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Towards the Modelling of Mathematical Metacognition

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Metacognition has been accorded a role in both mathematical problem solving and in the learning of mathematics. There has been consistent advocacy of the need for the promotion of metacognitive activity in both domains. Such advocacy can only be effective if the advocated process is well understood. In this paper we have four goals: to describe a *multi-method* technique developed to study student mathematical metacognition; to set out the structural elements and configuration of a coherent model of metacognition in the domain of mathematical problem solving; to report on the empirical utility (and validity) of this model; and, to report the insights into student mathematical metacognition arising from the research.

Introduction

In order to optimise the use of a process, one must first understand it. The term metacognition often features in conversations about educational improvement. Like many other popular terms in education, the prevalence of a term is no indication of the extent to which it is understood or the degree of consensus as to its meaning. There is danger in the blind advocacy of a poorly understood process. In this paper we situate the term theoretically and offer a model of metacognition that we have found useful in researching the metacognitive behaviour of sixth-grade students solving mathematics problems. The results of this research are reported, both for the insights offered into student metacognitive behaviour and to demonstrate the viability of our model of metacognition. Over the years, metacognition has been linked to improved student outcomes (Biggs, 1987; Birenbaum, 1996; Brown & De Loache, 1983; Wilson & Wing Jan, 1993, 1998; Wittrock, 1986). In the field of mathematics, researchers have coupled metacognition with successful mathematical performance (Goos, 1994; Schoenfeld, 1987; Stacey, 1991). A strong advocate of metacognition, Silver (1985) argued that failure or success in mathematical problem solving can be due to use of metacognition. Similarly, Cardelle-Elewar (1992) reported that students having difficulties in mathematics do not use a range of cognitive or metacognitive strategies.

What is needed are details of what students actually do metacognitively when learning mathematics and when they solve mathematics problems, the function of metacognition in both these domains (learning and problem solving), and valid and reliable strategies for monitoring and promoting metacognition. A coherent and viable model of metacognition is essential if we are to identify appropriate methods for studying and monitoring metacognition. In this article, we propose a model of metacognition and a method by which to study the metacognitive behaviour of children. The method reported here was used to study the use of metacognition by grade 6 students in the curriculum domain of mathematics. The research results highlight key aspects of the students' metacognition, and raise important methodological issues related to the validity of research into student thinking.

What is 'Metacognition'?

In attempting to define *metacognition*, three major obstacles have occupied much research time. These include: conceptualising the main aspects of metacognition, establishing the relationship between these aspects, and distinguishing between cognition and metacognition. In this article these difficulties are addressed explicitly.

The terms *metacognition* and *reflection* are often used interchangeably and, we would suggest, with imprecision and uncertainty. Many curriculum documents assert the importance of reflection, but these may indeed be referring to the virtues of metacognition. We do not consider reflection and metacognition to be synonymous. Reflection, in our view, is a more general term than metacognition and, in its broadest use, seems to refer to almost any instance of purposeful thought. Reflection is not the concern of this article. We leave the precise definition of reflection to others. We do provide a definition of metacognition that has proved useful in our work.

An early definition of metacognition by Flavell (1976) has become regularly quoted in the literature. He used the term to refer to an individual's awareness, consideration, and control of his or her own cognitive processes and strategies. Since Flavell, a variety of meanings have been given to the term metacognition. Nevertheless, despite this diversity, reference is frequently made to two aspects of metacognition: knowledge about cognition, and self-regulation of cognition (Brown, 1987). The confusion over the term metacognition can be blamed, to some extent, on metacognition having these two separate, but related, aspects (Garofalo & Lester, 1985; Schoenfeld, 1992).The neat division of metacognition into knowledge and regulatory components ignores two key non-regulatory functions of metacognition. These are: individual *awareness* of thought processes, and individual *evaluation* of these thought processes. In this article, we suggest that the function that Flavell characterised loosely as *consideration* (which suggests little more than reflection) can be more usefully and specifically identified as evaluation.

In our work, metacognition is used to refer to the *awareness individuals have of their own thinking; their evaluation of that thinking; and their regulation of that thinking* (Wilson, 2001). This definition is consistent with existing literature but also extends that literature. These three functions of metacognition: awareness, evaluation and regulation require careful specification.

Any such specification must acknowledge differences in the nature of metacognition according to whether the domain of metacognitive activity is *learning problem solving*. As an aside, it may be that research into the use of metacognition in problem solving may offer insight into the constructive nature of the learning process. It is possible that learning can be usefully conceived as a process of continual problem solving, and that metacognition is the key to resolving apparent differences in the two processes. In this case, a coherent model of metacognition would make an important contribution. For the moment, the specification of a metacognitive function must be considered in relation to whether learning or problem solving is the domain in question. The functions of metacognition, as defined in this study, are now explicated.

Metacognitive awareness relates to individuals' awareness of where they are in the learning process or in the process of solving a problem, of their content-specific knowledge, and of their knowledge about their personal learning or problem solving strategies. It also includes their knowledge of what needs to be done, what has been done, and what might be done in particular learning contexts or problem solving situations. Metacognitive awareness encompasses an individual's cumulative knowledge of acquired competencies and on-going knowledge of mental processes in progress.

Metacognitive evaluation refers to judgements made regarding one's thinking processes, capacities and limitations as these are employed in a particular situation or as self-attributes. For example, individuals could be making a judgement regarding the effectiveness of their thinking or of their strategy choice. Such an evaluative function assumes some awareness of the individual's thinking processes and anticipates the possible regulation of those processes.

Metacognitive regulation occurs when individuals make use of their metacognitive skills to direct their knowledge and thinking. Metacognitive regulation draws upon individuals' knowledge (about self and strategies, including how and why they use particular strategies) and uses *executive* skills (such as planning, self-correcting, setting goals) to optimise the use of their own cognitive resources.

When thinking metacognitively, learners reflects on their existing knowledge or thought processes. Individuals may be aware of, evaluate and/or regulate their own thinking. While the completion of a mathematical task is basically a cognitive process utilising cognitive strategies (e.g., adding up or using percentages), metacognitive behaviour deals with the selection and use of these cognitive strategies (e.g., This strategy is not working; what do I know about the task to help me work it out?).

It is acknowledged that metacognition is employed within a social context (for example, a classroom) that is personally experienced, and that other aspects of the individual's experience of this context, such as prior knowledge, abilities, preferred ways of learning, values and expectations, and volition (Corno, 1993) affect the process (and, therefore, the products) of learning and problem solving. The importance of such personal attributes is recognised but not addressed explicitly in this article. Such attributes may facilitate or hinder the metacognitive activity of the learner/problem solver or even provide the focus for that activity. Seen in this light, these attributes are not active agents in that activity, but may provide the matter on which metacognitive activity is undertaken. In this article, the nature of metacognitive activity itself is addressed.

Difficulties with Monitoring Metacognition

Objections have been raised regarding the legitimacy of researching metacognition, and about techniques used for the study of metacognition. Many arguments relate to the validity of verbal reports. Self-reporting, commonly used in research about metacognition, has been questioned (Nisbett & Wilson, 1977). One of the major criticisms of verbal reports is that this process may alter the cognitive thoughts being studied (Cavanaugh & Pelmutter, 1982; Meichenbaum, Burland, Gruson, & Cameron, 1985). Brown (1987) claims that the most recurrent and serious concerns relate to the accessibility, veridicality, and completeness of verbal reports. Verbal report problems are even more problematic when dealing with children who may have limited linguistic abilities (Cavanaugh & Pelmutter, 1982), or who simply lack the vocabulary needed to describe their thought processes.

Another problem for researchers in this field is that students may not be able to recall their metacognition because some aspects of their problem solving behaviour have become automatised (Ericsson & Simon, 1980; McKoon & Ratcliff, 1992). If students do not report cognition or metacognition, it is difficult to determine whether the absence is actual or due to automisation, lack of motivation, or other factors. Ericsson and Simon (1980) claim that in trying to say what one was thinking, the subject may not remember, might misremember, or might invent memories, for example, describing strategies that have just occurred to them.

Methods can be employed that increase the likelihood that self-reporting is valid and reliable (Ericsson & Simon; 1980, Newfield, 1980). Such methods have been utilised in this study to combine individuals' self-reports with corroborative data from other sources. In this research, video-stimulated recall was used to optimise the self-reporting process; *thinking cues* were also developed to support the participating children in describing their thinking processes. In support of these two methodological strategies, Randhawa (1994) suggested that video recording of think-aloud protocols, along with clinical interviews, can capture the cognition of the problem solver. Haynes (1997) suggested the provision of cues for discussing thinking: "How can one be metacognitively aware or reflective without a language in which to think about oneself?" (p. 6) whilst Artzt and Armour-Thomas (1992) utilised student conversations in small groups to assess students' metacognition.

Hacker, Dunlosky, and Graesser (1998) argued that if metacognition is defined as conscious and deliberate thoughts about one's own thinking, then these metacognitive thoughts are potentially controllable and reportable, and therefore accessible to the researcher. In our research we have combined methods to capitalise on the individual strengths and avoid the disadvantages of the above approaches. On this basis, a *Multi-Method Interview* (MMI) was designed to meet the challenge of researching metacognition and to implement recommendations for new research methods to assess metacognition (Cavanaugh & Perlmutter 1982; Dunlosky, 1998; Garofalo & Lester, 1985; Meichenbaum et al., 1985; Mulcahy, Short & Andrews, 1991; Randhawa, 1994). The methodological issues associated with this new composite technique and the key elements of the technique itself are discussed in detail elsewhere (Wilson, 2001). A brief overview of our method is given below.

A New Technique for Monitoring Metacognition

An MMI was developed to assess metacognition. It included a problem-based clinical interview including self-reporting and a think-aloud technique (where chosen by the participant), observation, and audio and video recording. The main feature of the clinical interview was a card-sorting procedure by which the children reconstructed their thought processes during a problem solving episode just completed (adapted from a technique employed by Clarke, 1989).

Fourteen metacognitive action statements, each associated with one of the three metacognitive functions (awareness, evaluation, and regulation, discussed earlier), were listed individually on playing cards. One of the major concerns in any research interview is the risk of *putting words into the mouths* of the interviewees. In this study, significant effort was expended to minimise this possibility. The statements on the cards (e.g., *I thought about what I already know)* were drawn initially from the literature on metacognition, field-tested with grade 6 children, and subsequently revised for use in this study. The essential feature of the card statements was that the words used were those generated by students during field-testing. Cards listing cognitive behaviours were also provided and were generated by the same process of field-testing and refinement. In addition, blank cards were provided in order that students could record their own descriptions of a particular metacognitive activity, when they felt this was not already described on any of the prepared cards. When students used these blank cards the comments were easily coded into the three metacognitive functions. The use of video to stimulate students' reflections on the card sequence they had already constructed served as a validating strategy for the sequences. As will be discussed, nearly all of the students made changes to their card sequences as a result. This confirmed the significance of the checking procedure. While the videos were examined by the researcher for insight into student behaviour, their role in stimulating students' reconstructions of their thought processes was undoubtedly their most important and useful function.

Throughout this article, we make reference to *metacognitive actions*. Our intention is to associate metacognitive activity with purpose and agency. It is not intended to suggest that metacognitive (or cognitive) actions are directly observable, but the students in this study were certainly able to identify and describe the distinct types of activity we report. Use of the term *action* in this context reflects the sense of purposeful activity captured in such student statements as "I made a plan to work it out."

After attempting a mathematics problem, students were asked to sort and sequence the cards in order to construct a visual account of their thinking in solving the problem. The same action statement card could be used several times in one sequence. The opportunity to think-aloud was offered to the students in this study and was found to impose an additional burden that distorted the problem solving process. Not a single student maintained their use of the think-aloud process.

The students were videotaped during each problem solving attempt and each subsequent card-sorting activity. The video of the problem solving attempt was replayed to students after they finished the problem and the card sorting task. While watching the video, students checked to ensure that their card sequence was an accurate representation of their thinking. Students could add, delete or change cards. They most often added cards after they viewed the video. Video use was considered central to issues of validity and reliability. Students' changes to their reconstruction of their thought processes when assisted by the video record of their problem solving activity raise concerns about research methods that do not offer students this form of support (see Wilson, 2001). The revised card sequences were then used to analyse the nature of a student's metacognitive activity.

Subjects And Tasks

Ninety interviews were conducted and analysed. Grade 6 students were recruited from six different classes (five students from each of the two classes from each of three different types of schools in Victoria, Australia). The three schools included: one small, inner city, multi-cultural school; a suburban, high socio-economic, technology-rich school; and, an outer-suburban, low socio-economic school community. Teachers volunteered their classes as possible subjects for this study. The teaching styles in these classrooms varied from conservative to progressive. The participating teachers completed a brief questionnaire regarding their students' use of metacognitive strategies. All of the children in those classes were invited to take part in the study. From those children who volunteered, a group was identified in each school that balanced gender and mathematical ability. The teachers rated each student's mathematical ability.

Each participating student completed a familiarisation task prior to three different types of experimental tasks: numerical, spatial, and logical. These three tasks are set out in the appendix to this article. Each student was interviewed three times, once for each task. The problems were non-routine and were intended to provide the participants with a similar level of challenge to other problem solving tasks they may have encountered during school mathematics classes. The tasks were chosen to provide students with opportunities to engage in metacognitive activity in contexts with different numerical, logical and visual features.

Metacognitive Action Card Statements

The action statements on the cards were adapted from an inventory of metacognitive behaviours compiled from the literature. These were field tested with a smaller cohort of grade 6 students, and were modified to match their language and to ensure that the full range of likely metacognitive activities was anticipated in the set of cards available to students. The 14 metacognitive action cards were:

Awareness:	*I thought about what I already know*
	I tried to remember if I had ever done a problem like this before
	I thought about something I had done another time that had been helpful
	I thought 'I know what to do'
	I thought 'I know this sort of problem'
Evaluation:	*I thought about how I was going*
	I thought about whether what I was doing was working
	I checked my work
	I thought 'Is this right?'
	I thought 'I can't do it'
Regulation:	*I made a plan to work it out*
	I thought about a different way to solve the problem
	I thought about what I would do next
	I changed the way I was working

While the definition of each of the three metacognitive functions is given earlier in the article, the practical categorisation of the action statements above reflects the use of similar phrases elsewhere in the literature. In essence, the *awareness* statements connect past experience to the problem in hand, while the *evaluation* statements involve reflection on an activity just undertaken. *Regulation*, by its nature, anticipates particular problem solving actions.

The cognitive cards varied according to the task, for example, the card: *I turned a shape over* was used on the spatial task but not for the other problems. Other cognitive cards, for example, included: *I added* and *I drew a diagram* according to the type of mathematical activity typically required for each task, and based on the earlier piloting of the tasks. The cognitive cards were not intended to provide fine-grained probes into the nature of a student's cognitive activity. Our need was to distinguish between cognitive activity and the metacognitive functions that were the focus of our investigation.

The Structure of Metacognition

In seeking to model mathematical metacognition, we take model to signify a representation of structure. At this stage in our research we are able to report empirical details of the elements of such a model and their structural associations. The mechanisms by which elements are linked to each other and to cognition continue to be the subject of on-going research. In this sense, the model must be seen as under development (as is every model, whether in the domain of physics, geography or economics).

The schematic configuration of the model represented in Figure 1 is not an inevitable consequence of either the definitions outlined earlier in this paper or the research method employed. In point of fact, our initial hypothesised sequence of awareness, evaluation, and regulation was challenged by the empirical data generated in this study. This challenge to our initial hypothesis provides some reassurance that our results were not dictated by either the form of our methods or by the operationalisation of the key terms of our theory. In addition, confidence in the data is argued because of the links to literature, the trialing procedures, the implementation of different tasks, the varied levels of students' abilities and school contexts, and the use of various video and other checking techniques.

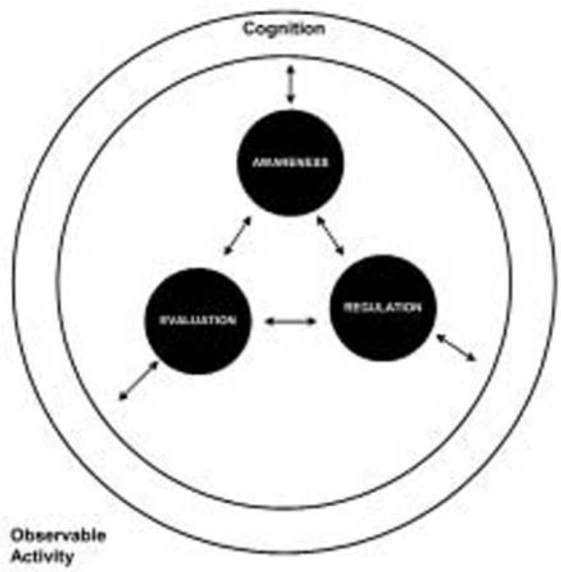

Figure 1. The structure of a model of *metacognition.*

The model that emerged is the result of consistent findings in this empirical research study. Some features of Figure 1 can also be argued on the basis of theory:

- the objects on which metacognition acts are cognitive objects;
- it is via cognition that we interact purposefully with the world; and
- the overt actions that might be recorded on a videotape are the result of cognitive activity that is, itself, influenced by metacognitive activity.

This coherence of the emergent model with theory is also reassuring.

For the purposes of our research, cognition mediates between metacognitive activity and those events that we might videotape or observe. This accounts for the basic topography of Figure 1. The arrows in Figure 1 do not suggest any particular process or any favoured sequence. The model is itself a product of the analysis conducted in this research study. Equally, the model, once established, provides a graphic illustration of the relationship between the key aspects of student metacognitive and cognitive activity, and a ready means to identify any patterns in that activity.

What can be said about the arrangement of the three metacognitive functions? Given the definition of metacognition and of its constituent functions (as defined earlier), it seemed reasonable to expect that evaluation would always be preceded by awareness and that every regulatory action would be preceded by an evaluative action. A simple notation of this sequence would be AER and, if such a sequence were universally prescriptive, it would also prescribe the configuration of Figure 1. A simple AER card sequence would be: *I thought 'I know what to do'* [awareness]; *I added* [cognitive activity]; *I thought about how I was going* [evaluation]; *I changed the way I was working* [regulation].

Empirically, however, the students also reported other sequences. The model shown in Figure 1 also suggests the plausibility of the alternative sequence ARE. For example, *I tried to remember if I had ever done a problem like this before* [awareness]; *I made a plan to work it out* [regulation]; *I counted* [cognitive activity]; *I thought about whether what I was doing was working* [evaluation]. Both AER and ARE were evident in the empirical data, although embedded within sequences that repeated individual elements (e.g., A, A, A, C, R, E, C, C, E, where cognitive activity [C] is omitted in identifying the metacognitive pattern ARE).

Not surprisingly, the empirical data reveal the solution of a mathematical problem to be a complex process involving a continual alternation between metacognition and cognition. This alternation is accommodated in the two-way arrows that link each of the metacognitive functions to cognition and to each other. Later in this article, the sequence A, A, E, R, R, C, C, R, C, E, E, E, is discussed. This sequence shares certain features with many other sequences documented in this study. For example, in terms of metacognitive actions, most students reported that they started with the awareness function (68 times of a possible 90 interviews). In addition, nearly all reported sequences ended with evaluation (87 times of a possible 90). These results suggest a high level of consistency in student metacognitive behaviour and provide compelling evidence that at least some of the structural characteristics of metacognition are accommodated in the schematic detail of Figure 1.

An example from the data is useful here. The action cards selected by Janice[1] on the second task (spatial) are listed below in order.

Action card 1:	*I thought about something I had done another time that had been helpful* (awareness)
Action card 2:	*I tried to remember if I'd ever done a problem like this before* (awareness)
Action card 3:	*I thought about what I already know* (awareness)
Action card 4:	*I tried to see if a shape would fit* (cognition)
Action card 5:	*I thought about what I would do next* (regulation)
Action card 6:	*I thought about how I was going* (evaluation)
Action card 7:	*I tried a different shape* (cognition)
Action card 8:	*I moved a shape around* (cognition)
Action card 9:	*I knew I done it wrong* (evaluation – written on blank card)
Action card 10:	*I thought about what I would do next* (regulation)
Action card 11:	*I checked my answer as I was working* (evaluation)

This card sequence can be represented simply as A, A, A, C, R, E, C, C, E, R, E. Janice consistently commenced with one or more awareness actions, followed by regulation and concluded with evaluation on all three tasks. Although Janice completed the first and last tasks, none were completed successfully. It is interesting to compare her sequences on the three tasks.

Task 1 (logic) sequence:	A, A, R, A, A, R, C, C, E, R, E, E.
Task 2 (spatial) sequence:	A, A, A, C, R, E, C, C, E, R, E.
Task 3 (number) sequence:	A, C, R, C, R, R, E, E.

There are advantages in displaying such sequences diagrammatically as this emphasises the fundamentally non-linear structure of the model itself. Figure 2 provides a schematic basis for the shorthand representation employed in this article to describe the empirical sequences revealed in this research. A represents awareness, E represents evaluation and R represents regulation. The positions of the three metacognitive functions are the same even when these letters are not shown in the figure.

[1] Pseudonyms are used in this paper

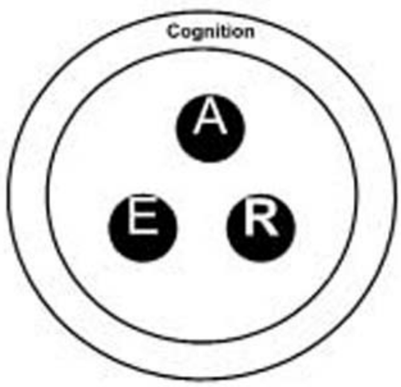

Figure 2. Schematic shorthand for the model of metacognitive activity.

Two pathways (example sequences) have been diagrammatically represented below (Figures 3 & 4) to demonstrate two possible student problem solving sequences. They are feasible, logical, and consistent with the data, and they include cognitive acts. Pathway One (Figure 3) shows the problem solver commencing with a cognitive action, followed by awareness, a cognitive act, evaluation of cognition, regulation, further cognition, and then the sequence is finished with an evaluative action.

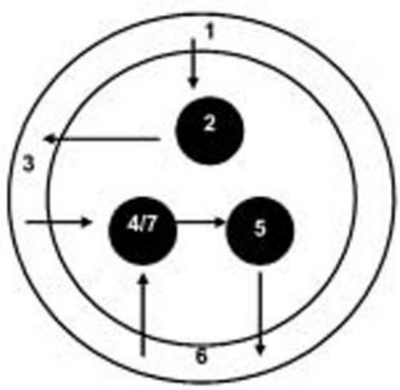

Figure 3. Example: Pathway One.

Pathway One has face validity, in that it conforms to expectations of how problem solvers might intersperse cognitive with metacognitive acts. There are many documented instances of students who reported sequences like this one. In this sense, Pathway One is both theoretically coherent and empirically grounded.

One of the grade 6 students interviewed was Lucas. Pathway One is similar to Lucas' successfully completed problem sequence. His sequence is longer but starts with the same basic metacognitive pathway (A, E, R, E), if each set of repeated functions is represented by a single letter. For example, Lucas reported the use of awareness four times at the beginning of the sequence followed by two evaluative acts, regulation, two more evaluative acts, and finished with evaluation. The complete sequence was: A, A, A, C, A, E, E, R, C, C, C, E, E, R, E. For the purpose of representing the structure of Lucas' metacognitive activity, cognitive action statements can be omitted, and the metacognitive action statements clustered and shown as A, E, R, E, R, E. This clustering of action cards related to the same metacognitive functions. For the purposes of applying the model to Lucas' account of his thinking, the clustering of same-function action statements focuses the analysis on the transitions between metacognitive function categories and cognition, and on the sequence of these.

A complete card sequence and accompanying transcript for another of Lucas' sequences is listed below to provide an example of student working and thinking. The problem solving number task required Lucas to use the digits 1-9 (once only) to fill a cross shape (see appendix). The sum of the numbers placed across the shape had to equal the sum of the numbers down. The action cards selected by Lucas in this interview are shown below in the order in which he sequenced them.

Action card 1: *I tried to remember if I'd ever done a problem like this before* (awareness)

Action card 2: *I thought about something I had done another time that had been helpful* (awareness)

Action card 3: *I thought about a different way to solve the problem* (regulation)

Action card 4: *I thought about what I already know* (awareness)

Action card 5: *I made a plan to work it out* (regulation)

Action card 6: *I counted* (cognition)

Action card 7: *I thought about what I would do next* (regulation)

Action card 8: *It didn't work so I tried a new strategy* (written by Lucas on a blank card, evaluation, regulation)

Action card 9: *I added* (cognition)

Action card 10: *I checked my answer at the end* (written by Lucas on a blank card, evaluation)

Part of the interview transcript [*number task* – see Appendix] demonstrates Lucas' understanding of his thinking during problem solving.

Lucas: All those first four I was sort of thinking about at the same time. *[Points to 1st – 4th cards: I tried to remember if I'd ever done a problem like this before, I thought about something I had done another time that had been helpful, I thought about a different way to solve the problem, I thought about what I already know].*

JW[2]: All around the same time?

Lucas: Yep, the plan was separate. *[Points to card 5]* They went together *[Groups cards 6 and 7]*: I counted, I thought about what I'd do next, and then 8, 9 and 10 together. It didn't work so I tried a new strategy, I added, I checked my answer at the end.

JW: So where were you up to when you made the plan?

Lucas: Oh well I made the plan in my head.

JW: Before you started writing or after?

Lucas: I made the plan [in my head] and then put the plan down to see if it would actually work and it did so I used it.

JW: But you only did part [of the plan] over there. *[Points to the paper draft]*

Lucas: I know because after I started off doing it that way and I realised it wouldn't work so the other part of the plan was to do it a different way, and then I sort of checked it mentally by putting a 2 there. And I could see it was all going to work because each of these pairs add up to 10 so I just put 5 in the middle.

JW: I'm interested because you said you could see it wasn't going to work. I can't see it because you haven't even written it down. Where would you see it? In your head?

Lucas: Yeah. *[Laughter]*

JW: Because the 2 and the 4…

Lucas: I suppose you could do it another way. No, I don't think you could because with the 2 and the 4 if they were both together. I was thinking to put the 5 in the middle and then go 6, 8 down here but that wouldn't work because then you'd have to put the odd numbers across here and they'd add up to less.

JW: When you went straight from that one [original draft] to this one *[second and final draft]* I thought hang on a minute he's not doing the same so he's obviously thought of something in between to make him realise this is the way to do it. Because you didn't even check the answer.

Lucas: That's true.

JW: Because you just know?

Lucas: Yeah, well you sort of, if you think about these types of problems, you sort of just know that they're *[short pause]*, you can just see that it's going to work because there's lots of 10s and that number *[points to the centre box]* could be anything, 18 or something and it'd still work.

JW: You've done these sort of problems before have you?

Lucas: Ah, no, not this one, not too close to this, but where you have to put the numbers and boxes and make everything add up.

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[2] JW was the interviewer

This example of Lucas' thinking and Janice's sequence are consistent with the general notion of the ARE metacognitive sequence shown in Pathway Two (see Figure 4). To a greater extent than the other students interviewed, Lucas was aware that he approached mathematical tasks in a similar way regardless of problem type. This awareness of his own thought processes was evident in his interview responses. Pathway Two (see Figure 4) shows the problem solver commencing with a cognitive action which is followed by awareness, regulation, cognition, evaluation, cognition and finished with evaluation. Pathway Two resembles Pathway One except for the relative location of the first regulation and evaluation acts in the sequence. In Pathway One, regulation is a consequence of the need for a decision on the part of the problem solver as to how best to proceed; that is, regulation based on evaluation (Pathway One) seems plausible. In Pathway Two, regulation occurs on the basis of retrieval procedures. Like Pathway One this second pathway is theoretically coherent and empirically grounded. Several students reported the metacognitive sequence shown as Pathway Two.

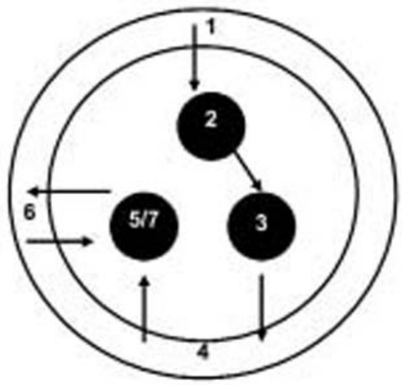

Figure 4. Example: Pathway Two.

Pathways One and Two are included as examples only. Many different pathways were reported. These various pathways shared specific structural features. As has already been noted, most pathways started with awareness and finished with evaluation. Evaluation was the most frequently reported metacognitive function. Metacognitive actions associated with the regulation function were reported as the next most frequently used, and awareness was reported the least. It is possible that the low frequency of student reporting of awareness may derive from lack of student recognition of awareness as a metacognitive activity, due to its less obvious character (in the sense of conscious *calling-to-mind*) when compared with evaluation and regulation. In drawing conclusions from these data, it must be assumed that metacognitive activity is likely to be under-reported by children. Nonetheless, when particular structural features are common across the majority of 90 interviews, the significance of those structural features can be inferred with some confidence.

The relationship between various aspects of metacognition has rarely been explicated. Indeed, researchers may have been thwarted in their attempts to monitor the intricacies of transition between metacognitive and cognitive activities by their use of traditional research techniques. The MMI allowed us to distinguish between various metacognitive functions as used during different stages of the problem solving process. The identification of metacognitive function alone does not necessarily suggest how its use might be optimised. The function of metacognition in promoting successful problem solving is more problematic and requires further investigation.

Metacognition and Success

Student problem solving is a complex interplay between cognition and metacognition (Artzt & Armour-Thomas, 1992; Dunlosky, 1998; Lester, Garofalo & Kroll, 1989; Schoenfeld, 1992). A key issue concerns what is "available to awareness" (Steffe, 2002, p. 2). Since our interest is in the optimisation of learning and problem solving as purposeful activities, it seems appropriate to study those processes that are available to awareness in order that we might optimise their use.

The proposition that frequent use of metacognition is advantageous to the problem solver is generally accepted, but not much is known about the type of metacognitive behaviour that is beneficial. It is reasonable to suppose that optimising the use of metacognitive behaviour can be addressed in terms of:

- the appropriate use of a relevant metacognitive function;
- the optimal frequency of metacognitive actions relative to cognitive actions; and
- the optimal sequencing of metacognitive actions and functions.

These distinctions are only meaningful within the framework of the model (Figure 1). It was noted earlier that in order to optimise the use of a process, you must first understand it. Having established the empirical plausibility of the model (and its theoretical coherence), we have a framework with which to investigate its optimal use to support problem solving and learning.

There is no doubt that students are using metacognition, but regardless of how many metacognitive acts are used, success is not guaranteed. Certainly, our data demonstrate that frequent use of metacognitive activity does not necessarily equate with problem solving success.

Many classroom tasks require minimal metacognition. For example, a routine task triggering an automatic response involves little metacognitive activity. Metacognition becomes essential when tasks are more challenging. Identifying associative links between successful problem solving and the successful deployment of metacognition is complicated by the individual student's knowledge base, which may or may not be adequate to the demands of the task. The knowledge demands of the tasks employed in this research should have fallen well within the capabilities of the participating students. Other intellectual capabilities also contribute to problem solving success (Krutetskii, 1976). In addition, affective attributes, such as motivation, self-esteem, and more transitory emotive factors may also influence a student's problem solving performance. Non-cognitive factors which are often linked to successful and unsuccessful problem solving, for example, student beliefs about themselves, schooling, learning and mathematics (Garofalo, 1989; Goos & Galbraith, 1996; Schoenfeld, 1987; Siemon, 1993), were not within the parameters of this research. It is important to note that student use of metacognition may be well-reasoned and yet not lead to successful completion of the problem. Similarly, Corno (2002) argues that students may proceed on one path (for better or for worse) without reconsidering their initial goals. Consider the approach taken by Therese who experienced a high level of challenge when faced with a spatial mathematical task for which she had to arrange shapes to make a rectangle. Therese reported the following sequence: A, A, E, R, R, C, C, R, C, E, E, E. This is not an unusual type of sequence. Therese used the following action cards to represent her task approach:

1. *I tried to remember if I had ever done a problem like this before* (A)
2. *I thought 'I know what to do'* (A)
3. *I thought 'Is this right?'* (E)
4. *I thought about what I would do next* (R)
5. *I changed the way I was working* (R)
6. *I tried to see if a shape would fit* (C)
7. *I moved a shape around* (C)
8. *I thought about a different way to solve the problem* (R)
9. *I tried a different shape* (C)
10. *I thought about whether what I was doing was working* (E)
11. *I checked my work* (E)
12. *I thought 'I can't do it'* (E)

Although Therese did not complete the task successfully, these metacognitive actions seem sensible enough. Some cognitive actions appear to have been unreported (that is, have gone unrecognised by Therese), nevertheless the progression from drawing upon known strategies (awareness), to evaluation and regulation of progress, and then further regulation and repeated evaluation could lead to the successful completion of some tasks. For some students experiencing difficulties (such as Therese), the repetition of regulation and evaluation went on for some time, often to no avail. The promotion of metacognitive activity will require the development of a language by which to discuss such activity in the classroom. Lacking such a language by which Therese might share her actions with the teacher, Therese's lack of success in problem completion might lead to lack of recognition of the basic rationality of her metacognitive activity, and consequent misdirection of the teacher's support.

The importance of metacognition, as cited often in the literature, cannot be ignored, but the interrelationship with various other cognitive and non-cognitive factors seems also to be highly significant. If cognitive capacity (Nelson, Kruglanski, & Jost, 1998) is low, metacognitive processes are likely to be limited. Students may not have the cognitive resources to complete the task, or their metacognitive judgements may be incorrect. For example, regulation usually relies on monitoring (evaluation); if the monitoring is inaccurate, this will impact upon subsequent actions.

In this research study, awareness was the least reported function, and evaluation was the most reported function, in both the successful and unsuccessful sequences. Successful sequences were more likely to be shorter. Longer sequences were more commonly associated with unsuccessful problem solving. However, longer sequences do not necessarily represent more or less effective use of metacognitive strategies, as students may lack the cognitive skills or confidence necessary to achieve closure, and students who are experiencing difficulties may need to employ metacognition for more prolonged periods. Before advocating curriculum reform based on the notional benefits of metacognition, more needs to be known about what is being promoted and why this is likely to be important. In particular, detailed information on student utilisation of metacognitive functions is needed in order that any teacher intervention or scaffolding can be informed and purposeful (see Holton & Clarke, 2002).

Characterising Student Mathematical Metacognitive Activity

Despite the different tasks used (logic, number, and spatial) in different classrooms and different schools, there was a high level of consistency in students' use of metacognition during problem solving activity. In general, the process could simply be described as almost always commencing by students reviewing what they know (awareness). Regulatory and evaluative actions were then sequenced in different ways. The process typically concluded with students reviewing what they had done (evaluation), whether they completed the task successfully or not. When difficulties occurred in mid-sequence, students sometimes reported re-using the awareness function. Cognitive activity was used at various stages during the problem solving task in a less predictable fashion than metacognition. This, in itself, is interesting. If problem solving is seen as the purposeful alternation between cognitive and metacognitive activity, then it is of interest if student engagement in metacognitive activity shows more apparent consistency of structure than student engagement in cognitive activity. Further research is required into the connections between specific metacognitive actions and the cognitive activities with which they are related. A more fine-grained analysis of cognitive actions may reveal a structure of relationship that was not accessible in this study.

Implications For Educational Practice

It is our hope that the structure of the model of metacognitive behaviour presented in this article, and the method employed in this research, will provide a basis for the further study of metacognitive activity. Of central concern is the ability of schools and teachers to provide a curriculum that promotes effective use of metacognition. This need is underpinned by the two key assumptions: that metacognition is of value, and that its use can be optimised (Baird, 1998; Baird & Mitchell, 1986; Baird & Northfield, 1992). The role of motivation and volition in *reflective driven learning* (Baird, 2001) deserves attention when considering how student activity might lead to successful achievement (see also Baird, 2002). Baird's explanation takes into account that learning involves emotions and self-directed behaviours (including motivation and volition).

The question of the impact that the classroom environment might have on students' metacognition is an interesting one. Kilpatrick (1985) suggested that school practice could indeed inhibit the development of metacognitive skills in some students. One reading of the analysis of Holton and Thomas (2001) is that prescriptive teacher guidance of student problem solving could appropriate student metacognitive activity and restrict its development. For reasons of methodological utility, the data analysed for this article were generated in clinical rather than classroom settings. The fact that the data were drawn from student problem solving activity supports the extrapolation of both the model and the findings to classroom problem solving. This is not to ignore the impact of the classroom environment on student metacognitive activity.

If the optimisation of student metacognitive activity is to become an instructional obligation, then teachers will need to be more informed in the provision of support for students' metacognitive activity, and need to consider the implications of their task selection from the perspective of promoting the development of metacognitive, as well as cognitive, capabilities. Holton and Clarke (2002) argue that both conceptual and heuristic scaffolding are of great importance for problem solvers. Classroom conversations that include statements such as those provided on the cards in this study, or the prompts provided by Holton and Clarke (2002), could be useful for increasing student self-control over their own learning. Research is needed to explore whether the scaffolding of metacognitive behaviour by the teacher promotes or restricts students' independent metacognition. Holton and Clarke (2002) raise the possibility that heuristic scaffolding may prompt students to internalise scaffolding questions and instructions as forms of metacognitive activity. More specific information about students' metacognitive behaviour is needed through further research into teachers' classroom utilisation of a model of metacognition such as the one proposed here.

Teachers may believe that metacognitive behaviour is valuable and they may actively reinforce metacognition as an important part of the curriculum, but the result is not necessarily that students learn to be metacognitively active. More detailed research needs to be undertaken into the extent to which students act metacognitively in a way that does or does not match their teachers' instructional intentions. The focus of this study was on student metacognitive behaviour, but another study could also include teachers' perceptions of metacognition and the strategies they employ to encourage metacognition.

Of concern is Borkowski and Muthukishna's (1992) argument that teachers themselves often lack metacognitive understanding. This is particularly worrying if the view of Hirabayashi and Shigematsu (1987) is accepted. They argued that students copy their teacher's behaviour and develop matching metacognitive practices. Therefore the question of how to improve teachers' understanding, use, and promotion of metacognition is important and requires further investigation.

Conclusions

There are a number of issues raised during this study that have implications for theory, for research methodology, and for the development of a *metacognitive curriculum.*

Researching Metacognition

The effectiveness of the multi-method interview for the study of student metacognitive behaviour has been demonstrated in the context of grade 6 mathematics. The general utility of the multi-method interview as a research tool requires further investigation with students of other ages and in other contexts.

Of particular significance was the consistency with which access to the video record prompted students to change their initial accounts of their problem solving attempts. This raises important concerns regarding the results of studies that depend for their data on students' accounts of their thought processes obtained without the additional stimulus of a video recording a means by which the students' reconstructive accounts can be tied more closely to their observable actions in the situation being studied.

Metacognitive Behaviour

The structure of metacognitive behaviour proposed in this paper appears to be both theoretically coherent and empirically well-grounded. It has the potential to inform more detailed research into the relationship between metacognitive functions and cognitive activity.

The utilisation of metacognitive activity to promote student learning and successful problem solving behaviour must be based on a more detailed understanding of metacognition, other non-cognitive factors and the structure of student metacognitive activity. Our model (Figure 1) was developed from the analysis of problem solving data and is intended to contribute to this understanding. The components of Figure 1 have been variously discussed in the literature, however the particular schematic configuration we have used offers a graphic illustration of the relationship between these components. One of the most important functions of a model is its capacity to suggest possible relationships as yet unidentified. We feel that Figure 1 posits a multiplicity of metacognitive pathways and offers a useful framework for the analysis of children's problem solving.

The grade 6 students in this study displayed a marked consistency in the sequence of metacognitive functions employed. This consistency provides the starting point for improved teacher understandings and may lead to the development of a metacognitive curriculum.

A Metacognitive Curriculum

The promotion of metacognition within the curriculum must start with its legitimisation as a topic of classroom conversation. The card-sorting technique used in this study provides teachers with a tool to support their conversations with students about their metacognitive activity, and as a resource to teach students to monitor their own thinking. Its classroom use in an adapted form could stimulate students' metacognitive behaviour in other problem solving and learning situations. For example, students might be encouraged to use the cards to make their metacognitive activity visible for discussion with other individual students or with the whole class.

Several of the students in this study demonstrated an impressive awareness of their thought processes and, in particular, of their metacognitive activity (for example, Lucas, see above). It has been argued that learning is enhanced when students are aware of their self-regulatory abilities and use these to achieve their goals. Metacognitive research needs to focus on both the characteristics of thinking and the classroom conditions that contribute to such student behaviour.

Metacognition is undoubtedly a complex and multi-faceted phenomenon. In this article, we have put forward the structure of a model of metacognition that distinguishes three specific metacognitive functions and locates them in relation to each other (see Figure 1). We have applied this theoretical framework to the analysis of student metacognitive activity while solving mathematics problems. Our data collection technique *(the Multi-Method Interview)* seems a productive approach to the investigation of student metacognitive activity. We found that student metacognitive behaviour displayed a consistency of structure that should inform its promotion in classroom settings.

The findings reported in this article add detail to what is known about the nature of young mathematicians' metacognitive behaviour. If the education community values metacognition, then the further study of metacognition is vital. It is our hope that we have contributed tools by which this investigation might be carried forward.

Postscript

As a result of this study, a checklist and set of cards have been developed and implemented successfully by the first author as a tool for discussing and reflecting on metacognitive activity.

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Appendix

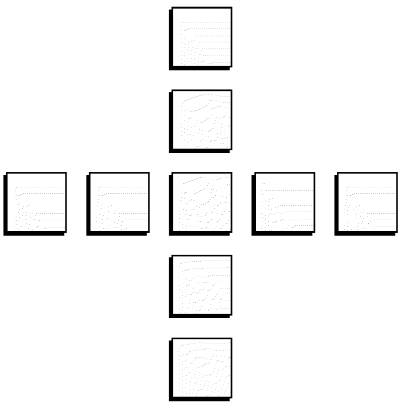
Multi-Method Interview Tasks

Logic task

The task required students to read and solve the following problem:
I have twelve animals. They are either dogs or chickens. When I look out my window I see 30 legs. How many are dogs and how many are chickens?

Number task

This task required students to use the digits 1-9 (once only) to fill a cross shape. The numbers placed across the shape must equal the numbers down.



Using the digits 1-9 (once only)
make a cross where all the numbers across
equal the sum of the numbers down.

Spatial task

Students were given shapes (Tangram pieces) and asked to make a rectangle with the shapes.