IL NUOVO CIMENTO VOL. 20 D, N. 9 Settembre 1998

Obliquely propagating dust-acoustic solitary waves in a magnetized dusty plasma

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(ricevuto il 26 Agosto 1997; approvato il 9 Marzo 1998)

Summary. — A theoretical investigation has been made of obliquely propagating dust-acoustic solitary waves in a magnetized dusty plasma which consists of a negatively charged cold dust fluid, Boltzmann-distributed electrons, and nonthermally distributed ions. The reductive perturbation method has been employed to derive the Korteweg-de Vries (K-dV) equation which admits a solitary wave solution for small but finite amplitude limit. The solitary wave may change from compressive to rarefactive depending on the value of α (a parameter determining the number of nonthermal ions present in this plasma model). It is seen that for no background free electron there exist compressive (rarefactive) solitary waves when $\alpha \leq (>)0.155$ and that, as the number of the background free electrons increases, we need a higher value of α in order for rarefactve solitary waves to exist. The effects of obliqueness and external magnetic field on the nature of these compressive and rarefactive solitary waves are also discussed.

PACS 52.35.Fp – Electrostatic waves and oscillations (e.g., ion-acoustic waves).
PACS 52.35.Ra – Plasma turbulence.
PACS 52.35.Mw – Nonlinear waves and nonlinear wave propagation (including parametric effects, mode coupling, ponderomotive effects, etc.).

Nowadays, there has been a great deal of interest in understanding different types of collective processes in dusty plasms which are common in asteroid zones, planetary rings, cometary tails, as well as the lower ionosphere of the Earth. It has been found that the presence of static charged dust grains modifies the existing plasma wave spectra, whereas the dust charge dynamics introduces new eigenmodes in dusty plasmas. Rao *et al.* [1], for example, were the first to report theoretically the existence of extremely low phase velocity (in comparison with the electron and ion thermal velocities) dust-acoustic waves in an unmagnetized dusty plasma whose constituents are an inertial charged cold dust fluid and Boltzmann-distributed electrons and ions. Thus, in the dust-acoustic waves the dust particle mass provides the inertia, whereas the restoring force comes from the pressures of inertialess electrons and ions. A recent laboratory experiment [2] has conclusively verified the theoretical prediction of Rao *et al.* [1] and reported some nonlinear features of the dust-acoustic waves. Recently, motivated by the experimental observation [2] of the low phase velocity dust-acoustic waves, we have studied nonlinear dust-acoustic waves in a two-component dusty plasma consisting of a negatively charged cold dust fluid and Maxwellian [3] and non-Maxwellian [4] distributed ions. As the effect of the external magnetic field, which has not been considered in those earlier investigations [1, 3, 4], drastically modifies the properties of electrostatic solitary structures [5-7], we, in the present work, have studied the obliquely propagating dust-acoustic solitary structures in a magnetized three component dusty plasma which consists of a negatively charged cold dust fluid, elecrons having Boltzmann distribution and ions with fast particles.

We model the dust fluid as a cold single fluid, which consists of extremely massive, micron-sized, negatively charged inertial dust grains, in the presence of an external static magnetic field ($\mathbf{B}_0 \parallel \hat{\mathbf{z}}$). Thus, at equilibrium we have

$$n_{\rm i0} = Z_{\rm d} n_0 + n_{\rm e0},$$

where $n_{\rm i0}$, n_0, and $n_{\rm e0}$ are the unperturbed ions, dust, and electron number densities, respectively, and $Z_{\rm d}$ is the number of electrons residing on the dust grains. The dynamics of the low phase velocity (lying between the ions and dust thermal velocities, *viz.* $v_{\rm td} \ll v_{\rm p} \ll v_{\rm ti}$) dust-acoustic oscillations is governed by [3, 4]

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{u}) = 0, \tag{1}$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = \nabla\varphi - \omega_{\rm c}(\mathbf{u} \times \hat{\mathbf{z}}), \tag{2}$$

$$\nabla^2\varphi = n + \mu_0 e^{\sigma\varphi} - \mu_1(1 + \beta\varphi + \beta\varphi^2)e^{-\varphi}, \tag{3}$$

where n is the the dust particle number density normalized to n_0, $\mathbf{u}$ is the dust fluid velocity normalized to the dust-acoustic speed $C_{\rm d} = (Z_{\rm d} T_{\rm i}/m_{\rm d})^{1/2}$, with $T_{\rm i}$ the ion temperature (in energy units) and $m_{\rm d}$ the mass of negatively charged dust particulates, φ is the electrostatic wave potential normalized to $T_{\rm i}/e$, with e the magnitude of the electron charge. $\sigma = T_{\rm i}/T_{\rm e}$, with $T_{\rm e}$ the electron temperature (in energy units), $\mu_0 = \mu/(1-\mu)$ and $\mu_1 = 1/(1-\mu)$ with $\mu = n_{\rm e0}/n_{\rm i0}$, and $\beta = 4\alpha/(1+3\alpha)$, with α a parameter determining the number of nonthermal ions [4] present in our single fluid dusty plasma model with nonthermal ions. The time and space variables are in units of the dust plasma period $\omega_{\rm pd}^{-1} = (m_{\rm d}/4\pi n_0 Z_{\rm d}^2 e^2)^{1/2}$ and the Debye length $\lambda_{\rm Dd} = (T_{\rm i}/4\pi Z_{\rm d} n_0 e^2)^{1/2}$, respectively. $\omega_{\rm c} = (Z_{\rm d} e B_0/m_{\rm d})/\omega_{\rm pd}$ is the dust cyclotron frequency normalized to $\omega_{\rm pd}$.

To study dust-acoustic solitary waves in our dusty plasma model, we construct a weakly nonlinear theory of the dust-acoustic waves with small but finite amplitude which leads to the scaling of the independent variables through the stretched coordinates [8]

$$\begin{cases} \xi = \epsilon^{1/2}(l_x x + l_y y + l_z z - v_0 t), \\ \tau = \epsilon^{3/2} t, \end{cases} \tag{4}$$

where ϵ is a small parameter measuring the weakness of the dispersion, v_0 is the wave phase velocity normalized to $C_{\rm d}$; l_x, l_y, and l_z are the directional cosines of the wave vector $\mathbf{k}$ along the x-, y-, and z-axes, respectively, so that $l_x^2 + l_y^2 + l_z^2 = 1$. We can expand the perturbed quantities n, u_z, and φ about their equilibrium values in power of ϵ by following refs. [6] and [8]. To obtain the x- and y-components of dust electric field and polarization drifts, we can expand the perturbed quantities $u_{x,y}$ by following a standard technique [6] where the terms of $\epsilon^{3/2}$ are included. Thus, we can expand n, $u_{x,y,z}$, and φ as [6, 8]

$$\begin{cases} n = 1 + \epsilon n^{(1)} + \epsilon^2 n^{(2)} + \dots, \\ u_{x,y} = 0 + \epsilon^{3/2} u_{x,y}^{(1)} + \epsilon^2 u_{x,y}^{(2)} + \dots, \\ u_z = 0 + \epsilon u_z^{(1)} + \epsilon^2 u_z^{(2)} + \dots, \\ \varphi = 0 + \epsilon \varphi^{(1)} + \epsilon^2 \varphi^{(2)} + \dots. \end{cases} \tag{5}$$

Now, using (4) and (5) in (1)–(3), one can obtain the first-order continuity equation, z-component of the momentum equation and Poisson's equation which, after simplification, yield $n^{(1)} = l_z u_z^{(1)}/v_0 = -l_z^2 \varphi^{(1)}/v_0^2$ and $v_0 = l_z/\sqrt{\mu_0\sigma + \mu_1(1-\beta)}$. We can write the first-order x- and y-components of the momentum equation as $u_y^{(1)} = \frac{l_x}{\omega_c}\frac{\partial \varphi^{(1)}}{\partial \xi}$ and $u_x^1 = -\frac{l_y}{\omega_c}\frac{\partial \varphi^{(1)}}{\partial \xi}$. These, respectively, represent the x- and y-components of the electric-field drift. These equations are also satisfied by the second-order continuity equation.

Again, using (4) and (5) in (2) and (3), and eliminating $u_{x,y}^{(1)}$, we obtain the next higher-order x- and y-components of the momentum equation and Poisson's equation as

$$u_y^{(2)} = -\frac{l_y v_0}{\omega_c^2}\frac{\partial^2 \varphi^{(1)}}{\partial \xi^2}, \tag{6}$$

$$u_x^{(2)} = -\frac{l_x v_0}{\omega_c^2}\frac{\partial^2 \varphi^{(1)}}{\partial \xi^2}, \tag{7}$$

$$\frac{\partial^2 \varphi^{(1)}}{\partial \xi^2} = n^{(2)} + \frac{l_z^2}{v_0^2}\varphi^{(2)} - \frac{1}{2}(\mu_1 - \mu_0\sigma^2)[\varphi^{(1)}]^2. \tag{8}$$

The first two of these equations denote, respectively, the y- and x-components of the dust polarization drift. Similarly, following the same procedure, one can obtain the next higher-order continuity equation and z-component of the momentum equation as

$$\frac{\partial n^{(1)}}{\partial \tau} - v_0\frac{\partial n^{(2)}}{\partial \xi} + l_x\frac{\partial u_x^{(2)}}{\partial \xi} + l_y\frac{\partial u_y^{(2)}}{\partial \xi} + l_z\frac{\partial}{\partial \xi}[u_z^{(2)} + n^{(1)}u_z^{(1)}] = 0, \tag{9}$$

$$\frac{\partial u_z^{(1)}}{\partial \tau} - v_0\frac{\partial u_z^{(2)}}{\partial \xi} + l_z u_z^{(1)}\frac{\partial u_z^{(1)}}{\partial \xi} - l_z\frac{\partial \varphi^{(2)}}{\partial \xi} = 0. \tag{10}$$

Now, using (6)–(10), one can eliminate $n^{(2)}$, $u_z^{(2)}$, and $\varphi^{(2)}$, and obtain

$$\frac{\partial \varphi^{(1)}}{\partial \tau} + A\varphi^{(1)}\frac{\partial \varphi^{(1)}}{\partial \xi} + B\frac{\partial^3 \varphi^{(1)}}{\partial \xi^3} = 0. \tag{11}$$

This is the K-dV equation with the coefficients A and B given by

$$\begin{cases} A = -\frac{3}{2} l_z S_\alpha \sqrt{\sigma\mu_0 + \mu_1(1-\beta)}, \\ B = \dfrac{1}{2}\dfrac{l_z}{[\mu_0\sigma + \mu_1(1-\beta)]^{3/2}}\left[1 + \dfrac{1 - l_z^2}{\omega_c^2}\right], \\ S_\alpha = 1 - \dfrac{\mu_1 - \mu_0\sigma^2}{3[\mu_0\sigma + \mu_1(1-\beta)]^2}. \end{cases} \tag{12}$$

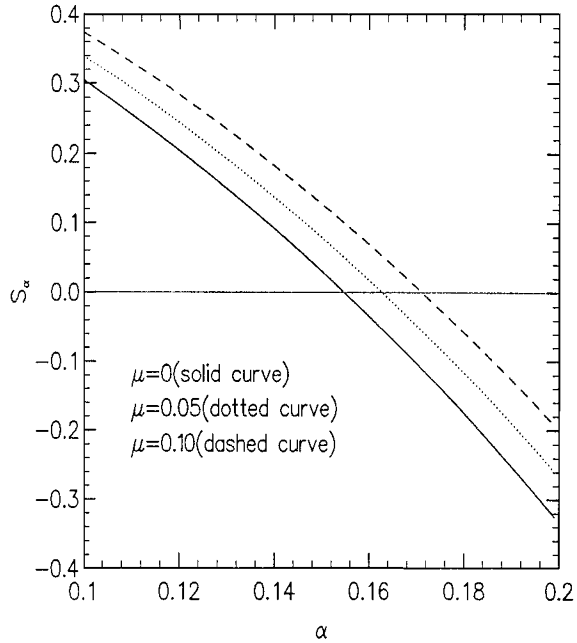

Fig. 1. – S_α is plotted against α for $\mu = 0$ (solid curve), $\mu = 0.05$ (dotted curve) and $\mu = 0.10$ (dashed curve).

The steady-state solution of this K-dV equation is obtained by transforming the independent variables ξ and τ into $\eta = \xi - u_0\tau$ and $\tau = \tau$, where u_0 is a constant velocity normalized to C_d, and by imposing the appropriate boundary conditions, *viz.* $\varphi \to 0$, $\frac{\mathrm{d}\varphi^{(1)}}{\mathrm{d}\eta} \to 0$, $\frac{d^2\varphi^{(1)}}{d\eta^2} \to 0$ at $\eta \to \pm\infty$. Thus, one can express the steady-state solution of this K-dV equation as

$$\varphi^{(1)} = \varphi_m^{(1)}\mathrm{sech}^2[(\xi - u_0\tau)/\delta], \tag{13}$$

where the amplitude $\varphi_m^{(1)}$ and the width δ (normalized to λ_Dd) are given by

$$\begin{cases} \varphi_m^{(1)} = 3u_0/A, \\ \delta = \sqrt{4B/u_0}. \end{cases} \tag{14}$$

It is found that either compressive ($\varphi_m^{(1)} < 0$) or rarefactive ($\varphi_m^{(1)} > 1$) solitary waves may exist depending on whether A is positive or negative. As β is always less than 1, it is obvious that there exist compressive solitary waves when $S_\alpha > 1$ and rarefactive solitary waves when $S_\alpha < 1$. Figure 1 shows the α-value range for which compressive and rarefactive solitary waves exist for different values of μ. It is seen that for $\mu = 0$, *i.e.* for no background free electron, there exist compressive (rarefactive) solitary waves when $\alpha < 0(>)0.155$. As μ (background free electrons) increases, we need a higher value of α, *i.e.* more nonthermal ions, in order for rarefactive solitary waves to exist. It is obvious that the amplitude of both the compressive and rarefactive solitary waves decreases with

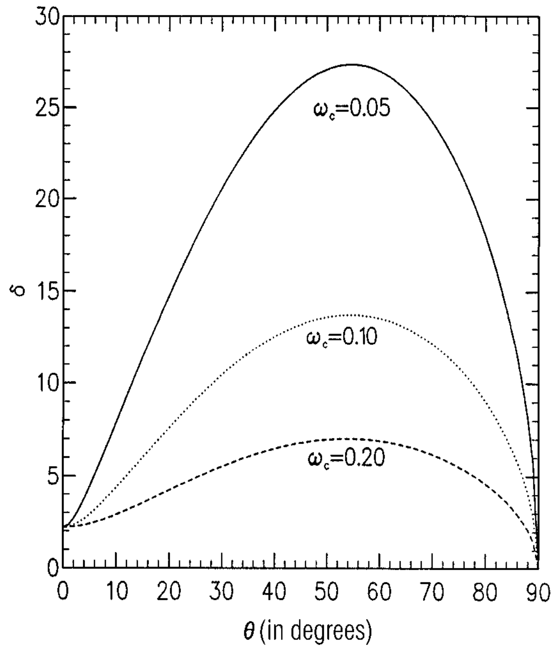

Fig. 2. – δ is plotted against θ for $\sigma = 0.01$, $u_0 = 1.0$, $\alpha = 0.2$, $\omega_c = 0.05$ (solid curve), $\omega_c = 0.10$ (dotted curve) and $\omega_c = 0.20$ (dashed curve).

the rise of l_z ($l_z = \cos\theta$, with θ the angle between the directions of the wave propagation vector $\mathbf{k}$ and the external magnetic field $\mathbf{B}_0$). Figure 2 shows how the width (δ) of these solitary waves changes with the obliqueness (θ) and the magnitude of the external magnetic field (ω_c). It is observed that the width (δ) increases with θ for its lower range (*i.e.* from $0°$ to $\sim 45°$), but decreases for its higher range (*i.e.* from $\sim 45°$ to $90°$). Though in fig. 2 the variation of δ with θ has been shown for any value of θ between $0°$ and $90°$, our perturbation method, which is only valid for small but finite amplitude limit, is not valid for large θ which makes the wave amplitude large. The range of θ, where the present analysis can be applied, can be found by considering (5), (12) and (14) as

$$\cos\theta > \left|\frac{2u_0\epsilon}{S_\alpha}\sqrt{\mu_0\sigma + \mu_1(1-\beta)}\right|. \tag{15}$$

This condition implies that the range of θ for which our analysis is valid does not only depend on the plasma parameters (α, σ and μ) but also on ϵ (a small parameter measuring the weakness of the dispersion) and u_0 (constant velocity normalized to C_d).

It is seen that the magnitude of the external magnetic field has no effect on the amplitude of the solitary waves. However, it does have an effect on the width of these solitary waves. It is shown that, as we increase the magnitude of the magnetic field, the width of these solitary waves decreases, *i.e.* the external magnetic field makes the solitary structures more spiky.

It may be stressed here that the results of this investigation should be useful in understanding the nonlinear features of the localized electrostatic disturbances in laboratory and space plasmas, in which negatively charged dust particulates, free electrons, and ions with fast particles are the plasma species.

To conclude, it may be added that the time evolution and stability analysis of these solitary structures are also problems of great importance but beyond the scope of the present work.

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The author is greateful to Profs. P. K. SHUKLA, R. A. CAIRNS, M. SALIMULLAH, and Dr. R. BINGHAM for their various critical suggestions and stimulating discussions during the course of this work.

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