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A new approach to studying ENSO predictability: Conditional nonlinear optimal perturbation

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Abstract **A new approach, the conditional nonlinear optimal perturbation (CNOP) is introduced to study the predictability of El Niño-Southern Oscillation (ENSO) using a theoretical coupled ocean-atmosphere model. The differences between CNOP and linear singular vector (LSV) are demonstrated. The results suggest that the nonlinear model and CNOP are superior in determining error growth for studying predictability of the ENSO. In particular, the CNOP approach is used to explore the nature of the 'spring predictability barrier' in ENSO prediction.**

Keywords: nonlinear, optimal perturbation, predictability, ENSO model.

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Determination of the fastest growing perturbation is of central importance in the predictability of numerical weather and climate prediction. Usually, it is assumed that the initial perturbation is sufficiently small such that its evolution can be governed by the tangent linear model (TLM) approximately. In TLM approach, the fastest growing perturbation of TLM corresponds to linear singular vector (LSV), which was first introduced by Lorenz[1] to study the predictability of atmospheric motions. Recently, TLM approach was used to explore the predictability of the coupled ocean-atmosphere model[2].

To investigate the nonlinearity of the atmospheric and oceanic motion, Mu[3] proposed the concept of nonlinear singular vector (NSV) and nonlinear singular value (NSVA). The two-dimensional quasi-geostrophic model has been used by Mu and Wang[4] to study the NSV and NSVA. The results demonstrate that for some types of basic states, there exist local nonlinear optimal perturbations, which usually possess larger norm and smaller growth rate compared to the first NSV. In this case, the local nonlinear optimal perturbations could play a more important role than the first NSV in the study of the predictability.

The nonlinear optimal perturbation can be used to estimate the prediction error[5]. Given the information of the initial observation, the nonlinear optimal perturbation should be less than an upper bound of the initial observational error. But the local nonlinear optimal perturbations could exceed it in some cases. This weakness suggests that we should investigate the nonlinear optimal perturbation with constrained conditions. For this purpose, the concept of conditional nonlinear optimal perturbation is introduced in this paper.

1 Conditional nonlinear optimal perturbation

Let M_T be the propagator of the nonlinear model from 0 to T[5]. $\boldsymbol{u}_0$ is an initial perturbation superposed on the basic state $\boldsymbol{U}(t)$, whose initial value is $\boldsymbol{U}_0$.

For a chosen norm $\|\bullet\|$, an initial perturbation $\boldsymbol{u}_{0\delta}^{*}$ is called the conditional nonlinear optimal perturbation (CNOP) with constraint condition $\|\boldsymbol{u}_0\| \leqslant \delta$, if and only if

$$J(\boldsymbol{u}_{0\delta}^{*}) = \max_{\|\boldsymbol{u}_0\|\leqslant\delta} \| M_T(\boldsymbol{U}_0+\boldsymbol{u}_0) - M_T(\boldsymbol{U}_0) \|. \qquad (1)$$

In the above, the constraint condition is given by a chosen norm. Obviously, we can also investigate the situations that the initial perturbations belong to some kinds of functional set, or satisfy some physical laws.

On the basis of the practical demands, Mu et al.[5] classified three predictability problems. The second problem is closely related to CNOP, i.e. if $\boldsymbol{U}_0$ in (1) is taken as initial observation, (1) gives an upper bound of prediction error caused by the initial observational errors.

2 Applications of CNOP to the predictability for ENSO

In this section, we consider the following theoretical coupled ocean-atmosphere model for ENSO[6], hereafter WF96 model.

$$\begin{cases} \dfrac{\mathrm{d}T_E}{\mathrm{d}t} = a_1 T_E - a_2 h_E + \sqrt{\dfrac{2}{3}}T_E(T_E - a_3 h_E), \\ \dfrac{\mathrm{d}h_E}{\mathrm{d}t} = b(2h_E - T_E), \end{cases}$$

where $a_1 = (\Delta T_0' - \overline{u}_1' + \overline{T}_x' - \alpha_s')|_{x_E}$, $a_2 = (\mu + \delta_1)\overline{T}_x'|_{x_E}$, $a_3 = (\mu+\delta_1)$, $b = \dfrac{2}{\delta(1-3\varepsilon)}$. The parameters, $\Delta T_0'$, $\overline{T}_x'$ and $\overline{u}_1'$, are determined by the climate mean state, which vary with time and reflect the annual cycle of the basic state. For detailed physical meaning the readers are referred to the paper of Wang and Fang[6]. This model describes the interanual variation of SST and thermocline depth anomaly in Niño-3 region.

In this paper, the WF96 model is integrated by fourth-order Runge-Kutta scheme with dt = 0.01, which represents one day. The numerical algorithm adopted is LBFGS method[7].

(i) Conditional nonlinear optimal perturbation. Let $\boldsymbol{U}_0$ be the initial value of the basic state $\boldsymbol{U}(t)$, $\boldsymbol{u}_0$ its initial perturbation. For the chosen norm $\|\boldsymbol{u}\| = \max\{|T_E|$,

$|h_E|\}$, the CNOP $\boldsymbol{u}_{0\delta}^*$ of $\boldsymbol{U}(t)$ can be obtained by solving (1). To facilitate the following discussion, we define $\lambda_\delta = \dfrac{J(\boldsymbol{u}_{0\delta}^*)}{\|\boldsymbol{u}_{0\delta}^*\|}$, which represents the growth rate of CNOP.

In the following, we choose El Niño and La Niña events in WF96 model as two basic states, denoted as U^E and U^L, whose initial values are (−0.1, 0.1) and (0.1, −0.1) respectively. For $\delta \in [0.01, 0.08]$ and different time periods $[0, T]$, the differences between the growth rate of CNOP and linear singular value (LSVA) are investigated (Fig. 1), where the linear singular value is given by $\lambda_L = \max\limits_{\boldsymbol{u}_0} \dfrac{\|\mathrm{M}(\boldsymbol{U}_0)(\boldsymbol{u}_0)\|}{\|\boldsymbol{u}_0\|}$ and $\mathrm{M}(\boldsymbol{U}_0)$ is the tangent linear model of WF96 model with respect to $\boldsymbol{U}(t)$. It follows that LSVA does not change with δ, but the growth rate of CNOP does. Consequently, when the initial perturbations are large, the differences between the growth rate of CNOP and LSVA are also very considerable. Besides, the optimization time also plays a role in determining the difference between them. For $T \leqslant 10$ months with initial time being January, the difference $\eta = \dfrac{|\lambda_\delta - \lambda_L|}{\lambda_\delta} \leqslant$ 13.57%. In this case, the WF96 model can be approximated by the TLM. But when $T > 10$ months, the TLM is a good approximation only for the sufficiently small initial perturbations. With δ increasing from 0.01 to 0.08, the value of η becomes more and more large, and in November and December they are up to 19.04% and 25.51% respectively. All these suggest that the TLM is not a good approximation to the nonlinear model for the large perturbations and the long optimization time periods. Hence, in the study of the predictability for ENSO, the nonlinear model, rather than the TLM, should be employed.

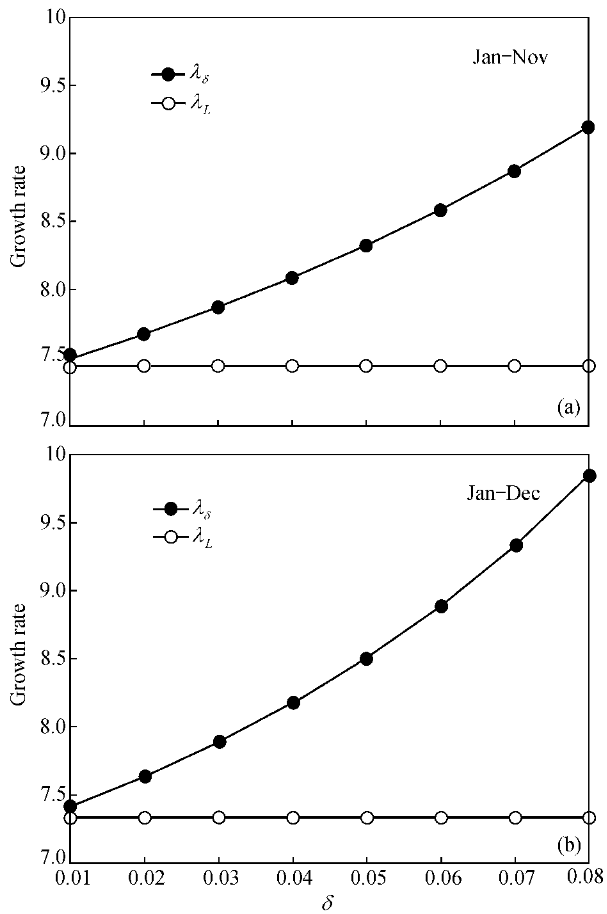

Fig. 1. λ_δ and λ_L, the growth rate of CNOP and LSVA of U^E.

(ii) Applications of CNOP to the study of spring predictability barrier for ENSO. A year is divided into four seasons starting with October to December (OND), followed by January to March (JFM) and so forth.

For T = 12 months with initial time being October, the CNOPs of the basic states U^E and U^L are computed respectively. We investigate the slopes of the curve $\gamma_\delta(t) = \dfrac{\|\boldsymbol{u}_{N\delta}(t)\|}{\delta}$ at different seasons, where $\|\boldsymbol{u}_{N\delta}(t)\|$ represents the nonlinear evolution of CNOP and the slope of the curve is denoted by κ, which measures the growth of CNOP normalized by δ at different seasons.

Table 1 The values of κ corresponding to El Niño event U^E

δ	OND	JFM	AMJ	JAS
0.01	2.5108	4.5505	16.6049	−3.2472
0.02	2.5113	4.5677	19.4686	−3.0822
0.03	2.5118	4.5839	23.1792	−0.6510
0.04	2.5124	4.5989	28.1567	1.6326
0.05	2.5129	4.6129	35.1483	4.1823
LSV	2.5101	4.5327	14.3367	−3.4097

Table 2 The values of κ corresponding to La Niña event U^L

δ	OND	JFM	AMJ	JAS
0.01	2.5658	1.7592	0.3080	−0.9755
0.02	2.5616	1.7710	0.3399	−1.0285
0.03	2.5575	1.7826	0.3634	−1.0757
0.04	2.5533	1.7942	0.3886	−1.1246
0.05	2.5491	1.8059	0.4154	−1.1749
LSV	2.5698	1.7477	0.2975	−0.9387

The positive (negative) value of κ corresponds to the increase (decrease) of the initial observational error, and the larger the absolute value of the κ, the faster the increase or the decrease of the initial observational error. The numerical results are shown in Tables 1 and 2. The values of κ related to LSV in Tables 1 and 2 are the slopes of the curve $\gamma_L(t) = \dfrac{\|\boldsymbol{u}_L(t)\|}{\|\boldsymbol{u}_{0L}\|}$ at different seasons, where $\boldsymbol{u}_{0L}$ and $\boldsymbol{u}_L(t)$ represent the LSV and its linear evolution. From Table 1, it is demonstrated that the largest growth of CNOP for U^E occurs during the AMJ season, which corresponds to the time of ENSO predictability barrier. Although the largest error growth during the boreal spring is also shown through the LSV approach, the values of κ corresponding to LSV and CNOP have remarkable differences for the large values of δ. If the TLM of WF96 model and LSV are used to estimate the error growth, the prediction error could be underestimated. So the usage of CNOP may provide a useful tool for the study of spring predictability barrier (SPB) for ENSO. Results shown in Tables 1 and 2 demonstrate that the error growth is enhanced during the spring of an El Niño event and suppressed for a La Niña event. A consequent question is what causes the SPB.

On the basis of the observational fact that the zonal SST gradient in the tropical Pacific is minimum during boreal spring, Webster and Yang[8] suggested that the SPB is linked with the annual cycle. Webster[9] further analyzes the cause of SPB. He suggested that the SPB is due to the weakly coupling between ocean and atmosphere during spring of the year. In the WF96 model, the coupling coefficient is invariant, but there also occurs the phenomenon of SPB. This indicates that the weakly ocean-atmosphere coupling in spring cannot explain the SPB in the WF96 model. On the other hand, Wang and Fang[6] analyzed the growth mechanism of the perturbation in their model. They suggested that the weakest easterly wind stress and equatorial upwelling during spring in the tropical Pacific, which is also entitled the strongest coupled ocean-atmosphere instability during spring of the year, is one of the reasons of the SPB.

To make sure this cause of SPB in the WF96 model, we compute the CNOPs of El Niño events under the conditions of strong and weak coupled ocean-atmosphere instability, where the initial value of the El Niño event in the WF96 model is $(-0.1, 0.1)$, the initial time is January, and the optimization time period T starts from October to the next September. The values of κ are shown in Tables 3 and 4. It is shown that for an El Niño event, the error growth during the boreal spring (AMJ) for the strong coupled instability is notably larger than that for weak coupled instability. With δ increasing from 0.01 to 0.05, the difference becomes progressively large. This indicates that the strong coupled ocean-atmosphere instability during boreal spring is one of the causes of the SPB for ENSO, and the larger the initial error, the more remarkable its impacts on the SPB.

Table 3 κ of El Niño event for strong ocean-atmosphere coupled instability

δ	OND	JFM	AMJ	JAS
0.01	2.5101	4.6612	23.9462	−2.1042
0.02	2.5107	4.6783	28.6207	1.8590
0.03	2.5112	4.6945	34.9999	2.0981
0.04	2.5116	4.7096	44.1814	4.2018
0.05	2.5121	4.7235	58.4372	7.5530

Table 4 κ of El Niño event for weak ocean-atmosphere coupled instability

δ	OND	JFM	AMJ	JAS
0.01	2.5114	4.4407	11.7789	−3.9069
0.02	2.5120	4.4577	13.6336	−3.0381
0.03	2.5126	4.4737	15.9483	−0.8393
0.04	2.5132	4.4889	18.9065	2.2881
0.05	2.5137	4.5030	22.8041	3.8150

3 Conclusion and discussion

In this paper, the concept of conditional nonlinear optimal perturbation is introduced to study the predictability of ENSO in terms of a theoretical coupled ocean-atmosphere model. We have shown the difference between conditional nonlinear optimal perturbation (CNOP) and linear singular vector (LSV). The results suggest that the CNOP approach is more applicable than LSV in studying the predictability problems. Besides, the spring predictability barrier (SPB) for ENSO is studied by CNOP approach and the preliminary results are presented.

Although the model adopted in this paper is simple, it grasps the nonlinear characteristic of ocean-atmosphere coupling. The CNOP approach reveals the impact of nonlinearity on the predictability for ENSO. It is reasonable to believe that for the more complicated model, CNOP is of more importance in the study of climate predictability. Of course, some difficulties will be faced, which are what will be solved in the future works.

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References

1. Lorenz, E. N., A study of the predictability of a 28-variable atmospheric model, Tellus, 1965, 17: 321—333.
2. Roger, M. S., Eli Tziperman, Instability of the Chaotic ENSO: The growth-phase predictability barrier, J. Atmos. Sci., 2001, 58: 3613—3625..
3. Mu, M., Nonlinear singular vectors and nonlinear singular values, Science in China, Ser. D, 2000, 43: 375—385.
4. Mu, M., Wang, J. C., Nonlinear fastest growing perturbation and the first kind of predictability, Science in China, Ser. D, 44: 1128—1139.
5. Mu, M., Duan, W. S., Wang, J. C., The predictability problems in numerical weather and climate prediction, Adv. Atmos. Sci., 2002, 19: 191—204.
6. Wang, B., Fang, Z., Chaotic oscillation of tropical climate: a dynamic system theory for ENSO, J. Atmos. Sci., 1996, 53: 2786—2802.
7. Liu D. C., Nocedal, J., On the memory BFGS method for large scale optimization, Mathematical Programming, 1989, 45: 503—528.
8. Webster, P. J., Yang, S., Monsoon and ENSO: Selectively interactive systems, Q. J. R. Meteorol. Soc., 1992, 118: 877—926.
9. Webster, P. J., The annual cycle and the predictability of the tropical coupled ocean-atmosphere system, Meteorol. Atmos. Phys., 1995, 56: 33.

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