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Pedagogic Discourse and Equity in Mathematics: When Teachers' Talk Matters

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In this paper, we discuss the role and nature of pedagogic discourse. We argue that teachers' talk plays a much more important role in students' learning than is often considered—particularly in the learning of racially, ethnically, and linguistically diverse students. We present one teacher who has a record of assisting her fifth grade Latino students to make significant academic gains in mathematics, and we examine the way she uses her talk in teaching and how students in her class develop control over the mathematics discourse. To help make our point, we contrast this teacher with another teacher whose instructional talk is not as mathematically rich.

Current reforms in mathematics education have placed communication at the heart of the learning process (National Council of Teachers of Mathematics, 1989, 2000). As a result there has been increased interest in issues and questions related to communication around mathematics (Elliot & Kenney, 1996; Steinbring, Bussi, & Sierpinska, 1998), in how the kinds of verbal interactions frame what mathematics learning occurs (Atweh, Cooper, & Kanes, 1992), and general linguistic conditions that exist in classrooms that affect children's learning through talking particularly in the case of second language learners (Khisty, 1996). Also research on classroom talk, in general, has suggested that teachers' talk often constrains students and students have few opportunities to contribute to the quantity of talk and the substance of talk (e.g., Mercer, 1995). Such findings can be interpreted to mean that teachers' talk is negative or irrelevant in the learning process. While mathematics classrooms have begun to shift away from a teacher-centred instructional model, we suggest that the teacher is still a critical element in the learning environment because he/she is still an actor in it, and consequently, is a participant in the communication that goes on. This is especially true in classrooms that include second language learners and where the medium of instruction is the students' weaker academic language.

In this paper we will present our analyses of a teacher's interactions with her fifth grade Latino students who are English language learners. Although the focus of our discussion is drawn from our own context, that is to say, of Latino English language learners, the concepts we present apply to any context where there are students who are learning mathematics through a language which is not native to them. It is our contention that understanding how the students in this particular classroom become proficient in mathematics depends on understanding the processes of interaction and the characteristics of talk in this classroom. The most salient patterns to emerge within those processes have to do with how an environment is created that is filled with words—rich words—that students appropriate as their own, use as tools for their thinking, and use as tools to communicate their thinking. We will demonstrate how the teacher, who we call Ms Martinez, begins by populating the environment with words and ideas. Eventually, the students also populate the environment with words.

We further suggest that the process is not so simple as to simply give words to students. The words represent meanings that are waiting to be developed and eventually internalised. Therefore, which words are presented to the students and how they are developed are vitally important. Just as important is that students have opportunities to use these words in their talk and as they work. However, it is the nature of the teacher's pedagogic discourse and the role it plays in students' learning that is the focus of our discussion. In essence, our argument rests in the distinction between communication in teaching versus teaching in communication.

Each of us has extensive experience with in-service teacher development and we have had many opportunities to visit classrooms in the large urban district in the U.S.A. where Ms Martinez works. We have known Ms Martinez in various professional contexts and recognise her reputation as a highly effective teacher in a school district that is noted for its record of overall low student achievement. In the district, she also is distinct for her instructional style which emphasises problem-solving and teaching for meaning consistent with the NCTM *Standards* (1989) (Khisty & Viego, 1999), and which represents a very different way of teaching minority students who too often are considered to need more remediation before handling challenging and higher level work (Lockwood & Secada, 1999).

For the last several years, both of us have been intrigued with Ms Martinez. We have found ourselves, independently and together, systematically attempting to understand how this teacher accomplishes such effective teaching. As we have observed her and reviewed videotapes of her teaching, we have come to hypothesise that part of her effectiveness rests on her rich and powerful use of talk. It is her instructional talk that is so striking. It is the relationship between what this teacher says and what students learn that will be discussed here. In light of this, we further argue that this is a critical perspective for equity given how common it is now to have in nearly every part of the world classrooms where there are multiple home languages represented. The phenomenon of students learning mathematics in their developing second language requires that we understand the nature of teaching in communication.

However, in order to fully understand this relationship, we also will present a second teacher who we will call Ms Tapia. Our intention is not to compare Ms Tapia's instruction with the instruction of Ms Martinez. Instead Ms Tapia's interactions and instructional talk provide a contrast that will highlight the concepts and issues we present.

We will begin by putting our discussion in a research context and setting forth our assumptions to understanding Ms Martinez and her talk, and then turn to examples from her teaching and that of Ms Tapia that bring us to our conclusions.

Background

Our analyses and understandings of Ms Martinez are grounded in assumptions of how language and other culturally significant symbolic systems mediate thinking. However, in this case, interest is not in the grammatical aspects of language but in human interaction, which is uniquely characterised by language. Interaction forms the social context in which children participate and which mediates their thinking and learning (Vygotsky, 1978). In this perspective, children's development of higher mental functions particularly occurs in language-rich social interactions with what Vygotsky (1978) called the "more capable other" or a person who enculturates the child through recurrent

meaning-making activities which occur in both informal and formal educational settings. If we look at families, we can easily see children being cognitively apprenticed (Rogoff, 1990) into cultural ways of thinking, meaning, and doing, and this is accomplished through language or talk. Therefore, language—including language use with a more mature or capable other—becomes the means for a thinking, conscious self (Wertsch, 1991). This perspective does not suggest interaction or activity as an imitative set of dynamics. On the contrary, if we observe everyday human interaction, we can see that there is joint participation and influence, where no one is unaffected. In fact, as persons interact, for example, teacher and students, they construct "an ecology of social and cognitive relations" (Erickson, 1996, p. 33).

In the context of learning mathematics, there is also what Bakhtin (1986) called a "social language". We can think of social language as that verbal resource we associate with, say, being a mathematician. Gee (1996) extends this idea into what he calls "*Dis*courses" which he describes as ways of talking, listening, even reading and writing—using social languages—together with ways of acting, interacting, and using tools and objects, in particular settings at specific times to display a particular identity, for example, being a mathematician. Moreover, in mathematics, we have specialized words, terms, and words from non-mathematics contexts that have special mathematical meanings. To understand discussions about mathematics—and to learn the subject, one needs to be able to comprehend this way of "speaking mathematically" (Pimm, 1987). More importantly, having recognition of capability, for example in mathematics, means being able to display control over this *Dis*course (Gee, 1996; Gee & Clinton, 2000).

But where does this knowledge, fluency, and control of the social language of mathematics come from? Since it is a specialised discourse that would not readily be heard in most social contexts, we can assume that it is not acquired in the same way as everyday language. If this is the case, how does this affect how students for whom English is a second language garner this specialised social language? English language learners (ELLs)—including a sizeable portion of Latinos and other students who come from homes where a language other than the dominant cultural language of schooling is used)—face a double task in regards to language development: one is acquiring the language that is used in most social contexts such as with friends or at the store, and the second is acquiring the academic language which is different from conversational language and which is used in schools particularly for more decontextualised work such as reading and writing (Cummins, 1994). The academic language of schoolwork takes students several years to develop and cannot be taken for granted—that is to say, it needs to be taught (Cummins, 1994). However, just as social language is not memorised, academic language is best developed in context through meaningful and active use of the new language. In this process, the most critical element is that students hear how the language is used in context (Krashen, 1994). In other words, someone must model the academic language and provide supports for the development of its meanings.

If we assume that language or discourse is at the heart of the social context in which learning occurs, and that there is ecology of cognitive relations in classrooms, then we must assume that an important issue in the mathematics learning of language minority students—in this discussion Latinos—would be the ecology that is created by the teacher and students. Further, as part of that ecology is the nature of the discourse that the teacher—as the more experienced other—brings to the process as part of the "natural environment". In essence, this

is what we wish to explore. We find in Ms Martinez's classroom an ecological environment that is richly coloured by her discourse, and seemingly because of this, her students are able to thrive mathematically.

The Classroom Context

Ms Martinez teaches fifth grade and typically has twenty-eight or more students in her class some of whom are identified as special education students. Most of her students enter her class one to two grade levels below in mathematics, but nearly all of them leave her class one to two grade levels above norm, including the students with special needs. In general, we can say that her students leave her class smart in mathematics as evidenced, for example, by performance on standardised tests.

Ms Martinez is part of the Bilingual Education program in the school but her students are advanced enough in English for this to be the primary medium of instruction in mathematics. Her mode of instruction is quite complex and some aspects of her instruction have been described in other papers (see, e.g., Chval, 2001; Chval & Khisty, 2001; Khisty & Viego, 1999). Consequently, we will not go into much detail about the way she organises and carries out her teaching. Nevertheless, in order to have some context for our discussion, it is suffice to say that she engineers a learning environment where students are actively engaged in problem solving and collaboration, oral and written communication and justification, and independent thinking. However, Ms Martinez is also an active participant in students' construction of knowledge through her questioning strategies and guidance. Her mathematics lessons are unusually long compared to the typical lesson in the school or even the district, but much of this is due to the time given to students to get a solution and to the discussion that focuses on meanings embedded in the whole process.

As noted earlier, both of us have systematically studied her: one through audiotapes and field notes for a whole year usually two or more times a week (Chval, 2001) and the other through videotapes gathered at least twice a week for four months and intermittent observations with field notes for the rest of the year (Khisty, 1998; Khisty & Viego, 1999). While we have independently pursued different questions about her instruction and classroom, we have had a common interest in her talk. For this discussion, we have pooled our audio and videotape data which are our primary sources.

Ms Martinez

Typically the school day begins with one or two "morning problems". These problems follow a theme such as triangles, circles, rectangles, or logic, and are the context not only for developing advanced mathematical concepts but also for enhancing basic skills. Typically, new concepts are introduced in situations highly contextualised with word problems, drawings and models, peer discussions, and even physical activity. The problems are on the board when students come in at the beginning of the day. When class starts, students are asked to solve them by working collaboratively. Usually the desks are arranged so that students can easily form pairs and consult with others as needed. Before the students begin working, some time is spent clarifying meanings of words in the problem if it appears that this is needed. As students work to solve the problems, Ms Martinez moves around the room listening to students, noting what needs clarification and how

students performed. When students have completed solving the problems, time is spent with the whole class with students presenting their solution strategies at the board. This time is really for the purpose of engaging the class in dialogue and meaning-making.

Ms Martinez frequently used mathematical words in her talk and students began using these same words in written and oral discourse. It was clear from analysing the transcripts and videotapes that students would not have used these words without the introduction and frequent use of Ms Martinez. The following class discussion illustrates the introduction of "quadrilateral" to students' repertoire. It should be noted that this transcript is from the first full day of school. Ms Martinez is indicated by "T" and a student is indicated by "S". We have italicised sections we wish to highlight. Relevant physical actions that accompany the dialogue are noted in brackets.

T: *It has four sides.* You know what? I'm going to put the word rectangle into a *category*. And I am going to *call this category, quadrilaterals.*
[Teacher writes "quadrilaterals" on the board.]
T: Do you recognise or at least listen to the sound of the word and see if there is any part of this word that you recognise. "Qua—dri—lat—er —al." "Qua – dri —lat —er —al." "Qua— dri—lat—er —al." Cuadro right. What is a cuadro?
S(1): A square.
S(2): A shape.
T: A shape that has?
S(3): Four sides.
T: A shape that has four sides. Look at your classroom. *Do you see a lot of shapes that are quadrilaterals?*

Ms Martinez immediately begins the first day to populate the lessons with what we call sophisticated words—words students would not usually use. She introduces the word and concept of "category", introduces the mathematical word "quadrilateral" and then puts the word in context in line 6 in a way that invites students to use it. It also is interesting how she implicitly capitalises on students' knowledge of Spanish (by connecting it to "cuadro") to have them construct a meaning for quadrilateral.

Here is an example taken from the fifth day of school. A class choral response is indicated by "Ch".

S(1): Multiplication.
T: Multiplication. We're not going to say *opposite. We're going to add a new word here.*
[Teacher writes "inverse" on the board.]
T: What's *the inverse* of multiplication?
[Pause]
S(1): Division.
T: What is the *inverse* of division?
Ch: Multiplication.
T: *Good you know another word.* Now let's look at another one. Everyone knows how to use the word length. Everyone knows how to use the word base, and the word height.

Notice how she again has students construct a meaning for "inverse" and how explicitly she recognises the newness of these words. In these two examples, we see how Ms Martinez attends right from the beginning of the school year to developing the mathematics discourse students need. These are words that students need if they are to function with problems on standardised tests or in the

curriculum. Moreover, she accomplishes this beginning development expertly and unobtrusively.

Near the beginning of the year, the students were given two problems. The first problem involved a rectangle with an area of 600 square metres and a width of 30 metres. On the chalkboard is a drawing of a rectangle with an area of 600 sq. m marked inside of it and 30 m marked on one side. The students were asked to find the missing length. The second problem involved a rectangle with a perimeter of 2 metres and a length of _ m also with a drawing, and the students were asked to find the missing width. The students have finished solving the two problems and now Ms Martinez asks for a pair to volunteer to solve one of the problems on the board. All the students eagerly raise their hands to be selected. Araceli (all names used are pseudonyms) and her partner are selected and as they go to the board they are reminded to speak loudly as they "talk through every step". The following dialogue is an example of how Ms Martinez uses her talk not only to extend students' thinking but also to model mathematical discourse.

Again, student choral responses are indicated by "Ch" and we have italicised sections we wish to highlight.

01	S(1):	We divide the six hundred by thirty.
	T:	*You divided the 600 square units* by 30. Why did you divide *the area by the side length?*
	S(1):	To find the side.
05	T:	To find *the side length*. Now, will that work? Can you give me a different kind of problem to show me why it will work? Can anybody show me a different problem so I can know why it works? OK, Rocky [Another student, Rocky, joins the others at the board].
	S(2):	With a different square [Rocky draws a rectangle and labels the corresponding parts 6 sq m and 3 m].
10	T:	How would you find the area of that square…really… rectangle?
	S(2):	Work it out.
	T:	*How are we going to find the side length? What's the idea behind* doing what we're doing?
	S(2):	You're going to divide.
15	T:	Why are you going to divide, Rocky? Will you explain?
	S(2):	Six divided by the three.
	T:	*Why is it going to work? What's the relationship …* like that between multiplication and division? Are they related?
	Ch:	Opposite.
20	T:	*One is the inverse of the other, the opposite of the other.* So if the area is multiplying, then what is the opposite of multiplication?
	Ch:	Dividing.
	T:	Dividing. So that's why this would work.

In this short episode, we see examples of how Ms Martinez guides her students through the use of questions to be more complete in their mathematical talk and to explain the meanings behind the calculations that are used. In the first line, Areceli demonstrates that she and her partner solved the problem correctly by describing the appropriate numbers and operation, but she is incomplete and vague about what she is saying. In the second line, Ms Martinez adds the units to the 600, but more importantly, she connects the words "area" and "side length" to the numbers that Areceli used, thus reinforcing mathematical meaning behind the numbers. At this early point of development, Ms Martinez's students demonstrate

a solution solely by relying on numbers and operations. As you will see in a later example in this paper, students elaborate on why they use specific operations and what the numbers represent in terms of the problem as the year progresses. However, in the above example, we see how Ms Martinez begins students' development of the appropriate academic and mathematical discourse by her modelling (in line 02) through her questions. She continues to model speaking mathematically in response to the second student (line 12) and again prompts her students to think about the big mathematical ideas in the problem (same line) with a reference to the "idea behind" their calculations. In line 17, we can see evidence of her developing mathematical thinking and the process of generalising. Finally in line 20, we see an excellent example of how she capitalises on students' understanding ("opposite" in the previous line) and then uses it to extend their understanding and connect it to the meaning of a specialised mathematical term ("inverse") which they encountered before but still do not use. This last example also demonstrates what is meant by teaching the academic second language through content (Mohan, 1990). It should also be noted that the questions and models by Ms Martinez do not significantly interrupt the flow of the students' talk or thinking.

In the following set of interactions taken from the sixth day of a different school year, the students are solving problems involving right triangles. Figure 1 depicts one problem that challenged her students: "A right triangle has an area of 500 sq. cm and a leg of 200 cm. Calculate the length of the other leg. Draw a sketch of the triangle."

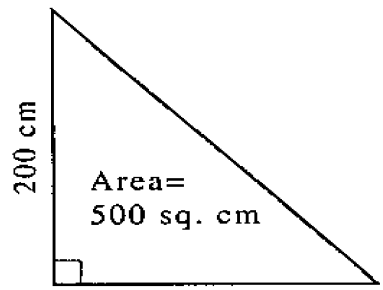

Figure 1. Right triangle problem.

Again, students have worked on this problem in groups and now the class is discussing it.

01	T:	There you have it. [A second, congruent triangle is drawn adjacent to the first triangle to create a rectangle.] Now what? How do you know that that triangle is 500 square centimetres?

	S:	Because it's congruent.
	T:	It's a *congruent triangle. What does congruent mean?*
	S:	The same.
05	T:	*Exactly the same*. So they must have the *same area.* Okay, so now having that there, what are you going to do with those two areas?
	S:	Add.
	T:	Show me how you add it.

This short interaction from the sixth day of the school year is only a small portion of a very long discussion. Students have encountered the word "congruent" many times by now—even within six school days. In fact, a student uses it appropriately (line 02) to respond to the question Ms Martinez asks. Ms Martinez accepts this response but rephrases the response to be more complete. She goes on to reinforce the word by asking the class what it means. One student offers an explanation that she accepts and incorporates into her own talk (line 05). As in the last example above, we see how Ms Martinez acknowledges students' own words and meanings thus making them viable partners in the interaction. At the same time, she seizes opportunities to have students hear how the discourse should be and reinforce their understandings through mathematical meanings. In essence, too, Ms Martinez, as we see in this episode does not take students' language development for granted.

Further in the discussion around the same problem, we see examples that some students have begun to appropriate the mathematics discourse and some still struggle with it.

01	T:	What does that number represent, Alejandro?
	Alejandro:	The, the…
	S(2):	I know. I know.
	Alejandro:	*It's the area of the whole rectangle.*
05	T:	Alejandro says yes, Alejandro go demonstrate to them that that is true. Come on. When I multiply 200 times 5, I'm going to get a thousand, yes?
	S:	Yes.
	T:	What does that one thousand represent?
	S:	The whole.
	T:	The area of a whole rectangle? The area of what?
10	S:	The whole.
	T:	The whole what?
	S(1):	Rectangle.
	S(2):	Congruent triangle..
	T:	Of a congruent triangle? No, if I build a congruent triangle, I'm going to have 500 right here. Why is that? What is this thing called?
15	S:	Rectangle.
	T:	This *whole thing is called a rectangle.* Alejandro explain to them why 200 times 5 is the area of a whole rectangle. [Pause.]
	Alejandro:	It's the…
	T:	It's the, what? Go ahead.
	S:	*It's the whole rectangle.*
20	T:	It's the whole rectangle. When you do 200 times 5 you are taking this leg and this leg, right? And if this leg was 2 centimetres and this leg was 3 centimetre, what's the area?
	S:	Six.
	T:	Six square centimetres. Why? Because I can do this model,

right?
S: *You multiply 200 times five.*
T: You multiply 200 times, you multiply one leg times the other leg and that gave you what?

At the beginning Alejandro struggles to get his answer out (line 02); however, with the time provided him by another student offering a response, he is able to formulate it into quite a complete sentence. Too frequently incomplete or one word responses are accepted by teachers and eventually this becomes the mode of talk in classroom discussions. Yet, Ms Martinez pushes her students to speak in complete sentences and some students begin to appropriate this habit as can be seen in lines 19 and 23. Nevertheless, some students still struggle but this may be simply because they are not comfortable speaking publicly in what may be their second language.

Also in lines 20 and 24, we see Ms Martinez repeating what a student has said. Some may argue that this strategy of repeating causes students to listen less to their peers since they know that the teacher will say it again; that this process keeps the teacher at the centre of the dialogue in mathematics rather than shifting the dialogue more to students. In this case, we need to recognise that one of Ms Martinez's objectives is to help students further develop their academic second language. It is an assumption of hers, given her training and experience as a Bilingual Education teacher, that her fifth grade Latino students still need to hear models of how the language is used. Students' talk may not always be comprehensible enough to be that model, and therefore, the teacher must provide it. Even if this is not the case, it is important to give students multiple opportunities to hear the language so they can acquire it. Ms Martinez's talk is very clear and easy to aurally comprehend, and her strategy of repeating does not seem unnatural, consequently she is practising a good second language development strategy (Collier, 1995; Krashen, 1994).

The reader may notice that in these episodes, there is relatively more teacher talk than student talk, and it may appear that students in her class do not engage in much dialogue. At the beginning of the year most of the talk is done by Ms Martinez. The students say few words. They speak in incomplete sentences. By the second half of the year, students speak in complete sentences and use the words correctly as demonstrated in the example below. What has transpired is a process of appropriation and internalisation. But to make the process work, students must have something to appropriate; ergo, Ms Martinez's own talk becomes essential, particularly at the outset. It also is important to keep in mind the context of these interactions. The episodes presented are those where students have volunteered to present their solutions to the whole class. In this situation, the students tend to be much more quiet and hesitant about speaking even though they are confident about their work. It is in sharp contrast to their active engagement while working at their combined desks with their peers and with Ms Martinez. More importantly, this interaction with students at the board is when Ms Martinez guides her entire class in thinking via her questions and when she provides oral examples of mathematical discourse - and in general, academic talk in their second language.

The discourse in Ms Martinez's classroom during the second semester is significantly different from the beginning of the year. The following example is taken from a lesson on April 12, near the end of the school year. The students had been given the following problem: The area of a three-quarter circle is 100 square centimetres. Calculate the perimeter of the three-quarter circle. After students have spent a great deal of time trying to solve the problem in groups, one student

volunteers to explain how she solved the problem. Violetta approached the chalkboard and drew a sketch of the three-quarter circle as shown in Figure 2.

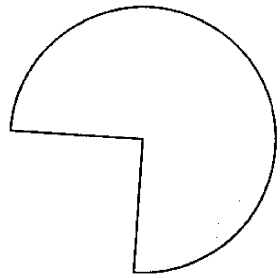

Figure 2. Three-quarter circle problem.

Violetta began to explain how she solved the problem as follows.

01: Violetta:	We are going to find the perimeter of the three-quarter circle. The area of the… The area of the three-quarter circle are 100 square centimetre. Now, we are going to go backward from the area to the perimeter. One hundred divided by three equals the area of one quarter-circle. Multiply by four *to get* the area of the whole circle. Divide by pi *to get* the area of the square built on the radius. You take the square root *to get* the radius. And then you multiply by two *to get* the diameter. Then we store it. [A reference to the student's calculator.] Then we multiply by pi *to get* the circumference of the circle. Then we divide it by four *to get* the quarter circle. Then we multiply by three *to get* the curvy part of the three-quarter circle. Then we sum it, sum it to memory. [Another reference to the calculator.] So we can get the circumference, the perimeter, of the three-quarter circle.
Chorus:	Forty-three point seventy-two, ninety-two, zero, two. [43.729202]

There are many outstanding qualities in this explanation of a solution to a multi-step problem, a problem that most fifth-grade students could not solve. It is organised, concise, and comprehensible. Violetta uses mathematical words, words that have been spoken by Ms Martinez many times. She uses complete sentences, modelled on a regular basis by Ms Martinez. Violetta explains the result of each operation (e.g., "multiply by pi *to get* the circumference"), again modelled by Ms Martinez every time she asked "why are you multiplying?" Violetta uses specific language that is not typical for fifth graders and that can be directly traced back to Ms Martinez (e.g. "area of the square built on the radius"). Violetta even corrects her mathematical talk in the last sentence when she begins to refer to the "circumference" of the figure and recognises it is not a circle so she correctly changes to the "perimeter" of the three-quarter circle. The importance of the teacher's talk is evident in Violetta's explanation. Without Ms Martinez's talk, Violetta would not have had *access* to the words necessary for this explanation. Beyond the mere length of Violetta's explanation, it is also striking that Ms

Martinez's voice is silent. At this point, her words were not needed.

Ms Tapia

In order to fully understand the nature of a teacher's pedagogic discourse, we will provide a contrast and describe a second teacher who was recommended by a principal and other members of the school district as a highly respected and competent teacher who is working on improving her teaching of mathematics. As noted in the beginning of our discussion, our intention is not to make a direct comparison between Ms Martinez and Ms Tapia. There are many differences between the two teachers. Ms. Martinez has been teaching for more than 20 years while Ms Tapia has been teaching for fewer years; they are in different places in terms of their professional growth and development. Ms Martinez teaches fifth grade and Ms Tapia teaches second. Ms Martinez teaches mathematics in the students' second language (English) while Ms Tapia teaches mathematics in the students' first language (Spanish). In addition, the transcripts from the two classes cover different mathematical content. Nevertheless, Ms Tapia provides a second example of classroom discourse in the mathematics context. This example serves to emphasise that we need to carefully examine what the teacher says and how it is said to determine if students have access to the academic discourse. Furthermore we need to examine if students have the appropriate scaffolds to develop and use that academic discourse in the classroom setting at every grade level.

Ms Tapia's mathematics teaching had been videotaped over several successive days and the videos are the source of data. Ms Tapia teaches second grade in a low-income neighbourhood in a different urban school district. She too is a Bilingual Education teacher, and in this case, because of the age and English proficiency of her students, Spanish is the primary language of instruction. For this discussion, we will present the translated version of dialogue.

Ms Tapia has participated in the district's funded projects to reform mathematics instruction in keeping with the NCTM *Standards* (1989). She organises her instruction so that after whole class instruction to introduce the day's lesson and review concepts, students form groups to work independently at learning centres. At each centre, students work on a different task that reinforces the day's lesson. The students work purposefully but do not seem to talk much about the task they are involved in. When they do talk, their talk is not filled with high level thinking about the mathematics in the task; they do not actively and extensively talk about how to get the solution, why one way works or does not work, or if the solution makes sense, and so on. It is clear from observing the various groups, that the students are interested and that they stay on task, but they simply do not engage in much mathematical talk among themselves. Furthermore, most students are able to do correctly the mathematics that they are given, but few are able to discuss the mathematics they are learning when they are probed with questions to explain it.

In this episode, Ms Tapia is reinforcing basic concepts of division. In Spanish, she had read with the students a currently popular book that has been highlighted for connecting mathematics to literacy. The story is about sharing a set number of cookies among an increasing number of people. Earlier, Ms Tapia had used active and engaging literacy development strategies with the students and they had comprehended the story very well, that is to say, they were able to re-read and retell the story with great detail and animation and clearly had enjoyed the story. This morning, she begins the lesson by retelling the story with the whole class,

often acting it out, and using plastic coloured chips to represent the story characters on the overhead projector and to demonstrate the way the cookies had to be shared among the characters.

The story begins with a group of cookies being shared between two people. Then the doorbell rings, and two more people come to the home and the same number of cookies has to be shared equally among all of them. The same situation re-occurs until each person gets only half a cookie. At selected moments, Ms Tapia asks the class a question to test comprehension and attention. During this entire interaction, the one question that was asked - and asked repeatedly - was: What happened next? To which the students en masse excitedly responded, "ding, dong," to represent the doorbell when new guests arrive to share the cookies. Ms Tapia's response to this is implied approval as she gestures positively and rearranges the plastic "cookies" to correspond to the new number of "guests."

Ms Tapia seemed to enjoy telling stories and the students equally seemed to enjoy her animation and her added cultural references. However, during the entire process Ms Tapia did not refer mathematically to the events of the story; she did not say things like "twelve cookies" or "divided among" or "half a cookie". When Ms Tapia finished reading the story, students moved to their assigned centres and began tasks related to dividing a set of objects equally into various groups just like in the story. Each task involved concrete objects and required students to work together and write or draw answers. After a set time, the student groups rotated to a different learning centre.

The dialogue that follows comes from the first group that is at the learning centre with Ms Tapia. However, nearly the same thing happens when each group comes to this centre. Ms Tapia uses the centre as a means to give more attention and support to individual students in the context of a small group. It is clear that she also wants students to better understand and to explain what happens mathematically when there are, say, six cookies shared between two people, and then six cookies shared between three, and so on. She tells the students in the group that they will role-play the story they just heard and gives them directions to use paper plates, to act out the sharing process with plastic chips, and to talk about what happens. She says to the group that she will simply listen.

01	T:	In the story, how many people are there ... at the beginning, how many do you remember?
	Ch:	Two.
		[There is a brief discussion about who will play what role and when.]
	T:	You two will start off. How many cookies should we use? You choose. Choose some. [The students seem very confused by this question and do not respond.] Do you want twelve, six? How many cookies?
	S(l):	Seven?
05	T:	Well, I think six. Let's start with six. Count out six [indicating the plastic chips in the middle of the group] and put them on your plates. [The two students who are to begin hesitate, but then, one selects six chips and puts them in a pile.] Grandmother has made six cookies ... OK; put them on your plates like in the story. [The students silently each take one cookie at a time until all the cookies are distributed between the two of them.] Good. What happened? [No one answers.] What happened? Look at your plates.
	S(2):	Three cookies.
	T:	Good. You each have three cookies. Now what happens?

S(3): [One of the other students at the table answers.] Ding, dong.
Ch: Ding, dong.
10 T: Come in. Two, no, one more come in. Now share the cookies again but for the three of you.
[One student hesitantly redistributes the cookies accordingly. No one talks as this goes on. This continues with the same type of dialogue between teacher and students as before with six players with one cookie each.]
T: Now, what happens if there are six more people and six cookies? Twelve people altogether and six cookies. What will happen?
Ch: Ding, dong.

Ms Tapia's talk is strikingly absent of mathematical words. She never says that the group of cookies is "divided equally" among the group so that each person gets "X amount." There are no typical mathematical questions such as: "How many cookies did each person get?" or "How did we share the cookies so each person got the same number?" Ms Tapia has engineered a potentially positive learning environment by contextualising the mathematics in a story that the students can readily understand, by having the students role play the mathematical concepts, and by having students work in small groups so that there is increased opportunity to engage in talk. However, what becomes evident as we observe this episode is that the students do not have the means for expressing the mathematics they are learning. They do not have sufficient ability or experience with mathematical talk to engage in its discourse in order to even indicate their misunderstandings. And, Ms Tapia does not provide many models of the talk so that students hear mathematical discourse. The students in this episode, like the others that followed to the learning centre, could talk about the story using a language arts discourse and were eager to talk about anything. But when it came to speaking mathematically about what transpired, they could not. There were many of the right things in their learning environment; but one thing was missing which only the teacher could have adequately provided: the mathematical discourse. Since she did not substantially speak mathematically in the classroom, the students did not have the opportunity to appropriate mathematical discourse.

Concluding Remarks

Ms Martinez and Ms Tapia present us with two very different examples of pedagogic discourse. Both are very competent teachers and both create positive learning environments for their students across a wide dimension of organisational possibilities. But, Ms Tapia overlooks one element in her consideration of good learning environments. She recognises that interaction among students and between students and herself is important. However, she neglects to consider how her own talk affects students' learning. Ms Martinez, on the other hand, deliberately populates the classroom environment with her own mathematical talk. She has made speaking mathematically a critical part of learning mathematics, as we have seen with her emphasis on having students use complete statements. Her language is rich with mathematical words that most students at fifth grade do not know. Yet her students not only come to know the words (e.g., "congruent"), but know what they mean. More importantly, her language goes beyond just words and form. What she says also serves to instil habits of mind as when she talks about a "category" that helps organise new concepts.

The purpose of our discussion has been to explore issues of the role and nature

of teacher's pedagogic discourse in the mathematics context. We set forth a premise that the teacher plays a critical role in the communication process that forms the context for learning since the teacher is obviously present in the classroom, is the "more capable other" (Vygotsky, 1978), and is the person who engineers the learning environment. Given this, we suggest that we cannot assume that children's learning occurs uninfluenced by the teacher's talk, and therefore, we must understand better teacher's talk in the interaction process. In one classroom, students do not develop their control of the discourse of mathematics. In the other classroom, control of the discourse was a clear learning objective and students became fluent and competent in the discourse—and gained in achievement (Chval, 2001). For researchers such as Freire (1970) and Gee (1996), language is much more than talk, conventions, or linguistic formations. The word plays a critical role in power relations making literacy the masterful control of discourses that are associated with social institutions such as schools (Gee, 1996). In essence, those with power are literate or in control of a discourse.

If we assume Gee's (1996) assertion that control of the discourse is related to power, then we cannot ignore issues of how students become literate in the mathematical discourse, and how the teacher's own pedagogic discourse influences the process. Clearly there is more to explore including how teachers come to recognise the power of discourse as part of an ecology of cognitive relations.

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References

Atweh, B., Cooper, T., & Kanes, C. (1992). The social and cultural context of mathematics education. In B. Atweh, & J. Watson, (Eds.), *Research in mathematics education in Australasia* (pp. 43-66). Brisbane: Mathematics Education Research Group.

Bakhtin, M. (1986). *The dialogic imagination.* Austin, TX: University of Texas Press.

Chval, K. B. (2001). *A case study of a teacher who uses calculators to guide her students to successful learning in mathematics*. Unpublished doctoral dissertation, University of Illinois at Chicago.

Chval, K. B., & Khisty, L. L. (2001, April). *Writing in mathematics with Latino fifth grade students.* Paper presented at the annual meeting of the American Educational Research Association, Seattle, WA.

Collier, V.P., (1995). *Promoting academic success for ESL students: Understanding second language acquisition for school.* Elizabeth, NJ: New Jersey TESOL - Bilingual Educators.

Cummins, J. (1994). Primary language instruction and the education of language minority students. In C. Leyba (Ed.), *Schooling and language minority students: A theoretical framework* (2nd ed., pp. 3-46). Los Angeles: Evaluation, Dissemination and Assessment Centre, California State University.

Elliott, P. C., & Kenney, P. (1996). *Communication in mathematics, K-12 and beyond.* Reston, VA: National Council of Teachers of Mathematics.

Erickson, F. (1996). Going for the zone: The social and cognitive ecology of teacher-student interactions in classroom conversations. In D. Hicks (Ed.), *Discourse, learning, and schooling* (pp. 29-62). New York: Cambridge University Press.

Friere, P. (1970). *Pedagogy of the oppressed.* New York: Continuum.

Gee, J. P. (1996). *Social linguistics and literacies: Ideology in discourses* (2nd ed.). London: Taylor & Francis.

Gee, J. P., & Clinton, K. (2000). An African American child's science talk: Co-construction of meaning from the perspectives of multiple discourses. In M. A. Gallego & S. Hollingsworth, (Eds.), *What counts as literacy: Challenging the school standard* (pp. 118-138). New York: Teachers College Press.

Khisty, L. L. (1996). Children talking mathematically in multilingual classrooms: Issues in the role of language. In H. Mansfield, N. Putnam, & N. Bednarz (Eds.), *Mathematics for tomorrow's young children: International perspectives on curriculum.* Boston, MA: Kluwer Academic Publishers.

Khisty, L. L., (1998, April). *The ecology of interaction in the effective mathematics classroom.* Paper presented at the Research Pre-session at the annual meeting of the National Council of Teachers of Mathematics, Washington, DC.

Khisty, L. L., & Viego, G. (1999). Challenging conventional wisdom: A case study. In L. Ortiz-Franco, N. Hernandez, & Y. De La Cruz, (Eds.), *Changing the faces of mathematics: Perspectives on Latinos* (pp. 71-80). Reston, VA: National Council of Teachers of Mathematics.

Krashen, S. D. (1994). Bilingual education and second language acquisition theory. In C. Leyba (Ed), *Schooling and language minority students: A theoretical framework* (2nd ed., pp. 47-78). Los Angeles: Evaluation, Dissemination and Assessment Centre, California State University.

Lockwood, A. T., & Secada, W. G. (1999). *Transforming education for Hispanic youth: Exemplary practices, programs, and schools.* Washington, DC: Centre for the Study of Language and Education, George Washington University.

Mercer, N. (1995). *The guided construction of knowledge: Talk amongst teachers and learners.* Clevedon, England: Cromwell Press.

Mohan, B. (1990). LEP students and the integration of language and content: Knowledge structures and tasks. In J. Gomez (Ed.), *Proceedings of the first research symposium on limited English proficient student issues*. Washington, DC: Office for Bilingual Education and Minority Language Affairs.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: NCTM.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: NCTM.

Pimm, D. (1987). *Speaking mathematically: Communication in mathematics classrooms.* New York, NY: Routledge.

Rogoff, B. (1990). *Apprenticeship in thinking: Cognitive development in social context.* New York: Oxford University Press.

Steinbring, H., Bartolini Bussi, M. G., & Sierpinska, A. (Eds.). (1998). *Language and communication in the mathematics classroom.* Reston, VA: National Council of Teachers of Mathematics.

Vygotsky, L. (1978). *Mind in society: The development of higher mental processes.* Cambridge, MA: Harvard University Press.

Wertsch, J. (1991). *Voices of the mind: A Sociocultural approach to mediated action.* Cambridge, MA: Harvard University Press.

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