Beliefs About Mathematics Teaching Held by Pre-service Teachers Involved in a First Grade Mentorship Program

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The study compared beliefs about mathematics teaching of four pre-service elementary teachers involved in an intervention experience with those of their non-involved peers. During this intervention, which was based on a socio-constructivist approach to mathematics instruction, the intervention group participated in regular, small-group teaching experiences supported by on-going seminars. The study also examined the relationship between professed beliefs and observed actions for the intervention group.

Although most pre-service teachers in this study seemed to attach some importance to children building their own knowledge through social interaction, the intervention group professed significantly stronger beliefs in a socio-constructivist instructional environment than the comparison group. Even though the intervention group strongly espoused socio-constructivist beliefs, they were not uniformly successful in translating these beliefs into instructional actions. Their actions appeared to be most consistent with a socio-constructivist perspective during the initial phase of an instructional episode, but in later phases their actions reflected more traditional beliefs about teaching mathematics.

Recent reform documents (National Council of Teachers of Mathematics, 1989, 1991; National Research Council, 1989; Leitzel, 1991) have described mathematics classrooms as places where students should be actively involved in constructing their own mathematical knowledge. In this setting, mathematics teachers are thought of not as transmitters of knowledge but as encouragers of mathematical thinking. Students are conceived as developing mathematical understanding by internalising and elaborating their mathematical ideas through interaction with peers and teachers.

Possibly because their own experience with mathematics was teacher-directed (Ball, 1990; Civil, 1990), teacher education students are slow to adopt a constructivist view of teaching. Instead they believe that teaching involves telling or showing students what to do (McDiarmid, 1990). Never having been a part of a mathematical community that analyses, tests, and challenges each others' mathematical conjectures (Ball, 1990), pre-service teachers seem to find it difficult to conceive of mathematics instruction that uses rich interactions between students to facilitate learning.

This study investigated whether extended intervention experience in an environment consistent with reform goals had an impact on pre-service elementary school teachers' beliefs about and actions in teaching mathematics. In particular, the study inquired whether the pre-service teachers involved in the intervention experience with children were more likely than their peers, with the same educational background but without the benefit of the intervention experience, to embrace and to implement the reform view of mathematics instruction. In view of the fact that actions are not always consistent with stated beliefs (e.g., Kesler, 1985; Parmelee, 1992), the study also examined the relationship between pre-service teachers' professed beliefs and observed actions.

The investigation was part of a larger longitudinal intervention project (Jones, Thornton, & Van Zoest, 1992; Jones, Thornton, & Putt, in press) which replaced the regular first grade instruction in multidigit number sense and place value with a program focusing on problem solving and cooperative learning activities. This larger project was based on two key assumptions: (a) children actively construct their own knowledge; and (b) children learn through interaction with others.

The first assumption, grounded in the constructivist epistemology of Piaget (1970), has been extended and amplified by recent research in mathematics education (Cobb, Wood, & Yackel, 1990; Steffe, Cobb, & von Glasersfeld, 1988; Steffe, von Glasersfeld, Richards, & Cobb, 1983). Of particular interest are studies which have used a constructivist orientation to investigate children's thinking in number and numeration (Bednarz & Janvier, 1988; Steffe et al., 1988; Jones & Thornton, 1993). The second assumption is rooted in the work of Vygotsky (Jones & Thornton, 1993; Vygotsky, 1978) and in the theoretical approaches to learning which have combined a constructivist orientation with a strong emphasis on social interaction (Cobb, Wood, Yackel, et al., 1991).

In accord with these assumptions, the project endeavoured to generate a socio-constructivist environment for teaching and learning where the pre-service teachers could participate in the implementation of instructional reforms. Because of their special role as adult facilitators of children's thinking over an extended period of time, the pre-service teachers who were part of the intervention project were referred to as *mentors*. This term will also serve as a convenient means of identifying the group throughout the paper.

Theoretical Background

Social Interaction

Two dimensions of social interaction become crucial in a classroom based on a socio-constructivist model: interactions between teacher and students, and interactions among students. The promotion of effective communication along the teacher-student dimension requires that teachers: (a) assume each student's mathematical ideas are personally meaningful; and (b) take responsibility for helping students to verbalise these ideas in a mathematically meaningful way (Yackel, Cobb, Wood, Wheatley, & Merkel, 1990). The communication patterns established by the teacher in interactions with students then become a model for interactions among students (Wood & Yackel, 1990).

According to this research, social interaction among students has the potential to enhance student learning in a number of ways. Through discussions with their peers, students can develop a better understanding of the problem, share their ideas about solutions, refine their solution attempts, and defend their mathematical understandings and strategies (Yackel, Cobb, & Wood, 1991). In fact, Richards (1991) proposes that such experiences provide a foundation for students to become "mathematically literate adults" (p. 27).

The present study used this research to identify four phases in problem solving instruction where social interaction plays a key role: (a) problem statement and clarification phase; (b) solution exploration phase; (c) impasse relief phase; and (d) solution presentation and interpretation phase. For each of these phases, actions consistent with a socio-constructivist perspective for both mentors (as teachers) and children are synthesised below.

The research of Cobb and colleagues suggests that, in the presentation of a problem, teachers adopting a socio-constructivist approach should provide just enough information to establish the background or intent of the problem (Cobb, Wood, & Yackel, 1991). In response, students would be expected to interpret and clarify the problem and attempt to construct one or, preferably, multiple solution processes. We refer to this as the *problem statement and clarification phase.*

As children initially approach a problem or task, Yackel et al. (1990) suggest that a socio-constructivist oriented teacher should encourage mathematical dialogue and consensus among the students. This would involve having the students share their strategies and solution attempts with each other as well as with the teacher. The students would be expected to explain their solution attempts and, in an ideal situation, negotiate solution strategies with their peers. We refer to this as the *solution exploration phase.*

Consistent with Wood, Cobb, and Yackel (1991), teachers at what we have termed the *impasse relief phase* would react to the children's frustration at not being able to solve a problem by encouraging them to persist toward a solution without offering substantive mathematical suggestions. Questions such as: "Could you think of the problem in another way?" or "Can you explain the problem to each other?" might be used. In response, the students would be expected to reinterpret the problem, generate a new strategy, or build on a previous approach with new insights.

When children are presenting and interpreting their solutions to a problem, Cobb et al. (1991) suggest that the teacher with a socio-constructivist orientation on learning should accept right and wrong answers in a nonevaluative way and probe solution methods regardless of their success (Cobb et al., 1991). According to these researchers, such a teacher would be expected to assist the students in verbalising their solution attempts in mathematically meaningful ways. Ideally, these discussions would result in students further refining their solutions. We refer to this as the *solution presentation and interpretation phase.*

These four phases in problem solving instruction provide a framework for assessing the degree to which pre-service mentor actions reflect a socio-constructivist perspective. The extent to which a mentor modifies entrenched perspectives and adopts this orientation toward problem solving instruction is dependent on how well a constructivist mode is "learned" and internalised.

Teacher Learning

Research on teacher learning suggests that children are not the only beneficiaries of instructional programs which generate a socio-constructivist classroom environment (Cobb et al., 1990). In a research project based on premises similar to those of the current study (Cobb, Wood, Yackel, et al., 1991), the participating teacher was reported to have changed her beliefs about the nature of mathematics, mathematics learning, and mathematics teaching. In particular, her view of teaching mathematics changed from "transmitting information to initiating and guiding students' development of knowledge" (Wood et al., 1991, p. 587).

A key factor in this second grade teacher's learning was recognising in her own classroom what had been presented in her sessions with the researchers (Cobb et al., 1990). Although the teacher had listened carefully and analysed research-based cognitive models, it was not until she implemented the approach advocated by the researchers in her own classroom that she "encountered situations that conflicted with her previous teaching and created a context for her to learn" (Wood et al., 1991, p. 597).

It was hypothesised that a similar scenario could be drawn for pre-service teachers. They may study in detail the theory and practice associated with a socio-constructivist perspective. They may even dutifully learn what steps they could take as teachers to ensure that a socio-constructivist environment exists in their classroom. Yet they may be unable to implement their learning in practice because it has not been internalised or integrated into their experience.

Opportunities for pre-service teachers to participate in instructional programs which reflect the intent of their methods classes provide a means for integrating theory and practice. Reviews of the literature indicate that although early field experiences have long been regarded as key components in teacher education programs (e.g. Farris, Henniger, & Bischoff, 1991), they have been used with varying degrees of success (e.g. McDiarmid, 1990; Ross, Hughes, & Hill, 1981; Scherer, 1979; Strawitz & Malone, 1986; Sunal, 1980). At best, pre-service teachers have been found to rethink and question their original limited perceptions of teaching. At worst, field experiences have cemented these same limited perceptions.

In a field experience carefully planned and executed to challenge pre-service teachers' traditional beliefs about teaching, McDiarmid (1990) reported evidence that the pre-service teachers had begun to consider what teachers needed to know about learners, subject matter, and pedagogy. On the other hand, field experiences can have the effect that Ball (1990) warns of when she writes "Experiences may inhibit open-mindedness, freeze ways of looking, or engender undesirable attitudes. Experiences can therefore limit our possibilities for continued learning." (p. 11).

Two distinctive aspects of the mentorship experience described in this paper were expected to generate a positive long-term effect on participating pre-service teachers. First, the experience was truly experiential as opposed to observational in that the mentors were actively involved in the instructional process. In this context, they were given freedom, as well as guidance, to make instructional decisions based on their knowledge of the individual children with whom they had been working.

Second, mentors belonged to a "community of learners." Along with observing and being a part of the socio-constructivist learning environment in the children's classroom, the mentors also participated in a learning community comprised of themselves, the classroom teachers, and the project staff. Consistent with Wood et al.'s (1991) proposition that "... the learning that occurs in the classroom as teachers interact with their students must be combined with opportunities for sharing these experiences with others involved in the same process" (p. 611), it was expected that weekly discussion sessions between mentors and project staff would provide potent opportunities for learning.

In summary, although it was not possible to provide mentors with the same experience as the second grade teacher (Cobb et al., 1990; Wood et al., 1991), salient characteristics of the experience were modified to fit the constraints of pre-service teacher education. The longitudinal nature of the mentorship experience, its emphasis on instructional decisions based on students' thinking, and the focus on communities of learners coalesced to provide an environment for pre-service teacher learning similar to that experienced by the second grade teacher. As the teacher's beliefs and actions were positively affected by involvement in a socio-constructivist teaching project (Cobb et al., 1990; Wood et al., 1991), it was hypothesised in this study that the beliefs and actions of the mentors would be positively influenced by participation in the mentorship program.

Teacher Beliefs

The distinction between teacher beliefs and teacher knowledge is not always clearly delineated in the literature (Thompson, 1992). Additionally, what some researchers call beliefs others may refer to as feelings (Ball, 1988), views (Civil, 1990), or attitudes (Leder, 1985). In this study, the meaning ascribed to teacher beliefs about the teaching of mathematics is consistent with that used by Ernest (1989). He describes teacher beliefs about the teaching of mathematics as encompassing three components: (a) the teacher's view or conception of the nature of mathematics; (b) the teacher's model or view of the nature of mathematics teaching; and (c) the teacher's model or view of the process of learning mathematics.

Similar components were recognised by Kuhs and Ball (1986), who identified four dominant and distinctive views of how mathematics should be taught: (a) *Learner-focused* (emphasising the learner's personal construction of mathematical knowledge); (b) *Content-focused with an emphasis on conceptual understanding;* (c) *Content-focused with an emphasis on performance* (emphasising student mastery of mathematical rules and procedures); and (d) *Classroom-focused* (using knowledge about effective classrooms). These views served as the foundation for the framework used in this study to analyse pre-service teachers' beliefs about the teaching of mathematics (Figure 1).

As there appeared to be a substantial overlap between the *classroom-focused* view and the two *content-focused* views (Kuhs & Ball, 1986), the framework was built upon the *learner-focused* and the two *content-focused* views. Given the importance of interactions among students, the learner-focused view was

Framework for Assessing Pre-service Teachers' Beliefs About Mathematics Teaching

1. Learner-focused with an emphasis on social interactions (*learner-interaction*)
2. Content-focused with an emphasis on conceptual understanding (*content-understanding*)
3. Content-focused with an emphasis on performance (*content-performance*)

Figure 1. Framework for assessing pre-service teachers' beliefs about mathematics teaching.

embellished to reflect a socio-constructivist orientation and was renamed *learner-focused with an emphasis on social interactions.* This extension captures the perspective of Kuhs and Ball (1986) that "the teacher aids the student by questioning, challenging, and offering experiences that reveal the inadequacy of inappropriate conceptions" (p. 5).

We claim that the three elements of the framework (Figure 1) provide a continuum to assess pre-service teacher beliefs about mathematics teaching with a socio-constructivist orientation at one end (element 1) and a performance-driven orientation (element 3) at the other extreme. The *content-focused with an emphasis on conceptual understanding* view of teaching was considered to be an intermediate point between the *learner-focused with an emphasis on social interactions* and the *content-focused with an emphasis on performance* views of teaching mathematics.

Many researchers have also stressed the importance of the relationship between teachers' beliefs and their instructional practice (e.g., Ball, 1990; Cobb et al., 1990; Thompson, 1992). Based on her review of the literature, Thompson (1992) has claimed that teachers' beliefs influence classroom practice and that teachers' beliefs are evaluated and reorganised through their own reflective acts.

Even though Thompson's conclusions were related to in-service teachers and their classrooms, it can be argued that a similar relationship exists between beliefs and experiences, both past and present, of pre-service teachers. In fact, she proposes that further research investigating the relationship between pre-service teachers' beliefs and their actions "would be valuable to reform efforts in mathematics teacher education" (p.135). Although this research has only began to hint at an understanding of the connections between teacher beliefs and actions, there is some evidence to suggest that actions may not always correspond to stated beliefs (Kesler, 1985; Parmelee, 1992). In particular, Parmelee (1992), in her study of four middle school mathematics pre-service teachers, found that they had difficulty implementing their stated beliefs during their student teaching experience.

Research Objectives

The present study sought to investigate the effect of a teacher learning environment set in the context of a socio-constructivist student learning environment on:

1. pre-service teacher beliefs about teaching mathematics; and
2. pre-service teacher classroom actions during mathematics instruction.

The study also attempted to probe the relationship between pre-service teachers' beliefs and actions.

In the case of pre-service teacher beliefs, pre-service teachers who had been involved in a special teacher learning environment were compared with a group of pre-service teachers who had the same educational and professional background but had not been involved in an intervention experience. The relationship between pre-service teachers' beliefs and their actions was examined only for the pre-service teachers involved in the intervention.

Method

Subjects

Four female college students who had served as mentors for at least one semester in a project investigating children's understanding of multidigit numbers (Jones, Thornton & Putt, in press) comprised the experimental group of pre-service teachers in the study. They were compared with a group of students (n = 103) who had reached the same stage in their teacher education program but had not been involved in the intervention.

Each of the four mentors was assigned to a pair of first grade students in each of two classrooms. Using a specially designed instructional program (Thornton & Bohn, 1992; Thornton, Jones, & Hill, 1993), the mentors met with the pairs of children for three 25-minute sessions per week. During these sessions the mentor's role was to facilitate the children's participation and learning and to provide a supportive environment for interactions among students.

For the purpose of this study, the four mentors will be called Alison, Nancy, Erin and Ilene. The first three were elementary education majors with a mathematics specialisation and were in their fourth semester of being mentors, while Ilene was a junior high/middle school mathematics major and was in her second semester as a mentor. All four were finishing their third year of a four-year teacher preparation program and, like those in the comparison group, had completed a mathematics methods course which incorporated a clinical experience. In this methods course the mentors examined many of the ideas about teaching and learning that they experienced in the mentorship program.

Instruments

A researcher-designed Beliefs About Teaching Mathematics (BTM) survey, interview protocol, and Video Analysis Teacher Action Scale (TAS) were the instruments used in this study. The BTM survey was a thirty-three item instrument designed to assess beliefs about mathematics teaching. The pre-service teachers responded to each item on a five-point Likert scale ranging from Strongly Disagree to Strongly Agree. The statements included in the beliefs survey were written to assess teaching qualities consistent with the framework presented in Figure 1. In particular, these statements were adapted from the "dominant views" of Kuhs and Ball (1986) and from a questionnaire developed by Cobb, Wood, Yackel, et al., (1991). A sample of these statements is included as part of the results (Tables 2 and 3).

A factor analysis based on data collected from 175 pre-service teachers revealed only one significant factor, which was labelled *socio-constructivist orientation* since it differentiated between responses which were high and low on this dimension. The Cronbach's coefficient alpha for the BTM instrument was 0.80.

To clarify the BTM survey responses of the four experimental subjects, a ten-question interview protocol was used to investigate relationships between the mentors' beliefs and their involvement in the ongoing number project. The interview questions are provided in Figure 2.

Interview Questions

1. As a mentor in the Children's Understanding of Number Project, what do you see as your main teaching responsibility?
2. What role do you feel questioning plays in teaching these lessons?
3. How do you react when it appears that there won't be time to cover all the activities in a project lesson?
4. While you've been involved with the project, have you been in a situation where children are struggling to solve a math problem? What did you do?
5. Have you been in a situation where working together did not seem to help the children? Please explain.
6. While teaching the project lessons, do you find yourself doing a lot of demonstrating, explaining, and/or describing?
7. How much of a priority is it for children to be able to justify the mathematical statements they make? Is it sufficient for them to be able to solve a problem or do they need to be able to explain their solution process?
8. Is teaching in this project what you had expected it would be? How is it similar? How is it different?
9. Has working in this project changed your ideas about the teaching and/or learning of mathematics? Please explain.
10. Is there anything else you would like to add to this interview?

Figure 2. Interview questions.

The video TAS (Figure 3) was designed to assess the extent to which pre-service teachers adopted a socio-constructivist orientation to instruction during the four problem-solving phases: (a) problem statement and clarification phase; (b) solution exploration phase; (c) impasse relief phase; and solution presentation; and

(c) interpretation phase. TAS incorporates a set of descriptors for each phase which delineate a continuum of instructional actions ranging from a content-performance orientation to a learner-interaction orientation. Although the mentors were not aware of the TAS form, the project seminars had included discussion of the idea of stages in problem solving and the importance of maintaining a socio-constructivist approach during the instructional process.

Video Analysis Teacher Action Scale

	Content-Performance	Content-Understanding	Learner-Interaction
1. Problem Statement & Clarification			
Teacher Actions	Directs towards a predetermined solution method.	Models the initial stages of one or more solution methods.	Provides just enough information to establish the intent of the activity (Cobb, Wood, & Yackel, 1991).
Student Actions	Begins to model predetermined solution method.	Begins to model and extend a solution method.	Begins to interpret and clarify the problem and attempts to construct one or more solution methods.
2. Solution Exploration			
Teacher Actions	Checks on progress and discourages deviation from the expected solution method.	Allows students to work individually on their chosen solution method then asks them to compare solutions.	Encourages mathematical dialogue and consensus with students sharing their strategies and solution attempts (Yackel, Cobb, Wood, Wheatley, & Merkel, 1990).
Student Actions	Executes the predetermined solution method with clarification from the teacher.	Continues to build on teacher's outline and then discusses solution with group members.	Explains and negotiates solution strategies and solution attempts with other group members.
3. Impasse Relief			
Teacher Actions	Intercedes immediately by redirecting to the predetermined solution method.	Intercedes fairly quickly with suggestions for the children to try.	Facilitates continuation of the dialogue without providing substantive mathematical suggestions. Encourages the children to persist to figure out the problem for themselves. (Wood, Cobb, & Yackel, 1991).
Student Actions	Revisits predetermined solution method through teacher review.	Attempts to follow through on teacher's solution suggestions.	Reinterprets the problem, generates a new strategy or builds on a previous approach with new insights.

Figure 3. Video analysis teacher action scale.

Video Analysis Teacher Action Scale

4. Solution Presentation & Interpretation			
Teacher Actions			
	Evaluates student answers as either right or wrong. If wrong reviews predetermined solution method. Ignores students solution methods.	Accepts right and wrong answers in a nonevaluative way while refraining from discussing unsuccessful solution methods.	Accepts right and wrong answers in a nonevaluative way and probes solution methods regardless of their success. Assists in verbalising all solution attempts (Cobb, Wood, & Yackel, 1991).
Student Actions			
	Passive acceptance of the teacher's assessment of their solution.	Presents solutions and listens to explanations of apparently successful solution methods.	Presents solutions and dialogues about solution strategies with the teacher and other students. May refine their solution method based on this discussion.

Figure 3. Video analysis teacher action scale. (Continued)

Procedure

Each Monday during the number project the four subjects along with other mentors met with project coordinators for a seminar to review project goals, to preview, discuss, and practice forthcoming lessons, and to discuss issues related to the thinking and progress of individual children. The BTM survey was given to the mentors during the first week of the study, which was the beginning of the second or fourth semester of their mentorship experience. Individual interviews were conducted during the following week.

During the second semester of the study, instructional sessions were videotaped on a rotating schedule. These videotapes provided an opportunity to examine the relationship between the mentors' expressed beliefs and their actions in an instructional context. Using TAS, four sessions for each mentor were analysed by the researcher and a research-associate who had been closely involved with the project from the outset and was experienced in observing and coding teacher actions.

Mentors' actions were coded on TAS using a five-point scale developed from three major categories: 1 (content-performance), 3 (content-understanding), and 5 (learner-interaction). After it became apparent that the mentors' actions were not always consistent throughout an instructional session, codes of 2 and 4 were introduced. These codes acknowledged instances where mentors exhibited actions from two adjacent category descriptions. The codes were arrived at using "double-coding" (Miles & Huberman, 1984, p. 60–63). The researcher and research-associate initially coded subjects independently, then clarified with each other any variations in coding until consensus was reached.

The BTM survey, the interview transcripts, field notes from project staff observations, reflective feedback sheets from mentors, and the TAS analysis enabled the process of triangulation to be applied. This approach increased the validity of the study by allowing the researchers "to consider alternative organisations and interpretations of data" (Eisenhart, 1988, p. 110).

Analysis and Results

Pre-service Teacher Beliefs

Independent *t*-tests were used to compare the mentor group and the comparison group both on overall beliefs scores (Table 1) and on scores from individual items (Tables 2 and 3). Welch's *t*-test statistic (Kirk, 1982) was used to compensate for heterogeneous variances and unequal sample sizes.

On the overall beliefs scores, the mentors were significantly higher than the comparison group ($p < 0.01$). The mentor group consistently responded "agree" or "strongly agree" to statements representing a *socio-constructivist orientation.* On the other hand, the comparison group generally responded less strongly and varied their responses across the full scale. The spread of scores in the comparison group is more clearly reflected in the standard deviations of individual items than in the standard deviation of their overall scores (Tables 2 & 3).

Table 2 presents the items in the BTM survey which showed significant differences between the two groups. These items focus on the teacher as encourager of mathematical thinking and the child's active and personal role in learning. For comparison purposes, Table 3 lists BTM questions on which there was no significant difference between the mentor and comparison groups. These questions relate to the teacher's role in listening to children's mathematical thinking and connecting mathematical ideas.

The follow-up interviews with the mentor group underscored their focus on the teacher as facilitator and the child as the constructor of mathematical ideas. This position is encapsulated by the responses of several mentors to the interview questions. The question that prompted each response is identified in parentheses (see Figure 2).

> *Ilene:*"The project helped make me to focus more on what *children* need to do in order to understand mathematics." (Question 9)
>
> *Erin:*"I say things like, 'How could we figure this out? What could we use to help us?' When they come up with an idea like using the cubes, I ask them, 'How would that help you?'" (Question 4)
>
> *Nancy:*"The way I was taught [in school] was to memorise facts, but don't bother about understanding. I think the way we are teaching in the project, the kids are learning a foundation to understand everything. They are figuring things out rather than having someone just tell them." (Question 9)

One mentor also reflected on the way in which the project had impacted on her beliefs about mathematics. In reflecting on her own school days, Erin commented: "I had bad experiences with math in school, so being involved in the project has changed my beliefs about mathematics and teaching mathematics to children" (Question 8).

Table 1
Means, Standard Deviations and t-Value for the Mentor and Comparison Groups on Beliefs About Mathematics Teaching Survey (BTM)

mentor group	$m = 4.32, s = 0.07$	$t = 4.92^*$
comparison group	$m = 4.09, s = 0.29$	

Note. $n = 4$ for the mentor group and $n = 103$ for the comparison group.
Welch's t-test statistic was used to compensate for the difference in sample sizes.
$^* p < .01$

Table 2
Means, Standard Deviations, and t-Values for BTM Items on Which the Mentor and Comparison Groups Differed Significantly

	Mean (Standard Deviation)		**Difference#**	
Question	Mentors	Comp-arison	*t*	*p*
A key responsibility of a teacher is to encourage children to explore their own mathematical ideas.	5.00 (0.00)	4.73 (0.46)	5.74**	< .001
Knowing how to solve a mathematics problem is as important as getting the correct answer.	5.00 (0.00)	4.37 (1.10)	5.81**	< .001
In mathematics, problems can be solved without using rules.	4.00 (0.00)	3.36 (1.02)	6.39**	< .001
Allowing a child to struggle with a mathematical problem, even feel a little tension, can be necessary for learning to occur.	4.75 (0.50)	3.80 (0.91)	3.56*	.025
Telling children the answer is an efficient way of facilitating their mathematics learning.[R]	5.00 (0.00)	4.38 (0.76)	8.35**	< .001
It is the teacher's responsibility to provide the children with clear and concise solution methods for mathematical problems.[R]	4.00 (0.00)	3.06 (1.08)	8.81**	< .001

[R] These items were reversed.
\# Because Welch's *t*-test statistic incorporates standard deviations in its calculation of degrees of freedom, *p*-values do not necessarily correspond to the magnitude of differences in means.
$^* p < .05$ $^{**} p < .01$.

Table 3
Means, Standard Deviations, and t-Values for BTM Items on Which the Mentor and Comparison Groups Showed Agreement

	Mean (Standard Deviation)		Difference#	
Question	Mentors	Comparison	*t*	*p*
Ignoring the mathematical ideas that children generate themselves can seriously limit their learning.	4.75 (0.50)	4.71 (0.56)	0.17	.873
A vital task for the teacher is motivating children to resolve their own mathematical problems.	4.50 (0.58)	4.44 (0.70)	0.21	.844
Teachers must be able to represent mathematical ideas in a variety of ways.	4.75 (0.50)	4.68 (0.55)	0.28	.800
Children always benefit by discussing their solutions to mathematical problems with each other.	4.25 (0.50)	4.17 (0.78)	0.32	.763
Although there are some connections between different areas, mathematics is mostly made up of unrelated topics.[R]	4.50 (0.58)	4.40 (0.63)	0.35	.751
It is not necessary for teachers to understand the source of children's errors; follow-up instruction will correct their difficulties.[R]	4.50 (0.58)	4.37 (0.95)	0.42	.698

R These items were reversed.
\# Because Welch's *t*-test statistic incorporates standard deviations in its calculation of degrees of freedom, *p*-values do not necessarily correspond to the magnitude of differences in means.
* $p < .05$ ** $p < .01$.

Pre-service Teacher Actions

Figure 4 presents the median ratings and ranges for the four mentors on the Video Analysis Teacher Action Scale (TAS) for each problem solving instructional phase. These ratings were based on the coding of four videotaped instructional sessions for each mentor. It is noteworthy that all four mentors received their highest ratings in the problem statement/clarification phase and that their ratings decreased as instruction proceeded through the other phases. This decrease was most obvious in the solution exploration and impasse relief phases, with some improvement occurring during the solution presentation/interpretation phase. In terms of the individual mentors, Erin most closely modelled the characteristics of the *learner-interaction* approach, and hence most often reflected her own stated beliefs and the philosophy of the project. Ilene reflected this position least and also showed the most variation across the four phases.

Although the quantitative ratings of the TAS form signalled both the shift in mentors' actions and differences among the mentors, qualitative data from the interviews and observations provided a clearer picture. Some idea of the change in mentor actions as students progressed through the problem-solving phases was evident in the mentors' conceptions of their role in the project. When questioned about their primary responsibility (Question 1, Figure 2), the mentors responded:

> Alison:"I think to help the kids understand the mathematical concepts, to help them figure things out without giving them the answer. To ask lots of questions. If that doesn't work after a while, model your thinking but don't model it so much that they can get the answer from it."
>
> Erin:"To be more of a guide. To present them with the materials but see if they can construct their own knowledge without me telling them what to do."
>
> Ilene:"I see myself as the person who stimulates learning in the classroom. I want interaction among the students. If they are stuck I ask questions more than give them the answers. I present the lesson then have them figure out what they can. I sit back and watch them do what they want to do. They don't learn anything when I tell them what to do. Mainly I like to see them do it for themselves."
>
> Nancy:"To get the general concepts and ideas for their future math across. To teach them to use a problem-solving approach so they can solve different problems even if they haven't seen them before. I used to tell the kids the answers and was impatient when they didn't get them right away. Now I am more facilitating, asking not telling."

The difference between Erin, whose actions were most consistent with a socio-constructivist orientation towards learning, and Ilene, who fell at the opposite end, were more clearly illustrated by their responses to Question 3 of the interview protocol (see Figure 2). Erin considered the goal of the lesson to be "to make sure they have the main points down. If they aren't grasping the main ideas it is more important to work on them than to get all the activities done." Ilene, on the other hand, felt "disappointed" if she wasn't able to complete the activities. She described her reaction to that situation as:

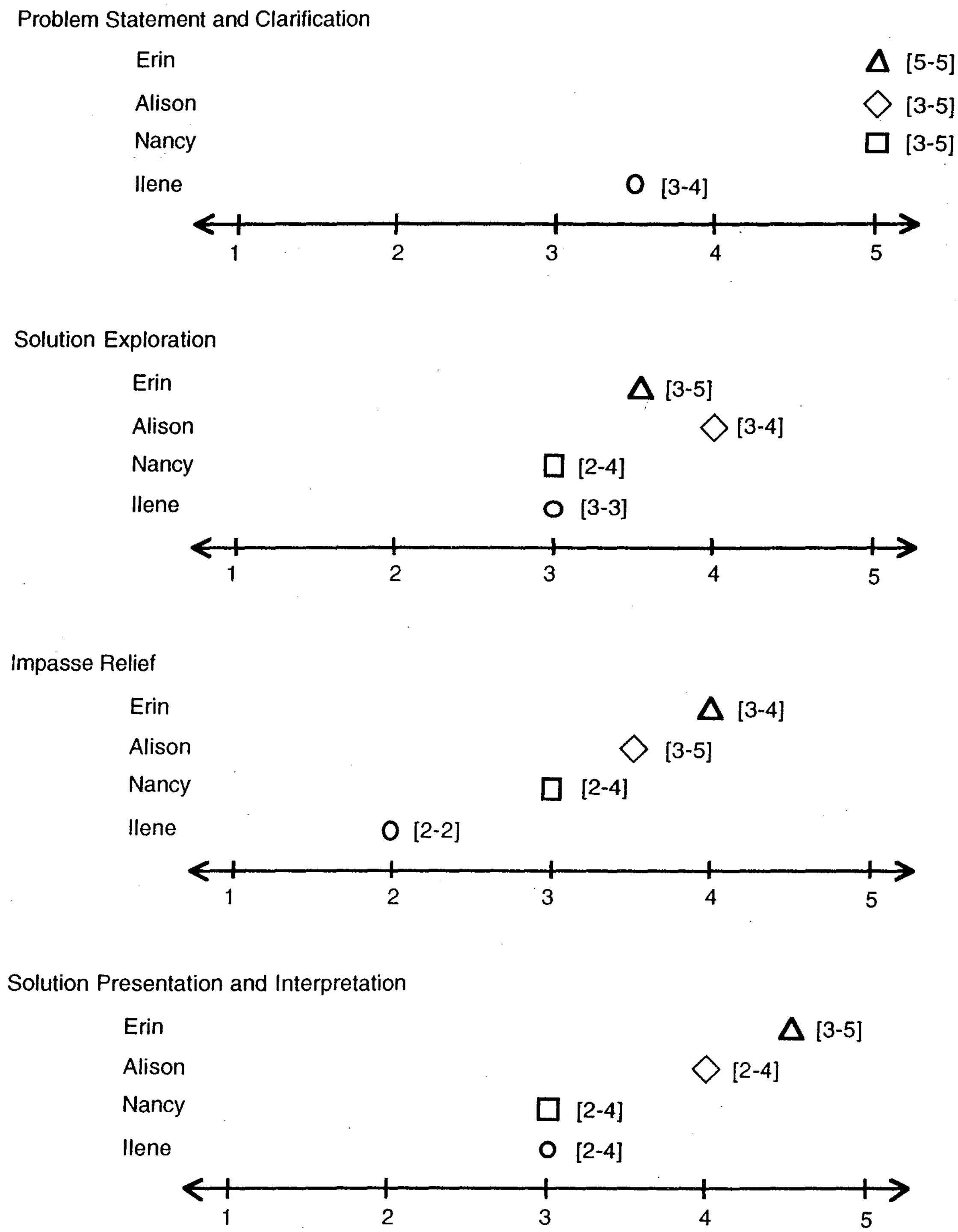
"I feel like I need to push them, feel like I need to tell them what to do. I try to put the responsibility to complete the lesson on the children: 'Mrs. J [the students' classroom teacher] will get upset if you don't get this done.' I feel like I should try to get through everything, but that it's better for them to learn some stuff well than rush through and not learn anything."

Alison's actions represented the middle-ground among the mentors. She responded that she felt "discouraged" when she wasn't able to complete all the activities and then described the balance she tried to maintain:

"One of my pairs is really slow. If they're going to understand, I won't be able to get through the whole lesson. So I don't want to go too fast, so that they don't get any of the concepts, but I want to get as much of the lesson done as possible."

The difference between beliefs and actions when an impasse situation was reached was illustrated by Nancy. At the first pause in the children's activity, she told them "If you want to look at your hundreds chart you can." Later in the same instructional session, she declared "I want you to figure this out for yourselves"

VIDEO ANALYSIS TEACHER ACTION SUMMARY



Key:

Symbols represent Medians [a-b] represents range

1: Content-focused with emphasis on performance.
2: Characteristics of both 1 and 3.
3: Content-focused with emphasis on conceptual understanding.
4. Characteristics of both 3 and 5.
5. Learner-focused with emphasis on social interactions.

Figure 4. Video Analysis Summary

but then reluctantly began to work them through the solution. This tension between optimal and practical was also evident in Ilene's comment: "I try to just ask questions and not tell. Usually I don't have to tell." (Question 6, Figure 2).

Discussion

This study investigated the impact of an intervention experience on pre-service elementary school teachers' beliefs about and actions in teaching mathematics. The intervention was based on a socio-constructivist approach to teaching and learning mathematics, and provided opportunities for pre-service "teacher learning" through regular, small-group teaching experiences supported by on-going seminars and written reflections on children's thinking. In particular, the study compared the beliefs about teaching mathematics of the mentor group with a group of their peers who had the same educational background but had not participated in the intervention.

With respect to this comparison, it was demonstrated that pre-service teachers who had a sustained experience in a socio-constructivist setting professed significantly stronger beliefs in this kind of instructional environment than a comparison group of their peers. Although the pre-service teachers from both groups valued the teachers' role in listening to children's mathematical thinking, they differed in the degree of importance they placed on children's construction of their own knowledge through social interaction. The mentor group professed this belief in a socio-constructivist environment with greater conviction.

It may well be that the strength of these beliefs was influenced by the intensity of the extended experiences with children, the supporting atmosphere of the project, and the opportunity to reflect on what they were doing. This in turn may have generated a much stronger focus on their students as learners rather than on themselves as teachers. There is also some evidence that their on-going experiences as mentors produced more positive beliefs about mathematics and countered some of the negative effects of their own school experiences in mathematics.

Although a number of studies have examined the relationship between teacher beliefs and instructional practice (e.g., Kesler, 1985; Parmelee, 1992; Thompson, 1992), agreement between teachers' beliefs and actions is not always evident. In this study, in spite of the fact that the mentors espoused the philosophy and beliefs of the project in their survey and interview responses, they were not uniformly successful in translating these beliefs into action. In particular, individual mentors' actions were not consonant with their beliefs during all phases of problem solving instruction.

As a general pattern, mentors' actions and beliefs were in harmony during the problem statement and clarification phase. However, in subsequent phases of the problem solving process, beliefs and actions diverged as their actions moved to a more conservative and traditional position. From a broad perspective one can attribute this shift to the mentor's lack of experience in facilitating the pedagogical challenges of the solution exploration and impasse relief phases of problem solving. In this study, there also appeared to be more specific reasons for the mentor's recourse to familiar ground.

In spite of the fact that the mentor's weekly seminars had consistently discussed key elements of a socio-constructivist environment, the resolution of pedagogical problems, and the need for reorganisation of knowledge about teaching (Cobb et al., 1991), there were challenges in the solution exploration and impasse relief phases that broke down the mentors' resolve to facilitate problem solving rather than model it. These included concerns about the tension produced for the children and themselves in problem solving situations, unease about the time the problem solving task was taking, and growing doubts about the ability of some children to solve the problem even in a collaborative environment.

In essence, the mentor's concerns were intense and short term and more representative of emotions than attitudes or beliefs (Mandler 1989; McLeod, 1989). Given the greater stability of beliefs and attitudes compared with emotions, the discrepancy between mentors' beliefs and actions is not untenable. Apparently it takes time for beliefs to supplant emotions and be translated into practice. Under these circumstances it is not surprising that the mentors relieved tensions in the solution exploration and impasse relief phases by modelling the problem solution rather than continuing to support the children's construction of a solution. The long term effect of emotions, like those experienced in this study, on beliefs and attitudes is clearly an area for future research.

Although the number of subjects involved in this intervention was small, it seems that on-going, supportive experience in a teaching environment consistent with recent reforms in mathematics instruction can be beneficial to pre-service teachers. In particular, such an experience appears to strengthen pre-service teachers' beliefs about the need to provide opportunities for children to construct their own mathematical knowledge in an interactive setting. An implication for teacher education is that there is value in incorporating these kinds of experiences in the pre-service program.

On the other hand, this involvement may not always result in pre-service teacher actions which are in harmony with stated beliefs. One of the limitations of this study was that the Video Analysis Teacher Action Scale was developed during the latter part of the teaching experiment. In hindsight, it may have been helpful to have used this scale to provide the mentors with immediate and continuing feedback on their teaching. The value of a feedback loop based on this instrument could be tested in further research, as it may prove useful in teacher education as an effective means of eliciting even greater change in pre-service teacher actions.

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