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RESEARCH ARTICLE

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Quality Mathematics Instructional Practices Contributing to Student Achievements in Five High-Achieving Asian Education Systems: An Analysis Using TIMSS 2011 Data

Abstract Although teaching quality is seen as crucial in affecting students' performance, what types of instructional practices constitute quality teaching remains a question. With the theoretical assumptions of conceptual and procedural mathematics teaching as a guide, this study examined the types of quality mathematics instructional practices that affect students' mathematics learning across five high-performing Asian education systems using the Trends in International Mathematics and Science Study (TIMSS) 2011 dataset. It found that no combinations of the components of conceptual and procedural mathematics teaching practices exist consistently across the five education systems. Results from the study provide important implications for practitioners and policy makers regarding how to improve mathematics teaching and learning in these Asian education systems as well as elsewhere.

Keywords conceptual teaching practice, procedural teaching practice, mathematics performance, international comparison

Introduction

Results from the 2011 Trends in International Mathematics and Science Study (TIMSS) showed that students in five Asian education systems, Republic of Korea (to be referred to as "Korea" for the rest of the paper), Singapore, Chinese Taipei, Hong Kong, and Japan, continued to outperform those in other education

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systems around the world (Mullis, Martin, Foy, & Arora, 2012). The mathematics achievement gap between these high-achieving Asian education systems and the other countries has persisted for decades (Fleischman, Hopstock, Pelczar, & Shelley, 2010; Medrich & Griffith, 1992). Mathematics teaching practices in these Asian education systems are often used to explain their students' higher mathematics achievements (Stevenson, Lee, & Stigler, 1986; Stigler, Gonzales, Kawanaka, Knoll, & Serrano, 1999; Stigler & Hiebert, 2009) following the assumption that teaching quality is a crucial driving force for improving student achievement (Barber & Mourshed, 2007; NCTM, 1989, 1991, 2000; National Mathematics Advisory Panel, 2008; OECD, 2004, 2005).

Then, what kinds of quality mathematics teaching practices teachers in these Asian education systems develop in the classroom, and whether and to what extent quality mathematics teaching practices exist consistently across the five Asian education systems are valuable questions to be examined. This will provide the insights necessary to identify actual teaching gaps between these systems and others. Thus, it will offer valuable suggestions for practitioners and policy makers regarding ways of improving mathematics teaching and learning in these educational systems as well as others around the globe (Ferrini-Mundy & Schmidt, 2005).

This study is designed to address these issues. Drawing on the international data from the Trends in International Mathematics and Science Study (TIMSS) 2011, it investigates, in particular, two research questions. First, what mathematics instructional practices are positively and significantly related to the overall mathematics achievements of students in the classrooms of high-performing teachers in each of the five education systems? Second, whether or to what extent are the effective mathematics teaching practices in these systems similar to or different from each other? While whether a common pattern of mathematics teaching can exist across different countries and regions and regions is still debatable, the assumption that certain kinds of mathematics teaching can improve students' learning has been an underlying component of mathematics teaching reform in many countries (Leung & Li, 2010). Thus, the examination of the above questions can be a valuable contribution to education policy.

Literature Bases

Theoretical Framework

The design of the study is informed by the Teaching Quality framework (Goe, 2007; Goe & Stickler, 2008), relevant theoretical assumptions about conceptual teaching (Hiebert et al., 1997; Romberg, 1990; Thompson, 2001) and procedural mathematics teaching (Geary, 1994; Greeno, Collins, & Resnick, 1996).

First, following the Teaching Quality framework, teaching quality is ultimately determined by what teachers actually do in the classroom while teacher qualifications, such as their education level, certification, credentials, and attitudes and beliefs only exert their influence on student learning outcomes through what teachers do in the classroom (Goe, 2007; Goe & Stickler, 2008). Thus, the examination of what teachers do in the classroom, especially teacher student interactions in the classroom where the actual teaching and learning takes place, should be the central focus of studies that examine teaching quality (Goe, 2007). Following this framework, this study identifies the types of in-classroom instructional practices that are used by mathematics teachers in the five Asian education systems and that can be conducive to students' mathematics achievement so as to inform practitioners and policy makers.

Second, this study also uses the assumptions of conceptual and procedural mathematics teaching to further help guide the examination of specific teaching practices. Conceptually oriented mathematics teaching emphasizes fostering students' higher-level conceptual understanding and their ability to solve complex, non-routine problems realized through student-centered learning, students' sharing their mathematical ideas and clarifying their understanding of problem solving processes, and relating problem-solving to their real life experience (Hiebert et al., 1997). This assumption is developed on the basis of the constructivist perspective of learning with an emphasis on the active role of the students in the construction of their own mathematical knowledge and skills that are important for their mathematics problem-solving in the social, economic, political life of the global world (Hiebert et al., 1997; Romberg, 1990; Thompson, 2001).

According to the proponents of conceptual teaching, problem-solving is considered as the process that engages students in finding a solution to a task by drawing on mathematical knowledge they have learned, especially those non-routine, complex problems that require students to make bold conjectures, propose multiple approaches, and search for solutions (Hiebert et al., 1997; NCTM, 1991). In learning how to solve such mathematics problems, students are required to properly represent the problem mathematically, justify their solution to a problem, explain the thinking processes behind their solutions, and check and examine others' ideas and solutions carefully (Hiebert et al., 1997; NCTM, 2000). Central to these learning activities is the development of students' conceptual understanding of mathematics. Thus, the ultimate focus of mathematics teaching should be on students' conceptual understanding via engagement in justifying and sharing mathematical ideas, clarifying their understanding of problem-solving processes using various representational tools, and relating problem-solving to their real life experience (NCTM, 2000).

Compared with conceptual teaching that was promoted during the past two decades or so, procedural teaching has been in existence for a much longer time and is still used by many mathematics teachers around the world. This type of teaching stresses students' solid memorization of algorithms, facts and rules, routine computational drill, procedural skill practice, and using algorithms, facts, rules, and concepts to solve simple routine problems (Geary, 1994; Greeno et al., 1996; Wu, 1999). It assumes that basic mathematics knowledge and skills play a foundational role in students' mathematics learning, even if they are required to learn how to use reasoning to solve complex problems (Geary, 1994; Greeno et al., 1996; Wu, 1999). The proponents of procedural teaching (Greeno et al., 1996) claim that the basic mathematics knowledge and skills constitute the sound foundation upon which high-level conceptual understanding of mathematics ideas and concepts can be developed and better mathematics problem-solving skills can be acquired (Geary, 1994; Wu, 1999). Scholars supporting this assumption (Geary, 1994; Greeno et al., 1996; Wu, 1999) argue that these basic mathematics knowledge and skills could be well developed through procedural mathematics teaching that focuses on solid memorization of algorithms, facts and rules, routine computational drill, procedural skill practice, and use of algorithms, facts, rules and concepts to solve simple and routine problems.

Although these assumptions conceptually capture the major attributes of two types of mathematics teaching practices, in reality mathematics teachers might use a hybrid of the two. Then, the question remains to be explored as to what combination of the two types of teaching would be most effective in improving students' mathematics achievement. This study helps examine whether and to what extent such assumptions of teaching are valid by investigating the various kinds of components of mathematics instructional practices as framed through the assumptions of conceptual and procedural oriented teaching using data from high-performing teachers in the five selected Asian education systems.

Empirical Bases

A search of the existing literature using key words such as "mathematics teaching" and "East Asia" in ERIC and other related databases showed that a small pool of studies have examined mathematics teachers' instructional practices in these five education systems. These studies provide useful information regarding the ways mathematics is taught in the classrooms of these systems.

First of all, the current literature reports that mathematics teachers in both Singapore and Hong Kong use a similar direct instructional approach that focuses on students' lower level thinking skills. Kaur and Yap (1998) observed teachers' in-class lessons and found that the 8th and 9th grade mathematics teachers in Singapore mainly used direct explanation and individual student seatwork, which is consistent with the procedural conceptions. Based on coding of classroom observations mainly in grades 5 and 9, Yeo and Zhu (2005, May/June) found that the mathematics teachers in Singapore predominantly engaged students with lower level thinking skills such as factual knowledge memorization and procedural computation. Furthermore, Kaur (2009) examined the instructional approaches of three competent 8th grade mathematics teachers in Singapore to identify the characteristics of good mathematics teaching in 8th grade classrooms and found that good mathematics teaching is characterized by teacher-centered instruction, which is achieved by teachers' active monitoring of students' seatwork and selective use of student work for whole-class discussion and clarification.

In the case of Hong Kong, studies also found a similar approach to mathematics instruction that focused on lower order thinking skills. Mok (2009) studied an 8th grade algebra model lesson and found that teacher talk and a directive approach were the major components of the lesson. The teacher's clear explanation and explicit guidance enabled students to follow along and master the content in an efficient way within a short period of instructional time. As reported in another study based on lesson observation and interviews with a 4th grade teacher (Mok, Cai, & Fung, 2008) although such instructional practices in the well-structured lesson provided ample opportunities for the students to learn specific basic procedural skills, the learning opportunities for fostering students higher-order thinking skills are missing. The 1999 TIMSS video study found that the lessons taught by Hong Kong teachers focused more on procedural skills (Hiebert et al., 2003, 2005; Stigler & Hiebert, 2004), which confirmed the above findings that procedurally oriented instruction is typically used in Hong Kong.

Second, in contrast to the teacher directed teaching approach that focuses on students' lower level thinking skills in Singapore and Hong Kong, the instructional practices in Chinese Taipei, Korea, and Japan are closer to that of conceptual oriented teaching. Lin and Li (2009) analyzed 92 lessons of six highly regarded mathematics teachers in Taiwan and found that these teachers used an approach that is conceptually focused and higher order thinking skills oriented. These teachers asked students to master fundamental concepts in a unit and regularly selected math problems from the textbook to engage students in using these concepts to develop multiple solutions either individually or in heterogeneous groups. In the meantime, the teachers observed their students' problem-solving processes to identify effective solutions and then asked students to present them to the whole class while encouraging students to explain and justify their solutions, and compare and contrast different solutions to enhance their understanding of the concept.

Based on the analysis of the video-recorded lessons taught by an elementary mathematics teacher over a year in Korea, Pang (2009) found that using the guided investigations was a major feature of her lessons, in which the teacher used one specific problem to elicit students' multiple solutions and then carefully orchestrated classroom discussion to lead the students towards higher-level mathematics thinking skills. The TIMSS 1995 video study that analyzed the lessons taught in 8th grade Japanese classrooms revealed that Japanese teachers went into greater depth to develop students' mathematical concepts and procedures leading to a focus on reasoning and proof in their lessons (Hiebert & Stigler, 2000; Stigler et al., 1999). The follow-up TIMSS video study in 1999 further revealed that Japanese teachers used more mathematical content connections and asked students to do more reasoning, namely, let students state how to use procedures instead of just going about using them in teaching (Hiebert et al., 2003, 2005; Stigler & Hiebert, 2009). Using data from TIMSS

2003, House and Telese (2008) found that Japanese teachers focus on having students solve routine and complex problems on their own using basic mathematical operations such as adding, subtracting, multiplying, and dividing without a calculator and using reasoning to explain their solutions, which contributed significantly to 8th graders' algebra learning. Also using TIMSS 2003 data, Hamilton and Martinez (2007) confirmed that students' independent problem-solving had a significant positive influence on students' mathematics achievement in Japan.

The above reviewed studies provide valuable information about the inconsistent characteristics of mathematics teaching practices in Asian education systems in light of conceptual and procedural teaching assumptions. However, most of these studies used qualitative method that involved a few participants, which limited their generalizability and thus could not provide an overall picture of the high-performing teachers' instructional practice in these education systems (Lin & Li, 2009; Kaur & Yap, 1998; Mok, 2009; Stigler & Hiebert, 2004; Yeo & Zhu, 2005, May/June). Additionally, most studies neither link teachers' specific instructional practice to their students' mathematics achievements, nor do they identify which types of teacher practices are correlated with students' higher achievement in these systems (Hiebert et al., 2003, 2005; Hiebert & Stigler, 2000; Stigler et al., 1999; Stigler & Hiebert, 2009).

Although two studies (House & Telese, 2008; Hamilton & Martinez, 2007) link teaching practices to students' performance in Japan, neither of them investigate the impact of teachers' instructional practices on student achievement in the high-achieving classes by comparing the five Asian education systems with each other. The current study aims to address this gap in the existing literature by taking into consideration all five high-achieving Asian education systems to investigate the high-achieving mathematics teachers' quality instructional practices that can contribute to students' overall mathematics achievement. It also identifies whether such quality instructional practices vary across the five different education systems.

Methodology

Data Source

The international datasets from TIMSS 2011 were used as the data source of the study for three reasons.

First, TIMSS 2011 adopted a two-stage, non-random sampling design to ensure the student sample was a nationally representative one (Martin & Mullis, 2012). At the first stage, schools were selected using probability-proportional-to-size sampling. To achieve a self-weighted student sample and reduce the chance of selecting smaller schools for cost efficiency, the probability of selection for the schools was based on the schools' measure of size (MOS) proportional to its share of student enrollment. At the second stage, one or two whole classes were randomly selected in each school sample (Martin & Mullis, 2012). The samples were drawn from students who were about to finish 8th grade in the above schools along with their teachers. Results from such a national sample have the potential to be generalized to larger student populations and used for policy recommendation (Schneider, Carnoy, Kilpatrick, Schmidt, & Shavelson, 2007).

Second, the teachers' questionnaire from the different participating countries in the TIMSS 2011 data set contains survey items about teachers' instructional practices that can be linked to students' mathematics achievements (IEA, 2011; Mullis, Martin, Ruddock, O'Sullivan, & Preuschoff, 2009). Such linkage is necessary to answer the research questions of this study. TIMSS 2011 assessed mathematics knowledge and skills that students have learned at school (Mullis et al., 2009), which is useful for investigating the impact of teachers' instructional practices on students' achievement. From students' achievement data, the overall mathematics achievement of students across the participating educational systems can be identified (Mullis et al., 2009).

Third, teachers' instructional practice survey items can be grouped by conceptual and procedural teaching styles by using concepts developed in the literature (Geary, 1994; Hiebert et al., 1997; Romberg, 1990; Wu, 1999). Thus, they help verify the theoretical assumptions of these two types of teaching. Although survey data cannot provide the same in-depth descriptions of teachers' instructional practice as on-site observations do, they are able to involve large populations of teachers to yield findings of external generalizability, which often can not be achieved in many qualitative studies (Schneider et al., 2007). In spite of the generally low reliability of self-reported data, the survey about teachers' instructional practices has been more reliable (Mayer, 1998, 1999; McCaffrey et al., 2001; Porter, Kirst, Osthoff, Smithson, & Schneider, 1993) as respondents were surveyed anonymously (Aquilino, 1994, 1998) and asked to describe their behaviors instead of judging the quality of their behaviors (Mullens & Gayler, 1999). In TIMSS 2011, teachers were asked to anonymously account for their classroom teaching rather than assess their teaching (Mullis et al., 2009). These features of TIMSS 2011 make it possible to be used to address the research questions of the current study.

Participants, Instruments, and Variables

From TIMSS 2011 international dataset, the five Asian education systems—Korea, Singapore, Taipei, Hong Kong, and Japan—which dominated the ranking in mathematics performance, were selected. To select the high-performing mathematics teachers and their students in these systems, I used the international average score of 500 on the mathematics mean of the class taught by the same mathematics teacher as a criterion. Due to the consideration of ensuring a sufficiently large sample size for the statistical analysis (Raudenbush & Bryk, 2002), I used this score instead of the higher benchmark score of 550. The teachers along with their students selected from the five education systems became the participants of this study. The specific sample size information for the teacher and student as well as the number of students taught by the same teacher is presented in Table 1.

Table 1 Sample Size for Selected Teachers and Their Students across Five High-Achieving East and Southeast Asian Education Systems in TIMSS 2011

Country or Region	Sample Size		Number of Students	Taught by the Same Teacher
	Teacher	Student	Minimum	Maximum
Korea	349	5,006	3	41
Singapore	294	5,311	11	19
Chinese Taipei	155	4,877	7	56
Hong Kong	122	3,601	4	43
Japan	166	4,418	4	45

The dependent variable of the study is the overall mathematics score of the 8th grade students. According to TIMSS 2011 assessment framework, the assessment items were designed with an aim to measure students' performance on knowledge and skills learned in the school curriculum (Mullis et al., 2009). The assessment framework consists of *content* and *cognitive* domains; the *content* domain covers number, algebra, geometry, and data and chance, while the *cognitive* domain includes *knowing*, *applying*, and *reasoning*. Standardized multiple-choice questions and open-ended assessment items were included in both domains. Students' overall mathematics score was simply calculated based on their performance on the two domains (Mullis et al., 2009). Using item response theory (IRT) model, TIMSS 2011 created five plausible values to represent students' overall mathematics score (Martin & Mullis, 2012), which were used as the dependent variable of the study.

The independent variables were the survey items about teachers' instructional practice that were selected according to the conceptual and procedural teaching framework (Geary, 1994; Hiebert et al., 1997; Romberg, 1990; Wu, 1999) and in consistence with prior studies (e.g., Desimone, Smith, Baker, & Ueno, 2005; Hamilton & Martinez, 2007; see Table 2 for details). The teachers' questionnaires in TIMSS 2011 were designed on the basis of a contextual framework that used Principles and Standards for School Mathematics of the National Council of

Table 2 Initial Coding and Recoding of TIMSS 2011 Items Indicating Mathematics Teachers' Instructional Practices

TIMSS Item Description (How often do teachers ask students to…?)	Original Coding	Recoding
1) Listen to me explain how to solve problems	1 = every or almost every lesson	1 = never
2) Memorize rules, procedures, and facts	2 = about half the lessons	2 = some lessons
3) Work problems (individually or with peers) with my guidance	3 = some lessons	3 = about half the lessons
4) Work problems together in the whole class with direct guidance from me	4 = never	4 = every or almost every lesson
5) Apply facts, concepts, and procedures to solve routine problems	8 = not administered	8, 9 = missing data
6) Work problems (individually or with peers) while I am occupied by other tasks	9 = omitted	
7) Explain their answers		
8) Relate what they are learning in mathematics to their daily lives		
9) Decide on their own procedures for solving complex problems		
10) Work on problems for which there is no immediately obvious method of solution		

Teachers of Mathematics (2000) as a guide for the survey questions (Ferrini-Mundy & Schmidt, 2005; Mullis et al., 2005, 2009). This ensured that teachers' questionnaire items about their instructional practice could be used to construct variables of conceptual and procedural teaching practices to relate to students' mathematics achievements. Such a use of individual items of each kind of teaching enables us to identify specific characteristics of two kinds of conceptualized mathematics teaching developed in each of the five places.

Conceptual teaching emphasizes fostering students' higher-level conceptual understanding and their ability to solve complex, non-routine problems realized through student-centered learning, students' sharing their mathematical ideas and clarifying their understanding of the problem-solving process, and relating problem-solving to their real life experience (Hiebert et al., 1997). As a result, the following five survey items were selected from TIMSS 2011 teachers' questionnaire as instructional practices indicating conceptual teaching. Specifically, these include the practices that: (1) teachers ask students to work problems (individually or with peers) while the teacher is occupied by other tasks, (2) ask students to explain their answers, (3) relate what they are learning in mathematics to their daily lives, (4) decide on their own procedures for solving complex problems, and (5) work on problems for which there is no immediately obvious method of solution.

As procedural teaching gives primary focus to solid memorization of algorithms, facts and rules, routine computational drill, procedural skill practice, and using algorithms, facts, rules and concepts to solve simple routine problems (Geary, 1994; Greeno et al., 1996; Wu, 1999), the following five items were labeled as procedural teaching practices, which include: (1) teachers ask students to listen to teachers explain how to solve problems, (2) memorize rules, procedures, and facts, (3) work problems (individually or with peers) with teachers' guidance, (4) work problems together in the whole class with direct guidance from the teacher, and (5) apply facts, concepts, and procedures to solve routine problems (see Table 2).

Each teacher responded to each of the above teaching practice items with one of the four frequency levels, at which each teaching practice is used in his or her mathematics teaching: 1) "in every lesson or almost every lesson," 2) "in about half the lessons," 3) "in some lessons," and 4) "never." To prepare for parametric analysis, these frequencies were recoded in a reversed manner so that higher numbers indicated higher frequency while lower numbers indicated lower frequency use of these instructional practices (see Table 2 for details).

To better understand the relationship between teachers' instructional practice and student achievement, this study used students' socioeconomic status (SES) as a control variable because studies found that a lion's share of the variance in students' achievement was accounted for by students' SES (Coleman et al., 1966; Hanushek, 1996, 1997; Hedges, Laine, & Greenwald, 1994). This SES variable also helps verify if teaching is the major or most important factor in shaping student learning. However, the TIMSS 2011 dataset did not provide sufficient information for constructing a reliable SES composite variable by following the four dimensions of indicators for SES, i.e., parental education, occupation, income (Hauser, 1994; Mueller & Parcel, 1981) and home location (Sirin, 2005). Therefore, only students' parental education level was used as a proxy measure for student SES.

Data Analysis

A two-level hierarchical linear modeling (HLM) was used for the data analysis as suggested for this context because of the nature of the data in which students clustered in classes (Raudenbush & Bryk, 2002). All HLM analyses were conducted using HLM 7 software (Raudenbush, Bryk, Cheong, Congdon, & Toit, 2011) that was capable of handling large dataset such as TIMSS (Rutkowski, Gonzalez, Joncas, & von Davier, 2010).

Before model building and estimations, the statistical assumptions of HLM were examined and violations were addressed to ensure the assumptions were met. The empty model was first build to partition the total variance in students' overall mathematics achievement into within- and between-classroom components in order to estimate the intraclass correlations (ICC) for assessing the pertinence of applying a multi-level modeling approach (Raudenbush & Bryk, 2002). Then, highest parental education level was put into the level-1 equation in the conditional model with grand mean centered to estimate the proportion of variance in students' overall mathematics achievement at level-1. The error term at level-2 for the two covariates was retained to keep the model fit as results from Chi-square tests showed significant difference between the level-1 models with random effects of parental education at level-2 and those models with the fixed effects of this covariate.

In the final model, the highest parental education level was retained in the level-1 equation with grand mean centered while the ten instructional practice variables were entered into a level-2 equation simultaneously with grand mean centered. In this way, the relative contribution of each instructional practice to the variance in students' overall mathematics achievement can be estimated across different classrooms. Finally, I calculated the change in proportion of variance in students' overall mathematics explained by the instructional variables at both level-1 and level-2 in the final model. The final model for the different education systems in TIMSS 2011 datasets are shown below:

Level-1 model:

$$Y_{ij} = \beta_{oj} + \beta_{1j}(ParentEd) + r_{ij}$$

Level-2 model:

$$\beta_{oj} = \gamma_{00} + \gamma_{01}(Listen) + \gamma_{02}(Memorize) + \cdots + \gamma_{010}(WorkOn) + u_{oj}$$

$$\beta_{1j} = \gamma_{10} + u_{1j}$$

In all the model analyses, separate data files were used for each of the five Asian education systems in TIMSS 2011. As HLM 7 has the capacity of handling plausible values, the five plausible values created by TIMSS 2011 to represent students' overall mathematics achievement were specified in HLM 7, which then automatically calculated the average for the perimeter estimates (Randenbush et al., 2011) to achieve accurate estimates. Based on the suggestion of Foy, Arora, and Stanco (2013), mathematics teacher weight was applied at level-2 in order to make the results to be generalized to the larger population.

Limitations

Several limitations are pertinent to this study. First, this study used 8th grade samples of the five education systems in TIMSS 2011. Thus, results from this study can only be generalized to such students along with their teachers in these systems. Second, this study only used teacher's self-reported instructional practices to indicate procedurally or conceptually oriented instructional practices. Even though such self-reported practices were perceived as reliable by some researchers (e.g., Mayer, 1998, 1999; McCaffrey et al., 2001; Porter et al., 1993), others did not regard them as highly reliable, especially when comparing them with in-classroom observations (Brophy & Good, 1986; Burstein et al., 1995). Nevertheless, studies using in-classroom observations tend to suffer from smaller sample size and thus lower level of generalization while large-scale studies such as TIMSS includes a nationally representative sample, which can ensure better generalization.

Results

Results from the HLM analysis revealed several findings relevant to the research questions regarding whether or to what extent procedurally or conceptually oriented instructional practices contribute to students' mathematics achievement in the high performing classrooms of the five education systems and whether or to what extent these vary across the different systems.

First of all, as indicated in Table 3, this study found that teaching practices in general play a more important role in shaping Japanese students' mathematics achievement, less for students in Chinese Taipei and Hong Kong, and the least for students in Korea and Singapore. Results from the analysis showed that the intraclass correlation (ICC) coefficients from the base models were .15, .20, .41, .47, and .66 respectively for Japan, Chinese Taipei, Korea, Hong Kong, and Singapore, all ps $<$ .001, which indicates that substantial variances exist at level-2 and a two-level HLM analysis was appropriate. In the full models, after controlling for parental education level, the between-classroom variances in the intercepts of student outcome score explained by level-2 instructional practice variables were 19% for Japan, 9% for Chinese Taipei and Hong Kong, and 5% for Korea and Singapore. These results suggest that teaching matters most for Japanese students' learning of mathematics, less for students in Chinese Taipei and Hong Kong, while least for students in Korea and Singapore.

Second, in terms of the conceptually oriented teaching practices, this study found the more frequently teachers in Japan asked students to work out problems by themselves either individually or with peers and teachers in Korea asked students to explain their answers, the more likely their students would have lower mathematics performance scores. In contrast, the more frequently teachers in Singapore and Japan asked students to decide on their own procedures for solving complex problems, the higher mathematics scores their students received.

Table 3 shows that after controlling for the effects of parental education level and the other instructional practice variables, the instructional practice variable that teachers ask students to work problems (individually or with peers) while the

Table 3 Results from Hierarchical Linear Modeling for Five High-Achieving East and Southeast Asian Education Systems

	Korea	Singapore	Chinese Taipei	Hong Kong	Japan
	Fixed effects (coefficient/standard error)				
Empty model γ_{00}	618.30^{***}/3.68	625.33^{***}/3.74	614.01^{***}/4.04	602.47^{***}/4.65	577.59^{***}/3.05
Control Model γ_{00}	618.94^{***}/3.42	625.08^{***}/3.73	616.50^{***}/3.53	601.77^{***}/4.62	577.95^{***}/2.75
γ_{10}	14.34^{***}/1.38	0.51/1.12	24.75^{***}/1.81	0.10/1.34	21.01^{***}/1.57
	Full Model				
INTRCPT1, γ_{00}	619.10^{***}/3.35	625.29^{***}/3.62	616.61^{***}/3.41	602.67^{***}/4.43	578.13^{***}/2.57
V1, γ_{01}	2.95/3.32	3.64/4.60	5.13/4.11	-10.72^{*}/5.17	−3.63/2.90
V2, γ_{02}	−5.75/3.46	-9.78^{*}/3.95	−0.59/4.11	1.48/4.43	7.66^{*}/3.17
V3, γ_{03}	5.88/3.83	−0.08/3.99	−5.85/4.03	10.10/5.59	0.93/2.94
V4, γ_{04}	−8.02/4.35	−5.86/4.42	−2.52/4.70	−8.12/5.09	−3.71/2.59
V5, γ_{05}	3.53/3.52	3.25/4.30	3.77/4.61	6.65/4.91	3.53/2.72
V6, γ_{06}	−0.91/3.82	−0.31/4.09	−4.52/3.86	−1.66/4.46	-5.39^{*}/2.68
V7, γ_{07}	-8.39^{*}/3.89	6.92/4.04	3.69/4.31	0.42/5.24	−1.47/2.97
V8, γ_{08}	4.09/4.29	−2.27/4.26	−2.26/4.35	−6.56/6.19	−0.58/3.11
V9, γ_{09}	4.39/4.88	10.37^{*}/4.59	−0.66/4.40	3.41/5.20	9.66^{**}/3.30
V10, γ_{010}	5.07/4.26	−7.10/4.77	5.32/4.21	3.10/4.64	−4.30/3.85
INTRCPT2, C1, γ_{10}	14.27^{***}/1.38	0.47/1.13	24.68^{***}/1.81	0.42/1.32	20.96^{***}/1.56

(*To be continued*)

(*Continued*)

	Korea	Singapore	Chinese Taipei	Hong Kong	Japan
	Random effects				
Empty model					
τ_{00}/σ^2	3118.38/4415.47	3301.95/1666.61	2073.29/8174.30	2210.71/2479.54	977.57/5679.80
$x^2(df)$	2837.57^{***}(323)	8192.69^{***}(282)	1296.12^{***} (152)	2489.06^{***} (116)	738.10^{***} (155)
Control model					
τ_{00}/σ^2	2665.79/4231.28	3287.89/1655.65	1429.97/7675.26	2186.27/2474.05	746.38/5298.17
$x^2(df)$	1893.85^{***}(309)	5829.77^{***} (282)	792.24^{***} (152)	1756.02^{***} (115)	513.52^{***} (154)
Full model					
τ_{00}/σ^2	2526.43/4231.33	3110.98/1655.80	1296.26/7677.73	1993.06/2479.58	605.14/5298.75
$x^2(df)$	1854.41^{***}(299)	5640.52^{***} (272)	735.49^{***} (142)	2324.95^{***} (106)	454.90^{***} (144)
	ICC				
	0.41	0.66	0.20	0.47	0.15
	Additional proportion of variance explained				
	level-2/level-1	level-2/level-1	level-2/level-1	level-2/level-1	level-2/level-1
Control model	15%/4%	0%/1%	31%/6%	1%/0%	24%/7%
Full model	5%/0%	5%/0%	9%/0%	9%/0%	19%/0%

Note. * $p < .05$, ** $p < .01$, *** $p < .001$. C1 refers to the control variable, i.e., parental highest education level. V1-V10 refer to the 10 teaching practice variables showed in the same order as in Table 2.

teachers are occupied by other tasks was significantly but negatively related to the overall mathematics achievement of Japanese students, $p < .05$. Additionally, asking students to explain their answers was significantly but negatively related to the overall mathematics achievement of Korean students, $p < .05$. However, the instructional practice asking students to decide on their own procedures for solving complex problems was significantly and positively related to the overall mathematics achievement of students in Singapore ($p < .05$) and Japan ($p < .01$). Nevertheless, one of the conceptual teaching practices was related to students' performances in Chinese Taipei.

Third, this study found that in light of procedural teaching practices, the more frequently teachers in Hong Kong asked students to listen to teachers explain how to solve problems, the lower mathematics scores their students received. Similarly, the more frequently teachers in Singapore teachers asked students to memorize rules, procedures, and facts, the lower mathematics scores their students had. In contrast, the more frequently teachers in Japan teachers asked students to memorize rules, procedures, and facts, the higher mathematics scores their students had.

As shown in Table 3, when the effects of parental education level and the other instructional practice variables were controlled, the instructional practice that teachers ask students to listen to teachers explain how to solve problems was significantly but negatively related to the overall mathematics achievement of students in Hong Kong, $p < .05$. The procedural teaching practice that teachers ask students to memorize rules, procedures, and facts was significantly and positively related to the overall mathematics achievement of the students in Japan, but negatively in Singapore, both *p*s $< .05$. Again, none of the procedural teaching practices were related with students' mathematics achievements in Chinese Taipei and Korea.

Finally, comparatively speaking, the level of parental education had an unequal influence on students' mathematics performances in the five education systems, with the largest influence on students' mathematics achievement in Chinese Taipei and Japan, less influence in Korea, and the least in Singapore and Hong Kong. As shown in Table 3, the percentages of variances in students' mathematics achievement at level-2 explained by parental education level are 31%, 24%, 15%, 1%, and 0% for Chinese Taipei, Japan, Korea, Hong Kong, and Singapore respectively after controlling the influences of all the other teaching variables.

Discussions and Conclusions

Setting it apart from other studies that often used the whole student sample in their analysis, this study examined the relationship between the instructional practices used by those high-achieving teachers and students' mathematics performance in the five Asian education systems, with an aim to identify the practices that are conducive to students' achievement in each of the five high-achieving Asian education systems. Results from this study lead to the following understanding about the research questions and relevant assumptions of conceptual, procedural teaching, as well as the role of teaching quality in improving student mathematics achievement.

First, this study found that three conceptually oriented teaching practices were related to students' mathematics performance in the opposite direction in some of the Asian education systems. The practice of teachers asking students to work on problems (individually or with peers) while the teacher was occupied with other tasks was negatively related to Japanese students' mathematics achievement. This result disagrees with the findings in the study by Hamilton and Martinez (2007). The practice of teachers asking students to explain their answers in Korea was also found to be negatively related to their students' mathematics achievements. It is also inconsistent with the findings in the study related to Japan (House & Telese, 2008). However, both teaching practices have been identified as popular teaching practices in Japan by TIMSS video studies (Hiebert et al., 2005; Stigler & Hiebert, 2004) as well as others (Hamilton & Martinez, 2007).

However, the practice of teachers asking students to decide on their own procedures for solving complex problems was found to be positively related to the mathematics achievement of students in Singapore and Japan. This result regarding the Japanese sample is consistent with House and Telese (2008) and somewhat consistent with the study by Hamilton and Martinez (2007), who revealed that this practice was positively, though not significantly, correlated with student mathematics performances in Japan. The minor discrepancy can be caused by the fact that the current study only included the high-achieving classes in these two countries while the study by Hamilton and Martinez (2007) used the whole sample of Japanese teachers.

As indicated in the analysis, not all of the five conceptually oriented practices were found to be conducive to students' mathematics achievement in the five education systems and some were actually found to have negatively influenced students' mathematics performances. This result in the study regarding conceptually oriented instructional practices raises a challenge to the assumed effectiveness of conceptually oriented mathematics teaching practices on mathematics learning (Hiebert et al., 1997; Romberg, 1990; Thompson, 2001). Thus, it is important for researchers to develop understanding as to why certain conceptual teaching practices are effective while others are not, by using exploratory qualitative research approaches. It is also important for policy makers to be cautious in using the theoretically assumed effective teaching practices to guide policy decision-making (Kember, 2000; Stevenson & Lee, 1995).

Second, this study identified two procedural teaching practices that were related to students' mathematics performance in the opposite manners in some of the Asian education systems. The practice that teachers ask students to listen to teachers' explanations of how to solve problems was found to be significantly related to Hong Kong students' mathematics achievement in a negative way but not significantly related to student mathematics achievements in the other four education systems. This practice, which quite obviously indicates a teacher-centered lecture style instruction, is supported by some scholars (Geary, 1994; Wu, 1999) but is not encouraged by the others due to its inability to motivate the students to actively discover and justify their own answers (Hiebert et al., 1997; Romberg, 1990). Some prior studies found that mathematics teachers in Hong Kong and other East and Southeast Asian education systems frequently use such instructional practice (Mok, 2009; Mok et al., 2008; Stevenson & Lee, 1995; Stevenson & Stigler, 1992; Stigler & Stevenson, 1991), but they did not link this practice to students' achievement. This result from the current study adds to our understanding that although this teaching practice is typically used by teachers in some of these systems, it might not actually help students to learn mathematics effectively since it was negatively related to Hong Kong students' mathematics achievement, and was not significantly related to students' mathematics achievement in Korea, Singapore, Chinese Taipei, or Japan. Such a result challenges the popular assumption in mathematics education reform in East and Southeast Asia as well as in the West that procedural teaching supports the learning of mathematics (Geary, 1994; Greeno et al., 1996; Wu, 1999).

In contrast, the other influential procedurally oriented practice—asking students to memorize rules, procedures and facts—was found to be significantly and positively related to the mathematics achievement of students in Japan, but negatively associated with students in Singapore. This practice is one of the major foci of procedural teaching as scholars (Geary, 1994; Wu, 1999) argued that students' solid memorization of algorithms, facts and rules help develop students' basic mathematics knowledge and skills necessary for them to solve complex problems. However, contrary to the perception in the western world that asking students to memorize rules, procedures and facts belongs to the lowest level in the learning hierarchy and thus, does not encourage the learners to engage in high-level and critical thinking (Krathwohl, 2002), this type of teaching has its popularity in the Asian education systems as prior studies reported that teachers in these places use such teaching practice quite frequently (Biggs, 1994; Kember, 2000; Yeo & Zhu, 2005, May/June). Nevertheless, the fact that the current study found this teaching practice was conducive to students' mathematics learning only in Japan, not in Korea, Chinese Taipei, Hong Kong, or Singapore suggests that this procedurally oriented teaching practice, though prevalent, might be helpful in influencing student performances in one Asian education system, but not in the others. Again, it challenges the assumption held by supporters of procedural teaching that procedural teaching plays an important role in building the necessary basic knowledge and skills for students' higher order mathematics learning (Geary, 1994; Greeno et al., 1996; Wu, 1999).

Together, the above results about the relationship between procedural teaching and students' mathematics learning suggest that it is important for the researchers to develop understanding about why certain procedural teaching practices are effective while the others are not for students' mathematics learning and how the contexts of school and students' backgrounds come into play using exploratory qualitative research approaches. It is also important for the policy makers to refrain from thinking either that, since these systems have the highest student achievement, any teaching practices their teachers use are useful in boosting students' learning outcome, or the theoretically assumed ineffective teaching practices will ultimately be ineffective for all kinds of students in any education systems and thus use such thinking to guide their policy decision making (Leung, 2001).

Third, although often seen as similar in the Western literature, mathematics teaching practices can be quite divergent and there were few shared patterns of effective teaching used by the high-performing teachers across these five education systems. According to Table 3, this conceptual practice is significantly and negatively related to student learning (−8.39*), so I think this practice can not be effective for student learning. This result seems consistent with the recommendation provided by the National Mathematics Advisory Panel (2008), who called for an end to the debate about the relative importance of conceptual and procedural teaching in helping improve students' mathematics learning as "conceptual understanding of mathematical operations, fluent execution of procedures, and fast access to number combinations jointly support effective and efficient problem solving" (p. xix).

Furthermore, as suggested in the literature (Leung, 2001), a certain type of practice might be perceived to have contrasting roles. A case in point is the practice of teachers asking students to memorize rules and procedures, which was regarded as an ineffective practice of "rote learning" in the U.S. literature (NCTM, 1989, 2000). However, this practice has been traditionally and widely used in several of the high-achieving East Asian education systems and has presumably helped students learn math well (Leung, 2001). Nonetheless, this study found rote learning to be conducive to students' mathematics learning in Japan, which provides evidence that variation exist in the high performing Asian education systems even for on type of teaching practice. Therefore, the conceptualization of quality mathematics teaching varies in these Asian education systems and also differs from that in the Western world such as the US. Results from the study support the effort to formulate an alternative conceptualization of the effective teaching used by East Asian mathematics teachers (Clarke, 2006; Clarke et al., 2008; Leung, 2001, 2005).

Finally, the study shows that the role of parental education in shaping student mathematics performance is also different among these five education systems. For example, the high percentage of variance in students' mathematics achievement explained by parental education level in Chinese Taipei, Japan, and Korea is consistent with findings in the western literature (Coleman et al., 1966; Hanushek, 1996, 1997). However, it is interesting to note the extremely low impact of parental education on student achievement in Singapore and Hong Kong and the comparatively higher impact of teaching quality on Japanese students' achievement. Future studies could look into such phenomena and gain a better understanding regarding the reasons why the roles of parental education level and teaching quality varied greatly among these systems.

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