An Investigation of U.S. and Chinese Students' Mathematical Problem Posing and Problem Solving[1]

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This study explored the mathematical problem posing and problem solving of 181 U.S. and 223 Chinese sixth-grade students. It is part of a continuing effort to examine U.S. and Chinese students' performance by conducting a cognitive analysis of student responses to mathematical problem-posing and problem-solving tasks. The findings of this study provide further evidence that, while Chinese students outperform U.S. students on computational tasks, there are many similarities and differences between U.S. and Chinese students in performing relatively novel tasks. Moreover, the findings of this study suggest that a direct link between mathematical problem posing and problem solving found in earlier studies for U.S. students is true for Chinese students as well.

Cross-national studies in mathematics have consistently reported that United States students do not perform as well as Asian students on tasks requiring the applications of mathematical knowledge and skills routinely learned in school[2] (Husen, 1967; Lapointe, Mead, & Askew, 1992; Robitaille & Garden, 1989; Stevenson et al., 1990; Stevenson & Stigler, 1992; Stigler, Lee, & Stevenson, 1990; U.S. Department of Education and National Center for Education Statistics, 1996). However, recent cross-national studies have shown that for tasks assessing relatively novel and complex problem solving, the performance differences between U.S. and Asian students are not so pronounced (Becker, 1992; Cai, 1995; Cai & Silver, 1995; Silver, Leung, & Cai, 1995). For example, Silver et al. (1995) analysed the responses of 206 Japanese fourth-grade students and 151 U.S. fourth-grade students to a task requiring multiple solutions. They found that Japanese students performed better than U.S. students with respect to the proportions of correct solutions. However, they also found many similarities in students' solution strategies and their explanations. For example, both U.S. and Japanese students were able to solve the problem with multiple solution strategies.

Cai (1995) conducted a cognitive analysis of the mathematical performance of 250 U.S. students and a comparable group of 425 Chinese students on tasks involving computation, simple problem solving, and complex problem solving. He

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[2] See Chapter 2 in Cai (1995) for a review of mathematical performance differences and factors contributing to the observed differences.

found that Chinese students outperformed U.S. students on tasks involving computation and simple problem solving, but not on tasks involving complex problem solving. Students' responses to complex tasks revealed many subtle differences and similarities in their thinking and reasoning. For example, almost every strategy that was used by U.S. students was also used by Chinese students, and vice versa. U.S. students tended to use visual or pictorial representations more frequently than Chinese students, and Chinese students used symbolic or notational representations more frequently than U.S. students.

These few cross-national studies not only contribute to our understanding about students' problem solving in different nations, but also establish the validity and feasibility of examining cognitive similarities and differences of students' mathematical thinking and reasoning. Yet, we are only beginning to uncover and understand the nature of these cognitive similarities and differences between U.S. and Asian students. According to Bradburn and Gilford (1990), the information from such cognitive analyses can play an important role in educational research and policy development. Such analyses should be informative, guiding both the interpretation of cross-national performance differences and the policy recommendations that emerge from large-scale cross-national studies such as the Third International Mathematics and Science Study.

The study reported in this paper was part of a continuing effort to examine U.S. and Chinese students' mathematical thinking and reasoning. This study extended earlier work (Cai, 1995, 1997; Cai & Silver, 1995) by focusing on complex problem solving and student-generated problem posing. The importance of this line of work is found in both cognitive psychology (Simon, 1989) and in mathematics education. Recent recommendations for the reform of school mathematics suggest an important role for problem solving and student-generated problem posing. For example, National Council of Teachers of Mathematics (1989) explicitly states that students should be given opportunities to solve mathematical problems using multiple solution strategies and to formulate and create their own problems from given situations.

This study also extended earlier work in mathematical problem posing (Silver & Cai, 1996) to examine the relatedness of mathematical problem solving and problem posing from a cross-national perspective. Given the importance of problem-posing activities in school mathematics, some researchers have started to investigate various aspects of the problem-posing processes. One approach has examined the link between problem posing and problem solving (Ellerton, 1986; Kilpatrick, 1987; Silver & Cai, 1996). Kilpatrick (1987) provided a theoretical argument that the quality of the problems subjects pose might serve as an index of how well they can solve problems.

Several researchers have examined this hypothetical link between problem posing and problem solving. For example, Ellerton (1986) compared the mathematical problems generated by eight high-ability young children with those generated by eight low-ability young children, by asking each to pose a mathematical problem that would be quite difficult for her or his friends to solve. Ellerton reported that the more able students posed problems that were more complex than those posed by less able students.

Silver and Cai (1996) analysed the responses of more than 500 middle school students to a task asking them to pose three questions based on a driving situation. The problems posed by students were analysed according their type, solvability and complexity. Silver and Cai used eight open-ended tasks to measure the students' mathematical problem-solving performance. They found that students' problem-solving performance was highly correlated with their problem-posing performance. Compared to less successful problem solvers, good problem solvers generated more mathematical problems and their problems were more mathematically complex.

Previous investigations of mathematical problem solving and problem posing (including Silver & Cai, 1996) have used tasks for problem solving that often are unrelated to tasks for problem posing. In the present study, U.S. and Chinese students' performance was examined through related problem-posing and problem-solving tasks. Thus, this study not only provided an opportunity to examine cognitive similarities and differences between U.S. and Chinese students in their complex problem solving and student-generated problem posing, but also provided an opportunity to examine if the hypothesised link between mathematical problem solving and problem posing holds across national and cultural boundaries.

Method

Subjects

A total of 181 U.S. sixth-grade students (87 boys and 94 girls) and 223 Chinese sixth-grade students (109 boys and 114 girls) participated in the study. The Chinese sample was from two typical schools in Xiaoshan city (Zhejing province). A group of Chinese mathematics teachers and educators judged students in these schools to be of average ability in mathematics. The U.S. sample was from one private school (29 students) and four typical public schools (152 students) in the Milwaukee metropolitan area. A group of U.S. mathematics teachers and educators judged students in these schools to be of average ability in mathematics. In each school in both countries, one of the sixth-grade mathematics teachers volunteered to participate in the study. All the sixth-grade students taught by these teachers were then tested.

Tasks and Administration

The tasks used in this study were from Cai and Silver (1994), with some minor wording changes. Each student received a booklet containing the following tasks, shown in Table 1: (a) four computational exercises; (b) a problem-posing task in which students were asked to pose mathematical problems based on a given figural pattern situation; (c) a division-with-remainder (DWR) story problem; and (d) a figural pattern problem.

The tasks were selected to reveal a range of students' mathematical performance: that connected with computational skills in the computational exercises; the generative aspects of mathematical thinking in the problem-posing

Table 1
Tasks Used in the Present Study

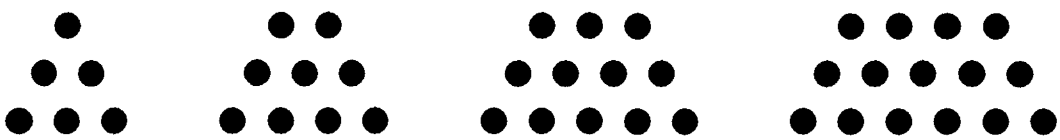
(a) Computational Exercises:

$\frac{1}{5} + \frac{2}{3} = ?$ $3480 \div 60 = ?$

$15.3 - 8.8 = ?$ $11.28 \div 3.6 = ?$

(b) Problem-posing Task:

Mr. Miller drew four of the figures in a pattern, as shown below.



(1) (2) (3) (4)

For his students' homework, he wanted to make up some problems according to this pattern. Help Mr. Miller by writing as many problems as you can in the space below.

(c) Division with Remainder (DWR) Problem:

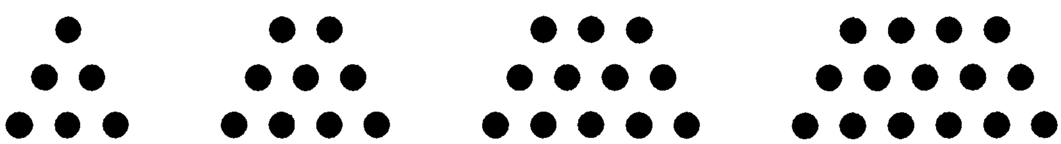
Students and teachers at Marquette Middle school will go by bus to have Spring sightseeing. There is a total of 1128 students and teachers. Each bus holds 36 people. How many buses are needed?

Answer:

Show all your solution processes below.

(d) Pattern Problem:

Look at the pattern below:



(1) (2) (3) (4)

A. Draw the fifth figure.

B. Draw the seventh figure.

C. Describe how you knew what the 7th figure would look like.

task; the interpretation of a solution in the DWR problem; and the generalisation and the simultaneous coordination of two dimensions in the pattern problem. The division involved in one of the computational exercises ($11.28 \div 3.6 = ?$) is the same as that involved in the solution of the DWR problem, although the former involves decimals and the latter involves whole numbers. The problem-posing task has the same mathematical structure as the pattern problem. Such a design allows for the examination of U.S. and Chinese students' performance through related problem-posing and problem-solving tasks.

Students had 30 minutes to complete the tasks, 15 minutes for the four computational exercises and the problem-posing task and another 15 minutes for the DWR problem and the figural pattern problem. Students were asked to stop even if they had not finished the four computational exercises and the problem-posing task after 15 minutes. They were not allowed to change anything about their responses to the four computational exercises and the problem-posing task while they were working on the DWR problem and the figural pattern problem even if they finished them early. Thus, they were unable to change their problem-posing responses after they had seen the figural pattern problem. Tasks were administered by students' regular mathematics teachers; students were not allowed to use calculators. The U.S. sample completed the tasks in English and the Chinese sample completed the tasks in Chinese. English back-translation was used to ensure the equivalence of the two language versions of the test used in this study. (See Cai, 1995, for details on the back-translation procedure).

The data consisted of students' written responses. For the computational exercises, only a final solution was required; for the DWR problem, students were asked both to provide a numerical answer and to write down their solution processes. For the problem-posing task, students were asked to write down problems they posed; for the pattern problem, students were asked to extend the pattern by drawing the fifth and seventh figures and describe how they knew what the seventh figure would look like.

Data Coding

Each response for the computational exercises was coded as correct or incorrect.

Student responses to the problem-posing task were coded along two dimensions. The coding scheme for analysing student responses to the problem-posing task was based on prior research in solving pattern problems in general (Simon, 1979) and on solving this specific figural pattern problem (Cai, Magone, Wang, & Lane, 1996). In solving pattern problems, one needs to induce a rule based on given elements of a pattern, then extend the pattern using the rule (Simon, 1979). Thus, the problems students posed in this study were first classified into extension problems, non-extension problems, or others. An extension problem is a problem questioning the pattern beyond the four given figures. A non-extension problem is a problem questioning the given figures in the pattern. Within these two types, problems can be factual, comparative, or rule-based. A factual problem is a problem questioning a certain figure in the pattern. A comparative problem is a problem questioning the relatedness of two figures in the pattern. A rule-based problem is a problem questioning the generality across a number of figures in the pattern. Table 2 shows examples of each type of problem. "Other" responses, not included in Table 2, include non-mathematical questions, such as "Why did Mr. Miller want to assign homework?", and mathematical questions irrelevant to the given pattern situation, such as "How many new students are there in Mr. Miller's class?"

Table 2
Sample Student Responses to the Problem-posing Task

	Extension problems	Non-extension problems
Factual	• What is the 10th figure? • How many dots are there in the 100th figure?	• How many dots are there in the 2nd figure? • What is the shape of the 4th figure?
Comparative	• How many more dots are there in the 100th figure than in the 99th figure?	• How many more dots are there in the 2nd row than in the 1st row in the 3rd figure?
Rule-based	• What is the rule by which each figure changes from the previous one?	[Not applicable: All rule-based problems are extension problems.]

Student responses for the DWR problem were coded using a classification scheme adapted from Silver, Shapiro, & Deutsch (1993). Each response was examined with respect to four distinct aspects: (1) solution processes, (2) execution of procedures, (3) numerical answers, and (4) explanations of the solutions. This classification scheme has been shown appropriate to code Chinese student responses to similar DWR problems (Cai & Silver, 1995).

Student responses for the figural pattern problem were coded using a classification scheme adapted from Cai et al. (1996). Each response was examined with respect to three distinct aspects: (1) correctness of the figures; (2) evidence of a description and solution strategies; and (3) drawing errors.

Results

The Computational Exercises

The results for each computational exercise are shown in Table 3. For each exercise, the differences in the proportions correct among the Chinese and the U.S. students was statistically significant ($p < 0.01$). A majority of the Chinese students (70%), but only about 30% of the U.S. students, had correct answers for all four computational exercises.

Chinese students were quite successful in solving the computational exercises. In three of the exercises, more than 90% of the Chinese students were correct. The last exercise (11.28 ÷ 3.6 = ?) was the most difficult one for the Chinese students. The most common error was a misplaced decimal point. Nearly 60% of the Chinese students who were wrong gave either 31.33 or 0.3133 as their answer. In contrast, the first computational exercise ($\frac{1}{5}+\frac{2}{3}=?$) was the most difficult for the U.S. students. About two thirds of the U.S. students missed this computational exercise, many of them adding numerators and denominators to find the sum.

Table 3
Percentages of Students with Correct Answers to Each of the Computational Exercises

Students	$\frac{1}{5}+\frac{2}{3}$	3480 ÷ 60	15.3 – 8.8	11.28 ÷ 3.6
Chinese (n=223)	95	93	94	83
U.S. (n=181)	36	78	64	58

The DWR Problem

The correct solution of a DWR problem requires not only correct execution of a division computation (computation phase) but also a correct interpretation of computational results with respect to a given story situation (sense-making phase). The majority of U.S. and Chinese students recognised the DWR problem as a problem which required division—89% of the U.S. students and 94% of the Chinese students selected division procedures. A few U.S. students used other appropriate procedures, such as repeated addition and repeated subtraction to solve the problem. No Chinese student used appropriate procedures other than division.

A larger percentage of the Chinese students (87%) than the U.S. students (66%) executed the procedures correctly (z = 4.99, $p < 0.01$). About 36% of the Chinese students and 42% of the U.S. students gave an answer of 32. Although about two thirds of the U.S. students and eight ninths of the Chinese students performed the appropriate computational procedures correctly, only about 38% of the U.S. students and 31% of the Chinese students provided appropriate and complete explanations for their final numerical answers. The vast majority of both the U.S. students and Chinese students with appropriate and complete explanations supported the final numerical answer of 32. Examples of appropriate explanations accompanying a final answer of 32 included: "If you need 31 buses. These 12 people need one bus. 31 + 1 = 32." A few U.S. and Chinese students provided appropriate explanations to support a final answer of 31: "If you have 31 buses, there are 12 people left over. You choose 12 buses to hold the 12 people, each bus holds extra 1 person, therefore, you just need 31 buses."

In solving the DWR Problem, a student response was considered as correct if the student provided 32 with or without an appropriate interpretation or an answer other than 32 with an appropriate interpretation. Based on this criterion of correctness, it was found that about 46% of the U.S. students and 41% of the Chinese students provided correct solutions for the DWR problem.

The Problem-posing Task

In total, the 181 U.S. students posed 861 problems (mean number: 4.76) and the 223 Chinese students posed 1588 problems (mean number: 7.12). On average, the Chinese students posed significantly more problems than the U.S. students ($t = 4.90$, $p < 0.001$). The proportions of the extension or non-extension problems for both samples were very similar. (Recall that an extension problem refers to a problem questioning the pattern beyond the four given figures. A non-extension problem refers to a problem questioning the given figures.) About 80% of the problems posed by each sample were non-extension problems and about 12% were extension problems. The Chinese students generated a total of 206 extension problems and the U.S. students generated 105 extension problems. About 32% (72 of 223) of the Chinese students and 28% (51 of 181) of the U.S. students generated at least one extension problem. In particular, 14 Chinese and 19 U.S. students generated only extension problems. About 5% of the problems posed by each sample were non-mathematical or irrelevant to the given pattern situation.

Seventy six percent (76%) of the problems generated by the U.S. and 68% of the problems generated by the Chinese students were comparative problems. The majority of the comparative problems involved the comparison of the number of dots in figures (e.g., "How many more dots are there in the fourth figure than in the third figure?). For each of the samples, most of the non-extension problems were comparative, but the extension problems were distributed across all three types (factual, comparative and rule-based). About 40% of both U.S. and Chinese students' extension problems were factual problems (e.g., "How many dots are there in the 10th figure?"), and 35% of U.S. students' and 30% of Chinese students' extension problems were rule problems (e.g., "What is the rule by which each figure changes from the previous one?").

The Figural Pattern Problem

Recall that students were asked to draw the fifth and seventh figures of the pattern shown in Table 1. A larger percentage of Chinese (85%) than U.S. students (61%) drew both the fifth and seventh figures correctly ($z = 5.49$, $p < 0.001$). About 20% of the U.S. and nearly 10% of the Chinese students drew both figures incorrectly. About 15% of the U.S. and 5% of the Chinese students drew the fifth figure correctly, but failed to draw the seventh figure correctly. Only a few U.S. and Chinese students drew the seventh figure correctly but drew the fifth figure incorrectly.

There were 71 U.S. students and 33 Chinese students who drew either the fifth or the seventh figure incorrectly. The majority of the drawing errors seemed to result from student difficulties in coordinating the two dimensions of the problem—the number of dots and the shape of the figure (a trapezium). About 65% of the U.S. students (46 of 71) and 55% of the Chinese students (18 of 33) correctly showed one, but not both, of these two dimensions. For example, many students correctly drew all 18 or 24 dots for the fifth or seventh figure, respectively, but did not maintain the trapezoidal shape. About 35% of the U.S. students' and 45% of the

Chinese students' incorrect drawings showed serious errors in both the number of dots and the shape of the figure.

Over 90% of the U.S. and Chinese students provided descriptions for their solutions. In some cases, students' solution strategies were readily apparent from their descriptions. In other cases, no strategy was apparent because the students' descriptions were either incomplete or unclear. There were seven different solution strategies used by at least one U.S. or Chinese student in solving this figural pattern problem. Table 4 describes these solution strategies and shows the percentage distributions of U.S. and Chinese students who used these strategies. A larger percentage of Chinese (83%) than U.S. (69%) students had clear indications of using one of the seven identified solution strategies ($z = 3.31$, $p < 0.01$). However, the percentage distributions of using these strategies between U.S. and Chinese students were quite similar. For example, Strategy 4 was the most frequently used strategy in both samples (about one third of the students in each case), and Strategy 6 was the least frequently used strategy in both samples.

Relation between Problem Posing and Problem Solving

Since the problem-posing task was designed to be structurally similar to the figural pattern problem, it is possible to examine the relation of problem solving and problem posing. Students in each sample were first divided into two groups (the extension group and the non-extension group) according to their problem-posing responses. The extension group consisted of those students who generated at least one extension problem for the problem-posing task; the non-extension group consisted of those students who did not generate any extension problems. Then, differences between the extension and non-extension groups in performance on the figural pattern problem were examined.

Table 5 shows the percentages of students who produced correct solutions (i.e., both fifth and seventh figures correct) as well as percentages of students whose explanations gave a clear indication of using an appropriate strategy. For both U.S. and Chinese samples, students in the extension groups both performed better than those in the non-extension groups and more frequently gave clear indications of using an appropriate solution strategy. For each of the samples, the differences between extension and non-extension groups in correctness of figures and in use of appropriate strategies were statistically significant. (The z-scores ranged from 1.96 to 3.97, $p < 0.05$ in all cases). Superior performance of the students in the extension groups is also evident by the fact that almost all of the 14 Chinese and 19 U.S. students who generated only extension problems drew both fifth and seventh figures correctly and provided a clear indication of using an appropriate solution strategy.

Although a significantly larger percentage of Chinese (85%) than U.S. students (61%) drew both the fifth and seventh figures correctly, the difference is not significant if the U.S. students in the non-extension group are excluded from the analysis. In fact, 84% of the U.S. students in the extension group drew both the fifth and seventh figures correctly, which is almost identical to that of Chinese students (85%). Similarly, although a significantly larger percentage of Chinese (83%) than U.S. students (69%) provided clear indications of using appropriate solution

Table 4
Percentage distributions of U.S. and Chinese Students' Solution Strategies

Strategy		U.S. (n = 181)	Chinese (n = 223)
1	Students focused on the number of dots in the three rows of each figure as a triplet and induced the rule of the triplets (Fig. 1: {1, 2, 3}; Fig. 2: {2, 3, 4}; Fig. 3: {3, 4, 5}; ...).	5	10
2	Students looked at the dots in each row as a sequence and found the rule of each sequence (Row 1: {1, 2, 3, 4, ...}; Row 2: {2, 3, 4, 5, ...}; Row 3: {3, 4, 5, 6, ...}).	6	12
3	Students looked at the figures diagonally and realised that each successive figure has one more sloping line of 3 dots. Students would then get the next figure by adding one more sloping line.	6	13
4	Students realised that, from figure to figure, each row has one more dot than the corresponding row in the previous figure. From the previous figure, students added one dot to each row individually to get the next figure.	31	26
5	Students focused on the total number of dots in each figure to describe how to get the next figure. The first figure has 6 dots, the second has 9, the third has 12, the fourth has 15, ..., therefore the seventh figure has 24 dots. In this case, the students did not mention the shape of each figure explicitly.	17	14
6	Students removed the first row of the previous figure, then added a new bottom row that had one more dot than the previous bottom row.	1	2
7	Students found that the number of dots in the first row of a figure was equal to the number of each figure. The number of dots on the second row was one more than the first row, and the number of dots on the third row was one more than the second row.	3	6
No clear indication of strategy used		31	17

strategies, the difference is not significant if the U.S. students in the non-extension group are excluded from the analysis. Indeed, 89% of the U.S. students in the extension group provided clear indications of using appropriate strategies.

To examine further the relatedness of students' problem posing and problem solving, students' solution strategies for solving the figural pattern problem and the problems they posed were compared. Recall that students used seven different

Table 5
Percentages of Students from Each Sample with Correct Solutions or Appropriate Strategies on Figural Pattern Problem

Sample	Group	Correct Solution	Appropriate Strategy
U.S. (*n*=181)	Extension (*n*=51)	84	89
	Non-extension (*n*=130)	52	61
Chinese (*n*=223)	Extension (*n*=72)	92	91
	Non-extension (*n*=151)	82	79

strategies to solve the figural pattern problem (see Table 4). Students' use of solution strategies appeared to be related to the kinds of problems they posed. For example, of those students who used Strategy 5, nearly 85% of both U.S. and Chinese students' posed problems explicitly questioning the numbers of dots in the figures, such as "How many dots are there in the 10th figure?" or "How many more dots are there in the fourth figure than in the third figure?" Of those who used one of the other strategies, only 48% of the problems posed by the Chinese students and 42% of the problems posed by the U.S. students explicitly questioned the number of dots in the figures. One possible reason for this difference is that Strategy 5 focuses only on the total number of dots in each figure. Students who used this strategy may actually have ignored the shape of the figures. In contrast, the other strategies attend to both the shape and the number of dots in each figure. Thus, for both samples, students who mainly focused on the number of dots to solve the figural pattern problem appeared to generate problems explicitly involving the number of dots for the problem-posing task.

Discussion

Similar to the results reported in Cai (1995), the present study reveals both similarities and differences between U.S. and Chinese students' mathematical problem solving and problem posing. That Chinese students performed significantly better than U.S. students on four computational exercises was not surprising, because prior comparative studies involving U.S. and Chinese students (Cai, 1995; Lapointe et al., 1992; Stevenson et al., 1990) have reported similar results. Of greater interest are the mixed results on the less routine problems.

In solving the DWR problem, similar percentages of U.S. and Chinese students chose the correct procedures but more Chinese students correctly executed these procedures. A similar percentage of U.S. and Chinese students provided appropriate interpretations of their solutions. Thus, the results suggest that Chinese students outperformed U.S. students on the computation phase of solving the DWR problem, but not on the sense-making phase. However, for both samples, students were more successful on the computation phase than on the sense-making phase. Thus, the results of this study suggest similar cognitive complexities of the DWR problem for both U.S. and Chinese students. These complexities derive not from computational requirements, but rather from the sense-making requirement included in interpreting the computational result (Cai & Silver, 1995; Silver et al., 1993).

Overall, Chinese students appeared to perform better than U.S. students in solving the figural pattern problem. In particular, a larger proportion of Chinese than U.S. students correctly extended the pattern to the fifth and seventh figures. In addition, a larger percentage of Chinese than U.S. students apparently used appropriate strategies. However, the kinds of drawing errors made by U.S. and Chinese students and the kinds of solution strategies used by U.S. and Chinese students were similar. For example, for both samples, the majority of the drawing errors seemed to result from student difficulties coordinating the two dimensions of the problem—the number of dots and the shape of the figures.

This study has extended previous cross-national studies by including an examination of U.S. and Chinese students' problem posing in mathematics. The results show that both U.S. and Chinese students are able to formulate mathematical problems based on a given situation. Chinese students generated more problems than U.S. students, but the proportions of each type of problems generated by U.S. and Chinese students were almost the same. For example, the proportions of the extension problems generated by both samples were very close. The proportions of the U.S. and Chinese students who generated at least one extension problem were also very close.

About one third of both the U.S. and Chinese students were able to see mathematical structures and formulate problems extending beyond the four given figures (i.e., extension problems). For both samples, the extension problems were only a small proportion of the total problems they posed. Why were so many responses non-extension problems, given that the majority of both the U.S. and Chinese students were able to correctly extend the figural pattern into the fifth and seventh figures? In the problem-posing task, the first four figures of the pattern were given. It would seem to be natural to pose a problem like: "What is the fifth figure?" or "What would the fifth figure look like?" However, only a few U.S. and Chinese students posed this kind of problems. Additional studies are needed to explore why this was so.

One plausible interpretation of this finding might be the novelty of problem-posing activities for both U.S. and Chinese students. In the U.S., although the mathematics education community (Brown & Walter, 1983; National Council of Teachers of Mathematics, 1989, 1991) has often called for integration of problem-posing activities into the school curriculum, problem-posing activities have rarely been implemented into classrooms (Silver, 1994). Chinese schools use national-unified mathematics textbooks which rarely include problem-posing activities. Problem-posing activities are also not often included in classroom instruction in China. Students in both countries clearly have much less experience posing problems than solving problems.

This study has also examined the relation between problem solving and problem posing from a cross-national perspective. The results show the same link between problem posing and problem solving in both countries. First, problems posed by both U.S. and Chinese students seemed to be related to their solution strategies for solving the figural pattern problem. Students who mainly focused on the number of dots to solve the figural pattern problem appeared to generate problems explicitly involving the number of dots for the problem-posing task. Second, the results suggest that, for each sample, students who generated extension problems performed better on the figural pattern problem than those who did not generate extension problems. Students who generated extension problems were also more likely to use appropriate solution strategies. The problem-posing task and the figural pattern problem contained the same figural pattern. Those who generated the extension problems for the problem-posing task might have mentally extended the figural pattern. They would then have been in a good position to solve the figural pattern problem because its main requirement is to extend the figural pattern. Hence, it seems natural that students in the extension group would perform better than those in the non-extension group in solving the pattern problem.

In summary, the findings of this study provided further evidence that Chinese students outperform U.S. students on computational tasks, but that there are similarities and differences between U.S. and Chinese students in performing less routine tasks. Moreover, the findings of this study suggest that a direct link between mathematical problem posing and problem solving from a cross-national perspective.

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