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Fuzzy Set Solutions for Optimal Maintenance Strategy Selection

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Abstract

Selecting optimal maintenance strategy under fuzzy environment is not a trivial task. This paper presents an illustration of multi-criteria maintenance strategy selection under fuzzy environment. Three most common maintenance strategies and eight maintenance decision criteria have been considered and most appropriate/ optimal strategy selection process is demonstrated using three different techniques/ methods. Fuzzy linguistic terms have been used to rate and weigh the maintenance decision criteria. Linguistic terms/ variables are represented by triangular fuzzy sets/ number and fuzzy set operations have been carried out using $\alpha -$ cut method. The basic technique used is rating and ranking method using fuzzy set theory wherein ratings of alternatives/ strategies is determined first and then ranking is carried out to decide the optimal strategy. Other methods i.e. ranking fuzzy sets using *cardinal utilities* and by *maximizing and minimizing sets* are also established to confirm our choice of optimal maintenance strategy.

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Keywords

Multi-criteria decision-making, Maintenance Strategy, Fuzzy Sets, Triangular fuzzy number, Rating and ranking

1. Introduction

Reliability of the system/ equipment can be enhanced with proper implementation of maintenance strategies. Various maintenance policies have been discussed and modeled in the literature. Some of the more celebrated common maintenance models include corrective or breakdown maintenance, preventive maintenance and predictive. Corrective maintenance is performed in response to unplanned or unscheduled downtime of the system. Preventive maintenance is periodical scheduled downtime wherein well-defined sets of activities like inspection, repair, replacement, cleaning, lubrication, adjustment, alignment etc. are performed. Preventive maintenance activities are further categorized into simple preventive maintenance and preventive replacement. Simple preventive maintenance changes the system reliability to some newer time. It does not return the system to its original condition. It is also known as *imperfect maintenance/ repair*. With preventive replacement, it is assumed that the component is returned to *as good as new*. This means that the age of the component is restored to time zero after each such maintenance. Predictive maintenance is a strategy wherein based on the previous data or information, experience etc. and also with sophisticated diagnostic tools and equipments for inspection, it is predicted that when part is near failure and maintenance is done just before it. The tools and equipments required for inspection includes non-destructive testing equipments, themographic monitoring, oil analysis, vibration analysis, sophisticated calibration and alignment measurement techniques etc.

Decision analysis is a technique that can be used to help decision-maker who is faced with a problem involving uncertainty. Much of the decision-making in the real world takes place in an environment in which the goals, the constraints and consequences of possible actions are not known precisely. To deal with quantitatively with precision, the concepts and techniques of probability theory, decision theory and information theory are usually employed. In doing so, we only consider the precision that is related to randomness only. Other major source of imprecision in many decision processes is fuzziness. By fuzziness, we mean a type of imprecision which is associated with fuzzy sets. Many techniques, methods and models have been developed over last few decades, discussion of those models are beyond the scope of this paper.

In many cases, it has become more and more obvious that comparing the different alternatives and determining the optimal solutions in decision problems cannot be done using single criterion. This has led to multi-criteria/ multi-attribute decision making (under fuzziness). The literature on this topic has grown tremendously in recent past and literature survey and review of the same has been outlined in [5,6,9].

Mechefske et al. [2] proposed fuzzy linguistic approach to select optimum maintenance and condition-based strategy. In their paper, a heuristic algorithm is developed using the fuzzy linguistic variables to characterize the capability of available maintenance strategies to satisfy a common set of maintenance goals and to select the best strategy from those available. Importance of each maintenance goal and capability of each strategy to achieve the maintenance goals have been assessed linguistically first. Then fuzzy set concepts, some operators and distance measures have been used to decide the best strategy. The paper also further demonstrates procedure to select the correct condition monitoring technique.

Our paper suggests a method to arrive at an optimal maintenance strategy depending on various criteria. Basic three types of maintenance strategies have been considered as different alternatives i.e. breakdown or corrective maintenance, preventive maintenance and predictive maintenance. Eight criteria (fuzzy in nature) have been considered to judge the optimal alternative.

Next section discusses brief basics of fuzzy set theory and triangular fuzzy number. Section 3 outlines fuzzy multi-criteria decision making methods used in this paper. First general problem statement is given and then three methods namely rating and ranking method (by Bass and Kwakernaak), ranking fuzzy sets using 'cardinal utilities' (by Baldwin and Guild) and ranking fuzzy sets by 'maximizing and minimizing sets' (by Chen) are described. Maintenance Strategy Selection Problem is developed in section 4 and in section 5 optimal solution strategy is found out using the three fuzzy multi-criteria decision-making methods listed above.

2. Fuzzy Set Theory and Triangular Fuzzy Number

L.A. Zadeh advocated the concept of grades of membership or the concept of possibility values of membership [3,4,7]. If $X = \{x\}$ represents a fundamental set and x are the elements of this fundamental set, to be assesed according to an uncertain postulation and assigned to a subset A of X, the set $A = \{x, \mu_A(x) / x \in X\}\}$ is referred to as the uncertain set or fuzzy set of X. $\mu_A(x)$ is the membership function of the uncertain set A. The membership function $\mu_A(x)$ for a fuzzy set A can be defined as: $\mu_A(x) : X \rightarrow [0, 1]$.

If the membership function of fuzzy number A is determined by:

$$\begin{aligned}\mu_A &= 0 && ;x < L \\ &= \frac{(x-L)}{(M-L)} && ;L \le x \le M \\ &= \frac{(R-x)}{(R-M)} && ;M \le x \le R \\ &= 0 && ;x > R\end{aligned} \tag{1}$$

then A is referred to as triangular fuzzy number, denoted by $A = (L, M, R)$ and is depicted in Figure 1. α-cut of this TFN is

$$A_\alpha = [L + \alpha(M-L),\ R - \alpha(R-M)] \quad ;\forall \alpha \in [0,1] \tag{2}$$

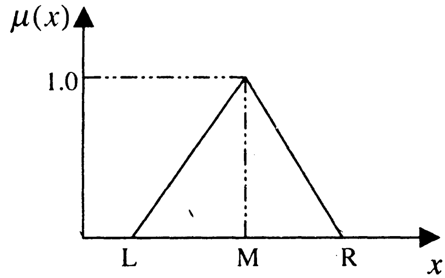

Figure 1. Triangular Fuzzy Number (TFN)

3. Fuzzy Multi-Criteria Decision Making

3.1 General Problem Statement

The basic problem is to choose between set of alternatives, given some decision criteria's. Let $A = \{a_i\}$; $i = 1,2,\ldots..n$ be the set of decision alternatives and $C = \{c_j\}$; $j = 1,2,\ldots..m$ be the set of (fuzzy) criteria's according to which the desirability of an alternative is to be judged. The aim here is to obtain the optimal alternative with highest degree of desirability with respect to all relevant criteria's. This problem is multi-criteria decision making problem that is tackled by many researchers [5,6] working in the area of decision-making in a fuzzy environment.

3.2 Rating and Ranking Method

Kahne [5] developed a technique of rating and ranking for stochastic models, which further on extended to fuzzy set theory by Bass and Kwakernaak. As per Kahne's model, let the rating of the alternative i with respect to decision criteria j be r_{ij}; $i = 1,2,\ldots n$; $j = 1,2,\ldots m$. The relative importance of decision criteria j is called weight and is denoted as ω_j. Therefore the ranking of the alternatives is performed according to their rank that is given by:

$$R_i = \frac{\sum_{j=1}^{m} \omega_j \cdot r_{ij}}{\sum_j \omega_j} \qquad ; i = 1,2,\ldots\ldots n \tag{3}$$

The optimal alternative is the one for which value of R_i is maximum. Bass and Kwakernaak [11] suggested an extension to the above model that has become almost classic in this area. It consists of two phases:

Phase 1: Determination of ratings of alternatives:

Let $A = \{a_i\}$; $i = 1,2,\ldots\ldots n$ be the set of decision alternatives and $\tilde{C} = \{\tilde{c}_j\}$; $j = 1,2,\ldots\ldots m$ be the criteria's that are fuzzy in nature. $\tilde{R}_{ij}$ be the fuzzy rating of alternative i with respect to criteria j and $\tilde{W}_j \in R$ is the weight or importance of criteria j. The rating of alternative i with respect to criteria j is fuzzy and is given by grade membership function $\mu_{R_{ij}}(r_{ij})$. Also, the relative importance (weight) of criteria j is given by a fuzzy set $\tilde{W}_j$ with grade membership function $\mu_{W_j}(\omega_j)$.

The evaluation of alternative a_i is therefore a fuzzy set, which is computed using R_{ij} and W_j:

$$g(z) = g(\omega_1, \ldots\ldots \omega_n,\; r_{i1}, \ldots\ldots r_{in}) = \frac{\sum_{j=1}^{m} \omega_j \cdot r_{ij}}{\sum_{j=1}^{m} \omega_j} \tag{4}$$

Membership function of μ_{zi} is then defined as

$$\mu_{z_i}(z) = min\,\{\min_{j=1}^{n}(\mu_{W_j}(\omega_j)),\ \min_{k=1}^{n}(\mu_{R_{ik}}(r_{ik}))\} \tag{5}$$

The final rating is therefore $\tilde{R}_i = \{(r, \mu_{R_i})\}$ given by a membership function $\mu_{R_i}(r) = \sup_{r} \mu_{z_i}(z)$.

Phase 2: Ranking:

Once final ratings have been obtained, ranking or rank ordering is done in phase 2. Bass and Kwakernaak established a measure to distinguish the 'preferable alternatives' from each other and rank them. If ratings R_i are fuzzy, then preference set $P_i = \{(p, \mu_{P_i}(p))\}$, which is also a fuzzy set, can be obtained using:

$$\mu_{P_i}(p) = \sup_{r} \mu_R(r_1, \ldots\ldots, r_m) \tag{6}$$

This fuzzy set can be effectively used to judge the degree of preferability of an alternative over all other alternatives.

3.3 Ranking Fuzzy Sets using 'Cardinal Utilities'

Baldwin and Guild [10] define a relation $\tilde{P}_{ij} = \{(r_i, r_j), \mu_{P_{ij}}(r_i, r_j)\}$ $;i \neq j$ with membership function as $\mu_{P_{ij}}(r_i, r_j) = f(r_i, r_j)$. This function expresses the 'difference' between the ratings of two fuzzy sets. Such set is defined as

$\tilde{O}(x_i) = \{x_i, \mu_O(x_i)\}$ with membership function

$\mu_O(x_i) = \sup_{r_i, r_j}\ min\,\{\mu_{R_i}(r_i), \mu_{R_j}(r_j), \mu_{P_{ij}}(r_i, r_j)\}$. This equation expresses the degree to which alternative x_i is preferable to its best rival alternative.

$\tilde{O}(x_i)$ correspondes to *max-min composition* of $\tilde{R}_i$, $\tilde{R}_j$ and $\tilde{P}_{ij}$. Without going into the mathematical particulars, we present the solution of the problem as:

$$\mu_O(x_i) = \min_j\,\{\tilde{\mu}_j\} = \min_j\left[\frac{\delta - \alpha}{1 + (\delta - \gamma) + (\beta - \alpha)}\right] \tag{7}$$

where the parameters of the above equations are depicted in the Figure 2.

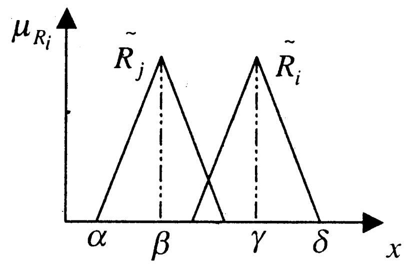

Figure 2. Membership Functions of Ratings $\tilde{R}_i$, $\tilde{R}_j$

3.4 Ranking Fuzzy Sets by 'Maximizing and Minimizing Sets'

A modification of ranking approach is suggested by Jain [5], which is further modified by Chen [8] for better discrimination of the ratings. He defines 'maximizing set (M)' and 'minimizing set (N)' whose membership functions are as follows:

$$\mu_M(r) = \left[\frac{r - r_{min}}{r_{max} - r_{min}}\right]^n \tag{8}$$

$$\mu_N(r) = \left[\frac{r_{max} - r}{r_{max} - r_{min}}\right]^n \tag{9}$$

where $r \in [r_{min}, r_{max}]$ is the real interval and n = 1 for linear, n = 2 for risk prone and n = 0.5 for risk averse membership functions.

To perform the ranking and get fuzzy optimal alternative obtain:

$$O(x_i) = \frac{R(x_i) + 1 - L(x_i)}{2} \tag{10}$$

where $R(x_i) = \sup_r \min\{\mu_{R_i}(r), \mu_M(r)\}$ and $L(x_i) = \sup_r \min\{\mu_{R_i}(r), \mu_N(r)\}$.

4. Maintenance Strategy Selection Problem

We consider three alternatives: corrective maintenance (A_1), preventive maintenance (A_2) predictive maintenance (A_3) and eight maintenance decision criteria's namely: low maintenance cost (C_1), improved reliability (C_2), improved safety (C_3), high product quality (C_4), minimum inventory (C_5), return on investment (C_6), acceptance by labour (C_7), enhanced competitiveness (C_8) by which to judge the three alternatives. The ratings and weights have expressed by linguistic terms (variables) i.e. as fuzzy sets. The grade membership for both the variables are considered as triangular fuzzy number (TFN). Figure 3 and Figure 4 shows TFN representations of linguistic variables related to rating and weights on the scale [0,1].

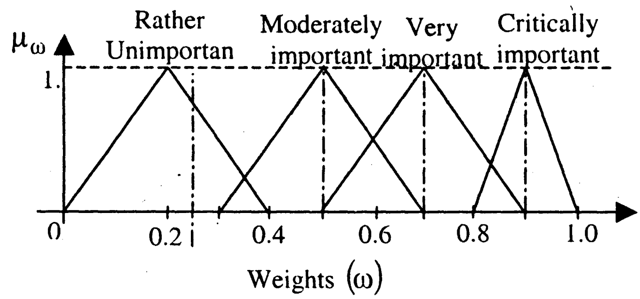

Figure 3. Fuzzy Sets Representing Weights in Linguistic Terms/ Variables

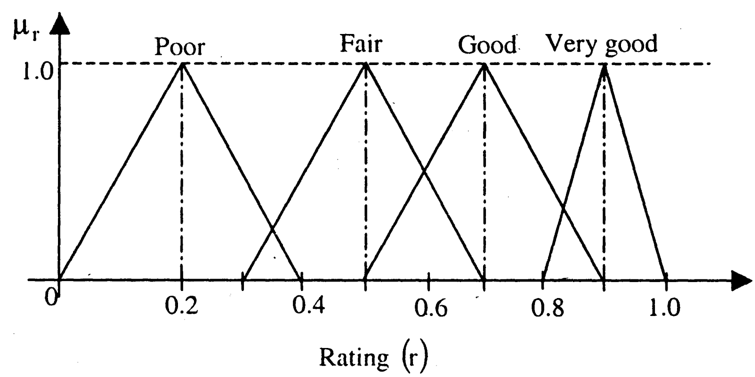

Figure 4. Fuzzy Sets Representing Ratings in Linguistic Terms/ Variables

The rating of the strategies with respect to decision criteria's (r_{ij}; i=1,2,3; j=1,2...8) is expressed in linguistic terms and is shown in Table 1. For example, rating of corrective maintenance (A_1) with regards to criteria low maintenance cost (C_1) is poor whereas rating of preventive and predictive maintenance (A_2 and A_3) is fair and good respectively with same criteria. Now our problem is to select the best strategy under the given criteria's considering the criteria not having equal weights. This is an additional constraint that criteria's are having unequal weights. Table 2 gives the weights (ω_j; j =1,2...8) of all eight criteria in linguistic terms/ variables.

Maintenance Decision Criteria's	**Alternatives/ Strategies**		
	Corrective Maintenance (A_1)	**Preventive Maintenance (A_2)**	**Predictive Maintenance (A_3)**
Low maintenance cost (C_1)	Poor	Fair	Good
Improved reliability (C_2)	Poor	Very Good	Very Good
Improved safety (C_3)	Fair	Good	Good
High product quality (C_4)	Poor	Good	Very Good
Minimum inventory (C_5)	Poor	Fair	Good
Return on investment (C_6)	Fair	Good	Very Good
Acceptance by labour (C_7)	Fair	Fair	Fair
Enhanced competitiveness (C_8)	Poor	Good	Good

Table 1. Rating of the Alternatives with respect to Maintenance Decision Criteria's

Maintenance Decision Criteria's	Weights
Low maintenance cost (C_1)	Very important
Improved reliability (C_2)	Critically important
Improved safety (C_3)	Critically important
High product quality (C_4)	Critically important
Minimum inventory (C_5)	Moderately important
Return on investment (C_6)	Very important
Acceptance by labour (C_7)	Moderately important
Enhanced competitiveness (C_8)	Very important

Table 2. Weights of Maintenance Decision Criteria's

5. Fuzzy Set Based Solutions

Evaluation of equation (4) is not simple as it involves arithmetic operations like addition, multiplication and division of fuzzy sets [1]. These operations are to be performed with the help of interval of confidence at each presumption level α (or popularly called as α -cut). Linear operations such as addition, subtraction etc. of two fuzzy numbers yields a fuzzy number. But this is not true for non-linear mathematical operations like multiplication, division of two fuzzy numbers. Therefore approximate method, that is a simplified method is usually used to carry out these operations. We have evaluated fuzzy ratings using exact as well as approximate methods in this paper and then in order to evaluate the approximation, left and right divergence is calculated. Divergence is the difference between exact and approximate values. Left divergence is the difference between lower values and right divergence is the difference between upper values of the rating set. If these differences are within some prescribed tolerances, the obtained approximate fuzzy number is considered as good approximation of fuzzy ratings.

All these results for different grade memberships (α -cuts) of ratings of the three maintenance strategies are given in Table 3. The membership functions of the final ratings obtained using Bass and Kwakernaak method is shown in Figure 5. It is evident from Figure 5 that preventive and predictive maintenance strategies completely dominate over corrective maintenance strategy. As there are two alternatives/ strategies in race for optimality now, we need to investigate the case still further to obtain the optimal strategy. To do this, we use one more decisive factor that is given in equation (6) that is able to distinguish between 'preferable alternatives'. We compute the preference sets to investigate preferability of one alternative over the other. Table 4 gives the preferability data for different grade membership values which is plotted in Figures 6 and 7. From the results it can be observed that predictive maintenance strategy is the optimal strategy under fuzzy decision criteria considered in the problem under investigation.

α -cuts of R_1 i.e. Rating of Corrective Maintenance						
α	**Exact Method**		**Approximate Method**		**Absolute Divergence**	
	Lower	**Upper**	**Lower**	**Upper**	**Lower**	**Upper**
0	0.106667	0.509859	0.106667	0.509859	0	0
0.1	0.126911	0.489756	0.1268624	0.4897352	4.86E-05	2.08E-05
0.2	0.147143	0.469649	0.1470578	0.4696114	8.52E-05	3.76E-05
0.3	0.167362	0.449538	0.1672532	0.4494876	0.0001088	5.04E-05
0.4	0.18757	0.429422	0.1874486	0.4293638	0.0001214	5.82E-05
0.5	0.207767	0.409302	0.207644	0.40924	0.000123	6.2E-05
0.6	0.227955	0.389177	0.2278394	0.3891162	0.0001156	6.08E-05
0.7	0.248133	0.369047	0.2480348	0.3689924	9.82E-05	5.46E-05
0.8	0.268303	0.348911	0.2682302	0.3488686	7.28E-05	4.24E-05
0.9	0.288466	0.328769	0.2884256	0.3287448	4.04E-05	2.42E-05
1	0.308621	0.308621	0.308621	0.308621	0	0

α -cuts of R_2 i.e. Rating of Preventive Maintenance						
α	Exact Method		Approximate Method		Absolute Divergence	
	Lower	Upper	Lower	Upper	Lower	Upper
0	0.504444	0.849296	0.504444	0.849296	0	0
0.1	0.520626	0.831349	0.521241	0.8316078	0.000615	0.0002588
0.2	0.536975	0.81345	0.538038	0.8139196	0.001063	0.0004696
0.3	0.553476	0.795604	0.554835	0.7962314	0.001359	0.0006274
0.4	0.57012	0.777812	0.571632	0.7785432	0.001512	0.0007312
0.5	0.586893	0.760078	0.588429	0.760855	0.001536	0.000777
0.6	0.603788	0.742405	0.605226	0.7431668	0.001438	0.0007618
0.7	0.620795	0.724798	0.622023	0.7254786	0.001228	0.0006806
0.8	0.637906	0.707261	0.63882	0.7077904	0.000914	0.0005294
0.9	0.655115	0.689798	0.655617	0.6901022	0.000502	0.0003042
1	0.672414	0.672414	0.672414	0.672414	0	0

α -cuts of R_3 i.e. Rating of Predictive Maintenance						
α	Exact Method		Approximate Method		Absolute Divergence	
	Lower	Upper	Lower	Upper	Lower	Upper
0	0.626667	0.921127	0.626667	0.921127	0	0
0.1	0.640216	0.905624	0.6408969	0.9059109	0.0006809	0.0002869
0.2	0.65395	0.890175	0.6551268	0.8906948	0.0011768	0.0005198
0.3	0.667853	0.874784	0.6693567	0.8754787	0.0015037	0.0006947
0.4	0.681912	0.859453	0.6835866	0.8602626	0.0016746	0.0008096
0.5	0.696117	0.844186	0.6978165	0.8450465	0.0016995	0.0008605
0.6	0.710455	0.828987	0.7120464	0.8298304	0.0015914	0.0008434
0.7	0.724917	0.813861	0.7262763	0.8146143	0.0013593	0.0007533
0.8	0.739495	0.798812	0.7405062	0.7993982	0.0010112	0.0005862
0.9	0.75418	0.783845	0.7547361	0.7841821	0.0005561	0.0003371
1	0.768966	0.768966	0.768966	0.768966	0	0

Table 3. Grade Membership Values of Rating of Three Maintenance Strategies

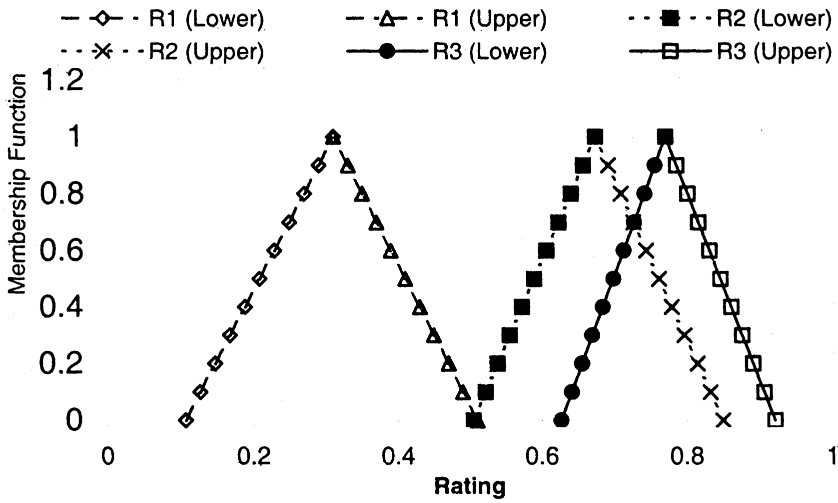

Figure 5. Final Ratings of Strategies

Preferability Data (P_3 over P_2) (P_1 not considered)			Preferability Data (P_3 over P_1 and P_2)	
α	Lower	Upper	Lower	Upper
0	-0.222629	0.416683	-0.0529105	0.6155715
0.1	-0.1907109	0.3846699	-0.0197746	0.5818592
0.2	-0.1587928	0.3526568	0.0133613	0.5481469
0.3	-0.1268747	0.3206437	0.0464972	0.5144346
0.4	-0.0949566	0.2886306	0.0796331	0.4807223
0.5	-0.0630385	0.2566175	0.112769	0.44701
0.6	-0.0311204	0.2246044	0.1459049	0.4132977
0.7	0.0007977	0.1925913	0.1790408	0.3795854
0.8	0.0327158	0.1605782	0.2121767	0.3458731
0.9	0.0646339	0.1285651	0.2453126	0.3121608
1	0.096552	0.096552	0.2784485	0.2784485

Table 4. Preferability Data

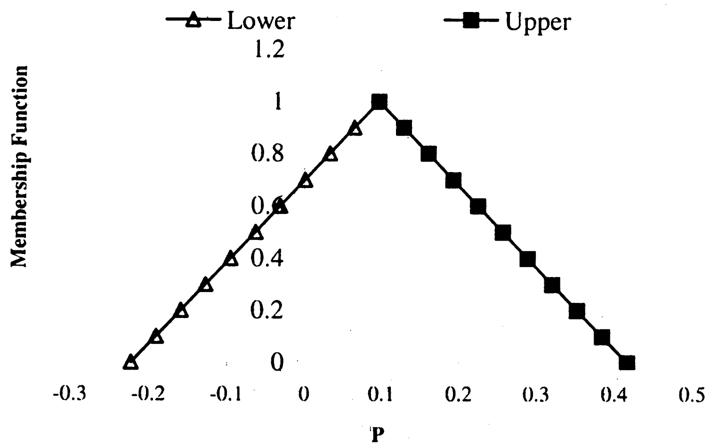

Figure 6. Membership Function of Preferability of Alternative 3 over Alternative 2

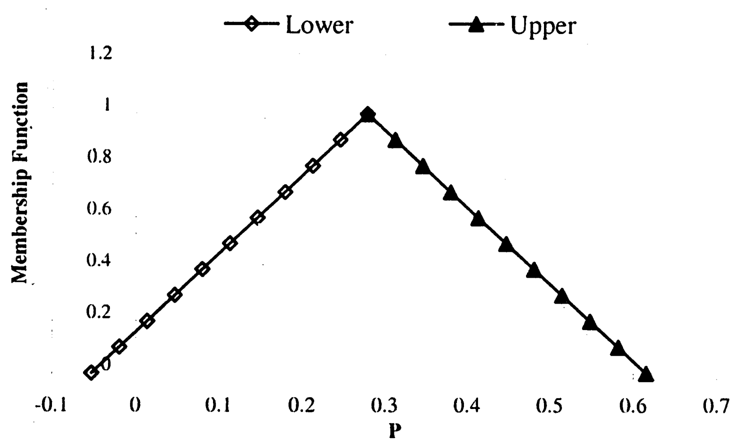

Figure 7. Membership Function of Preferability of Alternative 3 over Alternative 1 and 2

Now, we consider our problem of ranking the strategies using 'cardinal utilities' method by Baldwin and Guild. Optimal maintenance strategy is the one that has minimum μ_j value obtained through equation (7). As corrective maintenance strategy is not in the race for optimality, we compare the other two i.e. preventive maintenance (with rating R_2) and predictive maintenance (with rating R_3)

strategies for ranking purpose. From Table 5, we can observe that μ_j of strategy 3 is more than μ_j of strategy 2, i.e. rating of R_3 is greater rating of R_2. This means predictive maintenance strategy outperforms preventive maintenance strategy in optimality.

Parameters of Equation (7)	Maintenance Strategies	
	R_2	R_3
α	0.504444	0.626667
β	0.768966	0.672414
γ	0.672414	0.768966
δ	0.849296	0.921127
μ_j	0.239247289	0.245811865

Table 5. Values of Parameters Obtained by Baldwin and Guild Method

Let us now assess our problem of optimal maintenance strategy selection by method suggested by Chen. Ratings of two strategies namely preventive maintenance (with rating R_2) and predictive maintenance (with rating R_3) obtained by this method is depicted in Figure 8. The membership functions of the two fuzzy sets are:

$$\mu_{R_2}(r) = \frac{r - 0.504444}{0.16797} \quad ; 0.504444 \le r \le 0.672414$$
$$= \frac{0.849296 - r}{0.176882} \quad ; 0.672414 < r \le 0.849296$$
$$= 0 \quad ; \text{otherwise}$$

$$\mu_{R_3}(r) = \frac{r - 0.626667}{0.142299} \quad ; 0.626667 \le r \le 0.768966$$
$$= \frac{0.921127 - r}{0.152161} \quad ; 0.768966 < r \le 0.921127$$
$$= 0 \quad ; \text{otherwise}$$

We also compute maximizing and minimizing sets for $n = 1$:

$$\mu_M(r) = \frac{r - 0.504444}{0.416683} \quad ; 0.504444 \le r \le 0.921127$$
$$= 0 \quad ; \text{otherwise}$$

$$\mu_N(r) = \frac{0.921127 - r}{0.416683} \quad ; 0.504444 \le r \le 0.921127$$
$$= 0 \quad ; \text{otherwise}$$

Ranking values for both strategies are given in Table 6. As optimal ranking value ($O(x_i)$) of alternative 3 (predictive maintenance) is more, it is an optimal maintenance strategy as per this method.

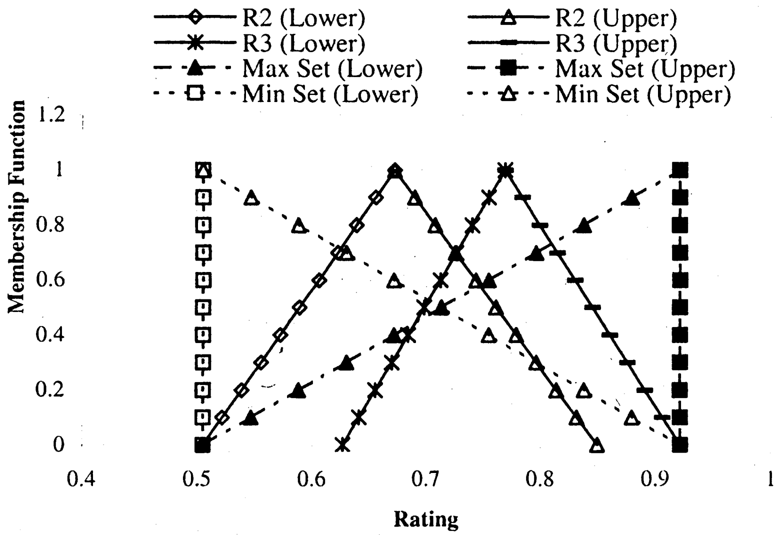

Figure 8. Final Ratings of Two-Alternatives

Parameters of Equation (10)	Maintenance Strategies	
	R_2	R_3
$R(x_i)$	0.589636	0.7333057
$L(x_i)$	0.7177234	0.5504817
$O(x_i)$	0.4359563	0.591412

Table 6. Values of Parameters Obtained by Chen Method

6. Conclusion

In this paper, three different methods have been established to select most appropriate or optimal maintenance strategy under fuzzy decision criteria. First maintenance decision strategies and criteria have been determined and then fuzzy linguistics is used to carry out the rating of the strategies with respect to the decided criteria. Then, linguistic weights are allotted to the decision criteria. Optimal maintenance strategy is then found out using rating and ranking method, ranking fuzzy sets using 'cardinal utilities' method and by 'maximizing and minimizing sets' method. In the devised example, predictive maintenance strategy turned out to be the optimal one.

7. References

(1) Verma, A. K., Srividya, A. and Prabhu Gaonkar Rajesh (2004), "Fuzzy Dynamic Reliability Evaluation of a Deteriorating System under Imperfect Repair", International Journal of Reliability, Quality and Safety Engineering, 11(4), 387-398.

(2) Mechefske, C. K. and Wang, Z. (2001), "Using Fuzzy Linguistics to Select Optimum Maintenance and Condition Monitoring Strategies", Mechanical Systems and Signal Processing, 15(6) , 1129-1140.

(3) Terano, T., Asai, K. and Sugeno, M. (1992), Fuzzy Systems Theory and its Applications, Academic Press, Inc., London.

(4) Kaufmann, A. and Gupta, M. M. (1988), Fuzzy Mathematical Models in Engineering and Management Science, Elsevier Science Publishers B. V., Amsterdam.

(5) Zimmermann, H.-J. (1987), Fuzzy Sets, Decision Making and Expert Systems, Kluwer Academic Publishers, Norwell, MA.

(6) Zimmermann, H.-Z. (1985), Fuzzy Set Theory and its Applications, Kluwer-Nijhoff Publishing, Hingham, MA.

(7) Kaufmann, A. and Gupta, M. M. (1985), Introduction to Fuzzy Arithmetic –Theory and Applications, Van Nostrand Reinhold Company, New York.

(8) Chen, S.-H. (1985), "Ranking Fuzzy Numbers with Maximizing Set and Minimizing Set", Fuzzy Sets and Systems, 17, 113-129.

(9) Machol, R. E. (Editor in Chief) (1984), Fuzzy Sets and Decision Analysis, Elsevier Science Publishers B. V., Amsterdam.

(10) Baldwin, J. F. and Guild, N. C. F. (1979), "Comparison Of Fuzzy Sets on the Same Decision Space", Fuzzy Sets and Systems, 2, 213-231.

(11) Bass, S. M. and Kwakernaak, H. (1977), "Rating and Ranking of Multiple-Aspect Alternatives using Fuzzy Set Theory", Automatica, 13, 47-58.