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Inventory Model for Damagable Items with Stock-Dependent Demand and Shortages

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Abstract

Items made of glass, ceramics, *etc*. get damaged during the storage due to accumulated stress of stocked items. Inventory models of these items are developed here with finite rate of replenishment and inventory level-dependent demand. Both linear and non-linear damage and demand functions are considered, shortages are allowed and fully backlogged. Optimum order quantities are determined using the profit maximization principle in integral form. The models are illustrated with numerical examples and a sensitivity analysis is presented.

1. INTRODUCTION

In inventory systems, nomrally four types of demands i.e. constant, time-dependent, stock-dependent and probabilistic demands are considered. According to Levin et.al [1972], "It is a common belief that large piles of goods displayed in a super market will lead to customer to buy more". Schary and Baker [1972] and Wolfe [1968] also established the impact of product availability for stimulating demand. For this reason, now-a-days, inventory models with stock-dependent demand are getting more attention and several authors have studied these models in depth. Gupta and Vrat [1986] assumed demand to be dependent on the initial stock level whereas Baker and Urban [1988] and Urban [1995] considered the polynomial form of on-hand inventory dependent demand. This type of models have been extended to include the deterioration of items. Normally, the rate of deterioration is taken to be deterministic and either constant or a function of time, *t*. Mandal and Phaujder [1989], Datta and Pal [1988], Padmanabhan and Vrat [1990], Pal, Goswami and Chaudhuri [1993] and others considered stock dependent demand for deteriorating items.

In real life, there are some items which get damaged during storage. Items made of glass, ceramics, etc. which are normally kept in warehouse one upon the other, break due to accumulated stress of piled stock. This stress is a function of both heaped inventory and time for which the stress is applied.

Hence, being encouraged by the findings of market research, a retailer is encouraged to display his goods in large numbers to motivate the customers to buy more and as a result, more and more items get damaged due to the accumulated stress of the piled items. So, for items made of glass, ceramics, etc., more stock invites more damage and a retailer thus faces a conflicting situation. Here, a general solution is suggested in this situation.

Works on deteriorating items mentioned above can also be applied to the damagable items only for the case where number of damaged items is linearly proportional to the current size of the heaped stock. But, till now, no separate attempt has been made to consider the problems of damagable items in general, specially with non-linear damage function.

In this paper, we propose inventory models with stock-dependent consumption rate for damagable items which get damaged due to the accumulated stress of stocked items. Although the accumulated stress is a function of on-hand inventory and time, for simplicity, we assume that the number of damaged items depend only on the current stock level. This dependency may be linear or non-linear. Here shortages are allowed and fully backlogged and demand may be linearly or non-linearly dependent on stock-level. We solve the models with the help of profit maximization principle in integral form. As a particular case, one of our model reduces to the conventional inventory model with constant deterioration. Models are illustrated with numerical examples and a sensitivity analysis is presented.

Model

A deterministic inventory model for damagable items with finite rate of replenishment and stock-dependent demand is developed under the following assumptions :

(*i*) Lead time is zero.

(*ii*) $T\ (= t_1 + t_2 + t_3 + t_4)$ is the duration of production cycle (Ref. fig. -1).

(*iii*) Consumption rate is $D(q)$ where $D(q)$ is a linear or non-linear function in q.

(*iv*) Number of damaged items is $B(q)$ per unit time where $B(q) = aq^{\delta}\ 0 \leq \delta,\ a \leq 1$.

(*v*) Production rate, $K\ (> D(q))$ is finite.

(*vi*) Shortage are allowed and fully backlogged.

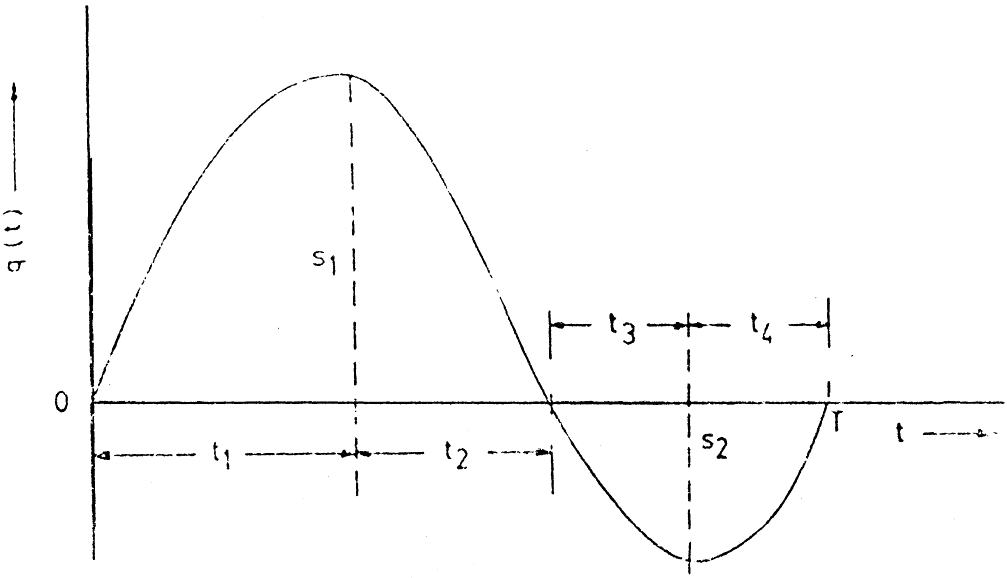

Fig. 1 Inventory model

(*vii*) Holding and shortage costs per item per unit time, C_1 and C_2, the set up cost, C_3, the unit production cost, p and the selling price, s are known and constant during each production cycle.

(*viii*) Even during the shortage period, demand rate D(q) is non-negative.

2. MATHEMATICAL ANALYSIS

With the above assumptions, we have

$$\begin{aligned} dq/dt &= K - D(q) - B(q), \quad 0 \le t \le t_1, \\ dq/dt &= -B(q) - D(q), \quad t_1 \le t \le t_1 + t_2, \\ dq/dt &= -D(q), \quad t_1 + t_2 \le t \le t_1 + t_2 + t_3, \\ dq/dt &= K - D(q), \quad t_1 + t_2 + t_3 \le t \le T. \end{aligned} \tag{1}$$

$T = t_1 + t_2 + t_3 + t_4$

$$= \int_0^{s_1} \frac{dq}{K - D(q) - B(q)} + \int_0^{s_1} \frac{dq}{D(q) + B(q)} - \int_0^{s_2} \frac{dq}{D(q)} + \int_{-2_2}^{0} \frac{dq}{K - D(q)}$$

$$= F(s_1, s_2), \text{ (say)} \tag{2}$$

where s_1 and s_2 are respectively maximum stock and backlogged levels. The total inventory holding cost is $C_1 G(s_1)$

$$\text{where } G(s_1) = \int_0^{s_1} \frac{qdq}{K - D(q) - B(q)} + \int_0^{s_1} \frac{qdq}{D(q) + B(q)} \tag{3}$$

and the total number of items produced during the time period, $T = K.(t_1 + t_4)$

$$= K \left\{ \int_0^{s_1} \frac{dq}{K - D(q) - B(q)} + \int_{-s_2}^{0} \frac{dq}{K - D(q)} \right\}$$

$$= \psi(s_1, s_2), \text{ (say)} \tag{4}$$

Similarly, the total shortage cost is $C_2 \phi(s_2)$, where

$$\phi(s_2) = \int_0^{-s_2} \frac{qdq}{D(q)} - \int_{-s_2}^{0} \frac{qdq}{K - D(q)} \tag{5}$$

and the total number of damaged items over the period (O.T)

$$= \int_0^{s_1} \frac{B(q)\,dq}{K - D(q) - B(q)} - \int_0^{s_1} \frac{B(q)\,dq}{B(q) + D(q)} = \theta(s_1) \quad (say) \tag{6}$$

Hence, average profit $Z(s_1, s_2) = ((s-P)\psi - C_1 G - C_3 - C_2\phi - s\theta)/T$ (7)

For maximum average profit, optimal values of s_1 and s_2 are the solutions of $\frac{\partial Z}{\partial s_1} = 0$ and $\frac{\partial Z}{\partial s_2} = 0$, provided

$$\frac{\partial^2 Z}{\partial s_1^2} < 0, \quad \frac{\partial^2 Z}{\partial s_2^2} < 0 \text{ and } \frac{\partial^2 Z}{\partial s_1^2} \frac{\partial^2 Z}{\partial s_2^2} - \left[\frac{\partial^2 Z}{\partial s_1 \, \partial s_2}\right]^2 > 0.$$

$\frac{\partial Z}{\partial s_1} = 0$ and $\frac{\partial Z}{\partial s_2} = 0$ lead respectively to

$$(s-p) F \frac{\partial \psi}{\partial s_1} + \left[-(s-P)\psi + C_1 G + s\theta + C_3 + C_2\phi\right]$$

$$\frac{\partial F}{\partial s_1} - C_1 F \frac{\partial G}{\partial s_1} - sF \frac{\partial \theta}{\partial s_1} - C_2 F \frac{\partial \phi}{\partial s_1} = 0 \qquad (8)$$

and $$(s-p) F \frac{\partial \psi}{\partial s_2} + \left[-(s-p)\psi + C_1 G + s\theta + C_3 + C_2\phi\right] \frac{\partial F}{\partial s_2}$$

$$- sF \frac{\partial \theta}{\partial s_2} - C_2 F \frac{\partial \phi}{\partial s_2} = 0 \qquad (9)$$

We solve the non-linear equation (B) and (9) by Newton-Raphson method to yield the optimal values – s_1^*, s_2^*, t_1^*, t_2^*, t_3^*, t_4^* and the corresponding optimal profits for particular demand and damage functions.

3. NUMERICAL RESULTS

To illustrate the model numerically, we assume the following values for the inventory parameters : $K = 250$, $\alpha = 100$, $p = Rs\ 8$, $C_1 = Rs.\ 1.5$, $C_2 = Rs.\ 6$, $C_3 = Rs.\ 100$, $a = .2$, $\beta = .3$, $S = Rs\ 10$.

Model - 1 : Both demand and damage functions are linear.

Let, $D(q) = \alpha + \beta q$, $B(q) = aq$, $0 < a < 1$.

For this case, the equations in (1) are exacly the same with those of the conventional inventory model with stock dependent demand and constant deterioration of Mandal and Phaujdar [7].

From equations (2), (3), (4), (5) and (6), we have

$$F = \{-\ln(1 - s_1 (a+\beta)/(K-\alpha)) + \ln(1 + s_1(a+\beta)/K)\}/$$

$$+ (a+\beta) + \{-\ln(1 - \beta s_2/\alpha) + \ln(1 + \beta s_2/(K-\alpha))\} / \beta$$

$$G = -\{(K-\alpha)\ln(1-(a+\beta)s_1/(K-\alpha)) + \alpha\ln(1+ (a+\beta)s_1/\alpha)\} / (a+\beta)^2$$

$$\psi = K\ \{-\ln(1-(\alpha+\beta)s_1/(K-\alpha))/(a+\beta) + \ln(1+\beta_1/(K-\alpha))/\beta\}$$

$$\theta = -a\{(K-\alpha)\ln(1-(a+\beta)s_1/(K-\alpha)) + \alpha\ln(1+ (a+\beta)s_1/\alpha)\} / (a+\beta)^2$$

$$\phi = -\{\alpha\ln(1-\beta s_2/\alpha) + (K-\alpha)\ln(1+\beta s_2/(K-\alpha))\}/\beta^2$$

With these expressions and above numerical values, we solve the equations (8) and (9) and the optimal values are :

$z = (s_1^*, s_2^*) =$ Rs. 51.44, $s_1^* = 59.42$, $s_2^* = 22.51$, $t_1^* = 0.44$, $t_2^* = 0.52$, $t_3^* = 0.23$, $t_4^* = 0.15$.

Model - 2 : Demand is linear and damage function non-linear.

Let, $D(q) = \alpha + \beta q$ and $B(q) = a\sqrt{q}$, $0 < a < 1$,

Proceeding as in model - 1, we obtain

$$t_1 = [-\ln\{1-x_1/(K-\alpha)\} + a/x_2 \ln\{(Y_1-x_2)(a+x_2)/((Y_1+x_2)(a-x_2))\}]/\beta,$$

$$t_2 = \{\ln(1+x_1/\alpha) - 2a/Y_2 \tan^{-1}(\sqrt{s_1}\ Y_2/(2\alpha + a\sqrt{s_1}))\}/\beta,$$

$$t_3 = -\ln(1-\beta s_2/\alpha)/\beta,$$

$$t_4 = \ln\{1+\beta s_2/(k-\alpha)\}/\beta,$$

$$G(S_1) = \{(K-\alpha)t_1 - \alpha t_2\}/\beta - \theta(s_1),$$

$$\psi = K(t_1 + t_4),$$

$$\phi = -\{\alpha\ln(1-\beta s_2/\alpha) + (k-\alpha)\ln(1+\beta s_2/(k-\alpha))\}/\beta^2,$$

$\theta = I_1 + I_2$,

$I_1 = \{ a^2 t_1 - 2a (k - \alpha) / x_2 \ln ((Y_1 - x_2) (a + x_2) / ((Y + x_2) (a - x_2)) \} / \beta$

$I_2 = - \{ a^2 t_1 - 4 a \alpha / Y_2 \tan^{-1} (\sqrt{s_1} Y_2 / 2\alpha + a \sqrt{s_1}) \} / \beta$,

$x_1 = \beta s_1 + a \sqrt{s_1}$, $x_2 = \sqrt{4 \beta (k - \alpha) a^2}$,

$y_1 = 2 \beta_1 \sqrt{s_1} + a$, $y_2 = \sqrt{4 \alpha \beta - a^2}$,

Solving equations (8) and (9) with these expressions, we get, $Z (s_1^* , s_2^*) =$ Rs. 115.42, $s_1^* = 74.67$, $s_2^* = 73.79$, $t_1^* = .53$, $t_2^* = .656$, $t_3^* = .83$, $t_4^* = .46$.

Model - 3 : Demand is non-linear and damage function linear.

Let, $B (q) = aq$; $D (q) = \alpha + \beta \sqrt{q}$, $q \geq 0$

$= \alpha$, $q < 0$

Proceeding as above, we get

$t_1 = \{ - \ln \{ 1 + x_1 / (k - \alpha) \} + \beta / x_2 \ln \{ (y_1 - X_2) (\beta + X_2) / ((Y_1 + X_2) (\beta - X_2)) \}) / a$,

$t_2 = \{ \ln (1 + x_1 / \alpha) - 2 \beta / Y_2 \tan^{-1} (\sqrt{s_1} Y_2 / 2\alpha + \beta \sqrt{s_1}) \} / a$,

$t_3 = s_2 / \alpha$, $\quad t_4 = s_2 / (K - \alpha)$,

$\phi (s_2) = k s_2^2 / 2 \alpha (k - \alpha)$,

$G (s_1) = \{ (k - \alpha) t_1 - \alpha t_2 \} / a - (I_1 + I_2)$,

$\psi (s_1 , s_2) = K (t_1 + t_4)$,

$\theta (s_1) = a G (s_1)$,

$$I_1 = \{\beta^2 t_1 2\beta(k-\alpha)/x_2 \ln\{(y_1 - X_2)(\beta + X_2)/((Y_1 + X_2)(\beta - X_2))\}/a,$$

$$I_2 = -\{\beta^2 t_2 - 4\beta\alpha/y_2 \tan^{-1}(\sqrt{s_1}\, Y_2/(2\alpha + \beta\sqrt{s_1}))\}/a,$$

$$x_1 = as_1 + \beta\sqrt{s_1},\ x_2 = \sqrt{4a(k-\alpha)+\beta^2}$$

$$Y_1 = 2a\sqrt{s_1} + \beta,\ Y_2 = \sqrt{4\alpha a - \beta^2}.$$

With these expressions, equations (8) and (9) are solved and optimal values are :

$t_1^* = .321$, $t_2^* = .437$, $t_3^* = .281$, $t_4^* = .182$, $s_1^* = 46.26$, $s_2^* = 27.98$,

$Z(s_1^*, s_2^*) = Rs.\ 32.10$.

Model - 4 : Both demand and damage functions are non-linear

Let $D(q) = \alpha + \beta\sqrt{q},\ q \geq 0,$

$= \alpha, \qquad q < 0 \qquad$ and $\qquad B(q) = a\sqrt{q}$.

Proceeding as above, we get

$$t_1 = -\frac{2\sqrt{s_1}}{(a+\beta)} - 2(K-\alpha)/(a+\beta)^2 \ln\{1 - (a+\beta)\sqrt{s_1}/(K-\alpha)\},$$

$$t_2 = \frac{2\sqrt{s_1}}{(a+\beta)} - \frac{2\alpha}{(a+\beta)^2}\ln\{1 + (a+\beta)\sqrt{s_1}/\alpha\},$$

$$t_3 = s_2/\alpha,\ t_4 = s_2/(K-\alpha),$$

$$T = t_1 + t_2 + t_3 + t_4,$$

$$G(S_1) = \{-Ks_1 + (k-\alpha)^2 t_1 + \alpha^2 t_2\}/(a+\beta)^2,$$

$$\theta(s_1) = a\{(k-\alpha)t_1 - \alpha t_2\}/(a+\beta),$$

$$\psi(s_1) = K(t_1 + t_4),$$

$\phi(s_2) = s_2^2/2\ \{1/\alpha + 1/(K-\alpha)\} = Ks_2^2/2\alpha\ (K-\alpha)$.

Now, we solve equations (8) and (9) with above expressions and get $t_1^* = .527$, $t_2^* = .753$, $t_3^* = .209$, $t_4^* = .139$, $s_1^* = 77.49$, $s_2^* = 20.38$, $z(s_1^*, s_2^*) = Rs.\ 74.95$

4. SENSIVITY ANALYSIS

For the above mentioned models, two types of sensitivity analysis are performed. First, the effect of changes in selling price, 's' on the average profit while holding the other parameters at their optimal values and next, the effect of change in the parameters - 'β', shape of the demand curve and 'a', co-efficient of damaged function keeping the other parameters constant. The results are presented graphically for the models -1, 2, 3 and 4.

(a) Effect of selling price on average profit (Fig - 2)

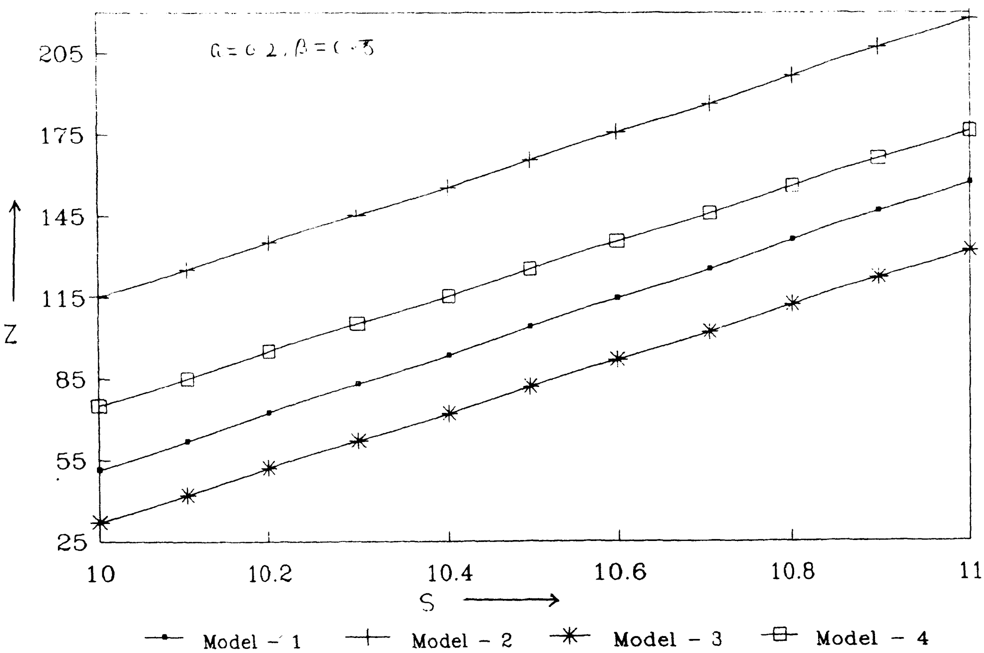

Fig 2. Effect of *a* on *Z*

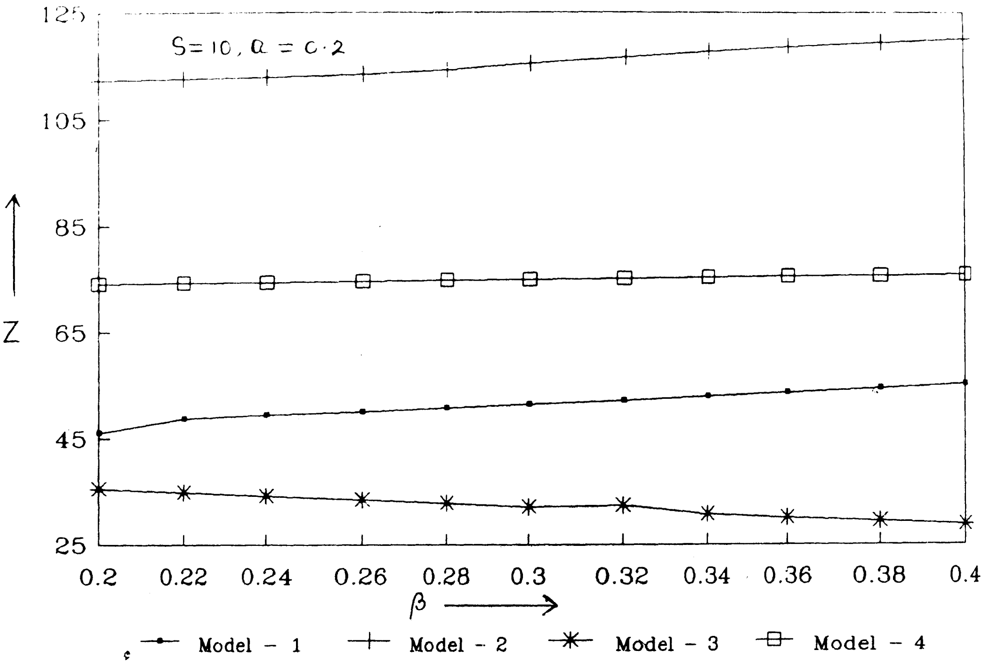

Fig. 3 Effect of β on Z

For all models, profit increases with selling price. When S is changed by 10%, profit rate jumps up approximately by 20% for model - 1, 8.62% for model -2, 23.78% for model - 3 and 11.92% for model - 4.

(b) Effect of demand function on average profit (Fig - 3)

When the shape of the demand curve i.e., β changes by 10%, profit in models - 1, 2 and 4 increases approximately by 1.43%, 1%, 0.17% respectively where it decreases by 1.92% in the case of model - 3.

(c) Effect of damage function on average profit (fig - 4).

When the co-efficient of damage function i.e., a is changed by 10%, the profit goes down approximately by 6.43%, 0.55%, 6.96%, 1% for models 1,2,3 and 4 respectively.

The above behaviour of profit expression is in tune with the assumptions and expectations for the inventory models.

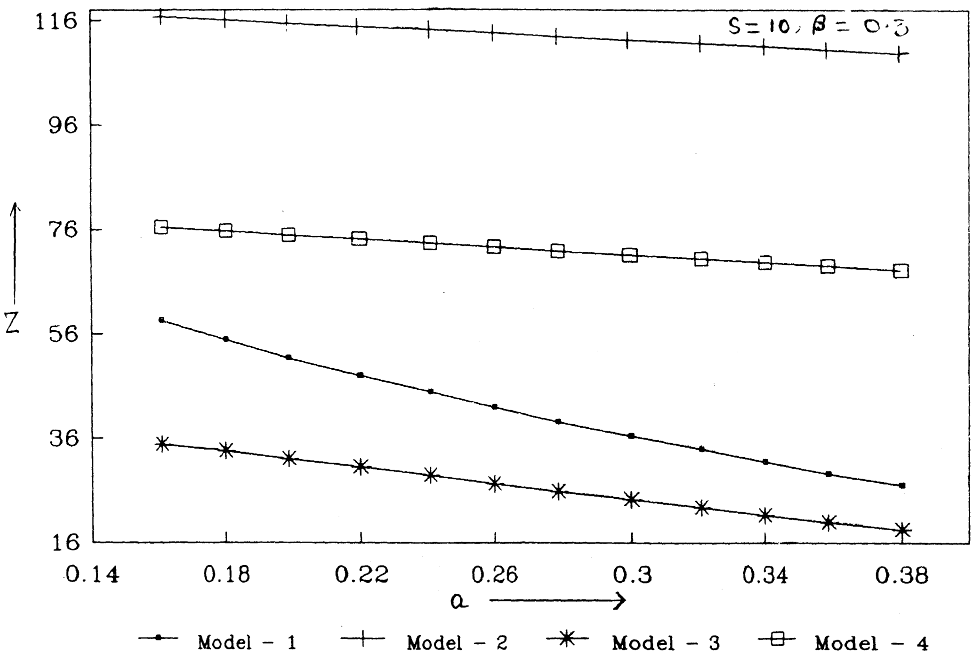

Fig 4. Effect of *a* on *Z*

5. CONCLUSION

Here, we have presented the solution of the inventory models of damagable items with fully back-logged shortages for both linear and non-linear damage and demand functions on on-hand inventory. For the first time, non-linear damage function on q has been considered for analysis. As the present analysis is in integral form, it can be applicable for any type of non-linear damage function. Present formulation can be easily extended to multi-item inventory models with/without constraints, multi-objective inventory problems, etc. These models can also be formulated in stochastic and fuzzy environments. Here, accumulated stess for which items get damaged, has been assumed to be function on hand inventory. Actually it is dependent on both inventory and time. Consideration of such realistic accumulated stress may be a topic of further investigations.

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