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IMC: A Method for Interval Calculus in Matrix

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Abstract. Time representation is important in many applications, such as temporal databases, planning, and multi-agents. Since Allen's work on binary interval relations (called interval algebra), many researchers have further investigated temporal information processing based on interval calculus. However, there are still some limitations, e.g. constraint satisfaction is a NP-hard problem in interval calculus. For this reason, we propose a new interpretation for interval relationships and their calculus in this paper, which establishes a new method to transform interval calculus into matrix calculus. Our experiments show that this method propagates temporal relations faster than interval algebra.

Keywords: Planning, temporal logic, interval algebra, interval calculus, temporal reasoning.

1 Introduction

In 1983, Allen [1] proposed a time world model of interval algebra (IA). This model has been successfully applied to multi-agents [7], temporal databases [6], simulations [8], and concurrent systems [2]. However, there are still some limitations; for instance, constraint satisfaction is an NP-hard problem in interval calculus. For this reason, we establish a new method of describing interval calculus in this paper, in which temporal relationships are represented with matrices (called as IMC). This method can enrich Allen's interval algebra and exhibit a new interpretation to interval relations. Our experiments show that it propagates temporal relations faster than interval algebra. For example, assume $I_1 > I_2$, I_2 m I_3, and I_3 o I_4, then we can solve the possible relations between I_1 and I_4

by propagation law in IA. This method requires one to access the temporal relation table two times: (1) for calculating $(I_1 > I_2) \odot (I_2 \text{ m } I_3) = I_1 \ \alpha \ I_3$, we need to determine α by accessing the temporal relation table and, (2) for calculating $(I_1 \ \alpha \ I_3) \odot (I_3 \text{ o } I_4) = I_1 \ \beta \ I_4$, we need to determine β by accessing the temporal relation table. But in our matrix model, we only need to compute $M_{I_1,I_2} \circ M_{I_2,I_3} \circ M_{I_3,I_4}$ and access the temporal relation table once. Our method is clearly superior where the propagation of temporal relations are concerned.

The rest of this paper is organized as follows. We begin in Sec. 2 by representing interval relationships in a matrix. In Sec. 3, we show the temporal relational calculus model in matrix. In Sec. 4, we present the rules of propagating temporal relationships. Finally, a summary of this paper is presented in the last section.

2 Representing Interval Relationships in Matrix

We begin by briefly defining the temporal model and relations in matrices. For simplification, let $U = [0, NOW]$ be the universe of time, where "NOW" is undetermined, and used to represent the current time.

A time interval I is an ordered pair (I^-, I^+) such that $I^- < I^+$, where I^- and I^+ are interpreted as points on the real line. An interval interpretation or I-interpretation is a mapping of a time interval to pairs of distinct real numbers such that the beginning of the interval is strictly before the end of the interval.

Definition 1 *Let $I \subseteq U$. If $a \leq b \leq c \wedge a \in I \wedge c \in I \longrightarrow b \in I$, then I is called a* convex interval *over U.*

2.1 Interval Algebra

Allen's time world model of interval algebra [1] is based on thirteen possible relationships between two time intervals as follows.

In interval algebra, unions of the basic interval relations are used to express the uncertain information. There are 2^{13} unions of binary interval relations and the set of all binary interval relation unions is denoted by $\mathfrak{R}$. The elements of $\mathfrak{R}$ are denoted by γ, β, α in the following. In particular, the null relation is denoted as $\emptyset$ and the universal relation is denoted as Ω. An atomic formula of the form

$$I \ \{\beta_1, \cdots, \beta_n\} \ J$$

is called interval formula and it is denoted by φ.

Allen's interval algebra consists of the set $\mathfrak{R} = 2^{\Omega}$ of all binary interval relations and operators: $-1, \cap, \oplus$, where -1 denotes the operation unary converse, $\cap$ denotes binary intersection, and $\oplus$ denotes binary composition. Using Allen's interval algebra, some forms of the constraint propagation algorithm have been proposed for reasoning in this framework [1]. However, there are still some limitations. For instance, constraint satisfaction is an NP-hard problem in interval calculus. For this reason, we propose a new interpretation for interval calculus in this paper.

Table 1. The thirteen basic relations ($I^- < I^+$ and $J^- < J^+$)

Interval Relation	*Symbol*	*Endpoint Relations*
I before J	$<$	$I^+ < J^-$
I after J	$>$	$I^- > J^+$
I meets J	m	$I^+ = J^-$
I met-by J	mi	$I^- = J^+$
I overlaps J	o	$I^- < J^-, I^+ > J^-, I^+ < J^+$
I overlapped-by J	oi	$I^- > J^-, I^- < J^+, I^+ > J^+$
I during J	d	$I^- > J^-, I^+ < J^+$
I includes J	di	$I^- < J^-, I^+ > J^+$
I starts J	s	$I^- = J^-, I^+ < J^+$
I started-by J	si	$I^- = J^-, I^+ > J^+$
I finishes J	f	$I^- > J^-, I^+ = J^+$
I finished-by J	fi	$I^- < J^-, I^+ = J^+$
I equals J	$=$	$I^- = J^-, I^+ = J^+$

2.2 A New Representation

In this subsection, we establish a new method of representing the above temporal relationships.

For a convex interval I, it can be fallen U into I^L, I^-, I^1, I^+, I^R, where, I^L, I^-, I^1, I^+, I^R are the set of the left outer-points of I, the set of the left end-point of I, the set of the inner-points of I, the set of the right end-point of I, the set of the right outer-points of I, respectively. In other words, I is uniquely determined by I^L, I^-, I^1, I^+, I^R, and $I^L \cup I^- \cup I^1 \cup I^+ \cup I^R = U$, for $a, b \in \{I^L, I^-, I^1, I^+, I^R\}$, if $a \neq b$, then $a \cap b = \emptyset$. For example, let $I = [5, 11.2]$, then $I^L = [0, 5)$, $I^- = \{5\}$, $I^1 = (5, 11.2)$, $I^+ = \{11.2\}$, $I^R = (11.2, NOW]$. We will apply these elements of intervals to determine temporal relations between any two intervals. In order to do so, some needed operators are defined as follows.

Definition 2 *Let I, J be two convex intervals, the operator "$\bullet$" is defined as,*

(a) $I^* \bullet J^@ \in \{0, 1\}$, *and*
(b) $I^* \bullet J^@ = 1$ *iff the intersection of I^* and $J^@$ is non-empty, otherwise,* $I^* \bullet J^@ = 0$.

where $*, @ \in \{L, -, 1, +, R\}$.

For example, let $I = [3, 11.2]$ and $J = [8, 17]$, then $I^L \bullet J^L = 1$ because $I^L \cap J^L = [0, 3) \cap [0, 8) \neq \emptyset$, also $I^- \bullet J^- = 0$, $I^1 \bullet J^1 = 1$, $I^+ \bullet J^+ = 0$, $I^R \bullet J^R = 1$.

Lemma 1 *Let I, J and K be three convex intervals, the operator "$\bullet$" satisfies the following rules,*

(R1) *idempotent law:* $\overbrace{I^* \bullet I^* \bullet \cdots \bullet I^*}^{n} = I^* \bullet I^*$,
(R2) *commutativity law:* $I^* \bullet J^{@} = J^{@} \bullet I^*$,
(R3) *associativity law:* $(I^* \bullet J^{@}) \bullet K^{\%} = I^* \bullet (J^{@} \bullet K^{\%})$,
(R4) *absorptive law:* $I^* \bullet (I^* \bullet J^{@}) = I^* \bullet J^{@}$,
(R5) $U \bullet I^* = I^* \bullet U = I^*$,
(R6) $\emptyset \bullet I^* = I^* \bullet \emptyset = \emptyset$.

where $*, @, \% \in \{L, -, 1, +, R\}$.

Proof. This can be proved directly by using the properties of sets.

Furthermore, we can define temporal relationships between two convex intervals as follows.

Definition 3 *Let I and J be two convex intervals, the temporal relationships between I and J can be described in the following matrix $M_{I,J}$:*

$$M_{I,J} = \begin{bmatrix} I^L \bullet J^L & I^- \bullet J^L & I^1 \bullet J^L & I^+ \bullet J^L & I^R \bullet J^L \\ I^L \bullet J^- & I^- \bullet J^- & I^1 \bullet J^- & I^+ \bullet J^- & I^R \bullet J^- \\ I^L \bullet J^1 & I^- \bullet J^1 & I^1 \bullet J^1 & I^+ \bullet J^1 & I^R \bullet J^1 \\ I^L \bullet J^+ & I^- \bullet J^+ & I^1 \bullet J^+ & I^+ \bullet J^+ & I^R \bullet J^+ \\ I^L \bullet J^R & I^- \bullet J^R & I^1 \bullet J^R & I^+ \bullet J^R & I^R \bullet J^R \end{bmatrix}$$

For example, the temporal relationship between $I = [3, 11.2]$ and $J = [8, 17]$ is as follows:

$$M_{I,J} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Based on Definition 3, the thirteen different possible temporal relations between two intervals can be transformed into thirteen matrices as follows. Let μ and γ be the intervals.

(1) μ BEFORE γ (or $\mu < \gamma$) if and only if

$$M_{\mu,\gamma} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

This matrix is denoted as M_1.

(2) μ AFTER γ (or $\mu > \gamma$) if and only if

$$M_{\mu,\gamma} = \begin{bmatrix} 1\,0\,0\,0\,0 \\ 1\,0\,0\,0\,0 \\ 1\,0\,0\,0\,0 \\ 1\,0\,0\,0\,0 \\ 1\,1\,1\,1\,1 \end{bmatrix}$$

This matrix is denoted as M_2, and M_2 is the converse of M_1, or $M_2 = M_1^{-1}$. Note that M_1^{-1} means, for $M_2 = (a_{ij})_{5\times5}$, $M_1 = (b_{ij})_{5\times5}$, then $M_2 = M_1^{-1} \longrightarrow a_{ij} = b_{ji}, i, j = 1, 2, 3, 4, 5$.

(3) μ MEETS γ (or μ m γ) if and only if

$$M_{\mu,\gamma} = \begin{bmatrix} 1\,1\,1\,0\,0 \\ 0\,0\,0\,1\,0 \\ 0\,0\,0\,0\,1 \\ 0\,0\,0\,0\,1 \\ 0\,0\,0\,0\,1 \end{bmatrix}$$

This matrix is denoted as M_3.

(4) μ MET BY γ (or μ mi γ) if and only if

$$M_{\mu,\gamma} = \begin{bmatrix} 1\,0\,0\,0\,0 \\ 1\,0\,0\,0\,0 \\ 1\,0\,0\,0\,0 \\ 0\,1\,0\,0\,0 \\ 0\,0\,1\,1\,1 \end{bmatrix}$$

This matrix is denoted as M_4, and $M_4 = M_3^{-1}$.

(5) μ OVERLAPS γ (or μ o γ) if and only if

$$M_{\mu,\gamma} = \begin{bmatrix} 1\,1\,1\,0\,0 \\ 0\,0\,1\,0\,0 \\ 0\,0\,1\,1\,1 \\ 0\,0\,0\,0\,1 \\ 0\,0\,0\,0\,1 \end{bmatrix}$$

This matrix is denoted as M_5.

(6) μ OVERLAPPED BY γ (or μ oi γ) if and only if

$$M_{\mu,\gamma} = \begin{bmatrix} 1\,0\,0\,0\,0 \\ 1\,0\,0\,0\,0 \\ 1\,1\,1\,0\,0 \\ 0\,0\,1\,0\,0 \\ 0\,0\,1\,1\,1 \end{bmatrix}$$

This matrix is denoted as M_6, and $M_6 = M_5^{-1}$.

(7) μ FINISHES γ (or μ f γ) if and only if

$$M_{\mu,\gamma} = \begin{bmatrix} 1\,0\,0\,0\,0 \\ 1\,0\,0\,0\,0 \\ 1\,1\,1\,0\,0 \\ 0\,0\,0\,1\,0 \\ 0\,0\,0\,0\,1 \end{bmatrix}$$

This matrix is denoted as M_7.

(8) μ FINISHED BY γ (or μ fi γ) if and only if

$$M_{\mu,\gamma} = \begin{bmatrix} 1\,1\,1\,0\,0 \\ 0\,0\,1\,0\,0 \\ 0\,0\,1\,0\,0 \\ 0\,0\,0\,1\,0 \\ 0\,0\,0\,0\,1 \end{bmatrix}$$

This matrix is denoted as M_8, and $M_8 = M_7^{-1}$.

(9) μ DURING γ (or μ d γ) if and only if

$$M_{\mu,\gamma} = \begin{bmatrix} 1\,0\,0\,0\,0 \\ 1\,0\,0\,0\,0 \\ 1\,1\,1\,1\,1 \\ 0\,0\,0\,0\,1 \\ 0\,0\,0\,0\,1 \end{bmatrix}$$

This matrix is denoted as M_9.

(10) μ CONTAINS γ (or μ di γ) if and only if

$$M_{\mu,\gamma} = \begin{bmatrix} 1\,1\,1\,0\,0 \\ 0\,0\,1\,0\,0 \\ 0\,0\,1\,0\,0 \\ 0\,0\,1\,0\,0 \\ 0\,0\,1\,1\,1 \end{bmatrix}$$

This matrix is denoted as M_{10}, and $M_{10} = M_9^{-1}$.

(11) μ STARTS γ (or μ s γ) if and only if

$$M_{\mu,\gamma} = \begin{bmatrix} 1\,0\,0\,0\,0 \\ 0\,1\,0\,0\,0 \\ 0\,0\,1\,1\,1 \\ 0\,0\,0\,0\,1 \\ 0\,0\,0\,0\,1 \end{bmatrix}$$

This matrix is denoted as M_{11}.

(12) μ STARTED BY γ (or μ si γ) if and only if

$$M_{\mu,\gamma} = \begin{bmatrix} 1\,0\,0\,0\,0 \\ 0\,1\,0\,0\,0 \\ 0\,0\,1\,0\,0 \\ 0\,0\,1\,0\,0 \\ 0\,0\,1\,1\,1 \end{bmatrix}$$

This matrix is dented as M_{12}, and $M_{12} = M_{11}^{-1}$.

(13) μ EQUALS γ (or $\mu = \gamma$) if and only if

$$M_{\mu,\gamma} = \begin{bmatrix} 1\,0\,0\,0\,0 \\ 0\,1\,0\,0\,0 \\ 0\,0\,1\,0\,0 \\ 0\,0\,0\,1\,0 \\ 0\,0\,0\,0\,1 \end{bmatrix}$$

This matrix is denoted as M_{13}.

This means that we can use the above matrices to represent possible temporal relationships as Table 2.

Table 2. Interval relationships in IA and in IMC

IA	$<$	$>$	m	mi	o	oi	f
IMC	M_1	M_2	M_3	M_4	M_5	M_6	M_7
IA	fi	d	di	s	si	$=$	
IMC	M_8	M_9	M_{10}	M_{11}	M_{12}	M_{13}	

3 Temporal Relational Calculus

Allen pointed out that given the temporal relationships of every two events, we can obtain a time relationship table whereby all events are consistently satisfied. For example, supposed I_1, I_2, I_3 are three convex intervals, $I_1 < I_2$, and I_2 mi I_3. Then by propagation law, the relation of I_1 and I_3 is $I_1\{<, o, m\}I_3$. In order to propagate temporal relations like Allen's model, we now define the temporal relational calculus for the above matrix method.

Definition 4 *Let I, J and K be three convex intervals, the operators "$\circ$" and "$\uplus$" are defined as,*

(1) $J^! \bullet K^\% \circ I^* \bullet J^@ \in \{0,1\}$, and $J^! \bullet K^\% \circ I^* \bullet J^@ = 1$ iff both $J^! \bullet K^\% = 1$ and $I^* \bullet J^@ = 1$, otherwise, $J^! \bullet K^\% \circ I^* \bullet J^@ = 0$;

(2) $J^! \bullet K^\% \uplus I^* \bullet J^@ \in \{0,1\}$, and $J^! \bullet K^\% \uplus I^* \bullet J^@ = 1$ iff either $J^! \bullet K^\% = 1$ or $I^* \bullet J^@ = 1$, otherwise, $J^! \bullet K^\% \uplus I^* \bullet J^@ = 0$

where $*, @, !, \% \in \{L, -, 1, +, R\}$. *Intuitively,* $\circ$ *means logical "AND",* $\uplus$ *means logical "OR".*

Lemma 2 *Let I, J and K be three convex intervals, the operator "*$\circ$*" and "*$\uplus$*" satisfy*

(i) $J^! \bullet K^{\%} \circ I^* \bullet J^! = I^* \bullet K^{\%}$;
(ii) $J^! \bullet K^{\%} \uplus I^* \bullet J^! = J^! \bullet I^* \bullet K^{\%}$.

where $*, !, \% \in \{L, -, 1, +, R\}$.

Proof. This can be proved directly by using Lemma 1, Definition 4 and the properties of sets.

Lemma 3 *Let I, J, K and T be four convex intervals, the operator "*$\circ$*" and "*$\uplus$*" satisfy the following rules,*

(G1) *idempotent law:* $\overbrace{(I^* \bullet J^@) \circ (I^* \bullet J^@) \circ \cdots \circ (I^* \bullet J^@)}^{n} = I^* \bullet J^@$, $\overbrace{(I^* \bullet J^@) \uplus (I^* \bullet J^@) \uplus \cdots \uplus (I^* \bullet J^@)}^{n} = I^* \bullet J^@$.
(G2) *commutativity law:* $(I^* \bullet J^@) \circ (J^! \bullet K^{\%}) = (J^! \bullet K^{\%}) \circ (I^* \bullet J^@)$, $(I^* \bullet J^@) \uplus (J^! \bullet K^{\%}) = (J^! \bullet K^{\%}) \uplus (I^* \bullet J^@)$.
(G3) *associativity law:* $(I^* \bullet J^@) \circ ((J^! \bullet K^{\%}) \circ (K^{\#} \bullet T^?)) = ((I^* \bullet J^@) \circ (J^! \bullet K^{\%})) \circ (K^{\#} \bullet T^?)$, $(I^* \bullet J^@) \uplus ((J^! \bullet K^{\%}) \uplus (K^{\#} \bullet T^?)) = ((I^* \bullet J^@) \uplus (J^! \bullet K^{\%})) \uplus (K^{\#} \bullet T^?)$.
(G4) *assignment law:* $(I^* \bullet J^@) \circ ((J^! \bullet K^{\%}) \uplus (K^{\#} \bullet T^?)) = ((I^* \bullet J^@) \circ (J^! \bullet K^{\%})) \uplus ((I^* \bullet J^@) \circ (K^{\#} \bullet T^?))$, $(I^* \bullet J^@) \uplus ((J^! \bullet K^{\%}) \circ (K^{\#} \bullet T^?)) = ((I^* \bullet J^@) \uplus (J^! \bullet K^{\%})) \circ ((I^* \bullet J^@) \uplus (K^{\#} \bullet T^?))$,

where $*, @, !, \%, ?, \# \in \{L, -, 1, +, R\}$.

Proof. This can be proved directly by using Lemma 1, Lemma 2 and the properties of sets.

In order to propagate temporal relations by matrices, we define the operators "$\circ$" and "$\uplus$" of matrices in Definition 5 and Definition 6.

Definition 5 *Let I, J and K be three convex intervals,* $M_{I,J} = (c_{ij})_{5\times5}$, $M_{J,K} = (b_{ij})_{5\times5}$ *be the temporal relational matrices, we define "*$\circ$*" operator of matrices as follows:*

$$M_{I,K} = M_{J,K} \circ M_{I,J} = ((b_{i1} \circ c_{1j}) \uplus (b_{i2} \circ c_{2j}) \uplus \cdots \uplus (b_{i5} \circ c_{5j}))_{5\times5}.$$

Now we define the operator "$\uplus$" of matrices.

Definition 6 *Let I, J and K be three convex intervals,* $M_{I,J}$, $M_{J,K}$ *are the temporal relational matrices. The operator "*$\uplus$*" of matrices is defined as*

$$N = M_{I,J} \uplus M_{J,K} = (b_{ij})_{5\times5} \uplus (c_{ij})_{5\times5} = (b_{ij} \uplus c_{ij})_{5\times5} = (d_{ij})_{5\times5}$$

where $N = (d_{ij})_{5\times5}$, $M_{I,J} = (b_{ij})_{5\times5}$, *and* $M_{J,K} = (c_{ij})_{5\times5}$.

After defining the operators "$\circ$" and "$\uplus$" of the matrices, we can define the propagation of temporal relations in a matrix model as follows.

Definition 7 *Let I, J and K be three convex intervals, $M_{I,J}$, $M_{J,K}$ are the temporal relational matrices. Suppose $M_{I,K} = M_{J,K} \circ M_{I,J}$, then the possible relations of I and K are determined as: for $1 \le i \le 13$, if $M_i \uplus M_{I,K} = M_{I,K}$, then the relation denoted with M_i is a possible relation of I and K.*

Example 1. Let $I > J$ and J s K, then

$$M_{I,K} = M_{J,K} \circ M_{I,J} =$$

$$\begin{bmatrix} 1\,0\,0\,0\,0 \\ 0\,1\,0\,0\,0 \\ 0\,0\,1\,1\,1 \\ 0\,0\,0\,0\,1 \\ 0\,0\,0\,0\,1 \end{bmatrix} \circ \begin{bmatrix} 1\,0\,0\,0\,0 \\ 1\,0\,0\,0\,0 \\ 1\,0\,0\,0\,0 \\ 1\,0\,0\,0\,0 \\ 1\,1\,1\,1\,1 \end{bmatrix}$$

$$= \begin{bmatrix} 1\,0\,0\,0\,0 \\ 1\,0\,0\,0\,0 \\ 1\,1\,1\,1\,1 \\ 1\,1\,1\,1\,1 \\ 1\,1\,1\,1\,1 \end{bmatrix}.$$

Because $M_i \uplus M_{I,K} = M_{I,K}$ for $i = 2, 4, 6, 7, 9$, and $M_i \uplus M_{I,K} \neq M_{I,K}$ for $i = 1, 3, 5, 8, 10, 11, 12, 13$, then the possible relations between I and K are $\{>, mi, oi, f, d\}$.

4 The Rules of Propagating Temporal Relation

In Example 1, according to $I > J$ and J s K, we estimated that the possible relations between I and K are $\{>, mi, oi, f, d\}$. In fact, $M_{I,K} = M_2 \uplus M_4 \uplus M_6 \uplus M_7 \uplus M_9$. In order to propagate temporal relations with matrices like Example 1, we now construct the temporal constraint propagation table as Table 3. Let "$\odot$" be the operator of temporal constraint propagation, and

$$\begin{aligned}
M_a &= M_1 \uplus M_2 \uplus \cdots \uplus M_{13}, \\
M_b &= M_1 \uplus M_3 \uplus M_5 \uplus M_9 \uplus M_{11}, \\
M_c &= M_2 \uplus M_4 \uplus M_6 \uplus M_7 \uplus M_9, \\
M_d &= M_2 \uplus M_4 \uplus M_6 \uplus M_9 \uplus M_{11}, \\
M_e &= M_7 \uplus M_8 \uplus M_{13}, \\
M_f &= M_5 \uplus M_9 \uplus M_{11}, \\
M_g &= M_1 \uplus M_3 \uplus M_5 \uplus M_7 \uplus M_{10}, \\
M_h &= M_{11} \uplus M_{12} \uplus M_{13}, \\
M_i &= M_6 \uplus M_7 \uplus M_9,
\end{aligned}$$

$$M_j = M_5 \uplus M_7 \uplus M_9 \uplus M_{11},$$
$$M_k = M_6 \uplus M_{10} \uplus M_{12},$$
$$M_l = M_1 \uplus M_3 \uplus M_5,$$
$$M_m = M_5 \uplus M_6 \uplus M_7 \uplus M_8 \uplus M_9 \uplus M_{10} \uplus M_{11} \uplus M_{12} \uplus M_{13},$$
$$M_n = M_5 \uplus M_7 \uplus M_{10},$$
$$M_o = M_2 \uplus M_4 \uplus M_6,$$
$$M_p = M_7 \uplus M_8 \uplus M_{13},$$
$$M_q = M_{11} \uplus M_{12} \uplus M_{13}.$$

The temporal constraint propagation table is as follows.

Table 3. The temporal constraint propagation

$\odot$	M_1	M_2	M_3	M_4	M_5	M_6	M_7
M_1	M_1	M_a	M_1	M_b	M_1	M_b	M_b
M_2	M_a	M_2	M_c	M_2	M_c	M_2	M_2
M_3	M_1	M_d	M_1	M_e	M_1	M_f	M_f
M_4	M_g	M_2	M_h	M_2	M_i	M_2	M_4
M_5	M_1	M_j	M_1	M_k	M_l	M_m	M_j
M_6	M_g	M_2	M_n	M_2	M_m	M_o	M_6
M_7	M_1	M_2	M_3	M_2	M_j	M_o	M_7
M_8	M_1	M_j	M_3	M_k	M_5	M_k	M_p
M_9	M_1	M_2	M_1	M_2	M_b	M_c	M_9
M_{10}	M_g	M_j	M_n	M_k	M_n	M_k	M_k
M_{11}	M_1	M_2	M_1	M_4	M_l	M_i	M_9
M_{12}	M_g	M_2	M_n	M_4	M_n	M_6	M_8
M_{13}	M_1	M_2	M_3	M_4	M_5	M_6	M_7

$\odot$	M_8	M_9	M_{10}	M_{11}	M_{12}	M_{13}	
M_1	M_1	M_b	M_1	M_1	M_1	M_1	
M_2	M_2	M_c	M_2	M_c	M_2	M_2	
M_3	M_1	M_f	M_1	M_3	M_3	M_3	
M_4	M_4	M_i	M_2	M_i	M_2	M_4	
M_5	M_l	M_j	M_g	M_5	M_n	M_5	
M_6	M_k	M_i	M_j	M_i	M_j	M_6	
M_7	M_p	M_9	M_j	M_9	M_o	M_7	
M_8	M_8	M_j	M_{10}	M_5	M_{10}	M_8	
M_9	M_b	M_9	M_a	M_9	M_c	M_9	
M_{10}	M_{10}	M_m	M_{10}	M_n	M_{10}	M_{10}	
M_{11}	M_l	M_9	M_g	M_{11}	M_q	M_{11}	
M_{12}	M_{10}	M_i	M_4	M_q	M_{12}	M_{12}	
M_{13}	M_8	M_9	M_{10}	M_{11}	M_{12}	M_{13}	

Now we demonstrate the use of the above table with an example.

Example 2. We solve $(M_3 \odot M_4) \odot M_3$ as follows.

$$\begin{aligned}(M_3 \odot M_4) \odot M_3 &= (M_4 \circ M_3) \odot M_3 \\ &= M_e \odot M_3 = (M_7 \uplus M_8 \uplus M_{13}) \odot M_3 \\ &= (M_7 \odot M_3) \uplus (M_8 \odot M_3) \uplus (M_{13} \odot M_3) \\ &= (M_3 \circ M_7) \uplus (M_3 \circ M_8) \uplus (M_3 \circ M_{13}) \\ &= M_3 \uplus M_3 \uplus M_3 \\ &= M_3.\end{aligned}$$

In order to present the inference rules, we now introduce a new operator "$\sqcap$" of the matrices as follows.

Definition 8 *Let $M = (a_{ij})_{5\times 5}, M' = (b_{ij})_{5\times 5}$ be temporal relational matrices, we define the "$\sqcap$" operator of M and M' as*

$$M'' = M \sqcap M' = (a_{ij})_{5\times 5} \sqcap (b_{ij})_{5\times 5} = (a_{ij} \sqcap b_{ij})_{5\times 5}$$

where, $a_{ij} \sqcap b_{ij} = 1$ iff both $a_{ij} = 1$ and $b_{ij} = 1$, $i, j = 1, 2, 3, 4, 5$.

We now provide some inference rules as axioms. Let I, J, K, T and S be five convex intervals, we then have the following sets of axioms **A**.

A1: $I\theta J \bigwedge J\theta K \to I\theta K$, or $M_i \circ M_i = M_i$;
where, $\theta \in \{<, >, f, fi, d, di, s, si, =\}$, $i \in \{1, 2, 7, 8, 9, 10, 11, 12, 13\}$.
A2: $I \text{ m } J \bigwedge I \text{ m } K \bigwedge T \text{ m } J \to T \text{ m } K$, or $M_3 \circ (M_4 \circ M_3) = M_3$;
A3: $I \text{ m } J \bigwedge J \text{ m } K \bigwedge I \text{ m } T \bigwedge T \text{ m } K \to J = T$, or $(M_4 \circ M_3) \sqcap (M_3 \circ M_4) = M_{13}$;
A4: $(I \text{ f } J \bigvee I \text{ f } J) \bigwedge I \text{ m } K \to J \text{ m } K$, or $M_3 \circ M_7 = M_3 \circ M_8 = M_3$;
A5: $I \text{ m } J \bigwedge (J \text{ s } K \bigvee J \text{ si } K) \to I \text{ m } K$, or $M_{11} \circ M_3 = M_{12} \circ M_3 = M_3$;
A6: $(I\theta J \bigwedge J = K) \bigvee (I = J \bigwedge J\theta K) \to I\theta K$, or $M_{13} \circ M_i = M_i \circ M_{13} = M_i$;
where, $\theta \in \{<, >, m, mi, o, oi, f, fi, d, di, s, si, =\}$,
$i \in \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$.

From the above, $A1$ demonstratess the propagation law of some temporal relationships, $A2 - -A6$ describe the uniqueness of the points $START, MEET$, and $FINISH$.

5 Conclusion

There are two most influential existing temporal formalisms featuring reified propositions. These are McDermott's event calculus [4], and Allen's time world model of interval calculus [1]. The latter has been widely accepted as a suitable temporal framework for planning and reasoning and has been successfully applied to multi-agents [3], and to temporal databases for manipulating their time relationships [6], and has also been used to specifically design simulations [8], and to specify the temporal properties of concurrent systems [2]. However, there are still some limitations. For instance, constraint satisfaction is a NP-hard problem in interval calculus. For this reason, we have proposed a new interpretation for interval calculus in this paper which establishes a new method of representing interval calculus and transforms interval calculus into matrix calculus. Summarizing the key points of this paper as follows, we have:

- established a new method (IMC) of representing interval calculus which transforms interval calculus into matrix calculus,
- presented the temporal relational calculus model in a matrix, and
- demonstrated the rules of propagating temporal relationships.

Our experiments showed that it is faster to propagate temporal relations using the above method. Therefore, we have proven that the matrix method may be conveniently applied to handle other temporal problems. Our future work on IMC will mainly concentrate on the constraint satisfaction problem in matrices.

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