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Transforming Ethnomathematical Ideas in Western Mathematics Curriculum Texts

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When ethnomathematical ideas, that is, the mathematical ideas of different cultural groups, are included in mathematics curriculum texts they can become part of the learning experience in various ways. Once included in western classroom mathematics texts, the ethnomathematical ideas become transformed. The transformations involve changes in form or purpose. The study reported here investigated how indigenous cultural practices are transformed when relocated from their original contexts to the western classroom mathematics curriculum. This paper describes the development of a conceptual model that illustrates five different modes of transformation that may occur when indigenous cultural practices are incorporated in mathematics curriculum texts. To illustrate aspects of the model the paper includes an example concerning the Australian Aboriginal practice of throwing a returning boomerang. The model provides a way for students and teachers to critically reflect on the ways that ethnomathematical ideas become transformed when used in the classroom. An awareness of how cultural practices are transformed may also allow teachers constructing their own curriculum texts to choose the most appropriate modes of transformation.

Introduction

This research (Dickenson-Jones, 2006) began by noting that there are differences between how indigenous cultural practices are represented in curriculum texts and how they are represented in the anthropological and ethnomathematical literature. This paper explores the transformations of indigenous cultural practices when included in western mathematics curriculum texts. Here, a transformation is defined as a change in the form or purpose of a practice. When an indigenous cultural practice is written into mathematics curriculum, its form and purpose may differ significantly from its form or purpose in its original context. Furthermore, since the cultural context of the western mathematics classroom may differ significantly from the original cultural context of the practice, it can be argued that both the indigenous cultural practice and the mathematics curriculum become transformed. This study, however, was concerned with the way indigenous cultural practices change.

The paper begins with a review of the literature that identifies the reasons for including ethnomathematical ideas in the curriculum. An account of the research methodology follows where the iterative process of collection, selection, and analysis of texts is described. The conceptual background to the study that drew on Bishop's (1993) notion of culture conflict in mathematics curriculum and on Kelly's notion of surface and core constructs (cited in Bishop, 1993) is also outlined. The analysis involved descriptive and comparative components across a range of criteria chosen to highlight the ways that the cultural practices are transformed.

The findings of the research are then synthesised as a relational model where the five different modes of transformation are identified relative to one another. By way of example, a curriculum text that incorporates the Australian Aboriginal practice of throwing a returning boomerang is described to illustrate the nature of three of the five modes of transformation.

A discussion follows in which the model is compared and contrasted with Adam's (2004) conceptualisation of ethnomathematical curriculum. Alangui's (2006) notion of mutual interrogation is also discussed in relation to the modes of transformation generated in this research.

The paper concludes with an account of the limitations of the study and implications for educators with reference to applications of the model in classroom practice and curriculum design.

Literature Review

Efforts have been made to conceptualise the ways that ethnomathematical ideas are included in the curriculum. One such attempt is Adam's (2004) classification of types of ethnomathematical curriculum identified in terms of the reasons for including ethnomathematical ideas. These ethnomathematical ideas may be included in mathematics curriculum texts for various reasons, for example mathematical games might be used as a motivational tool (Stillman, 1995; Zaslavsky, 1994) or mathematical ideas from the learners' culture might promote understanding of western mathematics (Knijnik, 2002). Adam (2004) and Begg (2001) identify five types of ethnomathematical curriculum based on the reasons for including ethnomathematical ideas.

One type of ethnomathematical curriculum seeks to make mathematics meaningful by presenting mathematics as a response to the needs of students (Adam, 2004). The ethnomathematical ideas of the students' cultures may be included in order to acknowledge and value their cultural backgrounds (Adam, 2004; Adam, Alangui, & Barton, 2003). Bishop (1988) also suggests that mathematical ideas from the learners' cultural background may be used to convey mathematical concepts in the classroom. This is based on the idea that students will recognise the relevance of mathematics if connections are made with activities that are familiar to them.

A second type of ethnomathematical curriculum might focus on the concept of mathematics as a human and socio-cultural construction and thus students examine the history of mathematical development in different cultures (Adam et al., 2003; Begg, 2001). For example, Hirsch-Dubin (2006) describes a mathematics education programme based on the development of the mathematical practices of the Ancient Mayans.

The third type acknowledges cognitive aspects of an individual's mathematical development. This type of curriculum is based on the idea that the development of mathematical thinking is a holistic process that occurs through concrete experience (Adam, 2004; Begg, 2001). One reason for this type of ethnomathematical curriculum is that knowledge should be connected so that students are given opportunities to recognise the links between mathematics and other forms of knowledge (Begg, 2001). In this type of curriculum, mathematics could be integrated with other disciplines (Begg, 2001). Mathematical ideas from the students' own cultures would be incorporated so that learning experiences would begin with concepts that are familiar to the child in order to connect existing knowledge with their own cultural mathematics and hence with world mathematics (Adam, 2004).

A fourth type of ethnomathematical curriculum is based on the premise that practices of the mathematics classroom constitute a distinct culture. In this conception of curriculum the way that mathematics is taught may be adjusted to make the classroom context more suited to the cultural backgrounds of the students (Bishop, 1988). This type of curriculum is concerned with how content is organised and presented as opposed to what mathematical ideas are taught (Adam, 2004).

The fifth type of ethnomathematical curriculum seeks to validate the mathematical ideas that form a part of the students' culture by integrating them with classroom mathematics (Adam et al., 2003). In this case, students would participate in mathematical activities from their own culture in order to encourage them to develop an understanding of "how mathematical ideas develop, how they are built into systems, how they are formulated, and how they are then applied in various ways within the culture" (Adam, 2004, p. 52).

In contrast to the pro-ethnomathematical views of Begg (2001) and Adam et al. (2003), Rowlands and Carson (2002) express concerns about the place of ethnomathematics in curriculum. While many mathematics teachers have begun to incorporate ethnomathematical ideas into the curriculum, there remain questions related to how they should be included. Rowlands and Carson (2002) identify four roles that ethnomathematics may take in mathematics curriculum. The four different positions Rowlands and Carson (2002) identify are as follows: "ethnomathematics should replace academic mathematics, ethnomathematics should be a supplement to the mathematics curriculum, ethnomathematics should be used as a springboard for formal academic mathematics and ethnomathematics should be taken into consideration when preparing learning situations" (p. 79). Rowlands and Carson base their argument on elitist interpretations of ethnomathematical curriculum and express concerns about affects on the position of what they refer to as "formal academic mathematics." The fact that they do not recognise formal academic mathematics as an ethnomathematics in its own right indicates that they have misunderstood the essence of ethnomathematical curriculum. In response to Rowlands and Carson's (2002) critique, Adam et al. (2003) argue that the displacement of formal academic mathematics is not the intention of ethnomathematics. They advocate a curriculum where both western mathematics and the mathematical ideas of different cultures may be equally valued (Adam et al., Barton, 2003).

These types of classifications of ethnomathematical curriculum are concerned with the reasons for incorporating the mathematical ideas of different cultures into the learning experience, but do not focus on what happens to the cultural practices when written into the western curriculum. The contribution that this study makes to existing research in the field of ethnomathematical curriculum is to highlight what happens to indigenous cultural practices when they are included in mathematics curriculum texts.

Ethnomathematical ideas are transformed rather than transplanted when included in the curriculum. Transformation occurs when western classroom mathematics intersects with the cultural practices of indigenous groups and may involve many issues that are related to preserving aspects of two different systems of knowledge.

An example that effectively illustrates how these transformations may occur is the Garma Maths curriculum written by and for the *Yolngu* people of Yirrkala in the Northern Territory of Australia (Christie & Yirrkala School Action Group, 1992; Watson-Verran, 1992). The Garma Maths curriculum was developed so that students could learn about *Yolngu* cultural practices as well as western classroom mathematics. One challenge was the need to reconceptualise the education process based on *Yolngu* understandings of space and time (Watson-Verran, 1992). Previous metaphors of Australian Aboriginal education were inadequate since they were based on western linear understandings of space and time and thus the previous metaphors conceptualised Aboriginal education as a two way road where *Yolngu* knowledge and western knowledge are seen as parallel systems.

For *Yolngu* people time spirals outwards from a central point as opposed to western conceptions of space and time as linear (Thornton, 1995). This means that objects or sites that are no longer physically present still appear in *Yolngu* maps of the land (Christie & Yirrkala School Action Group, 1992; Thornton, 1995) since it is possible for the past, present, and future to co-exist. The metaphor of the spiral was extended to conceptualise the philosophy of the Garma Maths curriculum as the place where the waters of a stream and the waters of the ocean spiral together and thus become one body of water (Christie & Yirrkala School Action Group, 1992). The stream represents *Yolngu* knowledge systems and the ocean symbolises western knowledge. Garma is the place where these two ways of knowing meet and are able to become one body of knowledge while both remain true to their origins. This conceptualisation of education sought to accord equal value to both of these knowledge systems and to illustrate the merging of different ways of knowing.

An understanding of the metaphors that informed transformations of *Yolngu* practices during the development of the Garma Maths curriculum supplied a rationale for considering transformations from the perspective of the indigenous cultural practices in the texts. The Garma Maths metaphors provided a way to view the transformations that occur with respect to aspects of the original contexts of the practices rather than only imposing the constructs of western mathematics.

Research Methodology

Data Collection and Selection

The data collection, selection, and analyses processes used in the study were iterative in the sense that one stage informed another in a cyclical manner. For example, preliminary analysis occurred during the data collection and selection phases to ensure that a range of different text types were represented and that sufficient literature was available to describe the practices in their original contexts.

Data collection was the first stage and involved the location of written mathematics curriculum texts that contained indigenous cultural practices. The collection of curriculum texts was limited to the countries of New Zealand, Papua New Guinea, and Australia. The texts were collected via library and Internet searches.

The second stage entailed reducing the set of texts collected to a manageable number. Criteria used here required that there be at least one text from each of the three countries, and that there were different types of curriculum documents and a range of indigenous cultural practices.

The third step involved the collection of supporting literature referring to either the indigenous cultural practices or mathematisations of these practices. There were three different categories of text consulted at this stage: ethnomathematical literature; anthropological writings about the practices; and other reference materials dealing with aspects of the particular practice. During this phase, analysis was undertaken in order to determine whether there was sufficient information to compare the representations of indigenous cultural practices in the supporting literature with their representations in the curriculum. Existence of supporting literature was crucial to the analysis in order to determine the transformations of the indigenous cultural practices that had occurred. In the absence of adequate supporting literature a replacement curriculum text was selected from the original body of data. Descriptions of the intended student group and intended outcomes were also required in order to make these comparisons. The data collection process continued to return to the data selection phase until the final data set comprised seven mathematics texts - two texts each from Papua New Guinea and New Zealand and three from Australia – and the supporting literature describing the cultural practices contained in these texts.

The seven texts finally selected were chosen to be indicative of a range of different ways in which indigenous cultural practices may be included in curriculum resources in the three countries. They were not intended to be definitive nor representative of the complete range of mathematics curriculum materials available. Such a task was not possible within the scope of this small research project.

Descriptive and Comparative Analyses

The analyses consisted of descriptive analysis followed by comparative analysis. Again, these processes were iterative given that one stage of analysis informed another. Both the intended student engagement with the practice and the intended outcomes of the text were crucial to the analyses since these were the criteria used to determine the form and purpose of the cultural practice when relocated into a curriculum text. Bishop's (1993) responses to culture conflict in the mathematics curriculum provided a starting point to describe the intended student engagement with the indigenous cultural practices in the texts. The analysis began with the concept of culture conflict but was eventually conducted in a manner that differed from this analytic frame.

The first phase of analysis was conducted to describe the curriculum texts in terms of the cultural practice in the text, the intended student engagement with the practice, intended student group, and the intended outcomes of the curriculum text. The criteria that framed the descriptive analysis were required so that these aspects of the texts could be compared in the comparative analysis phase.

Although he does not identify as an ethnomathematician, Bishop (1988) recognises that mathematical ideas are constructed by the societies that apply them as a way of coping with and understanding their world. Bishop (1993) asserts that a difference between a student's out-of-school contexts and the context of the mathematics classroom can potentially result in culture conflict. While his focus was on changes of western mathematics curriculum that may occur when including cultural perspectives, this research was concerned with transformations of the indigenous cultural practices.

Culture conflict is defined as a dissonance between the culture of the mathematics classroom and a student's out-of-school cultural context

(Bishop, 1988). Table 1 summarises the possible curriculum responses to culture conflict.

Table 1.

Curriculum Responses to Culture Conflict in Mathematics Education

Approaches to culture conflict	Curriculum
Culture-free traditional view	Traditional canonical
Assimilation	Some child's cultural contexts included
Accommodation	Curriculum restructured due to child's culture
Amalgamation	Curriculum jointly organised by teachers and community
Appropriation	Curriculum organised wholly by community

Source: Adapted from Bishop, A.J. (1993, July). *Conceptualising Cultural and Social Contexts in Mathematics Education* (p. 8-9). Paper presented at the Contexts in Mathematics Education Conference, Brisbane. Columns two, four, and five are omitted.

Clearly an indigenous cultural practice and classroom practice are embedded in distinct cultures. Thus, in order for an indigenous practice to be incorporated in western curriculum, a change must occur. Bishop's (1993) responses to culture conflict identify changes that may occur from the perspective of western mathematics curriculum in order to include indigenous perspectives. Although not concerned with how the indigenous cultural practices themselves become transformed, Bishop's categories provided a starting point for analysing the transformations that may occur when ethnomathematical ideas are combined with western mathematical concepts in the classroom. This study was concerned with the adjustments made to the indigenous cultural practices.

Kelly's (cited in Bishop, 1993) notion of surface and core constructs utilised in the study provided a basis for the comparative analysis. One's surface constructs are adjusted when a new idea or experience may be accepted into one's existing schema with relative ease. In contrast, one's deeper or core constructs are adjusted when a new or unfamiliar experience is accepted into one's existing schema.

The metaphors of surface and core constructs were utilised here to identify the transformations to which indigenous cultural practices may be subject when written into western mathematics curriculum texts. These transformations were determined by considering whether the surface or core constructs of the practices were adjusted when removed from their original contexts. Surface constructs were defined as the form or appearance of the practice while core constructs were concerned with the function of the practice in its original context.

The second stage of data analysis involved a direct comparison among the texts that formed the final data set. This analysis was conducted at two levels. Firstly, the form and purpose of the indigenous cultural practices in the original context were compared with their representations in the curriculum texts with reference to the intended student outcomes. Secondly, the texts were compared with one another according to the types of intended student engagement with the practice in the text. The types of student engagement were identified as: no engagement with the practice, engagement with aspects of the practice, engagement through theoretical comparisons, engagement through concrete comparisons, and performance of the cultural practice.

Ethical Considerations of the Research

The ethical considerations of the study related to the position of a non-indigenous researcher as the analyst of classroom mathematics curriculum texts based on indigenous cultural practices. An awareness of the potential to impose western mathematical constructs and thus trivialise the ethnomathematical ideas of different cultures was the main ethical factor (Ascher, 1991). An analysis of mathematisations of indigenous cultural practices requires that the researcher impose western understandings of mathematics on these practices. In doing so, it may appear that one form of mathematics is valued above different ways of knowing mathematically.

Also, in comparing representations of cultural practices, there is a danger that the research implies the superiority of a curriculum item where the practice in the text closely approximates its form and purpose in its original context. It is not the intention of this study to make value judgements concerning how the practices are transformed. Rather, the research seeks to illuminate how indigenous cultural practices may be transformed when included in western mathematics curriculum.

Modes of Transformation

The types of student engagement with the indigenous cultural practices that were identified in the study are generalised in Table 2 as five different modes of transformation. The modes are defined as disjunction, translation, integration, correlation and union. Each of the five modes of transformation reflects a different form of student engagement with the indigenous cultural practice in the text. Table 2 summarises the characteristics of the five different modes of transformation in terms of the intended student engagement with the practice and the outcomes of the curriculum text.

Table 2.
Different Modes of Transformation

Modes	Intended student engagement
Disjunction	Students do not need to engage with the cultural practice. Students are not required to engage with different ways of knowing the western classroom mathematics in order to achieve the intended outcomes.
Translation	Students interpret aspects of the cultural practice in isolation from its original form and in terms of western classroom mathematics. Students are not required to engage with different ways of knowing the western classroom mathematics in order to achieve the intended outcomes.
Integration	Students engage with the cultural practice through theoretical comparisons with western classroom mathematics. Students are required to think about different ways of knowing the western classroom mathematics in order to achieve the intended outcomes.
Correlation	Students engage with the cultural practice through concrete comparisons with western classroom mathematics. Students are required to experience different ways of knowing the western classroom mathematics in order to achieve the intended outcomes.
Union	Students engage with the indigenous cultural practice through performance of the practice at least for some purposes similar to the practice in its original context. Students are required to engage with the western classroom mathematics through the medium of the indigenous cultural practice in order to achieve the intended outcomes.

Disjunction describes the transformation where student engagement does not occur with any aspect of the indigenous cultural practice. There is no engagement with different ways of knowing the western classroom mathematics as the representation of the practice is entirely dislocated from its original cultural context. A text referring to Maori Weaving Patterns (New Zealand Ministry of Education, n.d.-b) is an example of this mode of transformation. The practice appears to be included only to provide a stepping-stone towards understanding patterns and algebraic relations.

Students are not required to engage with any aspect of the *tukutuku* weaving techniques.

Translation refers to the transformation where engagement is required with only some aspects of the indigenous cultural practice. Students engage with these aspects of the practice in order to interpret them in terms of the western classroom mathematics perceived in the practice. There is no student engagement with different ways of knowing. One example of this mode of transformation is a Samoan Art Patterns text (New Zealand Ministry of Education, 2004) which requires engagement with the practice in the sense that students are required to create their own *siapo* pattern using western classroom mathematical techniques.

Integration refers to the transformation where students are required to engage with the indigenous cultural practice through theoretical comparisons between the indigenous cultural practice and the western classroom mathematics. Student engagement with different ways of knowing occurs by theoretically comparing the cultural practice with the western classroom mathematics. An example of this mode may be seen in a text that incorporates PNG Measurement Systems (Ricky, Anea, & Miria, 2004). Students are required to discuss traditional measurements and make comparisons between traditional systems and metric measurement, but the intended student outcomes are concerned only with performance of the western classroom mathematics.

Correlation describes the transformation where engagement with the indigenous cultural practice is achieved through concrete comparisons with the western classroom mathematics. Students are required to experience aspects of the cultural practice as well as the corresponding classroom mathematics and to engage with different ways of knowing through concrete experience of both forms of mathematics. A curriculum item that illustrates characteristics of this mode of transformation is a Melpa Counting Systems text (Owens, 2003). The outcomes of this text require that students make concrete comparisons between Melpa systems of counting and the western system by performing both in a classroom situation.

Union refers to the mode of transformation that requires students to engage with the practice through performance of the indigenous cultural practice for at least some purposes that resemble those of the original context. Students are required to engage with the western classroom mathematics inherent in the curriculum text through a modified performance of the cultural practice. In this instance, the western classroom mathematics is presented as a different way of knowing mathematics. Of the seven texts only the *Yolngu* Kinship Relations text (Yirrkala Community School, 1991) was identified with the mode of union. In this case, the cultural practice written into the curriculum closely resembled the practice of *gurrutu* kinship relations in their original form. Students were required to relate to one another in a classroom setting according to the relationships defined by aspects of the *Yolngu* kinship system (Yirrkala Community School, 1991; Cooke, 1990). The outcomes of this text were concerned with the performance of the indigenous cultural practice as opposed to the western classroom mathematics.

The modes of translation, integration, and correlation are illustrated in more detail with reference to the Aboriginal Boomerang Dynamics text (New South Wales Board of Education, 2003) which is described in the following sections.

A Model to Map the Modes of Transformation

An extra dimension is added when these modes of transformation are represented relative to one another. Figure 1 is a model illustrating the different ways in which indigenous cultural practices are included in the mathematical curriculum texts that comprised the study. The model consists of five concentric circles with each annulus signifying a mode of representing indigenous cultural practices that was evident in the texts. The mode described as union is identified with the innermost circle where the representation of the cultural practice in the curriculum text most closely approximates the performance of the practice in its original context. The further the curriculum text is positioned from the centre of the model, the greater the difference between the representation of the indigenous cultural practice in the text and the practice in its original context. Concentric circles were used to highlight the non-hierarchical nature of the model and to illustrate the merging of two different systems of knowledge.

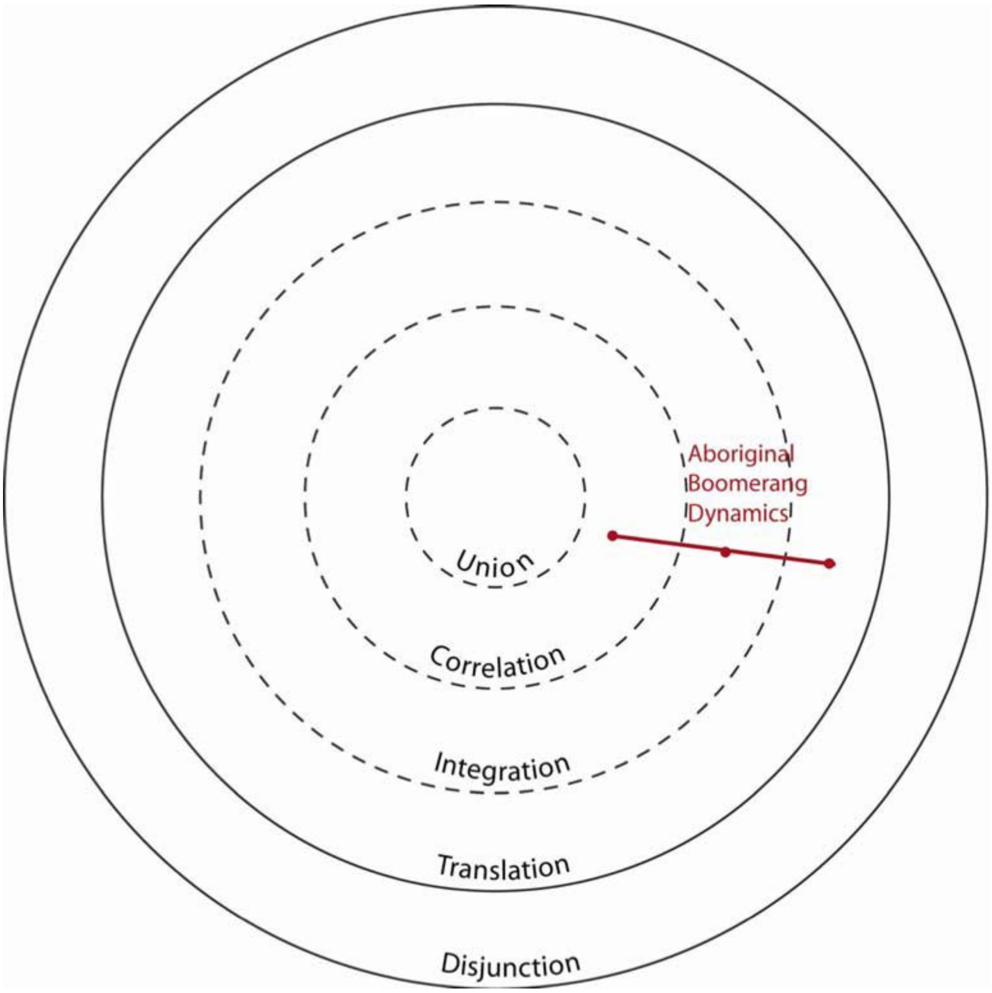

Figure 1. Modes of transformation of indigenous cultural practices.

A solid border encloses the annulus that represents disjunction to indicate that this mode of transformation is discrete. A text located in this zone can only be subject to disjunction as the practice is completely altered and thus assimilated into the western mathematics curriculum. In contrast, the circles that separate each of the four remaining modes are represented by broken lines to indicate that these boundaries are permeable. The curriculum texts can be assigned a position or positions on the model according to the various modes of transformation to which the cultural practice is subject when included in the text.

The study revealed that the curriculum texts might correspond to more than one mode of transformation. Aside from disjunction, the modes of transformation in this model are not mutually exclusive. For example, as indicated on the model, the Aboriginal Boomerang Dynamics text (New South Wales Board of Education, 2003) requires three different types of engagement from the student and is thus located in three rings of the model. A text may be located in one or more rings depending on the types of engagement required from the student with the indigenous cultural practice. When this was the case the modes were positioned in adjacent annuli in the concentric circle model. Although none of the texts in the study did so, it may be possible for a text to occupy multiple but non-adjacent modes of transformation. For example, it is possible that a text might correspond to the modes of union and translation without being subject to correlation or integration.

In the following section the Aboriginal Boomerang Dynamics text (New South Wales Board of Education, 2003) is used to illustrate the characteristics of the modes of transformation of translation, integration, and correlation.

Aboriginal Boomerang Dynamics

In this section the Aboriginal Boomerang Dynamics text (New South Wales Board of Education, 2003) is analysed in order to highlight the multi-modal nature of the model. Since the analysis used in the study relied on comparisons between the intended student engagement with the cultural practice in the mathematical text and the practice in its original context, it is necessary to describe the relevant aspects of the practice itself. This section therefore begins with a brief contextual description of the curriculum text followed by a description of the cultural practice itself. The ways in which the curriculum text illustrates the modes of translation, correlation, and integration are then detailed.

Curriculum Text Type and Location

The Aboriginal Boomerang Dynamics text (New South Wales Board of Education, 2003) is an integrated unit based on a number of traditional Australian Aboriginal cultural practices. The activities with which the analysis was concerned are centred on the cultural practice of throwing a returning boomerang. The unit was designed through collaboration between the New South Wales Board of Studies and primary and high school teachers in the town of Coonabarabran. The resource has been developed specifically for a school with a large Australian Aboriginal population and aims to cater for the learning styles of these students. The unit is intended for use with indigenous and non-indigenous students from grades 6 to 8.

The unit covers concepts associated with geometry and measurement including angles, length, and distance (New South Wales Board of Education, 2003). Activities that are related to returning boomerangs involve estimation of the angles between the two arms of boomerangs, flight paths, and the trajectory required to successfully throw a returning boomerang.

Cultural Practice: Returning Boomerangs

Boomerang is widely accepted as the name for a curved throwing stick that when thrown "traverses a flight path which differs considerably from that of a thrown stick" (Hess, 1975, p. 23). The word boomerang was adapted from the name in the *Turawal* language of the Aboriginal people of George's River near Sydney and although throwing sticks in other regions are identified by other names, this has become the accepted term to describe traditional throwing sticks from Aboriginal communities Australia wide (Barlow, 1994).

Returning boomerangs are used for recreational purposes rather than hunting and come in a variety of designs depending on the region of origin. Both the length of the wings and the angle of the curved wood contribute to the flight path of a boomerang. In order for the boomerang to return, it must be thrown with sufficient force, at the correct angle to an oncoming wind, and at approximately five degrees to the horizontal (Barlow, 1994; Cislunar Aerospace Incorporated, 1997). There is also a returning variety known as the cross boomerang which was traditionally used in the Cairns district of Australia (Hess, 1975). This variety consists of two pieces of straight wood tied together in the centre, so that all four arms are equal lengths and meet at right angles (Hess, 1975).

The making of boomerangs is an involved process. Traditionally, a piece of wood of the correct shape is cut from a tree using stone axes (Barlow, 1994). It is necessary that the chosen wood have a curve of the desired magnitude as this makes the boomerang stronger and ensures that it does not split easily if it strikes an object during flight. The rough shape is created using a stone axe and refined using a traditional adze with a metal blade. Boomerangs are then shaped and hardened over the fire before tools such as quartz flakes or bone and shell are used to complete the final design (South Australian Museum, 2000).

Translation

Translation refers to the mode of transformation where students engage with aspects of the indigenous cultural practice in isolation from its original context in order to achieve outcomes that are related to the western classroom mathematics. A learning activity that illustrates the mode of translation requires that students construct replica boomerangs in order to complete activities related to western classroom measurement. Students make boomerangs from paddle-pop sticks with four, five and six sides and throw these boomerangs, estimate then measure the distance travelled, and draw conclusions about the flight paths of the different boomerangs (New South Wales Board of Education, 2003). This activity is an example of translation since the indigenous cultural practice is included with the purpose of demonstrating aspects of the western classroom mathematics inherent in the practice.

Clearly, this activity refers to the practice of throwing returning boomerangs in order to teach about aspects of western classroom mathematics such as measurement. Although required to construct boomerangs, students do not need to use or possess knowledge of traditional techniques used to carve returning boomerangs. In addition, the purpose of throwing the boomerang is far removed from its original form and purpose since students are engaged in western classroom measurement rather than a recreational activity.

Integration

Integration refers to the mode of transformation that allows students to engage with the indigenous cultural practice through theoretical comparisons between the practice and the western classroom mathematics. The first learning activity in the Aboriginal Boomerang Dynamics curriculum text (New South Wales Board of Education, 2003) involves a WebQuest based on the practice of throwing and constructing returning boomerangs. Students may use a worksheet or an electronic stencil that provides the links to sites that give information on Australian Aboriginal returning boomerangs. Students are required to follow these links to locate information about the flight of a boomerang such as "the angle that the boomerang must be thrown into the wind" (New South Wales Board of Education, 2003, online). Although questions are ultimately concerned with the western classroom mathematics perceived in the cultural practice, activities also require that students learn about the practice itself in terms of the purposes for and construction of different types of boomerangs. In this way, they are required to learn about the practice in order to make theoretical comparisons between the practice in its original form and the western mathematical concepts to which it is connected.

Correlation

Correlation refers to the mode of transformation that allows students to engage with the cultural practice through concrete comparisons between the practice and the western classroom mathematics. As part of the 'Maths Day' at the Warrumbungles National Park Field Studies Centre students are required to experiment with throwing returning boomerangs (New South Wales Board of Education, 2003). The boomerang throwing activity focuses on "the mathematical dynamics of throwing a returning boomerang" (New South Wales Board of Education, 2003, online) while allowing students to experience the practice for themselves. This activity illustrates the mode of correlation as it requires that students perform the practice itself as well as experience concepts of western classroom mathematics related to the aerodynamics of the flight of a returning boomerang.

Discussion

The metaphor of surface constructs and core constructs (Kelly, cited in Bishop, 1993) described as part of the conceptual background to the analysis may be used to understand what each mode of transformation involves. The core constructs of an indigenous cultural practice are defined here in relation to the form and purpose of the performance of an indigenous cultural practice. Each mode of transformation can be considered in terms of adjustments to either surface or core constructs. With the exception of union, which requires performance of the practice for at least some purpose similar to its function in the original context, each of the transformations involved the adjustment of core constructs. The core constructs of the practices in the texts identified with the modes of disjunction, translation, integration, and correlation were adjusted to such an extent that the representations were very different from their original forms.

The concentric circle model depicting the five modes of transformation extends Adam's (2004) classification of ethnomathematical curriculum into five types. The modes of transformation identified go beyond reasons for implementing ethnomathematical curriculum and provide a framework for identifying the way that a cultural practice may change in form and purpose when included in curriculum texts. However, comparisons may be drawn between the five modes of transformation and Adam's (2004) five different types of ethnomathematical curriculum.

By way of example, the mode of transformation identified as union bears similarities to "ethnomathematical curriculum as an integration of the mathematical concepts and practices originating in the learner's culture with those of conventional, formal academic mathematics" (Adam, 2004, p. 52). Union is consistent with this type of ethnomathematical curriculum as it requires that students engage with the indigenous cultural practice through performance of the practice for at least some purposes similar to those of its original context. In a similar way the mode of transformation identified as disjunction may be compared with the type of ethnomathematical curriculum that Adam (2004) describes where examples from the learners' cultures are used in an attempt to ensure the mathematics is relevant to students. In this type of curriculum the ethnomathematical ideas may be used as "vehicles for communicating mathematical ideas" (Adam, 2004, p. 51). Characteristics of disjunction are evident in Adam's category since students are not required to engage with the cultural practice or different ways of knowing mathematics, but the classifications are not identical.

It is important to note that the model is depicted with permeable boundaries between the rings – excluding disjunction. This conveys the possibility that an indigenous cultural practice may be subject to more than one mode of transformation within the one curriculum text. In this way, the model adds another dimension to Adam's types of ethnomathematical curriculum by acknowledging that there may be more than one reason behind the inclusion of ethnomathematical ideas in the mathematics curriculum.

The model may be seen as applying Alangui's notion of mutual interrogation in a particular way. Alangui defines mutual interrogation as "the process of setting up two systems of knowledge in parallel to each other in order to illuminate their similarities and differences and explore the potential of enhancing each other" (2006, online). Conceptualising the interaction of two different knowledge systems in this manner, that is, "in parallel" implies that there is no intersection of different ways of knowing. If two different knowledge systems are set up in parallel, they never meet and therefore one way of knowing can never influence other way of knowing. In reality, it is inevitable that different systems of knowledge that exist in the one place meet so that one may potentially provide new ways of thinking about the other. The concentric circle model acknowledges different ways of knowing that may occur when two knowledge systems intersect.

Mutual interrogation is motivated by a desire to shift perspectives or to explore transformations of western classroom mathematics and cultural practices when the two intersect (Alangui, 2006). Bishop's (1993) approaches to culture conflict in the mathematics classroom result from interrogating a cultural practice from the point of view of the western mathematics curriculum. In contrast, the five different modes of transformation identified in the study came from interrogating the ethnomathematical ideas in the western mathematics curriculum from the perspective of the indigenous cultural practices.

In order to interrogate a curriculum text from the perspective of the indigenous cultural practice the concentric circle model borrows from a combination of metaphors central to the Garma Maths curriculum (Watson-Verran, 1992). One of these metaphors conceptualises the merging of two knowledge systems as the mouth of a river where the swirling salt and fresh water currents meet (Christie & Yirrkala School Action Group, 1992; Watson-Verran, 1992). The other metaphor conceptualises time as a spiral that moves outwards from the centre (Thornton, 1995). The model therefore seeks to acknowledge the dynamic nature of culture and cultural knowledge and the intersection of different ways of knowing that may occur when indigenous cultural practices are included in western classroom mathematics curriculum texts.

Limitations of the Study

Due to the small size of the final data set, the classification system developed is not presented as one in which it is possible to locate all mathematics texts that contain indigenous cultural practices. While every attempt was made to ensure that a range of different text-types were represented it cannot be assumed that the seven chosen were representative of the entire range of texts developed in the three countries. For example, the data set consisted only of those texts written in English and thus did not include the large number of texts that are written in Maori (Barton, Fairhall, & Trinick, 1998; New Zealand Ministry of Education, n.d.-a).

The criteria for the collection of the texts also produced a bias. A prerequisite for the inclusion of the texts was the availability of information about either the cultural practices or their mathematisations. A different set of data would have resulted had information been located about other indigenous cultural practices in the initial data set. Different modes of transformation may potentially have been identified from a different set of data.

Limiting the sources of data selection to only three countries may also have narrowed the general application of the classification system. The resources that the International Study Group on Ethnomathematics has collected on its website provide rich examples of ethnomathematics from ethnic groups including the Latino, Middle Eastern, and European (Eglash, n.d.), however these were not included in the data collection process as they did not meet the selection criteria.

Furthermore, the study was limited to mathematics texts on paper. There are other text-types, such as audio-visual materials and computer software, that contain indigenous cultural practices. For example, there are mathematics curriculum items available that encourage students to engage with simulations of Native American or African American cultural practices (Eglash, Bennett, O'Donnell, Jennings, & Cintorino, 2006). For practical reasons, these text-types were not included in the data analysed.

Implications for Educators

The model developed in this study may be used to identify the different transformations that indigenous cultural practices may undergo when included in mathematics curriculum texts. As the Aboriginal Boomerang Dynamics (New South Wales Board of Education, 2003) example showed, indigenous cultural practices may be transformed in a variety of ways within the one curriculum text.

The model provides a way of evaluating the suitability of mathematics curriculum resources that refer to indigenous cultural practices according to the modes of transformation of the practices involved. In this instance the different modes of transformation may be used to determine the intended outcomes and make decisions about the suitability of the text for its intended student group. For example, teachers would begin by identifying the student outcomes. The mode of transformation required to achieve these outcomes could then be determined by comparing the intended student engagement with the practices in the text to the types of student engagement corresponding to each of the modes of transformation. If the types of student engagement match the desired outcomes, then the curriculum text may be used without alteration. However, if there is a mismatch between the engagement with the representation of the cultural practice in the text and the intended student outcomes, the model provides a framework that allows teachers to modify the text so that students may reach the intended outcomes.

In addition, the model may assist teachers and curriculum writers to design relevant learning experiences that incorporate indigenous cultural practices. Once the intended student outcomes are identified, the ethnomathematical ideas can be presented utilising the most appropriate modes of transformation. For example, the outcomes may require that students engage with the cultural practice through concrete experience in order to experience different ways of knowing the western classroom mathematics. In this instance, the concentric circle model indicates that correlation is the mode of transformation that most closely matches these outcomes. Curriculum writers may then use the model as a framework to determine how the cultural practices should be transformed with reference to the types of student engagement required with the practice.

The model may also be used as an analytical tool to raise student awareness of how ehtnomathematical ideas are represented. An indigenous cultural practice in a curriculum item may teach about performance of the cultural practice in its original context, as in the modes of integration, correlation, and union. At the other end of the scale, the ethnomathematical content may be included only to teach about the western classroom mathematics perceived in the practice as in the modes of disjunction and translation. Furthermore, the model may be used so that students can analyse the different ways a cultural practice may be represented in a text.

Conclusion

When using ethnomathematical ideas in mathematics curriculum classroom teachers and curriculum writers should consider what student outcomes will be achieved by including a particular representation of an indigenous cultural practice. The concentric circle model of modes of transformation provides a framework that allows teachers and curriculum writers to match the intended student outcomes with the most appropriate modes of transformation, that is, the modes of disjunction, translation, integration, correlation, and union.

The model may also be used by teachers who wish to use existing mathematics curriculum texts that contain representations of indigenous cultural practices.

The aim of the model developed in this research is not to critique the potential educational value of ethnomathematical ideas in curriculum. Rather, the model provides a tool that may allow teachers of mathematics to identify the different kinds of representations of indigenous cultural practices in ethnomathematical curriculum and to choose those that are the most suitable for their own classroom environment.

The model produced in this research (Dickenson-Jones, 2006) provides a new way to approach Alangui's (2006) concept of mutual interrogation. Unlike Bishop (1993), Adam (2004), and Begg (2001), who interrogated ethnomathematical ideas from the perspective of the western mathematics curriculum, this study interrogated the ethnomathematical ideas from the perspective of the indigenous cultural practice. The contribution that this study makes to existing research into ethnomathematical curriculum is to suggest a framework by which the modes of transformation of indigenous cultural practices may be identified and critically reflected upon from the perspective of the cultural practice.

The strength of the concentric circle model lies in acknowledging the possibility that ethnomathematical ideas might be transformed in a variety of different ways within the one curriculum text. The model also provides a visual tool for illustrating each mode of transformation relative to the others.

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