ANALYSIS OF WELDED STRUCTURES WITH FAILED AND NON-FAILED WELDS BASED ON MAXIMUM LIKELIHOOD

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ABSTRACT

A maximum likelihood analysis method allowing for random censoring has been developed for statistical evaluation of fatigue data. The method is shown to be especially useful when analysing welded structures that have two or more potential failure locations. In those cases where fatigue cracking is observed at only one location, the method allows the non-failures to also be statistically considered. The method is illustrated using a fatigue data set from spectrum loaded welds made from 10 mm thick HTS 390 and VHTS 690 high tensile strength steel with three fabrication procedures. Each of the specimens tested had two or more failure locations and only a few of them showed significant fatigue cracking at both locations. As compared to the “usual” quality welded specimens from high strength steel, the use of a second weld pass followed by TIG dressing improved fatigue strength by about 25%. Full penetration and grinded welds from very high strength steel showed fatigue strength improvement of about 30% at $1x10^6$ cycles to failure but significantly more at longer lives. For both “usual” quality and grinded welds, spectrum loading with alternating tension and compressive mean stresses resulted in about twice the fatigue life as compared to spectrum loading with constant tensile mean stress. This indicates that the portion of the spectrum with compressive cycles was non-damaging.

IIW-Thesaurus keywords: *Fatigue strength; Fatigue cracks; Lifetime; Statistical methods; Welded joints; Low alloy steels; High; Strength; Multirun welding; GTA welding; Melting; Grinding; Penetration; Influencing factors; Failure; Weld toes; Stress; Compression; Practical investigations.*

1 INTRODUCTION

Design criteria for welded structures usually take into account the statistical variation in observed fatigue strength. Curves corresponding to mean fatigue strength minus two standard deviations are often defined, and standard deviation (std. dev.) values in $\log(\Delta\sigma)$ are often provided, enabling a designer to construct curves corresponding to virtually any probability of failure. Often new production technologies or material grades are examined in an attempt to measure potential improvements in fatigue strength. It is widely recognized that replicate tests must be performed but, when considering both mean value and scatter, it is not always immediately clear which set of specimens performed most favourably. A weld improvement method that produces both improved mean fatigue strength but also increased scatter may actually result in a design curve lower than for the non-improved set of specimens.

Different analysis methods can be applied to even a simple set of $\Delta\sigma$-N_f data, each resulting in somewhat different values for mean and std. dev. This is especially true in the case where the welded structure in the experimental test series has several potential failure locations or when a test is stopped due to failure at some other location in the structure. This issue has previously been addressed by Maddox [1].

A maximum likelihood analysis method allowing for random censoring has been developed at VTT by Wallin [4] for statistical evaluation of fatigue data. The method is shown to be especially useful when analysing welded structures that often have two or more potential failure locations. In those cases where fatigue cracking is observed at only one location, the method allows the non-failures to also be statistically considered.

2 ANALYSIS METHOD

A vast amount of fatigue testing has shown that $\log(\Delta\sigma)$ vs. $\log(N_f)$ for welded connections is well described by a straight line. The resulting equation is $C = N_f\,\Delta\sigma^b$ where C is often referred to as the fatigue capacity of the welded joint. For welds subject to variable amplitude loading the equivalent stress, $\Delta\sigma_{eq}$, is often substituted for $\Delta\sigma$. Linear regression analysis (LR) is the most common method used to determine the values of C and b as well as the variance of the linear fit. Often b is assumed to be 3 and LR is used to determine only C

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and the variance. Only failed data can be used in LR, so statistical information regarding the non-failed welds is usually ignored for welded structures or specimens containing multiple potential failure locations or specimens that are otherwise not tested to failure. For example, the I-beam specimen shown in Fig. 1 subject to bending fatigue loading will have two nominally identical potential failure locations. Occasionally fatigue cracks will occur at both weld toes, but in many cases the tests will be discontinued after one end of the gusset attachment fails and the beam can no longer sustain the required loads. Other specimen types have four or more failure locations.

In contrast to the widely used LR method, the randomly censored maximum likelihood estimation method (MML) provides a method for examining how well a certain probability density function describes an entire data set. Weld locations not belonging to the failure distribution function, f_c, belong to the related survival distribution function, S_c. The conditional probability is given as

$f_c(C)^{\delta i} \cdot S_c(C)^{1-\delta_i}$ where $\delta_i = 1$ for welds resulting in failure and $\delta_i = 0$ for welds where no fatigue cracking was observed at the end of the test.

The randomly censored MML expression for $\Delta\sigma$-N_f data has been presented by Wallin (1999). Equation 1 assumes that fatigue life at a given stress value is log-normally distributed. This technique has a benefit in that it is able to weigh also the contribution of run-outs to the statistical parameters. The equation can be presented as

$$\ln(L) = -n\ln(s\sqrt{2\pi}) - \sum_{i=1}^{n} \frac{\delta_i}{2}\left(\frac{\ln(\Delta\sigma_{eq,i}{}^{b}N_{f,i}) - m}{s}\right)^2 + \sum_{i=1}^{n}(1-\delta_i)\ln\left(\int_{\ln(\Delta\sigma_{eq,i}{}^{b}N_i)}^{\infty} \exp\left\{-\frac{1}{2}\left(\frac{x-m}{s}\right)^2\right\}dx\right) \quad (1)$$

where $\Delta\sigma_{eq,i}$ is the applied equivalent stress, $N_{f,i}$ is the failure life for a weld and N_i is number of cycles accumulated for a test that was stopped (censored). The terms m and s represent the mean and standard deviation of the logarithm of the random variable, $\Delta\sigma_{eq}{}^{b}N_f$. Values for m and s are iteratively selected such that the value of the estimator L obtains a maximum value. The last term on the right hand side of Eq. 1 is the standard normal cumulative distribution function, which is a standard library function for most spreadsheet or mathematical computation programs. This means that ln(L) is not difficult to compute. The $\Delta\sigma$-N_f slope, b, could also be fitted using this equation but for simplicity the slope was assumed to be $b = 3$.

In the event that all specimens result in failure, Eq. 1 reduces to a simple least squares fit, while if all specimens are run-outs it reduces to a deterministic model with $m = \ln(\Delta\sigma_{eq}{}^{b}N_f)$ / ^{max} for all specimens tested.

3 EXPERIMENTAL PROGRAMME

3.1 Test Specimens

The I-beam type specimens shown in Fig. 1 were manufactured from 10 mm thick HTS 390 and VHTS 690 steel plate. Weld quality was defined as "usual", i.e., corresponding to present shipyard standards, "grinded" requiring full penetration welding and grinding of the gusset to obtain a smooth transition radius between gusset and I-beam flange, and "optimised", which had a second weld pass followed by TIG-dressing.

3.2 Load Spectrum

Typically ship structural details are loaded by wide banded stress spectra. At certain ship details large, relatively slow blockwise variations in mean stress are superimposed on the spectrum loading. In highly stressed regions this causes reversed plastic deformation and affects mean stress and residual stresses, which change during the loading. The fatigue damage caused by small stress range cycles is different in spectrum loading from that in a constant amplitude loading, see e.g. Marquis [2]. The damaging effect of small fatigue cycles in the presence of large mean stress variation has not received much attention in the open literature.

Some criteria for a "realistic" ship load spectrum were defined and are here presented in Table 1. The total number of cycles in the spectrum, the ratio of the equivalent stress range and the maximum stress range and the overall distribution in stress cycles size were also important considerations.

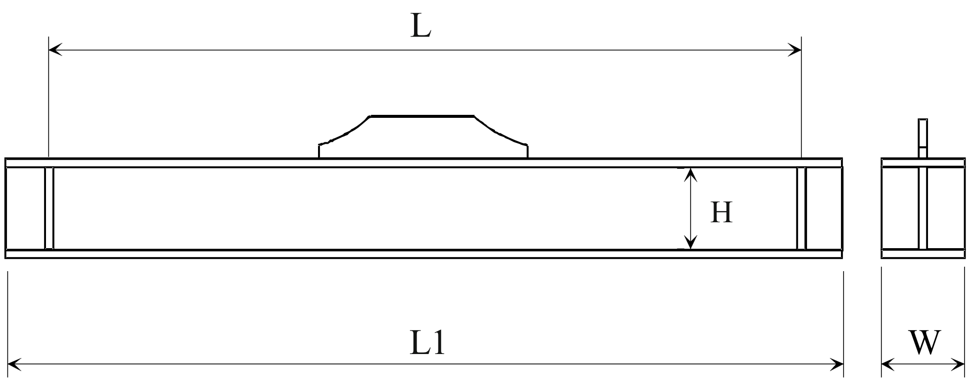

Fig. 1. I-beam specimen with two failure locations. W = H = 100 mm, L = 850 mm and L1 = 1000 mm.

Table 1. Definition of the “realistic spectrum”.

Parameter	Value	Note
Maximum stress	$\sigma_{hs,\,max} = R_{0.2}/1.5$	Maximum nominal stress at the hot spot is scaled to 2/3 of the yield stress.
Mean stress	$\sigma_{mean} = 0.5 \times \sigma_{max}$	Constant σ_{mean} or blockwise variation between $-\sigma_{mean}$ and + σ_{mean}.
Stress range	$\Delta\sigma_{max} = \sigma_{max}$ (constant mean stress) or $\Delta\sigma_{max} = 2\sigma_{max}$ (varying mean stress) $\Delta\sigma_{eq}$ = spectrum dependant	Maximum stress range in spectrum equals to maximum stress applied to specimen. Equivalent stress range depends on the spectrum chosen.
Type of spectrum	–	Based on WASH (Wave Action Standard History).

In all cases equivalent stress was computed using the root mean cube formula common for welded structures.

$$\Delta\sigma_{eq} = \sqrt[3]{\frac{n_i \Delta\sigma_i^3}{N_{total}}} \quad (2)$$

Blocks 12 and 13 of the WASH, Wave Action Standard History, spectrum (WASH TSC, 1989) [3] were chosen for testing. The final spectrum is defined by the stress ranges in the original WASH spectrum with either a blockwise varying mean stress, see Fig. 2a, or superimposed constant mean stress, see Fig. 2b.

Characteristics of the spectrum with blockwise changing mean are:
– Loading block consists of WASH sequences 12 and 13 that are both repeated with positive and negative mean.
– The largest cycle occurs twice during a single repeat of the spectrum.
– The equivalent stress range is 20.8% of largest cycle (root mean cube equivalent).
– The omission level is 13%, i.e., the stress range of the smallest cycle is 13% of the maximum stress range.
– the spectrum contains 46444 cycles before repeating.

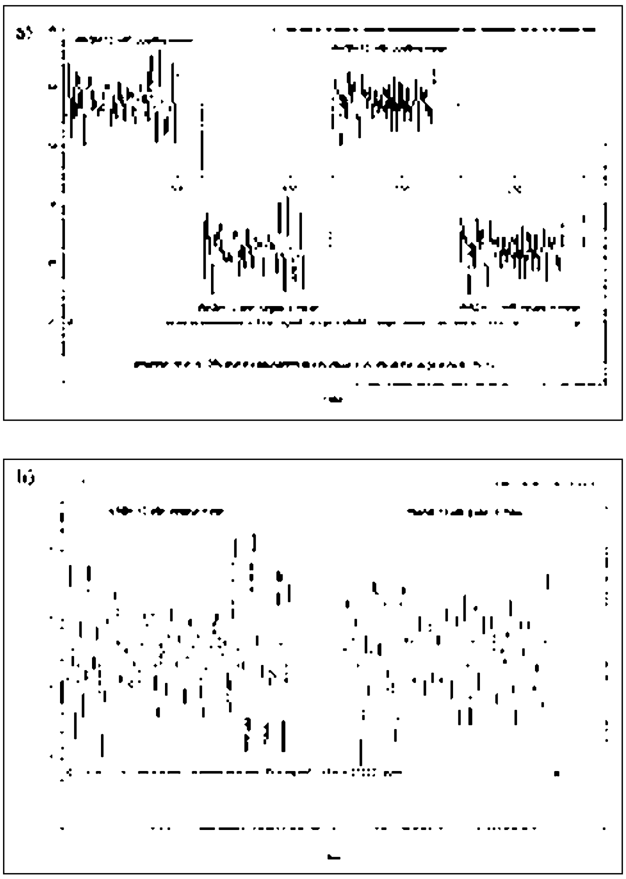

Fig. 2. Load spectra with a) blockwise varying mean stress, and b) constant tensile mean stress.

A second load spectrum with a constant tensile mean stress is virtually the same as that shown in Figure 2a, except that all cycles with negative mean stress are eliminated. Characteristics are:
– Loading block consists of WASH sequences 12 and 13 that are both repeated with positive load.
– The largest cycle occurs once during a single repeat of the spectrum.
– The equivalent stress range is 41.6% of largest cycle (root mean cube equivalent).
– The omission level is 26%.
– The spectrum contains 23222 cycles before repeating.
– Minimum applied stress is 0 MPa.

Load controlled three-point-bending was applied to the I-beam type specimens. When the load spectrum shown in Fig. 2a was applied, the loading was fully reversed meaning that the side of the beam with the weld attachment changed between tension and compression.

3.3 Fatigue Results

Fatigue data are given numerically in Tables 2-6 and are shown graphically in Figs. 3-7. In those cases where no initiated crack was observed at either of the weld toes at the stop of testing, a + sign follows the fatigue life in Tables 2-6 and an arrow is associated with the data point in Figs. 3-7.

Because each specimen had two nominally identical critical locations, one at each end of the welded gusset attachment, two data points were obtained from each specimen for MML. Failure in all cases was defined as the number of cycles when the crack penetrated through the thickness of the 10 mm flange. This corresponded to a surface crack length (2c) of about 35 mm. For MML the second critical location of the specimen was considered only to have a fatigue life longer than that of the weld toe for which failure was documented, i.e. when the first weld toe failed at N_{fi} cycles the second weld toe was considered a run-out (censored test) with $N_{i+1} = N_{fi}$. LR can only consider failures so only the life where specimen failure is observed is used. Behaviour of the second weld toe is ignored. Therefore, LR considers only specimen failure, while MML considers all nominally sim-

Table 2. Fatigue data, usual quality σ_{ys} = 390 welds.

Specimen	$\Delta\sigma_{eq}$	Spectrum mean	N_f	Comments
A20	58,7	constant tensile	3 400 000	weld toe failure
A12	58,7	constant tensile	4 000 000	weld toe failure
A16	117,3	constant tensile	470 000	weld toe failure
A18	58,7	constant tensile	4 400 000	weld toe failure
A05	58,7	constant tensile	3 800 000	weld toe failure

Table 3. Fatigue data, usual quality σ_{ys} = 390 welds.

Specimen	$\Delta\sigma_{eq}$	Spectrum mean	N_f	Comments
A06	117,3	changing	1 040 000	weld toe failure
A04	58,7	changing	7 400 000	weld toe failure
A09	58,7	changing	12 000 000	weld toe failure
A11	58,7	changing	7 300 000	weld toe failure
A15	58,7	changing	6 850 000	weld toe failure

Table 4. Fatigue data, optimised profile and TIG dressed σ_{ys} = 390 welds.

Specimen	$\Delta\sigma_{eq}$	Spectrum mean	N_f	Comments
D20	80,0	constant tensile	2 728 000	weld toe failure
D18	80,0	constant tensile	2 400 000+	no crack
D08	80,0	constant tensile	3 050 000	weld toe failure
D03	80,0	constant tensile	2 200 000+	no crack
D05	80,0	constant tensile	3 900 000+	no crack
D07	80,0	constant tensile	4 290 000+	no crack

Table 5. Fatigue data, Full penetration and grinded σ_{ys} = 690 welds.

Specimen	$\Delta\sigma_{eq}$	Spectrum mean	N_f	Comments
B01	75	constant tensile	12 000 000+	no failure
B13	91	constant tensile	4 250 000	gusset failure
B16	107	constant tensile	1 450 000	gusset failure
B11	107	constant tensile	1 540 000	weld toe failure
B14	107	constant tensile	1 400 000	weld toe failure
B32	107	constant tensile	2 150 000	gusset failure

Table 6. Fatigue data, Full penetration and grinded σ_{ys} = 690 welds.

Specimen	$\Delta\sigma_{eq}$	Spectrum mean	N_f	Comments
B07	107	changing	7 850 000+	no failure
B10	107	changing	3 700 000	gusset failure
B30	107	changing	5 400 000	gusset failure
B15	107	changing	5 800 000	weld toe failure
B09	107	changing	3 900 000	weld toe failure
B27	107	changing	4 900 000	weld toe failure

ilar welds within a specimen. In some cases cracks at the second weld were easily detected using magnetic particle examination. Total fatigue life for these locations could have been calculated based on linear fracture mechanics and would add valuable information that could be used for MML.

A summary of the statistical evaluation for all data contained in Tables 2-6 based on MML is given in Table 7 and for LR in Table 8. The value for *b* used in Eq. 1 and reported in these tables was assumed and not based on MML or LR. The mean curves presented in Figs. 3-7 were determined using the MML equation shown in Eq. 1. As can be seen, the MML analysis method allowed two data points per specimen to be considered even if the specimen did not fail, while LR permitted one data point for failed specimens and none for run-outs. In Figs. 3-7 the $\Delta\sigma$-N_f relationships from Tables 7 and 8 and the test data from Tables 2-6 is presented. These figures show the best fit mean lines computed using both MML and LR and the "design" line based on both analysis methods. The "design" line is defined as mean - 2 std. dev. It should be noted that this is not a true design line since both the mean and std. dev. are computed only for a relatively small sample out of a potentially large population. Standard deviation of a large population would probably be larger than that computed here.

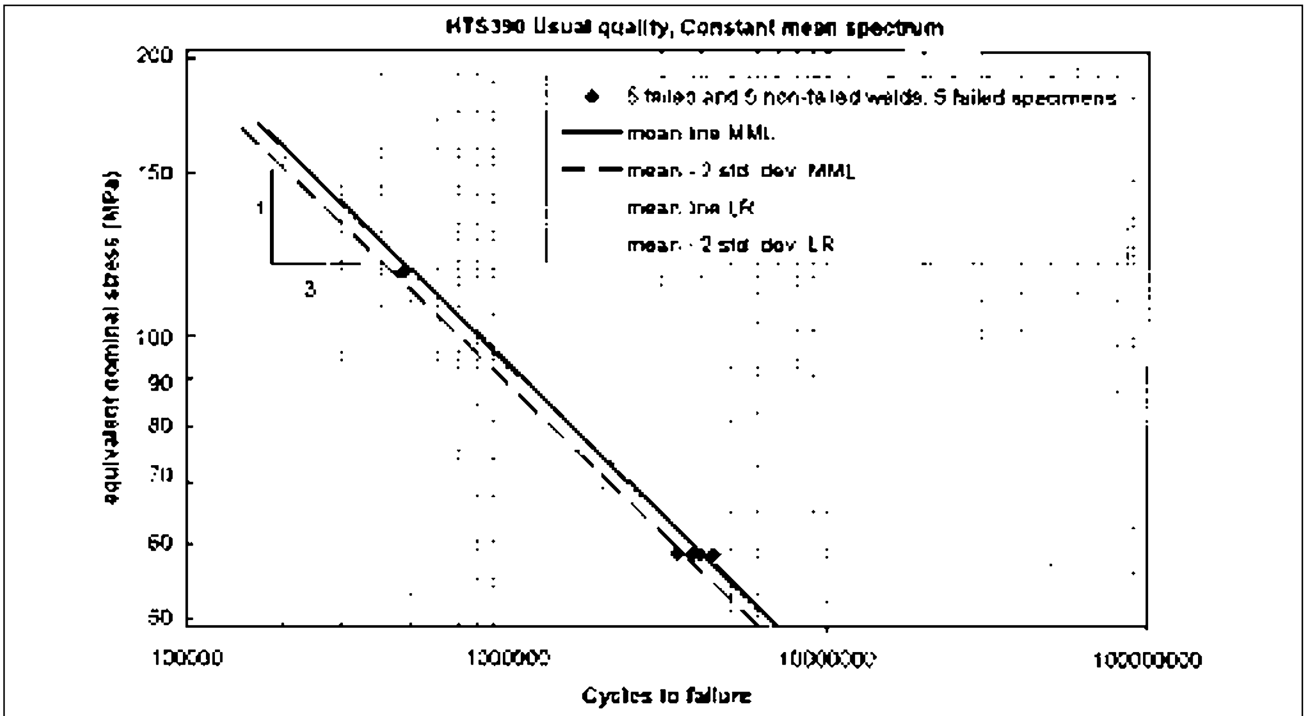

Fig. 3. Table 2 data and analysis.

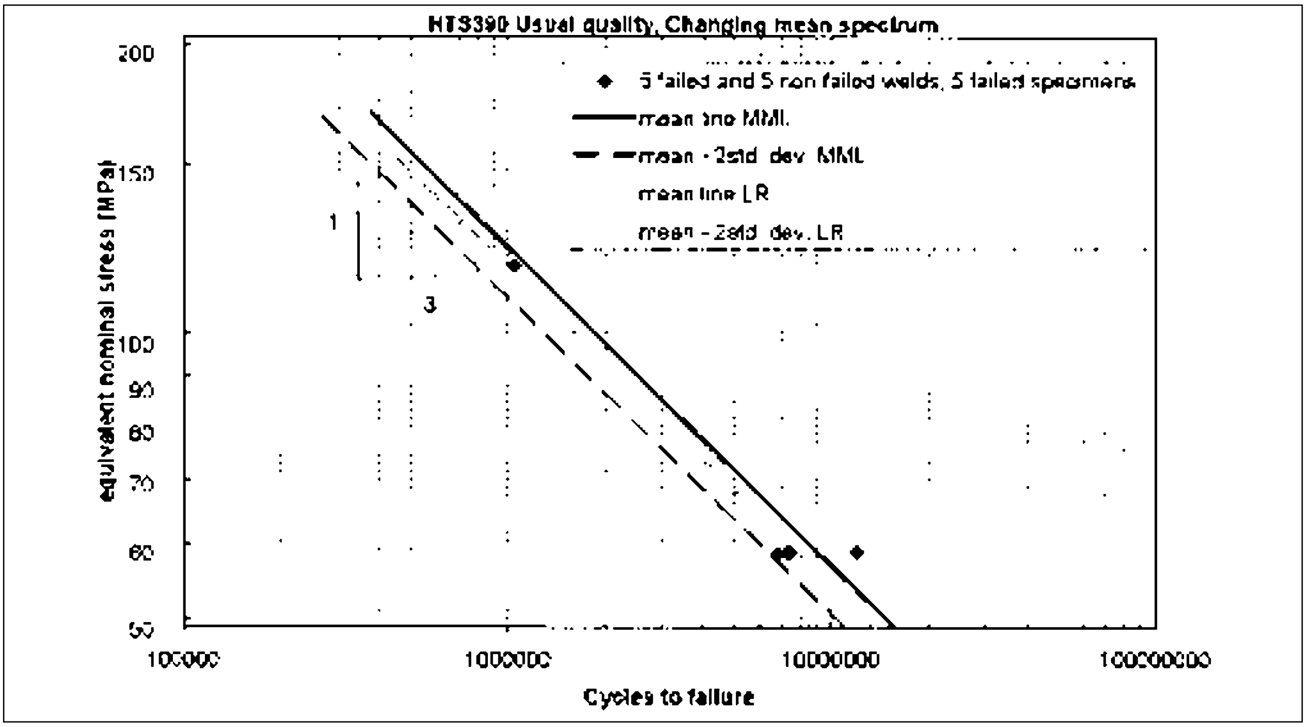

Fig. 4. Table 3 data and analysis.

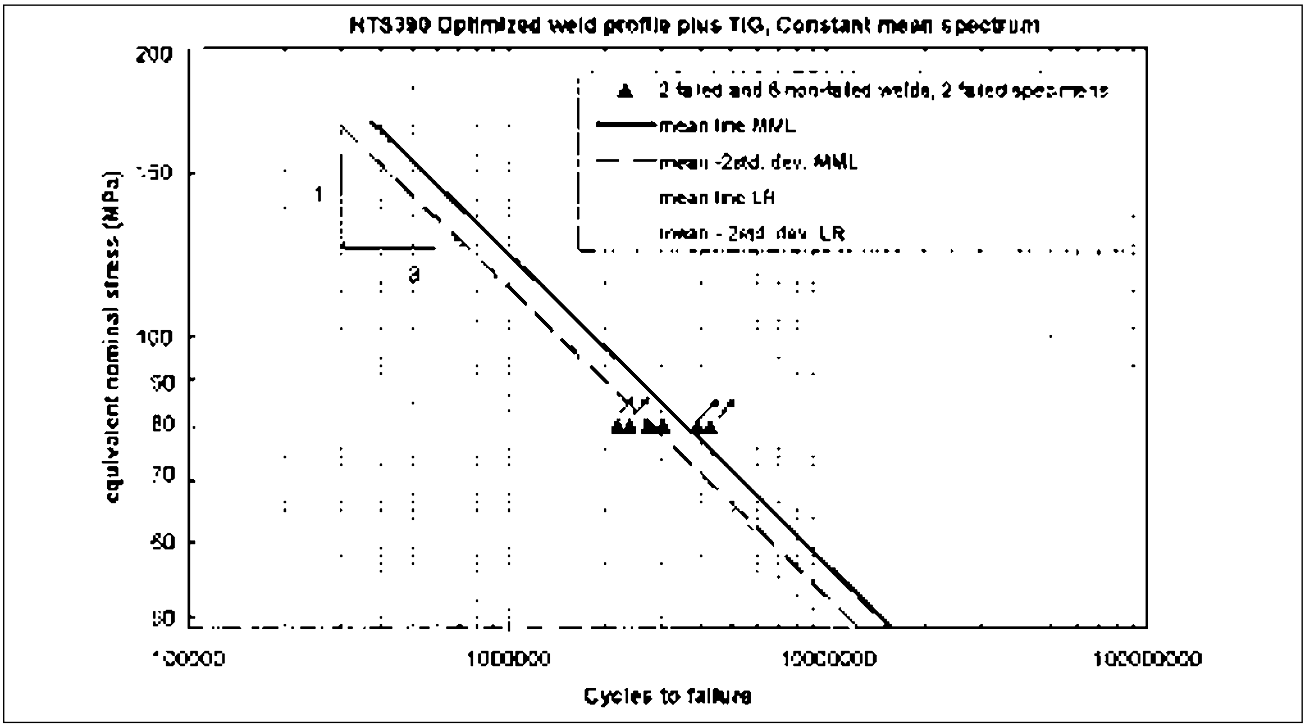

Fig. 5. Table 4 data and analysis.

Table 7. Data from Tables 2-6 evaluated according to MML (Eq. 1).

Table	*b* (assumed)	Data points in analysis	capacity, *C*	sample std. dev. in log (*C*)
2	3	10	8.2×10^{11}	0.032
3	3	10	1.8×10^{12}	0.078
4	3	10	1.8×10^{12}	0.053
5	5	12	2.7×10^{16}	0.064
6	5	12	8.2×10^{16}	0.099

Table 8. Data from Tables 2-6 based on LR, specimen failure only.

Table	*b* (assumed)	Data points in analysis	capacity, *C*	sample std. dev. in log (*C*)
2	3	5	7.8×10^{11}	0.041
3	3	5	1.6×10^{12}	0.098
4	3	2	1.4×10^{12}	0.064
5	5	5	2.4×10^{16}	0.080
6	5	5	6.4×10^{16}	0.086

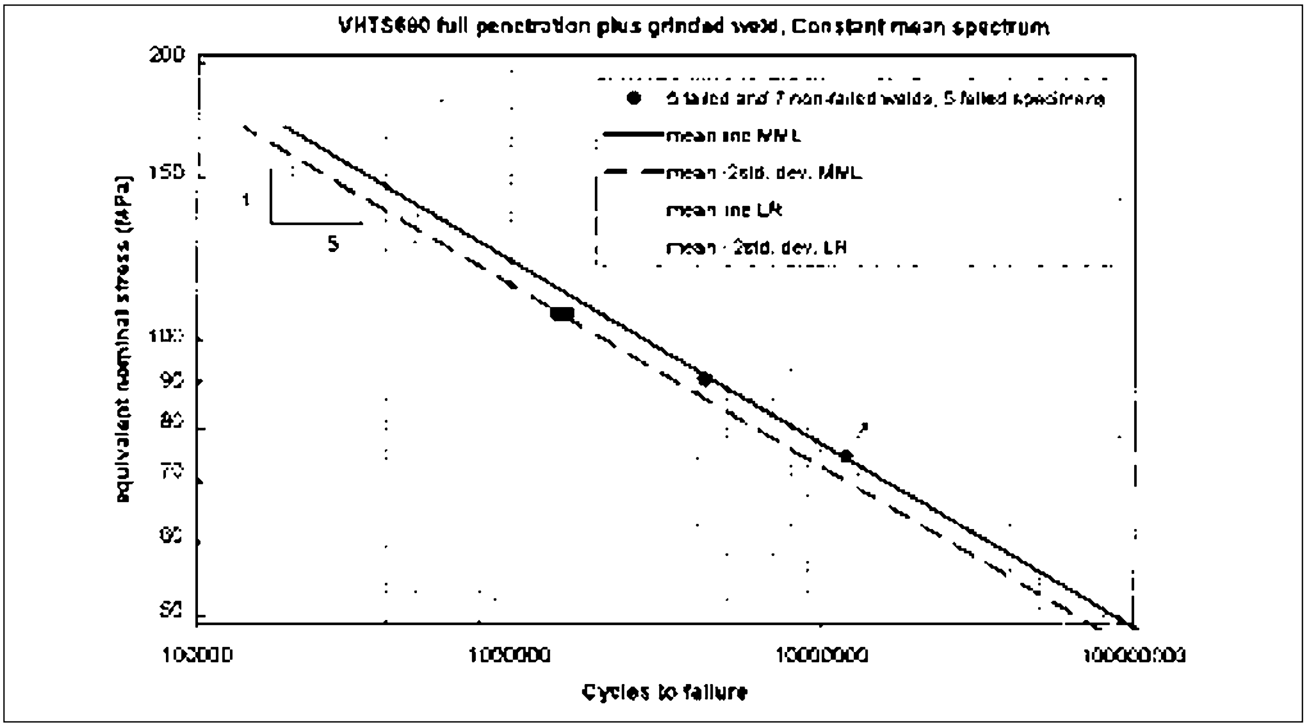

Fig. 6. Table 5 data and analysis.

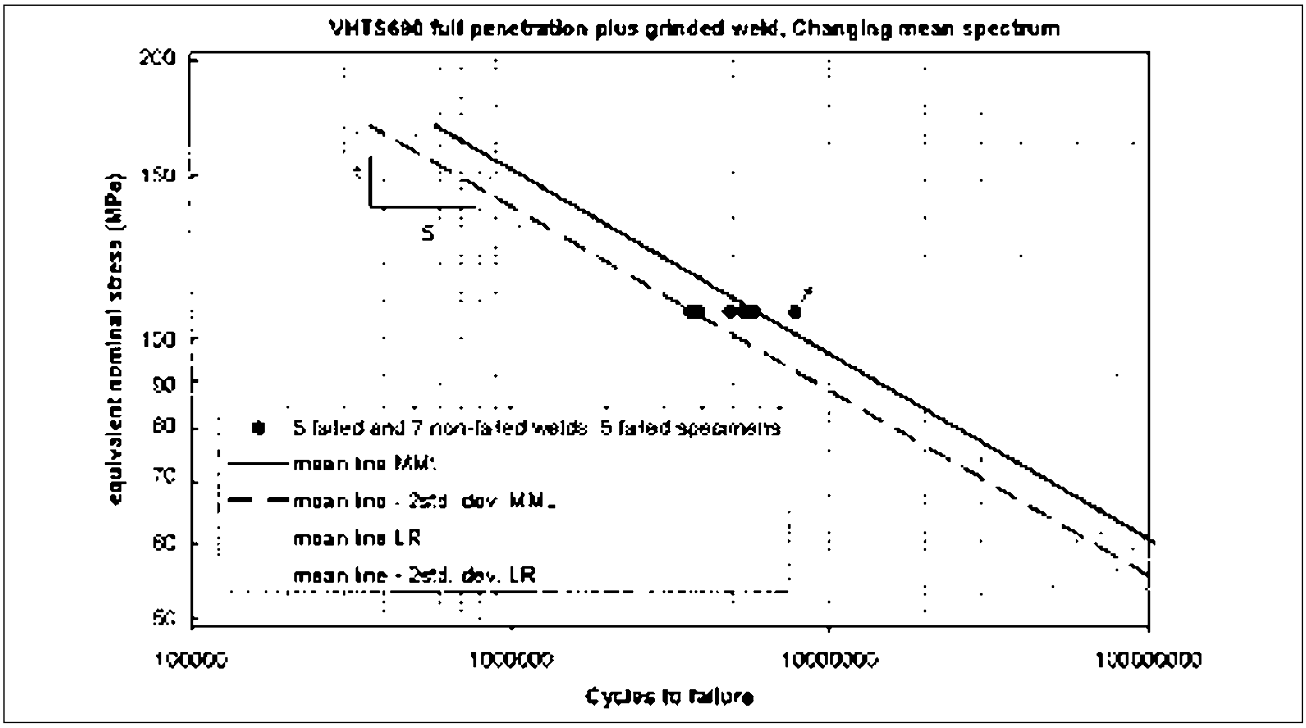

Fig. 7. Table 6 data and analysis.

4 DISCUSSION

For the TIG dressed weld joints, failure usually occurred at the intersection of the gusset and the first weld pass rather than at the TIG dressed weld toe, see Fig. 8. In only 2 cases out of 10 did cracking occur at the weld toe. From a production point of view fatigue failure from alternate locations shows possibly some difficulty with the welding process. Statistical analysis based on linear regression, which can only consider failed specimens, requires that most of the test information be discarded. However, from a statistical point of view it is valuable to know that the TIG dressed locations had a fatigue capacity greater than a certain value. This information cannot be considered in LR but can be included in MML. MML required only the assumption that the fatigue capacity of a specific TIG dressed joint was greater than what was applied during testing. No specific fatigue capacity was assumed. Even though the number of failures at the TIG dressed location was small, Table 7 and

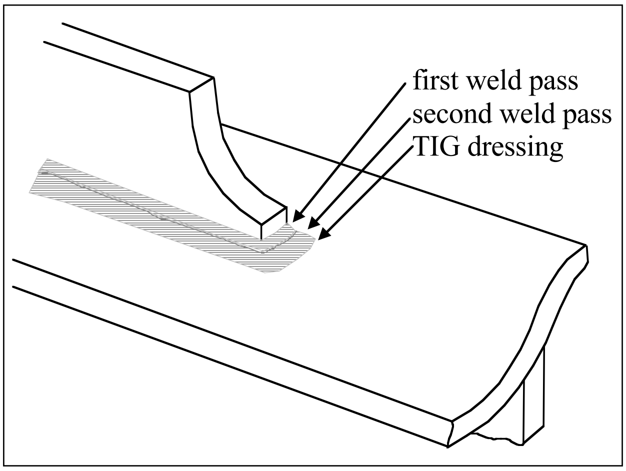

Fig. 8. Optimised weld profile and TIG dressed weld joint. Failure was most often at the seam between the gusset and the first weld pass.

Figs. 3 and 5 show that these welds had both a higher mean strength and higher "design" strength than the "usual" quality welds produced from the same material. The strength improvement in terms of stress due to TIG dressing was about 25%.

For the full penetration and grinded welds, Table 5 and 6, finite element analysis showed that the most highly stressed location was along the gusset. A nearly equal number of failures, however, were observed at the region of the weld toe that had been grinded away. These failure locations are indicated in Fig. 9. Fracture surface analysis revealed the presence of large pores along the weld fusion line, indicating that the weld toe had not been opened before accomplishing the full penetration welding from the second side. As compared to the "usual" quality HTS 390 welded specimens, the 690 VHTS full penetration and grinded welds showed and improved fatigue strength of about 30% in stress at $1x10^6$ cycles to failure. At longer lives the degree of improvement appears to be even greater due to the apparent difference in $\Delta\sigma$-N_f slope.

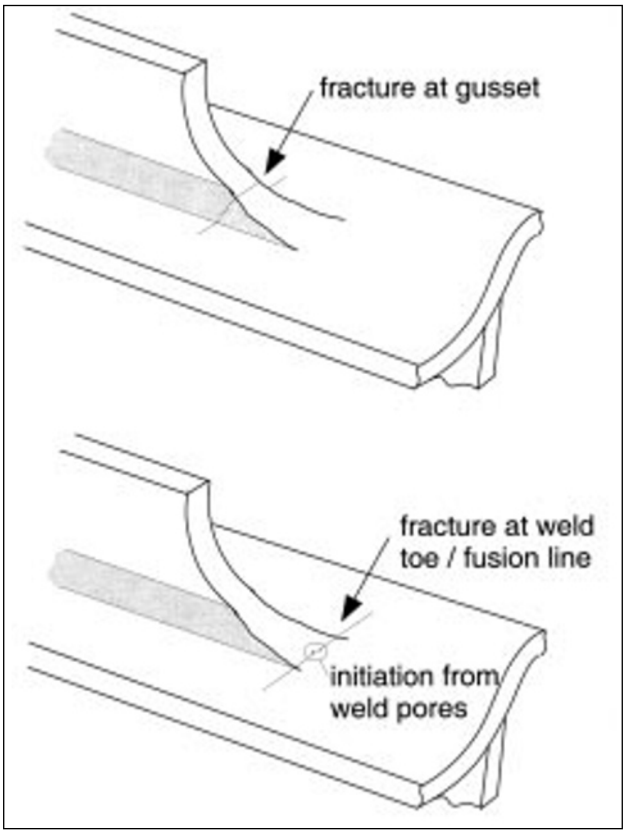

Fig. 9. Failure locations for the full-penetration and grinded VHTS 690 specimens.

As seen from Tables 7 and 8 and Figs. 3-7 the MML method predicts larger fatigue capacity in all cases as compared to LR analysis. This is not surprising since LR consistently ignores the welded connection in the structure with the longest fatigue life. Predicted scatter from the two analysis methods did not show such a consistent trend since it was dependent of several factors in the data. In all cases the mean - 2 std. dev. curve computed using MML was higher than for LR.

While the current analysis assumes a log-normal distribution of fatigue capacity, other distribution function can easily be assumed. [4] has presented a closed form solution for fatigue data assuming a two-parameter Weibul distribution. Similar procedures could also be used to add a threshold effect or fit data to a bi-linear $\Delta\sigma$-N_f curve.

It is interesting to note that for "usual" quality HTS welds, Figs. 3 and 4, the spectrum loading with blockwise alternating tension and compressive mean stresses resulted in about twice the fatigue capacity as compared to spectrum loading with constant tensile mean stress. A similar observation can be made from the VHTS grinded welds, Figs. 6 and 7. These observations clearly suggest that the portion of the spectrum with compressive cycles was completely non-damaging.

5 CONCLUSIONS

A maximum likelihood analysis method allowing for random censoring has been used to analyse welded I-beam specimens with welded gussets. The specimens were constructed from two high strength steels and involved "usual" quality welds and two alternate fabrication processes to obtain improved fatigue strength. Spectrum loading with both constant tensile and alternating tension / compression mean stresses were used. The maximum likelihood method was useful when analysing welded structures that have two or more potential failure locations. In those cases where fatigue cracking is observed at only one location, the method allows the non-failures to also be statistically considered. Simple linear regression analysis cannot account for run-out or otherwise interrupted tests, and therefore tends to underestimate the fatigue capacity of an individual weld.

As compared to the "usual" quality HTS 390 welded specimens, the 690 VHTS full penetration and grinded welds showed and improved fatigue strength of about 30% in stress at $1x10^6$ cycles to failure. At longer lives the degree of improvement appears to be even greater due to the apparent difference in $\Delta\sigma$-N_f slope. The use of a second weld pass followed by TIG dressing improved fatigue strength by about 25% in terms of stress.

For both “usual” quality HTS welds and VHTS grinded welds the spectrum loading with alternating tension and compressive mean stresses resulted in about twice the fatigue life as compared to spectrum loading with constant tensile mean stress. This indicates that the portion of the spectrum with compressive cycles was non-damaging.

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