Pressure Transient Analysis of Arbitrarily Shaped Fractured Reservoirs

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Abstract: Reservoir boundary shape has a great influence on the transient pressure response of oil wells located in arbitrarily shaped reservoirs. Conventional analytical methods can only be used to calculate transient pressure response in regularly shaped reservoirs. Under the assumption that permeability varies exponentially with pressure drop, a mathematical model for well test interpretation of arbitrarily shaped deformable reservoirs was established. By using the regular perturbation method and the boundary element method, the model could be solved. The pressure behavior of wells with wellbore storage and skin effects was obtained by using the Duhamel principle. The type curves were plotted and analyzed by considering the effects of permeability modulus, arbitrary shape and impermeable region.

Key words: Deformable reservoir, double-porosity reservoir, boundary element method, transient pressure, type curve

1. Introduction

Many methods use radial and circular systems to interpret unsteady state double-porosity reservoir flow problems (Chen and Jiang, 1980; Mavor and Cinco-Ley, 1979; Zhang and Zeng, 1992), but very little information is available for arbitrarily shaped reservoirs. For many cases, however, the double-porosity reservoir drainage shape is too complicated to be approximated by a circular shape. The existence of one or more impermeable regions further complicates the problem. So a numerical means is required (Britto and Grader, 1987; Kikani and Horne, 1988; Liu, *et al.*, 2001; Liu and Duan, 2004; Masukawa and Horne, 1988; Paulo and Abraham, 1988; Sato and Horne, 1993a; 1993b; Wang, *et al.*, 2000; Yin, *et al.*, 2005; Zhang and Zeng, 1992). Numerical techniques for solving partial differential equations describing various physical processes can be categorized into two distinct classes: the domain methods and the boundary methods. Finite difference and finite element methods fall in the first class, and the boundary element methods (BEM) constitute the second. The BEM is superior to the domain methods in several ways. The most notable advantage is the high degree of accuracy that results from its sound mathematical foundations. Flexibility in defining boundary geometries and conditions is another feature to be emphasized. However, the conventional boundary element method is not applicable to the problem of fluid flow in porous deformable media. In this paper, under the assumption that permeability varies exponentially with pressure drop (Kikani and Pedrosa, 1991; Ning, *et al.*, 2004), transient pressure response of wells in deformable double-porosity reservoirs was obtained. The type curves were developed and analyzed by considering effects of permeability modulus, arbitrary shape and impermeable barriers.

2. Mathematical model

For single-phase slightly compressible fluid flow in deformable double-porosity media, Ω, including n_s sinks of strengths q_l, the corresponding equations may be written as:

$$\frac{\partial^2 p_{fD}}{\partial {x_D}^2}+\frac{\partial^2 p_{fD}}{\partial {y_D}^2}-\beta\left(\left(\frac{\partial p_{fD}}{\partial x_D}\right)^2+\left(\frac{\partial p_{fD}}{\partial y_D}\right)^2\right)= \mathrm{e}^{\beta p_{fD}}\left(\omega\frac{\partial p_{fD}}{\partial t_D}+(1-\omega)\frac{\partial p_{mD}}{\partial t_D}\right)+ \mathrm{e}^{\beta p_{fD}}\sum_{l=1}^{n_s} q_{Dl}\delta(x_D-x_{Dl})\delta(y_D-y_{Dl}) \quad (1)$$

$$\lambda(p_{mD}-p_{fD})+(1-\omega)\frac{p_{mD}}{\partial t_D}=0 \quad (2)$$

$$p_{mD}=p_{fD}=0 \quad (x_D,\ y_D)\in\Gamma_{1-1}\in\Gamma_1 \quad (3)$$

$$\frac{\partial p_{mD}}{\partial n_D}=\frac{\partial p_{fD}}{\partial n_D}=0 \quad (x_D,\ y_D)\in\Gamma_{1-2}\in\Gamma_1 \quad (4)$$

$$\frac{\partial p_{mD}}{\partial n_D}=\frac{\partial p_{fD}}{\partial n_D}=0 \quad (x_D, y_D)\in\Gamma_i \quad (i=2,3,\cdots,m) \quad (5)$$

$$p_{mD}=p_{fD}=0 \qquad t_D=0 \quad (6)$$

where it is assumed that the flow is in a horizontal plane and that the Darcy's law is applicable; Γ_1 is the external boundary of reservoir, $\Gamma_{1\text{-}1} \cup \Gamma_{1\text{-}2}=\Gamma_1$ and $\Gamma_{1\text{-}1} \cap \Gamma_{1\text{-}2}=0$; Γ_i ($i>1$) is the boundary of impermeable regions, $\Gamma_i \cap \Gamma_j=0$, ($i \neq j$); m is the number of boundaries; δ is Dirac delta function. Subscripts f and m present fracture and matrix. The following dimensionless properties are introduced in the mathematical model:

$$p_\mathrm{D}=\frac{p_\mathrm{i}-p}{p_0}$$

$$x_\mathrm{D}=\frac{x}{\sqrt{A}},\quad y_\mathrm{D}=\frac{y}{\sqrt{A}}$$

$$t_\mathrm{D}=\frac{K_\mathrm{fi}t}{(\phi c)_\mathrm{f+m}\mu A}$$

$$q_\mathrm{D}=-\frac{\mu A}{K_\mathrm{fi}p_0}q_l$$

$$\lambda=\varepsilon\frac{K_\mathrm{m}t}{K_\mathrm{fi}}A$$

$$\omega=\frac{(\phi c)_\mathrm{f}}{(\phi c)_\mathrm{f+m}}$$

$$\beta=p_0\gamma$$

$$\gamma=\frac{1}{K}\frac{\partial K}{\partial p}$$

where p_i is initial formation pressure, Pa; p_0 is reference pressure, Pa; A is problem region area, m^2; μ is fluid viscosity, Pa·s; K is permeability, m^2; K_fi is initial fracture permeability, m^2; q_l is the strength of source well l, s^{-1}; c is compressibility, Pa^{-1}; ε is interporosity flow shape factor, m^{-2}; λ is the interporosity flow coefficient, i.e. the dimensionless matrix fracture permeability ratio; ω is the dimensionless fracture storage parameter; β is dimensionless permeability modulus; ϕ is porosity.

Eq. (1) is not written in a convenient form to be solved by using the boundary element method. With Pedrosa's substitution (Ning, *et al.*, 2004)

$$p_\mathrm{fD}=-\frac{1}{\beta}\ln(1-\beta\eta) \tag{7}$$

and after some algebraic manipulation, Eq. (1) can be transformed into

$$\frac{\partial^2\eta}{\partial x_\mathrm{D}^2}+\frac{\partial^2\eta}{\partial y_\mathrm{D}^2}=\frac{\omega}{1-\beta\eta}\frac{\partial\eta}{\partial t_\mathrm{D}}+(1-\omega)\frac{\partial p_\mathrm{mD}}{\partial t_\mathrm{D}}+\sum_{l=1}^{n_s}q_{\mathrm{D}l}\delta(x_\mathrm{D}-x_{\mathrm{D}l})\delta(y_\mathrm{D}-y_{\mathrm{D}l}) \tag{8}$$

where η is a dimensionless dependent variable.

Further assuming $\beta\eta<<1$ (Ning, *et al.*, 2004), by using the regular perturbation method (Kikani and Pedrosa, 1991) we obtain the zero order perturbation equation

$$\frac{\partial^2\eta_0}{\partial x_\mathrm{D}^2}+\frac{\partial^2\eta_0}{\partial y_\mathrm{D}^2}=\omega\frac{\partial\eta_0}{\partial t_\mathrm{D}}+(1-\omega)\frac{\partial p_\mathrm{mD}}{\partial t_\mathrm{D}}+\sum_{l=1}^{n_s}q_{\mathrm{D}l}\delta(x_\mathrm{D}-x_{\mathrm{D}l})\delta(y_\mathrm{D}-y_{\mathrm{D}l}) \tag{9}$$

Using the same method to manipulate Eq. (2), we have

$$\lambda(p_\mathrm{mD}-\eta_0)+(1-\omega)\frac{\partial p_\mathrm{mD}}{\partial t_\mathrm{D}}=0 \tag{10}$$

Transforming the corresponding equations to those in Laplace space, we obtain

$$\frac{\partial^2\overline{\eta}_0}{\partial x_\mathrm{D}{}^2}+\frac{\partial^2\overline{\eta}_0}{\partial y_\mathrm{D}{}^2}-f(s)s\overline{\eta}_0=\frac{1}{s}\sum_{l=1}^{n_s}q_{\mathrm{D}l}\delta(x_\mathrm{D}-x_{\mathrm{D}l})\delta(y_\mathrm{D}-y_{\mathrm{D}l}) \tag{11}$$

$$\overline{p}_\mathrm{mD}=\frac{\lambda}{\lambda+(1-\omega)s}\overline{\eta}_0 \tag{12}$$

$$\overline{p}_\mathrm{mD}=\overline{\eta}_0=0 \quad (x_\mathrm{D},\ y_\mathrm{D})\in\Gamma_{1\text{-}1}\in\Gamma_1 \tag{13}$$

$$\frac{\partial\overline{p}_\mathrm{mD}}{\partial n_\mathrm{D}}=\frac{\partial\overline{\eta}_0}{\partial n_\mathrm{D}}=0 \quad (x_\mathrm{D},\ y_\mathrm{D})\in\Gamma_{1\text{-}2}\in\Gamma_1 \tag{14}$$

$$\frac{\partial\overline{p}_\mathrm{mD}}{\partial n_\mathrm{D}}=\frac{\partial\overline{\eta}_0}{\partial n_\mathrm{D}}=0 \quad (x_\mathrm{D},y_\mathrm{D})\in\Gamma_i \ (i=2,3,\cdots,m) \tag{15}$$

where $f(s)=\dfrac{(1-\omega)\omega s+\lambda}{(1-\omega)s+\lambda}$ and s is the Laplace parameter.

3. Boundary element method

The zero order perturbation Eq. (11) is associated with the modified Helmholtz operator. The corresponding boundary integral equations in terms of the transformed variable η_0 can be expressed as

$$\alpha\overline{\eta}_0(x_\mathrm{D},y_\mathrm{D})=\sum_{i=1}^{m}\int_{\Gamma_i}(\overline{G}\frac{\partial\overline{\eta}_0}{\partial n_\mathrm{D}}-\overline{\eta}_0\frac{\partial\overline{G}}{\partial n_\mathrm{D}})\mathrm{d}\Gamma-\frac{1}{s}\sum_{l=1}^{n_s}\overline{G}(x_\mathrm{D},y_\mathrm{D};x_{\mathrm{D}l},y_{\mathrm{D}l};s)q_{\mathrm{D}l} \tag{16}$$

where $\alpha=\theta/2\pi$ and θ is the internal angle; $\overline{G}$ is the fundamental solution for the modified Helmholtz equation.

$$\overline{G}(x_D, y_D; \tilde{x}_D, \tilde{y}_D; s) = \frac{1}{2\pi} J_0(\sqrt{f(s)s} r_D) \qquad (17)$$

where $(\tilde{x}_D, \tilde{y}_D)$ and (x_D, y_D) are the arbitrary points over Ω; r_D is the dimensionless distance, $r_D = \left[(x_D - \tilde{x}_D)^2 + (y_D - \tilde{y}_D)^2\right]^{1/2}$; and J_0 is the zero order modified Bessel function of the second kind.

In order to evaluate the contour integral involved in the boundary integral equation, the boundary Γ is discretized into n_b elements. Nodes are allocated at the edges of elements, and boundary values are interpolated linearly in between. The node-numbering direction for outer boundary is the counterclockwise, and the clockwise direction is chosen for the inner boundary. A local (ξ, ζ) coordinate system is introduced for convenience's sake (Fig. 1). The origin of coordinates is at point P, from which the ξ axis is set parallel to the boundary element $\Delta\Gamma_j$ and in the opposite direction of node numbering. The ζ axis is defined by the right-hand rule.

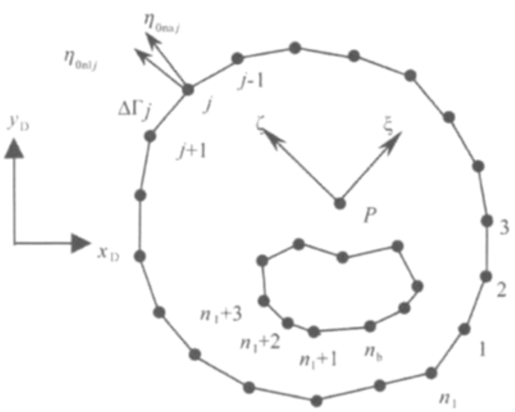

Fig. 1 Local (ξ, ζ) coordinate system

In the local (ξ, ζ) coordinate system, boundary values are interpolated as

$$\overline{\eta}_0 = \frac{(\eta_{0j+1} - \eta_{0j})\xi + \xi_{j+1}\overline{\eta}_{0j} - \xi_j \overline{\eta}_{0j+1}}{\xi_{j+1} - \xi_j}, \quad \xi_j \le \xi \le \xi_{j+1},$$

$$\frac{\partial \overline{\eta}_0}{\partial n_D} = \frac{(\eta_{0naj+1} - \eta_{0n1j})\xi + \xi_{j+1}\overline{\eta}_{0n1j} - \xi_j \overline{\eta}_{0naj+1}}{\xi_{j+1} - \xi_j}, \quad \xi_j \le \xi \le \xi_{j+1}$$

According to the previous instruction, the discretized form of Eq. (16) becomes

$$\alpha_i \overline{\eta}_{0i} = \sum_{j=1}^{n_b} (\overline{V}_{1ij}\overline{\eta}_{0n1j} + \overline{V}_{2ij}\overline{\eta}_{0naj+1} - \overline{W}_{1ij}\overline{\eta}_{0j} - \overline{W}_{2ij}\overline{\eta}_{0j+1}) - \frac{1}{s}\sum_{l=1}^{n_w} \overline{G}_{il} q_{Dl} \qquad (18)$$

where

$$\overline{V}_{1ij} = \frac{1}{\xi_{j+1} - \xi_j}\left(\int_{\Delta\Gamma_j} \xi \overline{G}_i \mathrm{d}\Gamma - \xi_{j+1}\int_{\Delta\Gamma_j} \overline{G}_i \mathrm{d}\Gamma\right)$$

$$\overline{V}_{2ij} = \frac{1}{\xi_{j+1} - \xi_j}\left(-\int_{\Delta\Gamma_j} \xi \overline{G}_i \mathrm{d}\Gamma + \xi_j\int_{\Delta\Gamma_j} \overline{G}_i \mathrm{d}\Gamma\right)$$

$$\overline{W}_{1ij} = \frac{1}{\xi_{j+1} - \xi_j}\left\{-\int_{\Delta\Gamma_j} \xi \left(\frac{\partial \overline{G}}{\partial n_D}\right)_i \mathrm{d}\Gamma - \xi_{j+1}\int_{\Delta\Gamma_j} \left(\frac{\partial \overline{G}}{\partial n_D}\right)_i \mathrm{d}\Gamma\right\}$$

$$\overline{W}_{2ij} = \frac{1}{\xi_{j+1} - \xi_j}\left\{-\int_{\Delta\Gamma_j} \xi \left(\frac{\partial \overline{G}}{\partial n_D}\right)_i \mathrm{d}\Gamma + \xi_j\int_{\Delta\Gamma_j} \left(\frac{\partial \overline{G}}{\partial n_D}\right)_i \mathrm{d}\Gamma\right\}$$

Eq. (18) can be solved for unknown boundary values in Laplace space by using the conventional BEM. Then interior solutions in Laplace space can be obtained. The Laplace space interior solution can be inverted to real space by using the Stehfest algorithm (Stehfesh, 1970). By using Duhamel principle (Kikani and Pedrosa, 1991; Yang and Zhao, 2002), the dimensionless bottom hole pressure considering wellbore storage and skin effect can be expressed as:

$$\overline{\eta}_{0w} = \left(\frac{s}{s\overline{\eta}_0 + S_1} + s^2 C_{D1}\right)^{-1} \qquad (19)$$

$$p_{wD} = -\frac{1}{\beta}\ln\left(1 - \beta L^{-1}\left[\overline{\eta}_{0w}(s)\right]\right) \qquad (20)$$

where $\overline{\eta}_{0w}$ is the Laplace space interior solution of zero order which does not consider wellbore storage and skin effect; L^{-1} is Laplace inversion transform operator; p_{wD} is dimensionless bottom hole flowing pressure. $S_1 = S\frac{q\mu}{2\pi K_f h p_0}$, and S is skin factor; $C_{D1} = C_D \frac{2\pi K_f h p_0 r_w^2}{q\mu A}$, and C_D is dimensionless wellbore storage coefficient, r_w is well radius, m; q is well flow rate, m^3/s; h is formation thickness, m.

4. Pressure transient analysis

A comparison of the analytical solution and numerical solution obtained with the boundary element method for the double-porosity reservoir with a closed circular boundary is presented in Fig. 2, which shows a very good agreement between the analytical and numerical solutions.

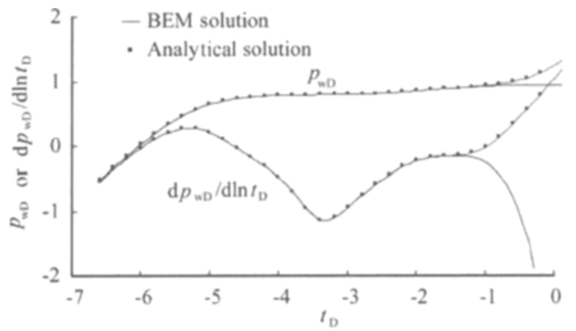

Fig. 2 Typical curves for pressure behavior in a closed circular deformable reservoir

One of the advantages of the boundary element method is the flexibility for the treatment of arbitrarily shaped boundary. Fig. 3a shows the transient pressure responses for arbitrarily shaped stress-sensitive reservoir with λ=400 and ω=0.04. The shape of reservoir and well location are shown in Fig. 3b. From Fig. 3a it can be deduced that the boundary condition of the reservoir has a great effect on pressure behavior. Compared with a constant pressure boundary, the piecewise constant pressure and piecewise closed boundary delay the decline time of time derivative of pressure. The pressure-derivative type curves are not on the 0.5 horizontal lines. As a result of the stress sensitivity of permeability, the derivative curves rise up and the slop increases with the increase in permeability variation coefficient.

Fig. 4a shows the effect of a single impermeable region on the pressure responses for stress sensitive reservoir with λ=200 and ω=0.02. The corresponding shape of reservoir and well location are shown in Fig. 4b. It can be seen that the impermeable region has a great influence on the pressure behavior, making the time of rise-up of the derivative curve earlier than expected in the case without impermeable regions.

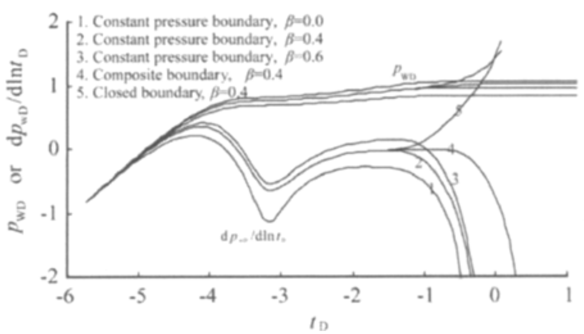

Fig. 3a Type curves for pressure behavior in arbitrarily shaped deformable reservoirs

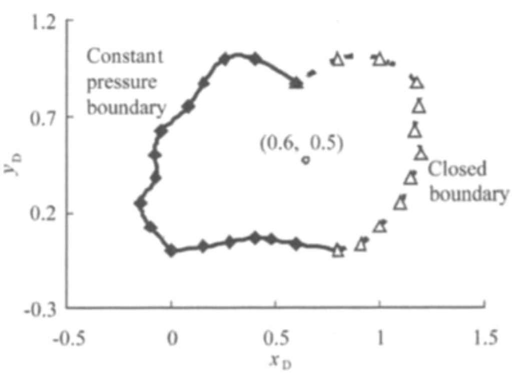

Fig. 3b Shape of deformable reservoir without impermeable regions and well location

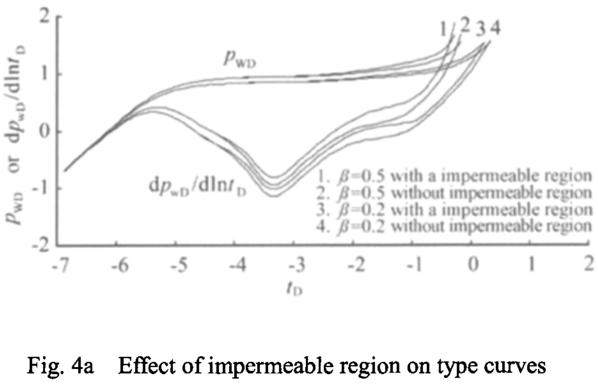

Fig. 4a Effect of impermeable region on type curves for pressure behavior in deformable reservoirs

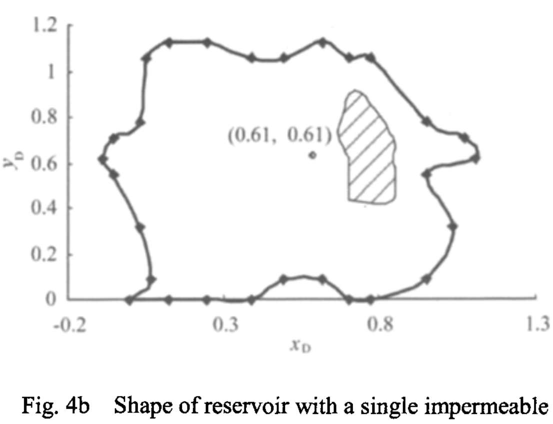

Fig. 4b Shape of reservoir with a single impermeable region and well location

5. Conclusions

1) The transient pressures of a circular reservoir with a constant pressure boundary and a closed boundary are analyzed by the boundary element method. Compared with the analytical solution this method is proved to be correct.

2) The impermeable region has a great influence on pressure and pressure-derivative type curves, making the time of rise-up of derivative curve earlier than expected in the case without impermeable regions.

3) Compared with constant pressure boundary, the boundary conditions of piecewise constant pressure and piecewise closed boundary delay the decline time of derivative of pressure.

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