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Principles of Practice and Teacher Actions: Influences on Effective Teaching of Numeracy

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Studies such as the Effective Teachers of Numeracy Study (Askew, Brown, Rhodes, Johnson, & Wiliam (1997) have contributed much to our understanding of what constitutes effective teaching of numeracy. This paper aims to build on these findings and to contribute a model that could be used to understand teachers' numeracy practices and the factors that influence these practices. Through a synthesis of the literature, the author has devised a set of principles of practice which encapsulates effective teaching of numeracy and has identified six teacher actions which can be used to enact these principles. Findings from case studies conducted with three teachers indicated that the model provided a useful framework for observing and understanding classroom numeracy practices.

The success of particular Asian countries in the Trends in International Mathematics and Science Study (TIMSS) studies conducted in 1995, 1998, and 2003 sparked an interest in describing commonalities between classrooms of particularly effective countries (Clarke & Clarke, 2002). In response to recommendations to redefine numeracy for teaching practice, much of the research in the last decade has focused on teachers' everyday practices (Groves, Mousley, & Forgasz, 2004), with a common theme of this research involving the description of characteristics of effective teachers of numeracy (e.g., Askew et al., 1997; Clarke et al., 2002).

Numeracy in this context refers to one's ability to "use mathematics effectively to meet the general demands of life at home, in paid work, and for participation in community and civic life" (Department of Employment, Education, Training & Youth Affairs (DEETYA), 1997, p. 15). Numeracy differs from mathematics in that it extends to people being able to use mathematics in everyday situations (Department of Education & Children's Services, 2003). Although the term 'numeracy' is used in primary school contexts alongside of, and sometimes in place of, mathematics (Department of Education & Children's Services, 2003), teaching students to become numerate is not about replacing mathematics with numeracy, but rather rethinking how mathematical knowledge is learned and its relevance to students' lives (Department of Education & Children's Services, 2003). This has implications for teaching practice in that traditional approaches are not sufficient for developing numeracy and some may even be counter-productive (McIntosh, 2002).

Askew et al. (1997) identified effective teachers of numeracy in a range of schools in the UK by looking at mean test scores of students over time.

Their identification of effectiveness was based on rigorous evidence of increases in pupil attainment, not on presumptions of "good practice" (Askew et al., 1997, p. 8). Data were collected from over 2000 pupils and evidence was gathered from a sample of 90 teachers from Year 1 to Year 6. While the authors' definition of numeracy differed slightly from that used in Australia in that it included number understanding and skills out of context as well as applications, they were able to identify characteristics that distinguished highly effective teachers of numeracy from other teachers. Among other factors they found that highly effective teachers held a particular set of coherent beliefs and understandings which underpinned their teaching of numeracy, including what it means to be numerate and the relationship between teaching and pupils' learning of numeracy; used teaching approaches that connected different areas of mathematics and different ideas in the same area of mathematics using a variety of words, symbols and diagrams; used pupils' descriptions of their methods and their reasoning to help establish connections and to address misconceptions; ensured that all pupils were being challenged, not just those who were considered more able; and encouraged purposeful discussion.

According to Stephens (2000), there appear to be no comparable studies of numeracy undertaken on a similar scale in other countries, but a study designed to identify the characteristics of effective teaching of numeracy by early childhood teachers was undertaken in Australia (Clarke et al., 2002), with the findings sharing commonalities with Askew et al.'s (1997) study. Clarke et al. (2002) were involved in a collaborative project as part of the Early Numeracy Research Project (ENRP) and conducted six case studies of early childhood teachers whom they identified as being effective teachers of numeracy. This identification was based on data that tracked students' mathematical growth over 2 years. The teachers whose students showed the greatest growth were selected for the study, involving detailed observations, questionnaires, and interviews. From the study, the researchers developed a set of common themes or features of what effective teachers did, including the following: maintained a focus on important mathematical ideas and made the mathematical focus clear to children; structured purposeful tasks that engaged children and enabled different possibilities, strategies, and products to emerge; used a range of materials/representations/contexts for the same concept; used teachable moments as they occurred and made connections to previous mathematical experiences; used a range of question types to probe and challenge children's thinking and reasoning, and encouraged children to explain their mathematical thinking/ideas; and had high but realistic expectations of all children (Clarke et al., 2002). Many of these characteristics are consistent with the findings from Askew et al. (1997), and relate to what Askew and his colleagues termed 'connectionist' teachers (Askew et al., 1997).

The findings from these studies are valuable in that they provide insight into the mathematical and pedagogical practices that teachers use to achieve effective teaching of numeracy, and a review of the literature has revealed that these two studies have been responsible for contributing in a major way to what is perceived to be effective teaching of numeracy. Askew et al.'s (1997) study in particular is widely cited in the literature (e.g., Anthony & Walshaw, 2007; Doig, 2007; Hurst, 2007; Carroll, 2005; Doig, McCrae, & Rowe, 2003; Coben et al., 2003) and their findings have been endorsed by similar research (e.g., Jones, Tanner, & Treadaway, 2001; Reynolds & Muijs, 1999; Thomas & Ward, 2002). The study addressed in this paper sought to build on the knowledge generated by these studies and to add to the research through the further identification of practices that could be directly observed in the classroom.

Principles of Practice for Effective Teaching of Numeracy

Identification of the Principles

As previously indicated, there were a number of characteristics of effective teaching of numeracy that were similar in both studies. As the intention was to examine the presence of these characteristics in the numeracy lessons conducted by the case study teachers, it was necessary to synthesise the findings from both studies into manageable elements. The term 'Principles of Practice' was adopted to identify these elements in order to encapsulate the meaning of 'principle' as being "a basic, primary or general truth, on which other truths depend" (Blair, 1982, p. 712). (Subsequent reading revealed that a similar phrase was used by Watson and De Gueest (2005) to describe teachers' intentions and actions and while somewhat similar in nature, Watson and De Gueest's (2005) principles focused on teacher beliefs, such as "mathematics can be a source of self esteem" and "all students should develop their reasoning and thinking in and through mathematics" (p. 225).) Table 1 identifies the principles of practice extrapolated from the findings of Askew et al.'s (1997) and Clarke et al.'s (2002) research and where appropriate, links have been made with other literature that also reported similar findings.

Table 1
Principles of Practice and Their Links with Askew et al. (1997) and Clarke et al. (2002)

Principle	Links with studies
Make connections	Used teaching approaches that connected different areas of mathematics and different ideas in the same area of mathematics using a variety of words,

	symbols, and diagrams (Askew et al., 1997) Used teachable moments as they occurred and made connections to previous mathematical experiences (Clarke et al., 2002) Supporting literature: Charles, 1999; McIntosh, 2002; NCTM, 2000.
Challenge all pupils	Ensured that all pupils were being challenged, not just those who were able (Askew et al., 1997) Had high but realistic expectations of all children (Clarke et al., 2002) Supporting literature: Jones et al., 2001; NCTM, 2000; Grouws & Lembke, 1996; Reynolds & Muijs, 1999; Thomas & Ward, 2002.
Teach for conceptual understanding	Had knowledge and awareness of conceptual connections between the areas they taught of the primary mathematics curriculum (Askew et al., 1997) Focused on important mathematical ideas and made the mathematical focus clear to children (Clarke et al., 2002) Supporting literature: Charles, 1999; McIntosh, 2002; NCTM, 2000.
Purposeful discussion	Used pupils' descriptions of their methods and their reasoning to help establish connections and to address misconceptions (Askew et al., 1997) Challenged pupils to think through explaining, listening, and problem solving (Askew et al., 1997) Encouraged purposeful discussion (Askew et al., 1997) Used a range of question types to probe and challenge children's thinking and reasoning and encouraged children to explain their mathematical thinking/ideas (Clarke et al., 2002) Used reflection to draw out key mathematical ideas during and/or after the lesson (Clarke et al., 2002) Supporting literature: Jones et al., 2001; Thomas & Ward, 2002.
Focus on mathematics	Focused on important mathematical ideas and made the mathematical focus clear to children (Clarke et al., 2002)

	Structured purposeful tasks that engaged children and enabled different possibilities, strategies, and products to emerge (Clarke et al., 2002)
	Emphasised the importance of using a variety of methods of calculation with a particular focus on mental computation (Askew et al., 1997)
	Supporting literature: Jones et al., 2001; Thomas & Ward, 2002).
Positive attitudes	Displayed very positive attitudes to mathematics (Askew et al., 1997)
	Confident in their own knowledge of mathematics at the level they were teaching and believed that mathematics can and should be enjoyable (Clarke et al., 2002)

Table 1 shows that the principles of practice identified as being central to effective teaching of numeracy were making connections, challenging all pupils, teaching for conceptual understanding, facilitating purposeful discussion, maintaining a focus on mathematics, and possessing and instilling positive attitudes towards mathematics. Although assessment techniques were mentioned in both studies they have not been included in the principles of practice. This decision was made partly because at the time of the study, assessment was a contentious issue in the state in which the study was conducted, and partly because the researcher believed that if the other principles were incorporated into teachers' practices, then assessment techniques could be developed to reflect this (rather than being a separate consideration). Although similar classroom and lesson organisation was found to be a contributing factor in the Clarke et al. (2002) study, and mentioned by other researchers (e.g., Jones et al., 2001; Reynolds & Muijs, 1999) no common form of classroom organisation was used by the effective teachers in Askew et al.'s (1997) study, hence it has not been included as a common 'principle' between the two studies.

Factors Influencing the Principles

According to Askew et al. (1997), understanding why some teachers were more effective than others required an examination of the relationship between teachers' beliefs, knowledge, and classroom practice, and recognition that the implicit beliefs or theories that teachers have, together with their knowledge, influenced the way that teachers interpreted classroom events.

Teacher knowledge. Shulman (1987) proposed that a teacher's base comprised seven types of knowledge. Although it is acknowledged that all knowledge types are important and interact with others to impact on effective teaching of numeracy, teachers' content knowledge and pedagogical content knowledge (PCK) are particularly applicable to the study reported on in this paper. According to Shulman (1987) the teacher has a special responsibility in relation to content knowledge and should possess depth of understanding in order to communicate what is essential about a subject and be able to provide alternative explanations of the same concepts or principles. Mewborn (2001), however, reported that studies have found that although elementary teachers generally have a command of the facts and algorithms that comprise school mathematics, they lack conceptual understanding of this mathematics. Although there is little evidence in the literature to identify how much or what type of content knowledge an elementary school teacher requires (Ball, 1990), many primary teachers express considerable lack of confidence in their own knowledge and understanding of mathematics (Stephens, 2000). Kanes and Nisbet (1994) found, for example, that 46% of primary teachers surveyed admitted to being insufficiently prepared, in terms of mathematics content, for the tasks of classroom teaching. Studies into pre-service teachers' content knowledge have revealed that the pre-service teachers' grasp of subject content knowledge was a factor in determining the way they dealt with children's responses, how they made connections with other mathematical concepts, and their choice of appropriate examples (Huckstep, Rowland, & Thwaites, 2003).

It seems, however, that having a well developed content knowledge does not ensure that one can teach it in ways that are meaningful for students (Mewborn, 2001). Pedagogical content knowledge (PCK) is of special interest because it represents "the blending of content and pedagogy into an understanding of how particular topics, problems or issues are organised, represented and adapted to the diverse interests and abilities of learners, and presented for instruction" (Shulman, 1987, p. 8). PCK also involves instruction, which Shulman (1987) described as the observable performance of teaching acts, involving interacting effectively with students through questions and probes, answers and reactions, and praise and criticism.

Constructivism. Within the seven types of knowledge, Shulman (1987) advocated that teachers needed to be familiar with the findings of empirical research in the areas of teaching, learning, and human development. The learning theory most closely associated with effective mathematics teaching is that of constructivism, whereby the learner constructs their own knowledge; each learner constructs a unique mental representation of the material to be learned and the task to be performed, selects information perceived to be relevant, and interprets that information on the basis of his or her existing knowledge (Shuell, 1996). The construction of an idea, therefore, will vary from individual to individual, even with the same teacher and within the same classroom (Van de Walle, 2004). The teacher has a responsibility to be aware of the implications of this when teaching and particularly needs to foster the connections between both different areas of mathematics and previous mathematics learning. This implies that to teach within a constructivist paradigm, one needs sound knowledge of the subject matter, along with knowledge of the pedagogical principles (PCK) needed to teach this knowledge to students. The connectionist teachers identified in Askew et al.'s (1997) study were found to hold beliefs that supported this premise, including the need to explicitly recognise and work on student misunderstandings.

Teacher beliefs. The final aspect to be discussed in relation to factors influencing the effectiveness of teachers of numeracy concerns teacher beliefs. While a concise definition of what constitutes beliefs cannot be agreed upon, Thompson (1992) distinguished them from knowledge in that they can be held with varying degrees of conviction and although independent of their validity, are valid for the individual who holds them. One of the major outcomes of Askew et al.'s (1997) study was the identification of three different sets of beliefs that were important in characterising and helping to understand the approaches teachers took towards the teaching of numeracy (Askew et al., 1997). Teachers' beliefs about what it is to be a numerate pupil, beliefs about how pupils become numerate, and beliefs about how best to teach pupils to become numerate were found to correspond with connectionist, transmission, and discovery orientations (Askew et al., 1997). While acknowledging that the orientations of connectionist, transmission, and discovery were 'ideal' types, Askew et al. (1997) found that some teachers were more predisposed to talk and behave in ways that fitted with one orientation over the others. For example, if a teacher believed that being numerate involves "the ability to perform standard procedures or routines" (p. 31), then pupil errors were more likely to be interpreted as the result of pupil carelessness or lack of attention (transmission belief). If, however, a teacher believed that pupils were trying to make sense of information, then errors may be interpreted as arising from misunderstanding, rather than carelessness (connectionist belief). It was found that "the teachers with a strongly connectionist orientation were more likely to have classes that made greater gains over the two terms than those classes of teachers with strongly discovery or transmission orientations" (Askew et al., 1997, p. 24).

Summary. The preceding discussion has identified six principles of practice that were derived from the commonalities between the major studies into effective teaching of numeracy. The ways in which these principles are enacted in classroom practice depend very much on a combination of the different types of knowledge held by the teacher and his/her own personal set of beliefs. When learning, students will construct their own knowledge and understandings and teachers need to be cognisant of this and adapt their instruction and practices accordingly. This requires the teacher to have both a strong content knowledge of the mathematics to be taught, along with the PCK required to make this knowledge comprehensible to students.

Teacher actions

The literature consulted to this point was useful in establishing general principles of practice that indicated effective teaching of numeracy, together with the importance of considering the influence of teachers' knowledge and beliefs on classroom practice. Although both Askew et al.'s (1997) and Clarke et al.'s (2002) studies included illustrative examples from classroom lessons to portray particular orientations or practices, these excerpts were generally brief in nature and did not detail specific teaching behaviours that could be observed. As the study discussed in this paper involved documentation of specific practices through observation of numeracy lessons, it was necessary to identify specific teaching actions that could be observed, described, and evaluated. While some of these teaching actions were identified following initial data collection, further review of the literature revealed that these teaching actions had been considered by other researchers and impacted upon teachers' effective teaching of numeracy. The teacher actions identified by directly observing case study teachers' lessons and from the literature were: choice of examples, choice of task, questioning, use of representations, modelling, and teachable moments.

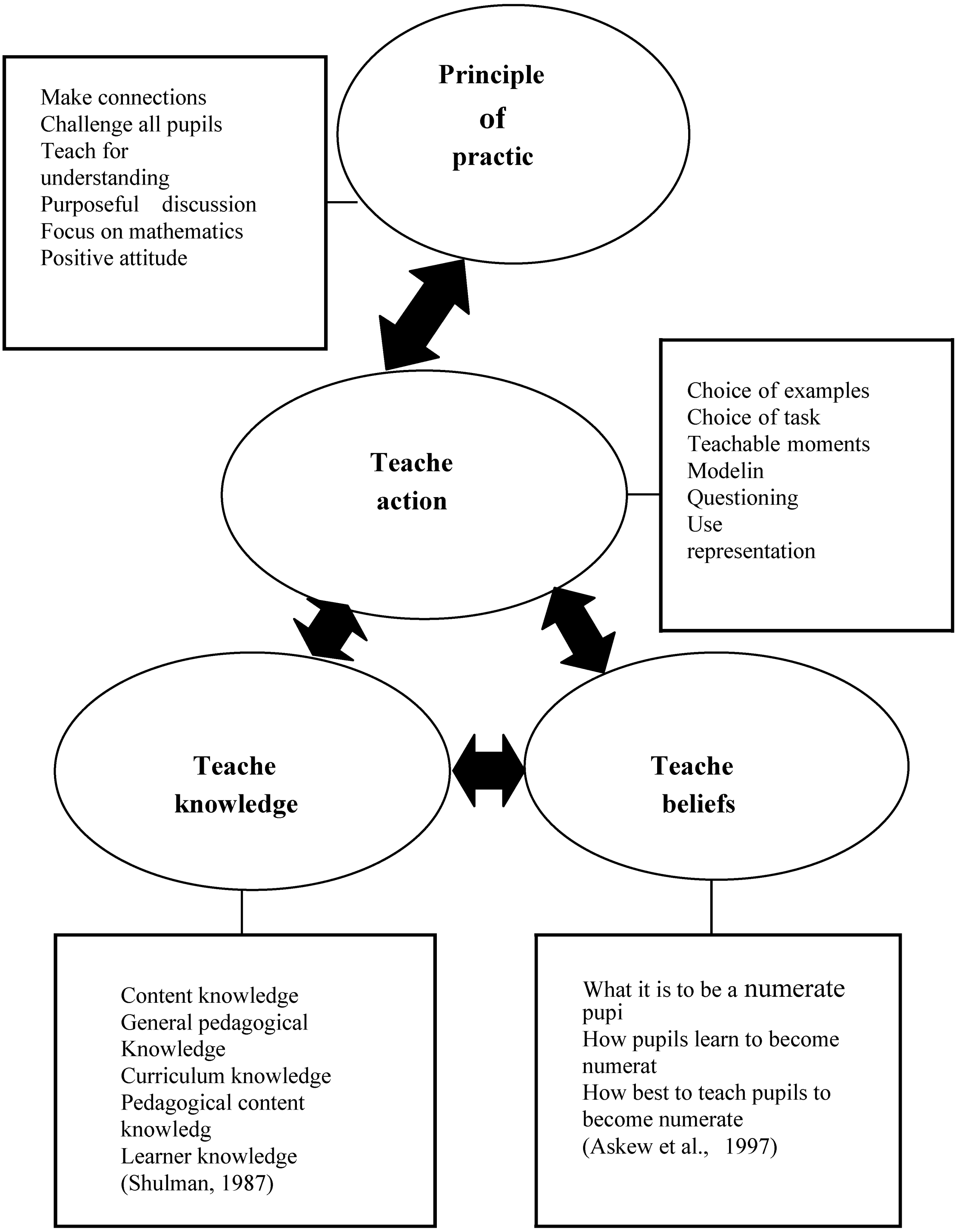

Figure 1. Teacher actions and their relationship to teacher knowledge, teacher beliefs and principles of practice.

The Proposed Model

Figure 1 shows a model, devised as part of the study, which was used to interpret and understand the numeracy practices enacted by the teachers in the study. It shows the relationship between teacher actions, teacher knowledge and beliefs, and the principles of practice. The principles of practice have been placed at the top of the model to indicate that the achievement of these principles is the goal, and that this goal is influenced by the circled factors below. Teacher knowledge and beliefs have been separated, but the two-way arrow indicates that, like the other factors, these two factors are inextricably linked. Knowledge and beliefs influence teachers' actions: For example, teachers' choice of examples is influenced in particular by their content knowledge and PCK and by their belief in how best to teach students to become numerate. The model was used in the study as a means of understanding each case study teacher's practice. Although 'teaching actions' were used as a starting point, the model allows for interpretation to occur at any of the circled factors. For example, if a particular teacher's practice was characterised by *making connections*, then the teacher's actions could be examined to determine which ones were particularly helpful in establishing these connections. Similarly, teacher knowledge and beliefs could be examined to determine what types of knowledge and beliefs led to the teacher's ability to *make connections*. As teaching actions were used as a starting point, the results and discussion section presented in this paper is structured around these actions. Following an overview of the methodology used in the study, the findings of the study in relation to four[1] of the teaching actions identified will be presented.

Methodology

A case study approach (Stake, 1995) was used to document the numeracy practices of three upper primary teachers. The teachers were selected using purposive and opportune sampling (Burns, 2000); years of teaching experience ranged from 6 to 18 and their classroom practice in general was highly regarded by the Principals in the different schools in which they taught. They were selected not because they were recommended as being particularly effective teachers of numeracy, but because they were considered 'good practitioners'; this was an important consideration as the study was focussed on factors specifically related to developing students'

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[1] Due to the large amount of data generated by the study, it was not possible within the confines of this paper to adequately address all six teaching actions and demonstrate how the model was applicable to understanding the actions observed; the author therefore selected four to serve as illustrative examples of this.

numeracy, rather than generic factors such as behaviour management. Between four and seven numeracy lessons (one each week) for each teacher were observed and videotaped. Parts of the lesson involving teacher-led class discussions were transcribed within hours of observation and field notes were also used to document aspects of the lessons that were not captured on videotape. Work samples were also collected and following each lesson, the video footage was viewed collaboratively by the researcher and the teacher, and the discussions audio-taped and transcribed. The transcripts of the lessons were manually analysed to identify the principles of practice in action, through the teachers' use of choice of examples, choice of task, teachable moments, modelling, questioning, and use of representations. Each of these actions was then further analysed and their effectiveness evaluated, often through the use of specific criteria. For example, teachers' choice of tasks was evaluated using a set of criteria which rated the tasks based on the level of cognitive demand they required (see Arbaugh & Brown, 2005). Categories were created which allowed for each type of question asked by the teacher to be classified, along with the students' corresponding responses. Frequency counts were conducted on the types of questions asked and the responses received. With the exceptions of questioning and teachable moments, further detail on the analysis of the teaching actions are provided in the results and discussion pertaining to each action.

Results and discussion

Choice of Examples

One instructional strategy that teachers can use to help students construct meaning and one that plays a central role in the learning of mathematics is the use of examples. Examples may include illustrations of concepts and principles, contexts that illustrate or motivate a particular topic in mathematics, and particular solutions where several are possible (Watson & Mason, 2002). Because examples are chosen from a range of possibilities, teachers need to recognise that some examples are 'better' than others (Huckstep et al., 2003). A good instructional example is one which is transparent to the learner, helpful in clarifying and resolving mathematical subtleties, and generalisable (Bills, Dreyfus, Mason, Tsamir, Watson, & Zavlavsky, 2006). Bills et al. (2006) maintain that the specific representation of an example or set of examples and the respective focus of attention facilitated by the teacher, have bearing on what students notice, and consequently on their mathematical understanding.

Evaluating the effectiveness of particular examples provided by the case study teachers was necessarily subjective as it was not possible to ascertain whether or not the example was perceived to be equally appropriate (or not appropriate) for all participants. As Askew (2004) found, "one cannot assume that individual pupil's interpretations … are either similar to each other's, or fit with the activity expectations of the teacher" (p. 74). Evaluation of the appropriateness of the examples provided was therefore based on classroom observations and the researcher's own pedagogical knowledge (see also Muir, 2007).

The data revealed that teachers' choice of examples was particularly relevant to developing the principles of *making connections* and developing *conceptual understanding.* The following provides an illustrative example of one teacher's choice of example, and its relative effectiveness in facilitating the making of connections and the development of conceptual understanding.

John, one of the case study teachers, used the example of sharing $15.00 between four people to review and model with the class how to represent remainders as decimals. This was deemed to be an appropriate example to use with the students, as an answer of 'three remainder three' would not have answered the question and observation of the students showed that they could work through the process accurately. As the class had previously been working on short division problems using single-digit divisors and as the aim of the lesson was to build on this and introduce division with two-digit divisors, John then wrote the following example on the board (in the form of the standard short division algorithm): 764 divided by 15. This was arguably a less effective example as the answer involved a recurring decimal which led to some confusion as the following students' statements show:

Trevor: But they can't be the same again because there's going to be a million zeros again and it's never going to be 93 again

Trudy: Would you put a recurring dot if it kept on going 9 3, 9 3?

Samuel: Does the remainder 5 of the 45 just go in the bin at the end?

Tr[2]: Just for our purposes, we stop at two decimal places – and yeah, it's just – it would probably go 3,3,3, but just for our purposes, it's such a small number, we'll just go to two decimal places

George: If there's a 9 at the start, and it goes 3,3,3, do you put the dot above the 3?

Tr: Um (pauses) I'll have to check that too – good question.

[2] Tr. refers to teacher throughout

John appeared uncomfortable with pursuing the line of questioning, with his response indicating a lack of content knowledge in regard to representing decimal remainders. The choice of example also indicated a lack of PCK in that John did not anticipate the possible misconceptions associated with representing remainders as decimals and adapt his instruction to cater for this. John later confirmed that this was the case:

> I think personally it's really poor on my part not being confident enough to use the specific mathematical terms, which I know is really important um cause we're trying to get the mathematical language through that um so that's not an oversight, but probably lack of planning on my part um and knowledge about using the recurring [decimals] and so it's something that I need to brush up or 'rebrush' up on make sure I know it for next time, so they've got those answers there which they're genuinely interested about … because you see that recurring – that was almost a flippant comment when we first started, and now they've picked up on it, so probably shouldn't even have introduced it until a later stage.

In this instance, then, the choice of example did not provide for students to *make connections* with previous learning and together with the teaching approach of following the standard procedure for short division, did not assist with developing *conceptual understanding.*

Choice of Task

The choice of task can be another determinant of the effectiveness of the numeracy lesson. Clarke et al. (2002) identified that the effective teachers in their study structured purposeful tasks that enabled different possibilities, strategies, and products to emerge, and that engaged children and maintained involvement. Research on types of tasks in which students engage indicate that a relationship exists between the level of thinking required by a mathematical task and the nature of students' learning (Arbaugh & Brown, 2005). Henningsen and Stein (1997) advocated that tasks needed to be appropriately challenging and motivating with a balance maintained between supporting students' reasoning and thinking without reducing the complexity and cognitive demands of a task, making 'choice of task' particularly applicable to the principle of *challenging all pupils*.

The criteria used to evaluate the case study teachers' choice of task involved rating the tasks according to the level of cognitive demands they required. The framework designed by Smith and Stein (1998, as cited in Arbaugh & Brown, 2005) provided a set of criteria to distinguish between tasks that involved higher-level demands (such as exploring and understanding the nature of mathematical concepts) and tasks with lower-level demands (such as tasks that involved memorization or procedures without connections to meaning).

A sequence of lessons that was conducted by Sue and based on problem solving was observed. Using the framework and its descriptors, all of Sue's tasks were classified as requiring higher-level demands. Example tasks involved solving a number of non-routine problems and carrying out investigations, such as identifying the total number of squares on a chessboard. The chessboard task was classified as requiring higher-level demands in that (potentially) it met the criteria of requiring students to use complex thinking and considerable cognitive effort, and demanded self-monitoring of one's own cognitive processes (Smith & Stein, 1998, as cited in Arbaugh & Brown, 2005). Observations and field notes indicated that students used various strategies and their engagement with the task illustrated how tasks could be used to illustrate the principle of *challenging all pupils*. On other occasions, although the tasks met the criteria for demanding higher-level cognitive demand, the instructions which accompanied the task led to students accessing it at a lower level. For example, students were given a number of problems to solve, but were instructed to only use 'guess and check' to solve them. Furthermore, they were required to place their answers in a table, with alternative strategies not encouraged:

Tr. Some people just worked it out in their heads, but that's not what I wanted you to do – I wanted you to draw up a table

This example highlights the potential limitations associated with using a set of criteria to evaluate classroom practices. While an overview of the tasks selected by Sue indicated that she was *challenging all pupils*, closer analysis revealed that this was not always the case.

In the case of Ronald, his choice of tasks provided an example of how the model proposed in Figure 1 could be used to understand his teaching actions. An analysis of Ronald's tasks showed a balance between tasks that required a higher-level cognitive demand and tasks that required lower-level cognitive demands. In a sequence of lessons on percentages, Ronald provided opportunities for students to carry out investigations, conduct their own surveys, and construct graphs. In one lesson, students constructed a 'placemat' to record their current understandings about percentages. The following excerpt details how the task was introduced to students:

Tr: So the questions on your placemat, OK, what do percentages tell us? What I want you to do is in your groups discuss what do percentages tell us? So you're thinking about in society and how they actually tell us. Probably the first thing that you need to do is decide what is a percentage – you can use your maths dictionaries for that or you can use discussion with your group. OK. Where are percentages used? That's where we start to think about sport, shops and things like that so I want you to find as many places as you can where they are used, and the top question is why and how do we

> use them, so when we're asking why and how we use them I want you to give me an explanation on why we use percentages instead of fractions sometimes, why do we use percentages instead of decimals. OK, you just need to talk to your group and maintain your group and then we'll go around and share them – any questions before we start?

The task provided an avenue for students to *make connections* with real life and between areas of mathematics, such as the link between percentages, decimals, and fractions. The task also allowed for various possibilities to emerge and provided Ronald with an insight into students' thinking that would shape future teaching directions. In this instance, the task then required higher-level demands and also produced higher-level responses in students. There were a number of occasions, however, when students were required to complete questions or problems involving percentages and were instructed to use the standard percentage algorithm to solve them. On these occasions, other teaching actions, such as choice of examples and modelling, were also incorporated into the choice of task. Ronald would typically model the algorithm with the whole class, with different students contributing at appropriate stages of the process. Students would then be given the task of completing a number of problems using the percentage algorithm.

In order to understand the perceived inconsistencies in Ronald's teaching practice, the model in Figure 1 was used to interpret Ronald's actions. Ronald consistently demonstrated a sound content knowledge of the mathematical topics he was observed teaching. Ronald also demonstrated strong PCK in relation to teaching the topic of percentages. This was evidenced in his provision of examples that involved numbers greater than 100 (anticipating the tendency for students to think of percentages only in terms of 100) and through his consistent emphasis on linking percentages with decimals and fractions. Ronald's teaching actions, however, could at least be partially explained through considering the influence of his beliefs. On the one hand Ronald could be said to have shared beliefs with the connectionist teachers (Askew et al., 1997) in that he was aware of the links between different aspects of the mathematics curriculum and often encouraged students to reason and justify their results. However, his predominant use of the percentage algorithm indicated an orientation towards the belief that being numerate involves primarily the ability to perform standard procedures or routines (Askew et al., 1997). Subsequent conversations with Ronald revealed that this latter belief was largely influenced by his perception that as a Grade 5/6 teacher, he had the responsibility of ensuring his students were "ready for high school", which he interpreted to mean that they needed to learn standard procedures.

Use of Representations

In this context, use of representations refers to the teachers' use of a range of materials, representations, and contexts for the same concept (Clarke et al., 2002). Askew et al. (1997) found that the connectionist teachers in their study encouraged the use of a variety of representations. Teaching practices consistent with this included using a variety of words, symbols, and diagrams to connect different areas of mathematics, with a particular emphasis on mental computation (Askew et al., 1997). Again while being applicable to all the principles, use of representations was found in this study to be particularly relevant to developing the principles of *conceptual understanding* and *making connections*. Although materials such as manipulatives (e.g., base-10 blocks, unifix cubes) and calculators can be used to enhance student understanding, the presence of materials is not enough to guarantee that students will learn appropriate mathematics from them (Kilpatrick, Swafford, & Findell, 2001), therefore it was necessary to examine not only what materials were used, but how and why they were used.

John was the only teacher in the study who explicitly incorporated concrete materials into some of his lessons. In one lesson, a range of materials including base-10 blocks and measuring equipment was made available for students' use (although students were not explicitly told or shown how to use them). In other lessons, students worked in groups to construct a cubic metre using newspaper and tape, and used base-10 unit blocks to construct shapes with different volumes. John indicated that in the past he had not usually utilised concrete materials and the following comment provides an insight into his beliefs regarding the use of concrete materials:

> Tr: … having the materials there for them to work with, having the concrete aids, which were one of the other things there [referring to effective characteristics of effective teaching of numeracy] which you know, I think I said last week, I tend not to use the concrete aids as much – it's been more number sort of oriented, it's been grade 5/6 – that sort of thing – which I want to change because um, I think I was just lazy in the class, because I don't believe it

John particularly utilised the materials after observing that students were finding it difficult to distinguish between area and volume, and found them useful for demonstrating the three dimensional nature of volume. Mini MAB were used by students to construct shapes with given volumes. John both explicitly modelled the use of the materials and monitored students' use of the materials. Lesson transcripts and field notes indicated that students appeared to be constructing an appropriate understanding of the three dimensional property of volume, indicating that the use of concrete materials, in this context at least, was purposeful and assisted students in making sense.

This teaching action could also be observed through teachers' acceptance of students' different approaches to completing tasks. At times, Sue, Ronald, and John all expected students to follow a set procedure or routine, thereby limiting the students' ability to select from a variety of representations. Ronald, for example, expected students to use the standard percentage algorithm to calculate percentages and Sue often instructed students to use a particular process, such as 'draw a table' to solve allocated problems. Often the teachers were quite explicit in directing students to solve a problem in a particular way:

> Tr: What we're going to do today is a particular way of doing maths – it's called a tree diagram so I'm going to give you a problem and you have to go back and sort it out using a tree diagram

As previously indicated, use of representations also included the provision of a variety of contexts. Ronald, in particular, was conscious of the need to provide relevant contexts for students to understand the purpose of learning about percentages. The following excerpt provides an example of this:

> Tr: Quite often you hear people when they're announcing results, um, of anything really, surveys, elections, whatever, that they actually quote percentages instead of the amount of people or the amount of votes that actually occurred. So if we were having a vote in Tasmania, and say there's 85 000 people who voted, and 60% of them voted Labor, why do you think they'd say 60% of them voted Labor, instead of saying for example, about 52 326 people voted Labor? Why do they use 60%?

Ronald reinforced his belief in the importance of this as he later explained:

> Tr: In everything I say to them, in everything we're doing, you can always question my practice, and if I can't explain to you why we're doing it, we won't do it, um and they do, particularly in first term, they said why are we doing this, and I explain to them why we're doing it, and there have been a couple of times over the years where I haven't been able to justify it, and we've stopped it and we don't do it, um, and the kids really appreciate that then because they know they're doing it for a purpose, not just doing rote learning or doing it for the sake of doing it

Some of the teacher practices observed in the study, therefore, could be classified as providing examples of 'use of representations'. Although some of these linked with other teaching actions, such as choice of task, choice of example, and modelling, it did warrant a separate category. As with other teaching actions, the way teachers encouraged (or did not encourage) different representations was likely influenced by their PCK and their own particular set of beliefs.

Modelling

Modelling in this context refers to the teacher showing students what to do and/or how to do it. This is not meant to imply that the teacher is 'telling' the students what to do, but rather that the teacher (usually) performs while the students are watching (Borasi & Fonzi, 1998). Students should take an active role at selected points under the guidance of the teacher and opportunities also need to be provided for students to model to their peers (Borasi & Fonzi, 1998). Modelling can be seen in the classroom through demonstrations given by both teachers and students, and can involve other teaching actions such as use of representations and choice of examples. Although no direct reference was made to the types of modelling observed by teachers in the Askew et al. (1997) and Clarke et al. (2002) studies, modelling provides the opportunity for the teacher explicitly to teach mathematical concepts and can be used particularly to develop the principles of *conceptual understanding* and maintain a *focus on mathematics*.

It was found that the teachers in this study used modelling either to demonstrate or explain a process or concept. This may have occurred when they were explaining a task to students and often involved the use of examples either to illustrate particular points or for students to put into practice what they had seen modelled. The following provides an illustrative example of how modelling was used by Ronald to demonstrate the process involved with using the percentage algorithm:

Tr: If I had 10% of 90 (writes on board) how would I set this out?

Mark: Ten over 90

Tr: Is it ten over 90? Remember 10% is ten out of what?

Mark: A hundred

Tr: A hundred. Times what?

Mark: Ninety over one

Tr: OK [writes on board] can we break that down at all? Dean?

Dean:	Ten into ten and ten into hundred
Tr:	Or ten into hundred and ten into ten – doesn't matter which way we do it – so how many tens in 90?
Dean:	9
Tr:	[crosses out and records on board]
Dean:	Um, ten times 9
Tr:	Is there anything else we can break down? We've got ten over ten? Is there anything else we can do?
Ben:	How many tens are in ten? One
Tr:	Yep
Ben:	How many tens are in ten? One
Tr:	Yep, so now we can multiply it out
Ben:	One times 9 equals nine and 1 times 1 is 1
Tr:	So the answer is?
Ben:	9

This process was repeated using different examples, and as the excerpt shows, the process was broken down into steps and it was clear that the intention was to show students how to carry out the process. In this instance, although the modelling process was clear and student contributions indicated that they could follow the process, it is questionable in terms of demonstrating the principle of *teaching for conceptual understanding*.

Conclusions and Implications

The results from the study indicated that there are a number of teaching actions that are observable in the classroom. Within each of these actions, there are qualitative differences which can affect effective teaching of numeracy and impact on students' understanding. The model presented in Figure 1 was useful in examining what types of actions teachers used, the influences which impacted upon the relative effectiveness of these actions, and how the actions themselves could be used to demonstrate the principles of practice associated with effective teaching of numeracy.

Observation in classrooms has often been characterised by quantitative accounts which record the frequency of particular behaviours (e.g., Galton, Hargreaves, Comber, Wall, & Pell, 1999; Galton, Simon, & Croll, 1980; Stodolsky, 1988). Although frequency counts formed part of the data collected for the study reported on in this paper, it was found that more accurate indicators of teachers' effective practices were found by examining the nature of the actions observed. For example, a frequency count may have indicated there were a number of incidences when the case study teachers incorporated the use of concrete materials when modelling a concept. While this could indicate an enactment of the principle *teaching for conceptual understanding*, closer examination of how the materials were used by both teachers and students revealed that this was not necessarily the case. This is consistent with findings from other research (e.g., Moyer, 2001; Ahmed, Clark-Jeavons & Oldknow, 2004) that it is not the presence of the materials themselves, but rather how they are utilised that can facilitate student understanding.

Through including 'teacher feedback' within the action of 'questioning' to encourage purposeful discussion, all of the teacher actions observed in the case study teachers' numeracy lessons were able to be categorised into at least one of the six actions identified. While some actions occurred simultaneously (e.g., questioning and modelling, modelling and use of representations, choice of example and choice of task), there were also incidences where actions occurred independently of others, warranting the six separate actions identified. It was also found that consideration of a number of actions was often required to demonstrate the principles of practice. Furthermore, although some actions were particularly applicable to some principles (e.g., questioning and encouraging purposeful discussion), all actions impacted to some extent on every principle. Teaching is a complex act and studying the interactions that occur between teachers and students implies that some form of organisation is required to make this process manageable and able to be reported to others. "The key seems to be in parsing the rich, interwoven, practical teaching experience into pieces that are small enough to be readily investigated, yet large enough so that their distinctively practical character is preserved" (Leinhardt, 1990, p. 20); linking the teaching actions with the principles of practice enabled this to occur.

In their final report, Askew et al. (1997) recommended that further research be conducted to validate the model of the three types of teaching orientation and to explore the nature of "connected" knowledge in more detail (p. 96). The study discussed in this paper found that together with teacher knowledge, the different types of beliefs held by the case study teachers influenced their teaching actions. The identification of the different sets of beliefs proved useful in interpreting teacher actions and helped account for the differences in teaching approaches observed.

Although there is no unique way of arriving at effectiveness in the teaching of numeracy (Askew et al., 1997), the implications from this study are that it begins with a close examination of observable classroom practices. The teachers in this study had the opportunity to view video footage of their numeracy lessons, engage in professional conversation with the researcher about their practice, and reflect on the effectiveness of the teaching approaches and interactions observed. Research into teacher change has often criticised approaches that focus on teacher practices, finding that teachers will select suitable 'bits', rather than make any substantial changes in their approach or teaching pedagogy (Hargreaves, 1996). However, significant changes in teachers' attitudes and beliefs can occur, after they have gained evidence of improvements in student learning (Guskey, 1995), or when teachers have the opportunity to explore new instructional strategies and ideas in the context of their own classroom practice (Borko, Mayfield, Marion, Flexer, & Cumbo, 1997). It is hoped that the model, principles of practice, and teaching actions described in this paper will be used by other researchers to interpret and understand effective teaching of numeracy, and by teachers themselves to examine their own numeracy practice.

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