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Bridging a Cultural Gap

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There has been a broad wave of change in tertiary calculus courses in the past decade. However, the much-needed change in tertiary pre-calculus programmes—aimed at bridging the gap between high-school mathematics and tertiary mathematics—is happening at a far slower pace. Following a discussion on the nature of the gap and the objectives of a potential bridging programme, this paper aims at demonstrating that the gap can be bridged, by presenting an ongoing modular bridging programme especially designed for the diverse types of student populations in teachers training colleges. We also present here some innovative teaching and assessment methods that were judged essential for the success of these programmes—focusing mainly on the "Questionnaire Based Instruction Method". Finally we suggest directions of follow up and research.

Introduction

There is a distinct cultural gap between school mathematics and tertiary mathematics. While school mathematics tends to concentrate on problem solving skills, tertiary mathematics is more abstract and emphasizes the inquisitive as well as the rigorous nature of mathematics. Many first-year college students find it difficult to adapt to a *culture* where concepts are *abstract*, yet require *rigorous* definitions; theorems have to be *proved*, and their assumptions meticulously *verified* before their results can be applied, etc. The gap seems especially difficult to bridge with students of non-university programmes like teachers colleges (see the section on target population below). In this paper, we will focus at potential transition courses aimed at bridging the gap for students of four-year secondary/high-school teacher training programmes. However, in our experience (with engineering colleges for example) many elements of the programme offered here can be applied to other populations as well.

This paper has 6 sections. First we discuss the rationale and objectives of an experimental two-stage transition programme that has been ongoing for the last four years. Next we describe our target population of students (and of teachers). In the following two sections we present, in detail, the two stages of the programme and their various components, and discuss some didactic aspects. The next section focuses on the innovative components of the programme, and, finally, we give some examples of common misconceptions of students that are revealed during the programme and the way that they are dealt with at each stage. Finally, in the conclusions section we report on students' reactions and suggest directions of follow-up and research.

The Rationale and Objectives of a Transition Programme

There has been a noticeable decline lately in the level of preparedness and sophistication of our teacher college students, also the degree of heterogeneity of students in our classrooms has increased (see next section). These factors made it increasingly more difficult to present advanced mathematical concepts in our classrooms. A discussion group devoted to this issue held at ICME10 (Copenhagen

2004) revealed that these difficulties are universal (see for example Gruenwald, Klymchuk & Jovanoski, 2004, Leviatan, 2006). It seems that the need for transitional programmes is by now widely acknowledged.

Traditionally the transitional stage in tertiary mathematics programmes consists of a cluster of *lower level courses*, such as linear algebra, analytic geometry, and introductory calculus, which basically reinforce school mathematics. Their stated purpose is to achieve a reasonably uniform level on which to base advanced courses in mathematics. Unfortunately such courses are not efficient in bridging the "cultural gap" described above.

As a partial response to the acute and growing need for "bridging courses", many *computer science programmes* around the globe have devised a general purpose, introductory, "Discrete Mathematics" course. Such a course typically starts with a short chapter on logic, aimed mainly at introducing the *language and notations* of advanced mathematics. The course then continues with a variety of subjects in advanced mathematics, typically: elementary number theory; combinatorics; and probability theory (see for example Epp, 2004). These presentations may be the only exposure of the students to these subjects.

Mathematics students in *teacher training colleges* are a particularly interesting population that requires special attention because those students will go out to teach mathematics in schools. So on the one hand they should set good examples in themselves, and on the other hand they would be in the best position to alleviate "cultural gaps' in their students. The transition programme suggested here has been designed specifically for this population, but it seems that many of its objectives and methods are universal.

The programme has six aspirational yet achievable objectives:

- *Identify* and *reinforce* previous "core school mathematics".
  We ended up with a detailed list of what we considered "must know" items.
- *Deepen and enrich the existing* knowledge by adopting a *more mature perspective* to school mathematics.
  Useful *basic* mathematical concepts are introduced, including for example different types of averages and their properties, properties of numbers, the pigeonhole principle and its useful applications, and combinatorial formulas with real life applications.
- *Introduce mathematical "culture"*: language, rules of logic, etc.
  This is important as experience shows that the special language and grammar of mathematics is not satisfactorily acquired just by studying advanced mathematical courses
- Get acquainted with typical *mathematical activities*: generalizations, deductions, definitions, proofs, etc.
- Re-introduce *central mathematical concepts* and *tools*: sets; numbers; functions; finite/infinite sequences; decimal expansions; etc.
  This is important as despite the fact that these ideas are used heavily in school, they were defined in a very descriptive, sometimes vague, way.
- Provide a rigorous, yet only semiformal, exposure to selected *new topics* in *advanced mathematics*.
  This includes for example an introductory chapter devoted to real numbers and convergence of sequences of real numbers—presented from a geometric viewpoint; a chapter on probability presenting various historical models with applications; and a set theory chapter presenting functions as

a special case of the more general idea of relations between sets, exhibiting other useful types of relations like equivalence and order.

Note that our advanced mathematics programme is aimed at students who have completed the highest-level high-school mathematics. Students who do not meet this criterion are required to *participate concurrently* in a non-credit first-year traditional programme consisting of three, year-long courses at the high-school level that make up the missing material in algebra, analytic geometry, and calculus.

Our Target Population

Generally speaking, our teachers college mathematics programmes train future K-12 teachers. The student body of the secondary and high-school programmes is composed of three types of populations:

(i) Regular students: students who only have a high school certificate are required to complete a 4-year BEd programme;

(ii) Academic trainees: students with a degree in sciences or engineering who wish to become mathematics teachers (a 3-4 semester *individually tailored* programme);

(iii) In-service teachers who need to complete their mathematical education for a certificate (a 3-4 semester *individually tailored* programme).

Students of the first category constitute almost 50% of our mathematics students. Along with their mathematics studies they have to choose another major (typically they choose computer science or linguistics). In addition to the two purely disciplinary programmes, regular students also participate in a 4-year teachers training programme that includes general education courses as well as pedagogy and didactic course (with a special emphasis on their majors). Students of categories (ii) and (iii) participate in some of those courses too.

Students of the last two categories have typically already taken tertiary calculus and logic courses, but, while possessing adequate computational skills, they usually have difficulties in their conceptual understanding of basic calculus concepts (real numbers, convergence, etc). In addition, students *of all three categories* typically have difficulties in applying simple rules of logic in their regular mathematical work. Both these difficulties ought to be properly addressed while designing a new intervention programme

The teachers of our disciplinary programme are all mathematicians who also teach at some research university, and have no special training in mathematics education. Didactic issues are dealt with mainly by the pedagogical instructors as part of the teachers- training programme. This may explain the fact that for many years, until not long ago, our disciplinary mathematics programme reflected a typical university one with an *inflexible first two years* of advanced level courses in linear algebra, analytic geometry, set theory, number systems, etc. Note that the calculus segment of the programme consists of two year-long advanced calculus courses, given during the second and third year

A Transition Programme—The Pre-Calculus Stage

The first step in building a transition programme was to reorganize and compress the pre-calculus courses of the traditional mathematics programme in order to enable addition of the new pre-calculus transition courses. Due to the heterogeneity of our body of students, the newly added programme was built to be modular. Its pre-calculus part consists of three components, which are later supplemented by two additional advanced post-calculus components.

Semester 1: Semi-formal "*Introduction to Advanced Mathematics*"—a lecture style course. The course is divided equally between basic concepts and tools in set theory and a systematic *survey* of all number systems as taught (or rather as should be taught) in school. Together these two topics constitute the first layer of advanced mathematics.

In this course concepts are rigorously defined, properties are formally stated (but not yet formally proved) and students are required to give "convincing arguments" to mathematical statements. Assignments are straightforward and follow directly from class activities and traditional methods of student evaluation are applied.

Cooperation between students—in class and in doing their assignments, is encouraged. The course involves a lot of class discussion aimed at revealing students' prior knowledge and intuition. Studies have shown that these preconceptions, which students have retained from previous studies, have a very strong influence on the learning process.

This introductory course is oriented mainly towards our regular students; the purpose is to create a common ground for all our students, to bring students of category (i) up to a more mature level, and to give them a taste of how mathematicians work from axioms and definitions through a chain of logical reasoning leading to theorems and proofs. Students of the two other categories who obtained their degrees long ago may also find this course useful, and allowing some of these students to participate in the course enriches classroom discussions with their wide range of professional experiences.

Semester 2: "*Reading, Writing and Reasoning in Mathematics*"—a workshop type course, which is undoubtedly the most crucial component of the programme. This is a "soft logic" course that has been designed as a response to the paradoxical situation in which logic, being perhaps one of the most formal mathematical subjects, is usually taught very late in traditional mathematics programmes, while its basic ideas and tools are essential to *all* advanced mathematics courses. The accompanying newly written material, designed to be used in a multitude of ways, includes textbooks for frontal presentations, material for workshops, self-learning materials and modules for creative exploration of specific topics -"discovering" and proving simple mathematical facts (see Leviatan, 2008). The course begins with basic concepts: statements, connectives, conditional statements, logical equivalences, quantifiers, etc. Examples and assignments are taken from the general literature, idioms, newspaper clippings, etc. The examples are geared to demonstrate the advantage of precise writing in everyday life, for clarifying intentions and avoiding ambiguities. Eventually the emphasis shifts to examples taken from high-school and first semester mathematics, leading finally to multi-quantified conditional statements of the type needed in our second-year advanced calculus course (to define the basic, yet logically complicated notion of a limit, for example). The "reasoning" component of the course is devoted to universal *rules of deduction*. We discuss valid/non-valid arguments, induction/deduction, etc. To make this topic effective, considerable emphasis is put on *common mistakes* and on well-known *fallacies* (see examples in the final section).

This course is open to students of all three categories, but in practice we encourage students of categories (ii) and (iii) who have a reasonable background in logic to officially start the logic section of their transition programme with the 4^{th} component (see the next section), and guide them in using materials from this 2^{nd} component as a background source for self study.

Semester 2 or 3: "*Number systems*"—a systematic and rigorous study of all number systems from natural numbers to complex numbers. Until recently it was a standard, very formal course in the spirit of Feferman (1964) or Little (2003). Now revised and designed especially for prospective middle/high school teachers with less formal, yet mathematically rigorous, presentations that can later on be passed on to future school students. Natural numbers are still presented via their characteristic properties, leading to Peano axioms, an approach that enables *mathematical proof* of "self-evident" properties of natural numbers. Special emphasis is put on the interrelations between recurrent definitions of arithmetical operations and the newly acquired method of proof by induction. The analysis reveals a mathematically efficient algebraic structure called a *number system*. Subsequent extensions to whole numbers, integers, and rational numbers (expressed as fractions) all follow the same procedure: motivated by application considerations and structural requirements, we begin by adjoining new symbols, representing "numbers", to an existing set of numbers. To give these symbols a numerical interpretation we formulate a minimal number of *basic characteristic properties* that express the desired quantitative properties. The familiar explicit definitions of arithmetical operations on the extended set of "numbers" are all derived from these basic properties (coupled with the prerequisite to *preserve the algebraic structure*). This whole procedure eventually leads to an extended number system, with additional structural properties (Leviatan, in preparation).

In the second part of the course we switch from an algebraic viewpoint to a *geometric* one and present a fresh and comprehensive approach to rational and real numbers which are expressed decimally. Such an approach, which can be seen in Gowers, (2004) and Leviatan, (2006), enables a mathematically sound *pre-calculus presentation* of real numbers that does not yet rely on the sophisticated concept of a limit. The course ends with the abstract concept of an ordered field.

A Transition Programme—The Post-Calculus Stage

The remaining components consist of post-calculus courses. For type (ii) and (iii) students they serve as *bona fide transition courses* and are thus strongly recommended. For regular students they may constitute a supplementary programme adding to the variety of advanced elective courses, but cleverly used they can serve as part of a "*transition out*" programme (see remarks below).

"Definitions and Proofs in Mathematics"—an elective post-calculus course aimed at demonstrating the usefulness of the reading, writing, and reasoning skills (acquired in the 2nd component) in mathematical work. Being able to *follow and analyze formal definitions and proofs* and being able to *write proofs* are at the heart of mathematics. Unfortunately, these abilities are not easily acquired. It is naïve, we believe, to assume that a student who participates in a regular tertiary mathematics programme will master such skills just by being routinely exposed to formal definitions and proofs, as it is hard to appreciate their style and structure while, at the same time, trying to grasp new mathematical concepts. Thus the idea behind this final logic component follows that of the pre-calculus 2nd component which uses current *tertiary* mathematical knowledge as the subject matter. Important mathematical statements (taken from a wide range of *advanced mathematics* topics) are categorized according to types and, accordingly, common methods of proof (and of disproof (are suggested and demonstrated. Again, in this course, a lot of emphasis is put on *misconceptions and common mistakes* related to proofs.

All students are advised to participate in this final-year course. Since the course is a direct continuation of the 2nd component, students of type (ii) and (iii) are guided, at the beginning of the course, to go over a shorter version of the written materials of that course. For our regular students the course also serves as a "polishing" stage just prior to students starting their practice in schools. The general concept behind this elective course is somewhat similar to that of another elective final-year course we offer entitled "Problem Solving Strategies in Mathematics". These two courses combined can serve as a "*transition out" programme* enabling students to appreciate an alternative viewpoint of the knowledge they have attained during their three years of advanced mathematics studies.

An advanced web-based final semester course entitled "*Topics in Advanced Mathematics*"—still in the process of being tested. The course consists of chapters from the following variety of advanced-mathematics subjects: probability, number theory; discrete mathematics; Euclidean/non-Euclidean geometry; and game theory. The emphasis in this component is on well-formulated definitions, proofs of basic theorems, small research projects that lead to new concepts, and historical background. Students are constantly asked to provide more details, to prove related results, to work on an example, to create new examples, or to experiment further. Each such unit is followed by a more significant *project*, to be worked on individually.

This course is *individually tailored* and is oriented only to students of categories (ii) and (iii). We assign to each such student three of the above units for guided self-learning to *complement* his/her individual mathematics programme.

Innovative Teaching and Assessment Methods

In implementing the above programme, *innovative* teaching and assessment methods have been applied: questionnaire based instruction (QBI), project based learning (PBL), self study, group study, workshops, etc.

The PBL method (see for example Frank, Lavy, & Elata, 2003), is applied heavily in both logic components. Throughout these two courses students are routinely requested to work, alone or in very small groups, on *projects* in which they apply the newly acquired logic tools in a large variety of mathematical contexts. The PBL method is also applied in the final component: topics in Advanced Mathematics.

To get a feeling of how the method works, we present here two typical accompanying projects, the first one is related to the general topic "conjectures and proofs (or disproofs)" and the second to the notion of a limit of an infinite sequence of real numbers (both geometrically and algebraically defined).

- *Pythagorean triples* (three positive integers k, m, l, such that $k^2 + m^2 = l^2$). Cut a 4×4 square out of a given 5×5 square. From the remaining 9 squares build a 3×3 square. Note $5^2 = \underbrace{(5-1)^2}_{=4^2} + \underbrace{(2 \cdot 5 - 1)}_{=3^2}$.

  Generalization: Use a $n \times n$ square to complete the equation $n^2 = (n-1)^2 + \square$.
  Search for examples of odd numbers that form a square number.
  Use the information to supply (many) examples of Pythagorean triples.

  (The odd numbers are $3^2, 5^2, 7^2, \ldots$ *each such number yields a Pythagorean triple.)*

  Now use the $n \times n$ square to complete $n^2 = (n-2)^2 + \square$.

Search for examples of numbers of the pattern in □ that form square numbers. Use them to supply more Pythagorean triples. Continue and generalize. Is success always guaranteed?

- Jug A contains 1 liter of wine, jug B contains 1 liter of water. Remove 1 cup of liquid from jug A and add it to jug B, and then remove 1 cup of liquid from jug B and add it to jug A. This process is repeated again and again. At the end of step n, measure a_n - the proportion of water in jug A and b_n - the proportion of wine in jug B. Compute a_1, b_1. Find a recurrent equation relating a_n to a_{n-1} and deduce computational formulae for a_n and b_n. Investigate properties of both sequences. Raise a conjecture about the limit l of the sequence $a_{1,} a_{2,} a_{3,} \ldots$. Consider small neighborhoods of l and look for a tail of the sequence contained in each such neighborhood. Prove that l is indeed the limit of the sequence.

Several other projects relate to famous mathematical problems e.g. the four colour problem, Fermat's theorem, Goldbach's conjecture etc. Note that some of these problems are yet unsolved. In this kind of project students are led to a deeper understanding of the given problem by experimentation and by searching for more information in the literature.

The *feedback-enhanced* QBI method was designed especially to highlight *potential conflicts* between students' intuitions and previous knowledge and the new theory about to be discussed. It is widely acknowledged that *a successful* resolution of such conflicts is a key ingredient in the process of the acquisition of knowledge. "The learner builds his or her knowledge on what is already known, but only in response to a challenge." (Smith, 1998, p. 782).

The use of questionnaires to study prior-concept images of students (Tall, & Vinner, 1981) is a standard *research tool*. Using questionnaires as a *learning method* involves a systematic use of preliminary questionnaires, preceding each new course subject. In class students are routinely requested to respond individually *in writing* to 7-10 carefully formulated very short assignments, which from experience tend to lead to conceptual difficulties arising from conflict with previous conceptions. These questionnaires are analysed together in class at the time the relevant theory is taught, thus helping to unveil and uproot misconceptions and conflicts (see the final section). The idea behind the method is to awaken the students' interest in the forthcoming topic, encourage self-reflection, and promote collaboration between students (see Barthel & Leviatan, 2006).

The idea of the QBI emerged originally as a method of dealing with the well-known phenomenon of *common misconceptions* in probability (see, for example Kahneman, Slovic & Tversky, 1982 and Leviatan, 1998). Later experience indicated that the method is of particular help in "transition courses", i.e. courses providing a bridge between prior mathematical knowledge and advanced mathematics studies.

To get a taste of questionnaire items, we present here some typical examples of prior-knowledge items. For more examples see the final section, and also Barthel and Leviatan (2006).

- Are the following two statements equivalent?: I eat what I like & I like what I eat.

(A good introduction to a common misconception - in real life and in mathematical work - of confusing a statement with its inverse statement.)

- In a casino, a roulette wheel stopped on a red number in the last 7 games, would you rather bet on red or on black in the next game?
  Is it a favourable policy to wait until there is a long run of reds and then bet on black?
  (*The well-known "gambler fallacy" observed by Kahneman & Tversky*, 1982).
- Find a fraction which is equal to 0.111….
  (It is a common knowledge that 1/3=0.333…. It is interesting to jointly discover that current state of knowledge does not lead to the trivial deduction that 1/9=0.111…. The need for direct numerical interpretation of infinite expansions becomes apparent.)
- Is 0.999… smaller than 1 (by how much?) or equal to 1 (how come?).
  (A conflict creating item, promoting discussion of order and of arithmetical operations on rational numbers expressed decimally.)
- Can you explicitly write down a decimal expansion of an irrational number?
  (A good introduction to real numbers expressed decimally. Such an explicit example is eventually constructed jointly in class.)

In most of the transition courses, as part of students' course requirements, all participants are asked to write *a detailed report* at the end of each course unit, including: their own individual (mathematical and didactical) *comments* on the written materials, suggested alternative *examples*, analysis of their own performance in the preliminary questionnaires, etc. The direct purpose of these reports is to serve as a method of individual assessment of students. But the reports play several important additional roles: they serve as feedback; they allow students to take part in the shaping of the written course materials; and last (but not least) it gives our prospective school teachers a chance to evaluate and report on written materials in mathematics, thus developing their critical thinking capabilities. Such a practice is rarely possible in standard mathematics courses.

Except for the preliminary "Introduction to Advanced Mathematics" course (in which a standard final exam is administered), in all other courses student evaluations are based on their performances on the accompanying *projects* and on the level of thoroughness of their *reports*. The *questionnaires* are not really used as an evaluation method.

Dealing with Students' Misconceptions

The preliminary questionnaires actively help students' misconceptions to *surface*. Bellow are some examples of misconceptions involved in the process of *negating* various types of composite statements as revealed in the 2nd component ("soft logic") of the programme. For each such misconception we present here a possible questionnaire item that helps *unveil* it.

- Negating a conditional statement by another conditional statement.
  Negate the following statement: If it rains I take an umbrella.
  The most common reply: If it does not rain I do not take an umbrella.
  (Using truth tables students are led to discover that a negation of a conditional statement can not possibly be a conditional statement. The process of finding the correct negation and uprooting the above misconception is not that simple.)
- Negating a compound "or"/"and" statement by another "or"/"and"

statement.
Negate the following statement: The dogs are barking and the convoy is passing.
The most common reply: The dogs are not barking and the convoy is not passing.
(This item is a good starting point to introducing the useful De-Morgan rules. Note that a comparison to the statement "The convoy is passing and the dogs are barking" can serve as a good basis to a discussion on the difference in meaning of compound statements in everyday language versus in logic. Another good example of exactly the same type is "They got married and she got pregnant.")

- Consistently failing to identify situations under which a conditional statement is true.
  Determine the truth value of each of the following two statements:
  - If water boils at 90°, then the sum of the angles of a triangle is 100°.
  - If a cat has nine lives, then 1+1 = 2.
  (Each of these items promotes a very serious discussion, which lead to useful discoveries.)
- Negating a universal statement by another universal statement.
  Negate the following statement: Fire engine cars are red.

All these relatively simple but tricky items are analysed together in class *at the time* the relevant theory is taught. As a follow-up action, students are requested later on in the course to individually respond to more complicated items (taken from everyday life and also from mathematics), for example:

- Negate the following statement: If I drink, I do not drive.
  (Many students negate this universal conditional statement by "If I don't drink, I drive ", thus combining two serious mistakes.)
- Negate: Babies are delivered by storks or downloaded from the internet (a universal connected statement)
  (At this stage many students correctly apply De Morgan rules, but still tend to negate the universal statement by another universal statement: "Babies are not delivered by storks and are not downloaded from the internet.)
- Negate: No good deed goes unpunished.
  (There are mixed replies to this negatively stated item, the most common replies are "Good deeds go punished/unpunished". It is interesting to compare those replies with the ones obtained when the same statement is conditionally phrased.)

The follow-up items are also discussed *together* in class and the process repeats. The final step in this long process is the *self analysis* of mistakes included in the individual report.

According to our experience, the discussion in class of students' replies encourages the students to review with a critical eye what they have retained from previous learning experiences, and to confront it with the new material. Moreover, this joint class analysis of the questionnaires stimulates active collaboration among the students. As is well known the emotional involvement of the students is a key to successful learning: "…we need to stimulate emotional connections to our subject matter if we expect it to transfer to long term memory." (Smith, 1998, p. 782).

Comments and Conclusions

One big advantage of the relatively small size of the classes in our experimental programme is that it enables us to closely monitor the transition programme and quickly incorporate the lessons learned from this experiment into improving the next version of the programme. In this respect, the single most important feedback channel is the feedback of the students themselves. Therefore, in their final report (see the section on innovations), students were asked to evaluate the effectiveness of each component of the programme, and their responses were recorded and fed back into the design of the programme. It would be too tedious and repetitious to quote the detailed reports and comments of the students, but the most frequent general comments (ordered by popularity) are:

- The whole programme gave me *self-confidence* (both as a student and as a prospective teacher).
- The programme helped a lot with other courses in the regular mathematics curriculum.
- Students appreciated the value of the sessions dealing with misconceptions and common mistakes.
- Students benefited from the accompanying *projects* (e.g. applying newly acquired tools to other mathematical subjects) and the chance to perform non-routine mathematical work in a closely guided way.
- Students particularly enjoyed playing the part of the *reviewers for newly written mathematical materials*—a job they took very seriously.

Here is a small sample of students' comments on the "soft logic" component:

> The course helped me a lot in understanding and even writing of mathematical proofs... It changed the way I read written materials in both my majors: mathematics and linguistics. It formed an amazing link between my two majors. (Student M)

> I always say in every math course I take that your course helped me understand the basic of mathematics, and it still helps me in other courses, and when I teach Math I feel I have deeper knowledge about the basics of math, and that helps me when I teach. (Student SA)

> I enjoyed to be challenged dealing with misconceptions... I suddenly discovered the importance of proper phrasing of words. I discovered that if I know" the rules of the game" I can easily win (not only in mathematics), it will all be very useful to me as a student and as a prospective teacher. (Student SH)

> It was very helpful to look at topics in mathematics from a different viewpoint… the course widened my mathematical horizons and gave me better insight into mathematics. (Student IR)

> It was a different type of course. In addition to coping with academic material almost by ourselves we were constantly asked to supply relevant examples and make both didactical and mathematical remarks on what we read. The criticism and reflection required for these tasks will help me cope in the future with academic material and teach math in a more lucid way. (Student SI)

> It was not easy, but it helped me understand the language of definitions, theorems and proofs in other math courses… (Student R)

Although it is not trivial to measure scientifically the success of a programme aimed at achieving all the goals we described in the first section, it seems like the interviews with the students can be seen as preliminary indication of the programme's success in improving the preparedness of students for tertiary mathematics. Another way to evaluate the success is to consult with teachers of advanced courses who can appreciate the difference between students who have undergone the programme and those who have not. This has also been done yielding encouraging feedback. All these types of (students' and teachers') feedback require a more systematic investigation.

During the course of our work on the experimental transition programme, a number of additional ideas came up, which may merit follow-up work:

1. A careful study of the *effectiveness* of the innovative teaching methods employed in this transition programme (like PBL and QBI).
2. A further exploration of the QBI method, which is used heavily in the transition courses and in our regular probability course, and which could effectively be adapted to other courses (in and out of mathematics).
3. Using questionnaires as a learning aid rather than as a research tool, and making the students into researchers of their own thought processes constitutes an interesting double reversal of roles worth exploring.
4. We presented here a transition programme tailored especially for prospective school teachers. We also described a popular transition programme designed for computer science students. It is interesting to explore the difference in their transitional needs of *various types of tertiary populations* and the effectiveness of their existing (or newly emerging) transitional programmes.
5. A follow up of students (of various populations) who have undergone their transitional programme can contribute to the effectiveness of such a programme. In our college we now start interviewing our final-year students (who have undergone the programme and are now actively involved in school teaching) on their views on the programme; it will be interesting to analyze their replies (see for example student SA above).
6. Exploration of the effectiveness of a potential "*transition out*" programme (as suggested in the post-calculus transition section) as a means of giving students a final polish on their knowledge base just prior to their embarking on a career in mathematics.

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