Risk Assessments – Part 2

Statistical Risk Estimations Dependent on the Failure Behaviour of Motor Vehicle Components

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Where there are frequent failures or safety-relevant field failures a rapid and assured estimation of the associated risk to the company is necessary. To achieve this the failure behaviour is recognised in the first step, ie whether the failures occur initially, randomly or as the result of wearout and ageing. The following determination of the expected failure quantity depends on the failure behaviour present. The newly developed models necessary allow not only the qualitative description of individual failure mechanisms, but also, for the first time, the quantitative calculation of the affected quantity for the various types of failure behaviour. This is especially important for risk assessments and can avoid wrong and consequently costly decisions.

1 Introduction

After recognising the failure behaviour in the first step [1] and confirming it via failure analyses and corresponding tests it is important to estimate how great the number of affected parts and thus the potential company risk will be. To do so, the concrete procedure and the methods to be applied strongly depend on the failure behaviour to be investigated.

A very important factor is the limitation of the quantities under consideration by identifying potential affected manufacturing lots via batch numbers, manufacturing dates, supplied parts or other relevant factors and influences. The better the possibility of separation is, the more accurately the maximum affected failure number can be determined.

When the failures occur randomly then the latent failure mechanism can lead to failures at any time. As this is valid over long periods the known methods can be applied for the kilometer and time-dependent lifetime prognoses, as will be briefly shown below.

If the failures occur systematically the above-mentioned models can not be directly applied for lifetime prognoses. These models assume the permanent existence of the failure type; this would result in a completely wrong assessment of the failure quantity. Instead, the systematic failure behaviour must be analysed and specifically considered, ie whether the failures occur initially or are caused by wearout or ageing.

A systematic initial failure behaviour means that the failure, if it exists, arises during a specifiable demand. This can happen within a certain time period or service time, up to a specific mileage or additionally during a number of switching cycles or temperature changes, etc. After this demand the failure type is no longer expected.

In the case of a failure behaviour caused by wearout or ageing the failure mechanism acts over the long-term and can in principle affect all vehicles. The time-dependent occurrence of the failure heavily depends on reaching the demand necessary; this varies strongly from individual to individual, as demonstrated by the mileage driven in practice. Additionally the continuous decline in vehicle population with increasing age has to be taken into account.

Often no detailed data are available for the required demand profiles in vehicle use, and so as a rule the distribution function of the mileage is resorted to. Both the mileage and the associated time requirement directly derived from it are used to estimate the maximum failure number. By additionally considering the failure distribution these estimations can be even more precise.

2 Reliability Characteristics Used

The time period between the start of operation and the failure of a system or component is generally denoted as its lifetime T. As a rule, this depends on random circumstances and can be described by a real random variable T. The associated distribution function (Eq. 1)

$$F(t) = P(T \le t) \tag{1}$$

is called lifetime distribution or failure function. In general the time point of the operation start is fixed as $t = 0$, whereby the lifetime can not become a negative value and $F(t) = 0$ is valid for all t † 0. The complement of the failure probability is the so-called survival probability or reliability function (Eq. 2).

$$\begin{aligned} R(t) &= P(T > t) = 1 - P(T \le t) = \\ &= 1 - F(t). \end{aligned} \tag{2}$$

When the distribution function $F(t)$ has a density $f(t)$, the so-called failure density, (Eq. 3)

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Risikobewertungen – Teil 2

Statistische Risikoabschätzungen in Abhängigkeit des Ausfallverhaltens von Kfz-Komponenten

$$f(t)=\frac{\mathrm{d}\,F(t)}{\mathrm{d}t}\text{ for }t>0 \quad (3)$$

is valid under the condition of continuity. Thus the associated hazard rate $h(t)$ can be easily calculated (Eq. 4):

$$h(t)=\frac{f(t)}{R(t)}=-\frac{\mathrm{d}\ln R(t)}{\mathrm{d}t}\text{ for }t>0. \quad (4)$$

These reliability characteristics are all equivalent in the sense that each can be transformed direct into the others [2]. In this way all characteristics are determined and calculable, even if only one of them is known.

Analogous to the mileage distribution in [1] the time requirement Z_s for a fixed distance $s > 0$ is a real random variable. The associated distribution function $A_s(t)$ reflects the probability of needing at most the time t for the distance s, as given by [3] (Eq. 5):

$$\begin{aligned}A_s(t) &= P(Z_s \le t)= \\ &= 1-L_1\left(\frac{s}{t}\right)\text{ for }t>0.\end{aligned} \quad (5)$$

The annual mileage S_1 is well described by a $LN(\mu, \sigma^2)$ distribution [3, 4]. It can then easily be shown that the time requirement Z_s has a $LN(\ln s - \mu, \sigma^2)$ distribution.

In the case of an unknown theoretical distribution function for the lifetime then the empirical reliability characteristics are determined on the basis of empirical data and are defined with regard to the content analogously to the theoretical characteristics and generally carry the same designation plus a tilde.

When, from a sufficiently large sample with size n_0 of equal components, a total of $n_a(t)$ parts failed up to time t and all remaining components are still in operation, the empirical failure function $\tilde{F}(t)$ is valid as (Eq. 6):

$$\tilde{F}(t)=\frac{n_a(t)}{n_0}. \quad (6)$$

By using other measurement units, such as the mileage s, the reliability characteristics are analogously defined and interpreted. Further characteristics and interrelations can be found in the extensive literature.

3 Statistical Risk Estimations

As already mentioned, the procedure and the methods for estimating the expected failure quantity are very strongly dependent on the failure behaviour under investigation. The operation of the failure mechanism as statistically recognised in the first step can vary considerably. This is clearly shown in **Figure 1** by the hazard rate in the form of the so-called bathtub curve [5].

Period I covers mainly the initial failures and can be modelled by DFR distributions (decreasing failure rate) as is well known. The period II is characterised by random failures and its nearly constant hazard rate can be easily incorporated by an exponential distribution. During the period III wearout and ageing are increasingly evident and so the hazard rate rises there and can be described by IFR distributions (increasing failure rate).

3.1 Systematic Initial Failure Behaviour

A systematic initial failure behaviour is shown by a steeply falling failure quota which approaches zero during a certain time period or demand and then disappears. The determination of this demand limit must be made specifically from case to case and is dependent on the affected components and the failure mechanism in operation. In addition to field data, it is possible to take into account results of test trials with series components, treated samples, sub-assemblies and elements and also theoretical results of simulation calculations, etc.

Often the demand limit, essential for the failure mechanism, can be denoted in the form of a distance s_0 up to which the failure mechanism will lead to a failure. In other words, all affected parts will fail before reaching this distance s_0. Vice versa the conclusion is that all components in vehicles with a mileage greater than s_0 are not affected by this failure.

Since the mileage driven in practice is individual and varies greatly, this must be considered in the determination of the remaining expected failures. This is done by using the mileage distribution $L_1(s)$ which is known or correctly determined for the affected vehicle platform, as described in the first Part [1].

When the quantity under investigation is in field service for a period of t_0 years, the question arises, how many vehicles have already reached the critical distance s_0 within this time period t_0, and how many of the remaining vehicles may yet be affected by failures. These proportions can easily be calculated with (Eq. 5) by using the time requirement. Consequently a proportion of $1 - A_{s_0}(t_0)$ has not driven the distance s_0, while the remaining proportion of $A_{s_0}(t_0)$ has already exceeded this mileage reading and is thus no longer endangered by this failure mechanism.

If a total of $n_{t_0}(s)$ parts fails due to this defect at the mileage s within the time period t_0, this failure quantity refers to the vehicles which have already reached this distance. Thus the expected km-dependent failure number at the distance s can be calculated analogously to [6] (Eq. 7):

$$n_k(s)=\frac{n_{t_0}(s)}{A_s(t_0)}. \quad (7)$$

This calculation is to be made for all distances s at which failures occurred. Their successive cumulation results in the total failure number $n_a(s)$ in dependence on the driven distance (Eq. 8):

$$n_a(s)=\sum_{\varsigma\le s} n_k(\varsigma)\text{ for }s\ge 0. \quad (8)$$

Following from this, the entire failure quantity n_a to be expected is valid as (Eq. 9):

$$n_a=n_a(s_0)=\sum_{\varsigma\le s_0} n_k(\varsigma). \quad (9)$$

Likewise, in order to establish when in terms of time the failures are to be expected, the time requirement is taken into account. This leads to the following simple estimation for the time-dependent failure number $n_a(t)$ (Eq. 10):

$$n_a(t)=n_a\cdot A_{s_0}(t)\text{ for }t>0. \quad (10)$$

Additionally, the failure occurrence concretely present can be considered (Eq. 11):

$$n_a(t)=\sum_{\varsigma\le s_0} n_k(\varsigma)\cdot A_\varsigma(t)\text{ for }t>0. \quad (11)$$

An extrapolation beyond the distance s_0 is not necessary, because the failure then no longer occurs. If the affected production quantity is in field service over a longer time period, the procedure presented here is applied selectively to subsets from shorter time periods.

3.2 Random Failure Behaviour

In the case of a random failure behaviour the defect can be latent and this is also reflected in the field data. The associated failure data arise from a large value range and permit the application of known models for reliability prognoses.

The expected failures are calculated again with (Eq. 7) and thereafter successively cumulated. Using the total affected production quantity n_0 as reference, the empirical distribution function is derived as (Eq. 12):

$$\tilde{F}_k(s) = \frac{n_a(s)}{n_0} = \frac{1}{n_0} \cdot \sum_{\varsigma \le s} n_k(\varsigma) \tag{12}$$
for $s \ge 0$.

After this the fitting is carried out using a theoretical distribution function $F_k(s)$, often in the case of random failures in the form of an exponential distribution. With the associated density $f_k(s)$ the time-dependent lifetime is then predicted (Eq. 13):

$$F(t) = \int_0^\infty f_k(s) \cdot A_s(t)\,\mathrm{d}s \text{ for } t > 0. \tag{13}$$

The exact derivations of (Eq. 12) and (Eq. 13) and also detailed explanations are given in [6] and [7]. The two lifetime distributions $F_k(s)$ and $F(t)$ are prognoses over longer periods in service and describe the operation of the failure mechanism in dependence on the driven distance and the time respectively. This leads to the time-dependent failure number (Eq. 14):

$$n_a(t) = n_0 \cdot F(t) \text{ for } t > 0. \tag{14}$$

3.3 Systematic Wearout Behaviour

If the failure is caused by wearout or ageing, a certain demand or time period is here necessary. Only after this point are failures to be expected. This late failure behaviour is reflected in the failure data and can in principle affect all components.

Analogously to systematic initial failures, the relevant demand limit is determined from case to case by using field data and results from tests and calculations. Likewise the demand limit can often be denoted as a distance s_0 which is the minimum that has to be driven so that the failure mechanism can lead to a failure. The varying driving behaviour during the period of t_0 years in field service is again included by the time requirement $A_s(t)$ using (Eq. 5).

Since all components can be affected by the wearout behaviour, the lifetime prognosis is made analogously to the procedure for random failures. The expected failure numbers are calculated using (Eq. 7) and the associated empirical distribution function using (Eq. 12). At the following fitting of a theoretical distribution function $F_k(s)$ the failure-free period up to the distance s_0 must be considered. Thus (Eq. 15) is valid for the time-dependent lifetime:

$$\begin{aligned} F(t) &= \int_0^\infty f_k(s) \cdot A_s(t)\,\mathrm{d}s = \\ &= \int_{s_0}^\infty f_k(s) \cdot A_s(t)\,\mathrm{d}s \end{aligned} \quad \text{for } t > 0. \tag{15}$$

For such long-time prognoses starting from a defined demand limit and extending towards infinity, the size of the existing vehicle population can exercise a significant influence. The population decreases continuously with increasing vehicle age and is dependent on car brand and model. This specific population factor $c(t)$ can be ascertained from registration statistics from transportation ministries, automotive associations, etc.

Finally the time-dependent failure numbers $n_a(t)$ can be calculated by taking into account the total production quantity n_0 and the declining vehicle population (Eq. 16):

$$n_a(t) = n_0 \cdot c(t) \cdot F(t) \text{ for } t > 0. \tag{16}$$

For all the models presented here, it is important that the mileage distribution must not be determined using the few failure data under investigation but must be based on a sufficiently large and representative data set.

4 Summary and Outlook

Starting with the statistical recognition of whether the failure behaviour is random or systematic, the models presented here in the second part permit a statistically assured prognosis of the failure quantity to be expected. These models are easily applicable and have proven themselves in real risk assessments. They are based on known and proven practical procedures in the automotive industry and they close a gap in the field of "Risk Assessment".

To prevent fallacious statistical prognoses based on faulty technical assumptions and thus to avoid potential misjudgements with high resulting costs it is vital to understand the technical nature of the defect and to correctly assess the fundamental operation of the failure mechanism. The extra expenditure of time, money and work for additional tests and calculations, etc., amounts at most to only a fraction of the costs arising from wrong decisions.

Statistical procedures together with the necessary technical analyses and tests provide the required confidence for judging and assessing potential risks.

References

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