

SKETCH

THE MATCHING LAW

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This article introduces the quantitative analysis of choice behavior by describing a number of equations developed over the years to describe the relation between the allocation of behavior under concurrent schedules of reinforcement and the consequences received for alternative responses. Direct proportionality between rate of responding and rate of reinforcement was observed in early studies, suggesting that behavioral output matched environmental input in a mathematical sense. This relation is termed “strict matching,” and the equation that describes it is referred to as “the matching law.” Later data showed systematic departures from strict matching, and a generalized version of the matching equation is now used to describe such data. This equation, referred to as “the generalized matching equation,” also describes data that follow strict matching. It has become convention to refer to either of these equations as “the matching law.” Empirical support for the matching law is briefly summarized, as is the applied and practical significance of matching analyses.

Key words: matching law, matching equation, generalized matching law, choice, concurrent schedule of reinforcement

Science is fundamentally quantitative—everyone knows that $E = mc^2$ —and many behavior analysts have endeavored to develop equations that relate behavioral outputs to environmental inputs. Richard Herrnstein was a pioneer in this area, and his research formed the foundation for the quantitative analysis of behavior. In his initial study, Herrnstein (1961) exposed three food-deprived pigeons to conditions where concurrent variable-interval (VI) schedules of food delivery were arranged on two response keys. In most conditions, a 1.5-s changeover delay (COD) was arranged. The COD ensured that no food could be delivered until at least 1.5 s had elapsed from the time a bird switched from pecking one key to pecking the other. In the

Portions of this manuscript are based on a prior introduction to matching by Poling and Foster (1993).

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absence of a COD, rapid switching between alternatives typically occurred. Several different VI-VI combinations were compared. Each VI schedule specified that food became available aperiodically, with the average time between successive food availabilities equal to the schedule value (e.g., 60 s under a VI 60-s schedule), and was delivered dependent on a keypeck response. Herrnstein recorded the number of pecks emitted and the number of food deliveries obtained under each alternative. When these measures were compared across schedule combinations, the relative proportion of responses emitted under a schedule matched the relative proportion of reinforcers (food deliveries) earned under that alternative. Said differently, Herrnstein found that in pigeons, relative rates of responding *matched* relative rates of reinforcement.

Herrnstein expressed this relation in a simple algebraic formula:¹

$$B_1/(B_1 + B_2) = R_1/(R_1 + R_2), \quad (1)$$

where B_1 is behavior (i.e., total responses) allocated to Alternative 1, B_2 is behavior allocated to Alternative 2, R_1 is the number of reinforcers received under Alternative 1, and R_2 is the number of reinforcers received under Alternative 2.

As McDowell (1988) indicated,

[Equation 1] is frequently referred to as the matching law. It constitutes a precise, deterministic, and mathematical understanding of choice behavior. According to the equation, organisms distribute their behavior across available response alternatives in the same proportion that reinforcers are distributed across those alternatives. (p. 99)

When data obtained under concurrent VI-VI schedules are entered into Equation 1 and portrayed graphically with values from the left side of the equation [$B_1/(B_1 + B_2)$] portrayed on the y-axis and values from the right side of the equation [$(R_1/(R_1 + R_2))$] portrayed on the x-axis, a regression line fitted through the data points has a slope of 1.0 and a y-intercept of 0 if matching is present. To understand what this means in terms of actual behavior, envision an experiment in which a pigeon is exposed for 1 hour each day to a concurrent VI-VI schedule of food delivery. Throughout the experiment, which comprises four conditions, the VI schedule in effect for pecks on the left key is VI 60 s. Across conditions, the schedule in effect for pecks on the right key is VI 60 s, VI 120 s, VI 30 s, and VI 240 s. Each schedule is in effect until responding is stable. Table 1 shows hypothetical data for the last session of exposure to each of these pairs of schedules. These data demonstrate what many authors refer to as "strict" (or exact) matching (e.g., Davison & McCarthy, 1988; Mazur 1991). Strict matching is present when Equation 1 provides a good description of obtained results.

1 There is no consistency in the letters used to refer to particular variables as equations were presented in the original articles. To reduce confusion, we have used the same letters in all equations.

Table 1
Hypothetical Data Showing Strict Matching

Schedules	Responses	$B_1/(B_1+B_2)$	Reinforcers	$R_1/(R_1+R_2)$
VI 60 s	2,000		60	
VI 60 s	2,000	0.5	60	0.5
VI 60 s	2,680		60	
VI 120 s	1,320	0.67	30	0.67
VI 60 s	1,320		60	
VI 30 s	2,680	0.33	120	0.33
VI 60 s	3,200		60	
VI 240 s	800	0.8	15	0.8

Strict matching often occurs under the conditions studied by Herrnstein (1961). However, Equation 1 does not provide a good description of the relation between responding and reinforcement parameters under some concurrent schedule arrangements. Obtained results deviate systematically from the predictions of Equation 1 in three ways: undermatching, overmatching, and bias (Baum, 1974, 1979). In undermatching, relatively more behavior than the amount predicted by Equation 1 is allocated to the alternative providing less reinforcement. In overmatching, relatively more behavior is allocated to the alternative providing more reinforcement. In bias, the amount of behavior allocated to one alternative is consistently higher or lower than the amount predicted by Equation 1, regardless of whether that alternative yields more or less reinforcement than the other response option. As Baum (1979) pointed out, these deviations from strict matching do not generate linear data when proportions are plotted. Overmatching will result in S-shaped functions with the data below the strict matching line at reinforcer proportions less than 0.5 and above it at reinforcer proportions greater than 0.5; undermatching will result in data above the strict matching line at reinforcer proportions less than 0.5 and below the strict matching line at reinforcer proportions greater than 0.5. A consistent bias will affect reinforcer proportions around 0.5 more than it will the extreme values (closer to 0 or 1.0), resulting in a curved data set either above or below the strict matching line, depending on the direction of the bias (Baum, 1974). Equation 1 does not provide a good description of such data.

The limitations of Equation 1 became clear as researchers investigated the effects of reinforcer amount and delay, reinforcer quality, maintenance schedules (i.e., concurrent schedules other than VI-VI), response parameters (force and topography), and other variables (see reviews by Davison & McCarthy, 1988, and McDowell, 2005). In an attempt to provide a better description of how response allocation is related to reinforcement parameters, several equations have been offered as alternatives to Equation 1. These equations provide alternative formulations to Herrnstein's initial matching law. All of them specify physical measures of behavior (time spent responding or responses emitted) on the left side of the equals sign and physical inputs to the animal (reinforcement variables) on the right side.

One equation that has generated considerable interest is the “generalized matching law.” This equation is based on the relation between the ratios of responses and reinforcers, rather than the proportions. When expressed in logarithmic form, which is common (Davison & McCarthy, 1988), the generalized matching law relates behavioral output to reinforcement according to the formula

$$\log (B_1/B_2) = a \log (R_1/R_2) + \log c, \quad (2)$$

where B_1 is behavior (or time) allocated to Alternative 1, B_2 is behavior (or time) allocated to Alternative 2, R_1 is reinforcement received under Alternative 1, R_2 is reinforcement received under Alternative 2, a is a slope of the regression line relating behavioral and reinforcement variables, and $\log c$ is the intercept of that line. In nonlogarithmic form, Equation 2 becomes Equation 3, which describes a power function:

$$(B_1/B_2) = c (R_1/R_2)^a, \quad (3)$$

where B_1 , B_2 , R_1 , and R_2 are as for Equation 2, a is the slope, and c is the y -intercept of the regression line relating input and output functions.

Figure 1 demonstrates how regression lines fitted to hypothetical data expressed in terms of Equation 2 would appear if matching, undermatching, overmatching, and bias were evident. Note that strict matching with no bias is present when the slope of the regression line (a in Equation 2) is 1.0 and the y -intercept ($\log c$ in Equation 2) is 0. Undermatching is evident if the slope (a) is less than 1.0, whereas a slope of greater than 1.0 indicates overmatching. A y -intercept greater or less than 0 is evidence of bias.

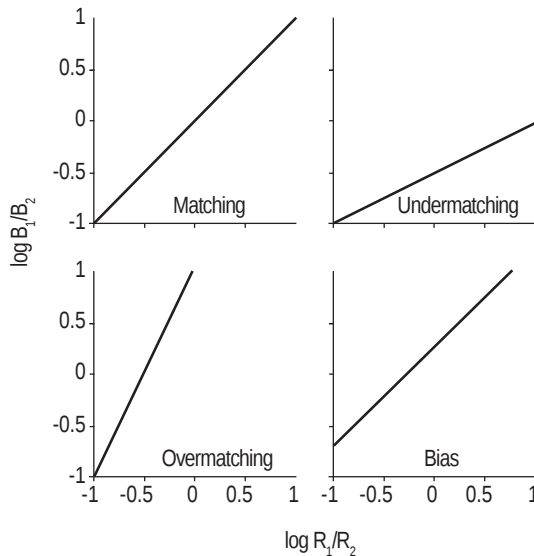


Figure 1. Regression lines demonstrating matching, undermatching, overmatching, and bias. These lines describe the relative allocation of responses or time to two alternatives (B_1 and B_2) as a function of the relative number of reinforcers earned under those alternatives (R_1 and R_2).

Data are transformed into logarithms (logs) when Equation 2 is used, and not everyone is familiar with this transform. In brief, a logarithm is the power to which a base number (typically, 10) has to be raised to produce

the number being expressed in log units. Table 2 shows several proportions, which can be envisioned as B_1/B_2 or R_1/R_2 , expressed as log values. Note that the logs for numbers less than 1.0 are negative. Simple on-line programs (e.g., www.1728.com/logrithm.htm, rechneronline.de/logarithm/) make log transforms easy, and spending a few minutes interacting with these programs makes logarithms easy to understand. The logarithms of ratios greater than 1.0 (e.g., 2.0, 3.0) will be the same distance from 0 as their inverses (here, 0.5 or 0.333), as $\log_{10} 2 = 0.301$ and $\log_{10} 0.5 = -0.301$, or $\log_{10} 3 = 0.477$ and $\log_{10} 0.33 = -0.477$. Logarithmic transforms shorten the number line needed to portray a set of observations and often transform functions that are curvilinear when the raw data are portrayed into linear functions, which are easier to describe and to remember. It is for these reasons that logarithmic transforms are common in many areas of science.

Table 2
Examples of Logarithmic Transforms (Base 10)

Proportion	Decimal equivalent	Log	Proportion	Decimal equivalent	Log
1000/1	1000	3.00	1/1000	0.001	-3.00
500/1	500	2.70	1/500	0.002	-2.70
250/1	250	2.40	1/250	0.004	-2.40
100/1	100	2.00	1/100	0.01	-2.00
50/1	50	1.70	1/50	0.02	-1.70
25/1	25	1.40	1/25	0.04	-1.40
10/1	10	1.00	1/10	0.10	-1.00
5/1	5	0.70	1/5	0.25	-0.70
1/1	1	0.00	1/1	1.00	0.00

The advantage of Equation 2 relative to Equation 1 is that the former provides an accurate mathematical description of the relation between measures of behavior and measures of reinforcement when the relation is not direct linear proportionality (i.e., strict matching does not occur). For example, Equation 2 provides a reasonable description of patterns of data demonstrating bias, as when concurrent VI variable-ratio (VR) schedules are arranged. Under such schedules, there is usually a consistent bias toward one alternative (Davison & McCarthy, 1988; Myerson & Hale, 1984).

Any pattern of data that is described well by Equation 1 is described equally well by Equation 2. When $a = 1$ and $\log c = 0$, Equation 2 is equivalent to Equation 1, and the data being described are similar to those portrayed in Table 1.

Equation 2 can be modified to take into account the effects of magnitude and delay, as well as frequency, of reinforcement. When all three are considered together, Equation 3 becomes the concatenated generalized matching law (Davison & McCarthy, 1988):

$$\log (B_1/B_2) = a \log (R_1/R_2) + b \log (A_1/A_2) + d \log (I_1/I_2) + \log c, \quad (4)$$

where I_1 and I_2 refer to immediacy (or, conversely, delay) of reinforcement under the two alternatives, A_1 and A_2 refer to the amounts or size of the reinforcers under the two alternatives, b and d refer to the sensitivity of behavior to changes in the amounts and immediacies, and all other values are as for Equation 2.

Matching equations suitable for describing behavior under what appear to be single-alternative situations, such as multiple schedules, have also been developed (Herrnstein, 1970). These equations are based on the notion that an organism always is faced with alternative responses, each reinforced under some schedule. This occurs even when only a single response class is systematically reinforced in an experimental setting. For example, a pigeon exposed to a simple VI schedule of food delivery for keypecking can emit a number of other responses that yield different reinforcers (e.g., grooming might reduce itching, turning might produce new sensory inputs). Herrnstein (1970) suggested that a simple description of the relation of these alternative sources of reinforcement to the rate of emission of the scheduled operant is provided by the equation

$$B_1 = k(R_1)/(R_1 + R_0), \quad (5)$$

where B_1 is behavior allocated to Alternative 1 (the scheduled, or known, operant), R_1 is reinforcement received under Alternative 1, k is the asymptotic rate of B_1 , and R_0 is the sum of all reinforcement other than that produced under Alternative 1. If it is assumed that R_0 and k are constant across conditions, Equation 5 predicts a hyperbolic relation between rate of responding and rate of reinforcement under a given simple (or multiple) schedule (Herrnstein, 1970; McDowell, 1988).

The Provenance of Matching

A large number of studies, reviewed elsewhere (Baum, 1979; Davison & McCarthy, 1988; de Villiers, 1977; Mazur, 1991; McDowell, 2005; Pierce & Epling, 1983; Williams, 1988), indicate that the generalized matching law typically provides a good description of data collected under concurrent VI-VI schedules of reinforcement. In many such studies, however, undermatching is evident (Robinson, 1992; Wearden & Burgess, 1982). When magnitude or delay of reinforcement is manipulated, rather inconsistent results have been obtained (Davison & McCarthy, 1988; Mazur, 1991); and when concurrent schedules other than VI-VI are examined, matching may or may not obtain. Finally, to date, matching analyses have been shown to apply only to contingency-shaped responding. None of the basic research from which the matching equations were derived dealt with rule-governed (or other verbally controlled) behavior. Moreover, absent from that literature are studies dealing with behavioral consequences delayed by more than a few seconds. Whether matching analyses can be usefully applied to outcomes that are long delayed (e.g., financial rewards in organizations) is unknown. Moreover, choice behavior under discrete-trials procedures is not adequately described by any version of the matching equation, which was developed to describe performance under free-operant arrangements. Under discrete-trial arrangements, in nonverbal organisms choice typically is directed exclusively to the alternative that provides more rapid access to the reinforcer (e.g., Hall-Johnson & Poling, 1984; Ragotzy, Blakely, & Poling, 1988). In contrast, verbal humans frequently maximize, directing essentially all of their choice responses to the alternative that produces the greatest amount of the reinforcer overall (e.g., Blakely, Starin, & Poling, 1988).

Despite the foregoing limitations, the generalized matching law with its two free parameters (a and c) provides a reasonable after-the-fact description of the relation between response allocation and environmental inputs

(reinforcement dimensions) under a range of conditions. Interestingly, it does so with respect to the behavior of groups as individuals (e.g., Baum & Kraft, 1998) as well as with respect to the responses of individual participants, and there is some evidence that matching occurs with minimal opportunity to learn (i.e., the tendency toward strict matching is innate) (e.g., Gallistel et al., 2007). With appropriate modification, the generalized matching equation will accommodate dimensions of reinforcement other than frequency. But it is important to recognize that the generalized matching equation, like all other versions of the matching law, allows us to make quantitative predictions about how an organism will allocate its time or behavior in a given circumstance only if we (a) have sufficient historical data to solve the equation and (b) know that current conditions are equivalent to those under which the equation was solved. The matching equation is not a "law" in the sense of providing an accurate a priori description of invariant relations between environmental inputs and behavioral outputs.

Equation 5, and the other equations developed by Herrnstein (1970) to extend matching to single- and multiple-schedule arrangements, have been tested in a wide variety of experimental settings. Davison and McCarthy (1988) noted that "Herrnstein's (1970) simple model has received extensive disconfirmation since it was proposed. Thus, it cannot be sustained in its original form" (p. 45). More recently, however, McDowell (2005) reached the opposite conclusion, stating that Equation 5 describes single-schedule data quite well.

Applied Behavior Analysis and Matching

From 1983 through 2003, two of Herrnstein's articles (1961, 1970) have been more frequently cited in the *Journal of Applied Behavior Analysis* than any other data-based article from the *Journal of the Experimental Analysis of Behavior* (Elliott, Morgan, Fuqua, Ehrhardt, & Poling, 2005; Poling, Alling, & Fuqua, 1994). This suggests that matching analyses can be extended to socially significant behaviors, and some data obtained in applied settings demonstrate that this is the case (e.g., Borrero & Vollmer, 2002; Borrero et al., 2007; Bradshaw & Szabadi, 1988; McDowell, 1982, 1988; Neef, Mace, Shea, & Shade, 1992; Neef, Shade, & Miller, 1994). For example, when McDowell (1981) examined the rate of self-injurious scratching by a 10-year-old boy as a function of the rate of verbal reprimands from family members, the relation was a hyperbola as predicted by Equation 5. Reprimands were shown to be serving a reinforcing function, and it was also demonstrated that the rate of scratching was directly (but hyperbolically) related to the rate of reinforcement.

Although a few attempts to apply matching analyses to the behavior of humans in their everyday environment have been little short of far-fetched, as Poling and Foster (1993) discussed, several studies of human behaviors of no direct clinical significance have demonstrated matching in noncontrived environments. For example, in the realm of sports psychology, three studies show that the generalized matching equation provides a reasonable description of the relation between the relative number of two- and three-point shots taken by basketball players and the relative number of points yielded by those shots (Alferink, Critchfield, Hitt, & Higgins, 2009; Romanowich, Bourret, & Vollmer, 2007; Vollmer & Bourret, 2000). A fourth study demonstrates that

the generalized matching equation does so with respect to play selection (runs vs. passes) and yards yielded in professional football games (Reed, Critchfield, & Martens, 2006). The practical significance of these findings is unclear—it is hard to see, for instance, how the findings pertaining to basketball could be used to improve the scoring of individuals or teams—but they do show an interesting relation between behavioral outputs (kinds of shots taken) and environment inputs (points earned from those kinds of shots). That, in and of itself, is noteworthy.

In addition to describing the allocation of human behavior in everyday and clinically significant settings, the matching equation can be used to index what is important to nonhuman animals, and hence relevant to their welfare (Foster, Temple, & Poling, 1998). As an example of this strategy, McAdie, Foster, Temple, and Matthews (1996) exposed domestic hens to concurrent VI-VI schedules of food delivery and demonstrated matching. Subsequently, pecks on one key intermittently produced the sounds of a poultry shed. This arrangement biased responding away from the key that produced the sounds, indicating that the sounds that many hens hear throughout their lives were aversive to the experimental subjects (as they are to humans exposed to them). Such findings certainly appear to have welfare implications.

Concluding Comments

Ever since the first version of the matching equation was introduced by Herrnstein in 1961, the quantitative analysis of choice responding has generated a great deal of empirical and conceptual interest among behavior analysts. To see evidence of the strength of this interest, one has only to attend the annual meeting of SQAB (Society for the Quantitative Analysis of Behavior), which accompanies the Association for Behavior Analysis International conference. The work of its members, and of other behavior analysts, makes it abundantly clear that behavior is lawfully related to environmental events—so lawfully related, in fact, that it is possible to write equations that accurately describe how the two covary. Herrnstein's initial matching equation (Equation 1) and the generalized matching equations (Equations 2 and 3, the logarithmic and power function versions, respectively) are arguably the best known of these equations. As revealed in a recent and comprehensive review by McDowell (2005), it is clear that whereas the generalized matching equation accurately describes concurrent-schedule data under a substantial range of conditions, Herrnstein's original equation fails to do so. Neither is a "law" in the sense of describing an invariant relation, but the generalized matching equation is a powerful descriptive tool that every behavior analyst should understand.

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