the ghost-DSE kernel does not prevent from obtaining a "*wrong*" scaling solution indeed not satisfying the coupled DSE system.

When approaching the critical value of the coupling, the gluon propagators obtained from the coupled DSE system in PT-BFM must be also thought to obey the same critical behaviour pattern as the ghost propagator. In the PT-BFM, the value at zero-momentum being fixed by construction [6, 23], one should expect that, instead of decreasing, the gluon propagator obtained for couplings near to the critical value increases for low momenta: the more one approaches the critical coupling the more it has to increase. This is indeed the case, as can be seen in figure 2(b). This implies that, near the critical value, the low momentum propagator does not obey eq. (2.6) and that consequently eq. (2.17) does not work any longer to describe the low momentum ghost propagator.[8]

Finally, one can pay attention to the critical value of the coupling, $\alpha_{\rm crit} = 0.1822$, and try to make a comparison with the physical strong coupling values in order get some idea of whether the current data can exclude or not this critical behaviour. Although the experimental PDG *world average* of the strong coupling in the $\overline{\rm MS}$ scheme, $\alpha_{\overline{\rm MS}}(M_Z) = 0.1184(7)$ [59], can be propagated from the Z^0 boson mass down to $\mu = 10\,\text{GeV}$ to give $\alpha_{\overline{\rm MS}}(10\ \text{GeV}) = 0.179(2)$, that incidentally lies on the right ballpark of the above critical value, such a comparison is meaningless because our coupling corresponds to one in MOM Taylor-scheme for zero number of flavours. One can use instead the available perturbative four-loop formula describing the running of the coupling in Taylor-scheme to estimate $\Lambda_{\rm QCD}$ in this particular scheme, then perform the conversion to $\overline{\rm MS}$ (see for instance eqs. (22,23) of the first reference in [14, 15]) and thus obtain the value quoted in table 3.2. Of course, it would be again meaningless to compare this last value with the one for $\Lambda_{\overline{\rm MS}}$ that can

[8] This is only true for the next-to-leading contribution of eq. (2.17), the leading one being only determined by the zero-momentum gluon propagator still works.

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$\Lambda^{N_f=0}_{\overline{\rm MS},{\rm crit}}$	$\Lambda^{N_f=0}_{\overline{\rm MS}}$ [61]	$\Lambda^{N_f=0}_{\overline{\rm MS}}$ [14, 15]	$\Lambda^{N_f=5}_{\overline{\rm MS}}$ [59]
434 MeV	238(19) MeV	244(8) MeV	213(9) MeV

Table 2. The critical value of $\Lambda_{\overline{\rm MS}}$ in pure Yang-Mills inferred from $\alpha_{\rm crit} = 0.1822$ (first column), lattice estimates for Yang-Mills $\Lambda_{\overline{\rm MS}}$ taken from literature (second and third columns) and the one obtained from the PDG value of $\alpha_{\overline{\rm MS}}(M_Z)$ by applying a four-loop perturbative formula for the running of $\alpha_{\overline{\rm MS}}$ with $N_f = 5$.

be obtained from the PDG value for $\alpha_{\overline{\rm MS}}(M_Z)$, also quoted in table 3.2, but we can refer the comparison to the lattice Yang-Mills determinations of the same parameter,[9] as for instance the two of them included in table 3.2. Thus, the lattice estimates of $\Lambda_{\overline{\rm MS}}$ appear to lie clearly below this critical limit for the PT-BFM DSE in pure Yang-Mills. However, as no quark flavour loops effect have been incorporated in our DSE, eq. (3.1), we cannot neither compare with the physical strong coupling nor conclude whether the critical limit can be allowed in the "*real world*".

4 Conclusions

The ghost propagator DSE, with the only assumption of taking $H_1(q,k)$ from the ghost-gluon vertex in eq. (2.3) to be constant in the infrared domain of q, can be exploited to look into the low-momentum behaviour of the ghost propagator. The two classes of solutions named "decoupling" and "scaling" can be indentified and shown to depend on whether the ghost dressing function achieves a finite non-zero constant ($\alpha_F = 0$) at vanishing momentum or not ($\alpha_F \neq 0$). The solutions appear to be dialed by the size of the coupling at the renormalization momentum which plays the role of a boundary condition for the DSE integration. The low-momentum behaviour of the decoupling solutions results to be regulated by the gluon propagator mass and by a regularization-independent dimensionless quantity that appears to be the effective charge defined from the Taylor-scheme ghost-gluon vertex at zero momentum.

In this note, we have studied the solutions of coupled ghost and gluon propagator DSE in the PT-BFM scheme and demonstrated that the asymptotic decoupling formula ($\alpha_F = 0$) successfully describes the low-momentum ghost propagator. The model applied for the massive gluon propagator is also verified to give properly account of the gluon solution, at least for momenta below 1 GeV (and for a coupling not very close to the critical point). Although we argued that a massive gluon propagator implies that the ghost dressing function takes a non-zero finite value at vanishing momentum, we also show that the zero-momentum ghost dressing function tends to diverge when the value of the coupling dialing the solutions approaches some critical value. Such a divergent behaviour at the critical coupling seems to be the expected one for a scaling solution (where, if the gluon is massive, $\alpha_F = -1/2$). If we consider the zero-momentum value of the ghost dressing

[9] It should be noted that the procedures for the lattice determination of $\Lambda_{\overline{\rm MS}}$ mainly work in the UV domain, where IR sources of uncertainties as the Gribov ambiguity or volume effects are indeed negligible. In fact, there are unquenched lattice determinations with $N_f = 5$ staggered fermions for the strong coupling [60] which are pretty consistent with the PDG value.

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function as some sort of "*order parameter*" indicating whether the ghost propagator low-momentum behaviour is suppressed ($\alpha_F = 0$ and finite ghost dressing function) or it is enhanced ($\alpha_F < 0$ and divergent ghost dressing function), the strength of the coupling computed at some renormalization point seems to control some sort of transition from the *suppressed* to the *enhanced* phases for the ghost propagator DSE solutions in the PT-BFM scheme. The last only takes place as some critical value of the coupling is reached. Neverteless, it can be proven that, as far as the gluon is massive, the scaling behaviour for the Yang-Mills propagators appear not to be a solution but an unattainable limiting case for the PT-BFM DSE solutions.

Acknowledgments

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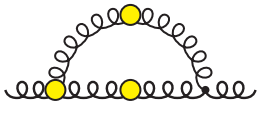
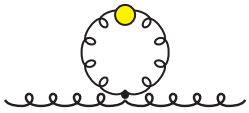
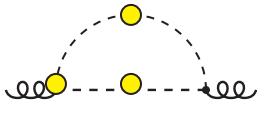
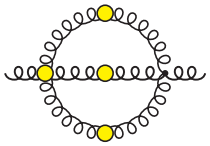
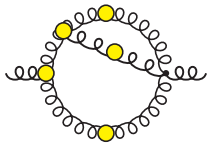
A No scaling solution with massive gluons

We consider the conventional gluon self-energy, $\Pi^{\mu\nu}(q)$, contributing to the gluon DSE:

$\Pi_{\mu\nu}(q) = \frac{1}{2}$  $+\frac{1}{2}$  $+$ 

$+\frac{1}{6}$  $+\frac{1}{2}$  , (A.1)

where the yellow bullets stand for full vertices and propagators.

After only assuming that the ghost-gluon vertex form factor H_1 is constant (the Taylor non-renormalization theorem tells us that the bare ghost-gluon vertex is finite), We showed in section 2 that a massive gluon propagator unequivocally implies a ghost dressing function diverging as $1/q$ at vanishing momentum. Then, the ghost-loop contribution to the gluon self-energy, eq. (A.1), with vanishing external momentum, k, is dominated by

$$
\begin{aligned}
g^T_{\mu\nu}(k) \times \quad & \sim \int \frac{d^4q}{(2\pi)^4} q^2 \left(1 - \frac{(k\cdot q)^2}{k^2 q^2}\right) \frac{F(q^2)}{q^2} \frac{F((q-k)^2)}{(q-k)^2} \\
& \sim \int_0^{q_0} dq \, \frac{q^2}{(q^2+k^2)^{3/2}} \int_0^{\pi} d\theta \, \frac{\sin^4\theta}{\left(1 - \frac{2kq}{q^2+k^2}\cos\theta\right)^{3/2}} \\
& \sim \int_0^{q_0} dq \, \frac{q^2}{(q^2+k^2)^{3/2}}
\end{aligned}
\qquad \text{(A.2)}
$$

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where q_0 is the momentum scale which the ghost dressing function is assumed to take its asymptotic infrared form below. For not to have to deal again with any UV regularization cut-off, we can consider the two momenta p, k such that $k^2 \ll p^2 < q_0^2$ and subtract the gluon DSE for these two momenta. Then, when $k^2 \to 0$ while p^2 is kept fixed, one would have for the transversal gluon propagator

$$\frac{1}{\Delta_R(k^2)} - \frac{1}{\Delta_R(p^2)} \sim \int_0^{q_0} dq \, \frac{q^2}{(q^2+k^2)^{3/2}} \ , \tag{A.3}$$

where we only account for the dominant part of the ghost-loop contribution. This contribution diverges thus logarithmically with a constant ghost-gluon vertex again as the only assumption made, while computing the contributions coming from other diagrams would leave us with the neccessity to make some new assumption about the full gluon vertices.

Thus, to avoid a diverging gluon self-energy, we need to invoke new contributions from the other diagrams in eq. (A.1), also diverging logarithmically as the external momentum vanishes, to cancel that from eq. (A.2). Otherwise, such a divergent behaviour of the gluon self-energy would lead the inverse of the gluon propagator to diverge, and the gluon propagator consequently to vanish, at zero-momentum.

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