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Mechanisms of particle vertical diffusion in sediment-laden flows

WANG Guangqian & FU Xudong

Key Lab for Water and Sediment Sciences of Ministry of Education, Tsinghua University, Beijing 100084, China
Correspondence should be addressed to Wang Guanqian (e-mail: dhhwgq@tsinghua.edu.cn)

Abstract **Diffusion coefficient of natural sediments and effects of lift force and gradient in particle velocity fluctuations were investigated through using a closed kinetic model. Comparison against experimental data of Einstein & Chien (1955) validated the model. The diffusion coefficient ε_{yy} of medium and large sediments distinctly exceeds fluid eddy viscosity ν^t_f, while ε_{yy} of fine sediments approximately equals ν^t_f. In the measured region of $0.03<y/H<0.4$, ε_{yy}/ν^t_f increases with the distance from the wall decreasing. Combined effects of lift force and gradient in particle velocity fluctuations change sediment gravitational settling remarkably below $y/H=0.2$, and need to be accounted for describing sediment diffusion in this region.**

Keywords: sediment, two-phase flows, kinetic model, diffusion coefficient, lift force.

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Sediment diffusion is of basic interest in mechanics of sediment transport, in which specification of sediment diffusion coefficient ε_{yy} is crucial. So far, ε_{yy} has been assumed in conventional advection-diffusion (AD) equation to equate fluid eddy viscosity ν_f^t in practical applications. Using measured concentration data, however, the determined ε_{yy} from the AD equation is usually greater than ν_f^t [1,2]. Although many efforts have been devoted to explain this inconsistency, e.g. Ni et al.[3—5] related ε_{yy} directly to fluid vertical fluctuating velocity and Czernuszenko[6] proposed a concept of sediment drift diffusion, it still remains in understanding the context of sediment diffusion theory. Based on two-phase flow formulation, it is found that the AD equation neglects effects of gradient in sediment velocity fluctuations[7] and lift force exerted by fluid[8,9], which not only determines formation of the upward non-monotonic-decreasing concentration profile, i.e. pattern Ⅰ in Wang et al.[4,8], but also results in the calculated ε_{yy} from the equation of an inaccurate one. Employing a kinetic model for two-phase flows, the authors[10] demonstrated that ε_{yy} of fine sediments approximately equals ν_f^t, but the calculated one is slightly greater. This study focuses on the diffusion coefficient of both fine and coarse sediments as well as the effects of lift force and gradient in particle velocity fluctuations.

1 Kinetic model for particle phase

The kinetic model for two-phase flows, due to its unique advantage in supplying a theoretical closure for macroscopic balance equations of the particle phase，has been developed and widely used in recent years[8—12]. This study is based on a kinetic model. A brief overview is given here. In turbulent two-phase flows, the Lagrangian equations for single particle motion read

$$\mathrm{d}R_{\mathrm{p}i}/\mathrm{d}t = v_{\mathrm{p}i},\ \mathrm{d}v_{\mathrm{p}i}/\mathrm{d}t = (u_i - v_{\mathrm{p}i})/\tau_\mathrm{p} + F_i + W_i, \quad (1)$$

where $R_{\mathrm{p}i}$ is particle position, $v_{\mathrm{p}i}$ is particle stochastic velocity, u_i is fluid velocity at $R_{\mathrm{p}i}$, τ_p is particle relaxation time, t is the time, F_i is the sum of the gravity and interphase forces except the drag, W_i is the force due to interparticle collisions, and i denotes the tensor index.

To transform Lagrangian variables in eq. (1) to Eulerian variables, particle probability density distribution function (PDF) is introduced in terms of the coordinates x_i and velocities v_i in phase space

$$f = \langle \delta(x_i - R_{\mathrm{p}i})\delta(v_i - v_{\mathrm{p}i}) \rangle, \quad (2)$$

where $\langle \bullet \rangle$ denotes averaging over an ensemble of all turbulence realizations and $\delta(\bullet)$ is the Delta function.

Ensemble-averaged concentration and velocities of particle phase are expressed through PDF as

$$C = \int f \mathrm{d}\boldsymbol{v},\quad C\langle v_i\rangle = \int v_i f\, \mathrm{d}\boldsymbol{v}. \quad (3)$$

Differentiating eq. (2) with respect to t, using a generalized Fokker-Planck diffusion operator for modeling interactions of a particle with fluid turbulence eddies along its path and a revised BGK model with corrected Prandtl number for particle-particle collisions, and converting the velocity coordinates from v_i to the fluctuating velocities $v_i' = v_i - \langle v_i \rangle$ in the phase space, a closed PDF-evolution equation is obtained

$$\frac{\mathrm{d}f}{\mathrm{d}t} + v_i'\frac{\partial f}{\partial x_i} - \frac{\partial f}{\partial v_i'}\left\{\frac{\mathrm{d}\langle v_i\rangle}{\mathrm{d}t} - \frac{\langle u_i\rangle - \langle v_i\rangle + \tau_\mathrm{p}F_i}{\tau_\mathrm{p}}\right.$$
$$\left. - \frac{\partial f}{\partial v_i'}v_k'\frac{\partial\langle v_i\rangle}{\partial x_k}\right\} - \alpha\langle u_i'u_k'\rangle\frac{\partial^2 f}{\partial x_i\partial v_k'} - \frac{\beta\langle u_i'u_k'\rangle - T\delta_{ik}}{\tau_\mathrm{p}}\frac{\partial^2 f}{\partial v_i'\partial v_k'}$$
$$+\frac{f_0}{2}\left(\frac{v_k'v_k'}{2T} - \frac{5}{2}\right)v_i'\frac{\partial \ln T}{\partial x_i}$$
$$= \frac{T}{\tau_\mathrm{p}}\frac{\partial^2 f}{\partial v_i'\partial v_i'} + \frac{1}{\tau_\mathrm{p}}\frac{\partial v_i' f}{\partial v_i'} - \zeta(f - f_0), \quad (4)$$

where $\mathrm{d}/\mathrm{d}t = \partial/\partial t + \langle v_i\rangle \partial/\partial x_i$, $\langle u_i\rangle$ and u_i' are fluid mean and fluctuating velocities, respectively, $T = \frac{1}{3}\langle v_i'v_i'\rangle$ is the pseudo-temperature of particles, $\zeta = \frac{96}{5}Cd_\mathrm{p}^{-1}\sqrt{T/\pi}$ is collision frequency, $f_0 = C(2\pi T)^{-3/2}\exp(-v'^2/2T)$ is the Maxwellian velocity distribution, d_p is the particle diameter, and α and β are two parameters describing particle-eddy interactions[12]

$$\alpha = St^{-1}(1+St)^{-1},\ \beta = (1+St)^{-1} \tag{5}$$

where St is the Stokes number defined as $St = \tau_p / T_{L_p}$, and T_{L_p} T_{Lp} is the Lagrangian integral time-scale seen by a particle.

Integrating eq. (4) over the whole velocity space, the conservation equations of mass, momentum and fluctuation kinetic energy of particle phase are derived

$$\frac{\mathrm{d}C}{\mathrm{d}t} + C\frac{\partial \langle v_i \rangle}{\partial x_i} = 0, \tag{6a}$$

$$\frac{\mathrm{d}\langle v_i \rangle}{\mathrm{d}t} - \frac{\langle u_i \rangle - \langle v_i \rangle + \tau_p \langle F_i \rangle}{\tau_p} + \frac{\partial \langle v_i' v_k' \rangle}{\partial x_k} + \frac{\varepsilon_{ik}}{\tau_p}\frac{\partial \ln C}{\partial x_k} = 0, \tag{6b}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{3}{2}T + \frac{1}{C}\frac{\partial}{\partial x_i}\left(C\frac{1}{2}\langle v_i' v_l' v_l' \rangle\right) + \langle v_i' v_l' \rangle \frac{\partial \langle v_i \rangle}{\partial x_l} + \frac{3T - \beta\langle u_l' u_l' \rangle}{\tau_p} = 0, \tag{6c}$$

where $\varepsilon_{ik} = \tau_p(\langle v_i' v_k' \rangle + \alpha \langle u_i' u_k' \rangle)$ is particle diffusion tensor. The corresponding closure relations are[13]

$$\langle v_i' v_k' \rangle = T\delta_{ik} - \frac{\tau_p T}{2+\zeta\tau_p}\left(\frac{\partial \langle v_i \rangle}{\partial x_k} + \frac{\partial \langle v_k \rangle}{\partial x_i} - \frac{2}{3}\frac{\partial \langle v_l \rangle}{\partial x_l}\delta_{ik}\right) + \frac{\beta}{2+\zeta\tau_p}\left(\langle u_i' u_k' \rangle + \langle u_k' u_i' \rangle - \frac{2}{3}\langle u_l' u_l' \rangle \delta_{ik}\right), \tag{7a}$$

$$\langle u_i' v_k' \rangle = \beta \langle u_i' u_k' \rangle, \tag{7b}$$

$$\langle v'^2 v_i' \rangle = -\frac{5\tau_p}{3+\zeta\tau_p}\left(\frac{3}{2}T\delta_{ik} + \alpha\langle u_i' u_k' \rangle\right)\frac{\partial T}{\partial x_k}. \tag{7c}$$

Eqs. (6) and (7) form a closed kinetic model for particle phase in turbulent two-phase flows. It accounts for both effects of fluid velocity fluctuations and interparticle collisions, and is close to that of Zaichik et al.[11]. As $St \to \infty$, $\alpha \to 0$, and $\beta \to 0$, the model reduces to that for rapid granular flows with interstitial fluid effects. As $St \to 0$, $\alpha\tau_p \to T_{L_p}$, $\beta \to 1$, and $\zeta\tau_p \ll 1$, particle motion is controlled by fluid turbulent fluctuations and eq. (7) correctly approaches the closure for fluid turbulence.

2 Sediment diffusion equation

Consider two-dimensional steady, uniform open-channel flows, i.e. $\partial/\partial t = 0$, $\partial/\partial x = 0$, and $\langle v_y \rangle = \langle u_y \rangle = 0$, where x, y denote the streamwise and vertical directions, respectively. The vertical force $\langle F_y \rangle$ consists of the effect tive gravity and a lift force F_L acting on particles

$$\langle F_y \rangle = F_L - (1-\rho_f/\rho_s)g, \tag{8}$$

where g is the gravitational acceleration, and ρ_f and ρ_s are fluid and particle densities, respectively.

The vertical momentum equation for particles reads

$$-C\tau_p\left(1-\frac{\rho_f}{\rho_s}\right)g + C\tau_p\left(F_L - \frac{\partial}{\partial y}\langle v_y'^2 \rangle\right) = \varepsilon_{yy}\frac{\partial C}{\partial y}. \tag{9}$$

The lift force on a spherical particle is comprised of two parts, one is due to fluid shear and the other the particle rotation. The shear-induced lift is usually more important than the latter one[14]. Thereafter, only the shear lift is considered. McLaughlin[15] extended Saffman's lift formula by removing the limitation of $Re_p = d_p|\boldsymbol{u}-\boldsymbol{v}|/\nu_f << Re_G^{1/2}$ and involving the effect of the presence of a wall, where Re_G is the Reynolds number defined in terms of d_p and local velocity gradient,

$$F_L = \frac{27}{2\pi^2}\frac{\rho_f}{\rho_s}\frac{\nu_f}{d_p}\sqrt{\frac{1}{\nu_f}\frac{\mathrm{d}\langle u_x \rangle}{\mathrm{d}y}}(\langle u_x \rangle - \langle v_x \rangle)J, \tag{10}$$

where ν_f is fluid kinematic viscosity; J is a function of particle slip velocity, fluid velocity gradient and the distance of particle center from the wall, with its maximum value of 2.225 corresponding to Saffman's formula (see McLaughlin for details[15]). Lataste et al.[16] demonstrated its good performance for heavy particle in a turbulent boundary layer.

Particle relaxation time τ_p is[11]

$$\tau_p = \frac{1}{18}\frac{\rho_s}{\rho_f}\frac{d_p^2}{\nu_f(1+0.15Re_p^{0.687})} \quad (Re_p < 1000). \tag{11}$$

Following its definition, ε_{yy} in eq. (9) is written as

$$\varepsilon_{yy} = T_{L_p}\left\langle u_y'^2 \right\rangle\left[1 + St\left(\left\langle v_y'^2 \right\rangle \Big/ \left\langle u_y'^2 \right\rangle - \beta\right)\right]. \tag{12}$$

3 Comparison with experimental data

Eq. (9) is compared with experimental data of Einstein et al.[17] for further analysis of the vertical diffusion of natural sands with different sizes. The balance equations of x momentum and fluctuation kinetic energy are solved simultaneously. Profiles of fluid velocity and eddy viscosity are determined with established empirical functions for clear-water because fluid turbulence modulation is dependent upon sediment concentration. The relations for the fluid structure proposed by Nezu et al.[18] are used[6]. Due to the difficulty in precisely specifying T_{L_p}, it is approximated with T_L[19].

Boundary conditions, formerly developed by Jenkins[20] and used by Fu et al.[21] in a horizontal duct flow, are assumed

$$\nu_s \left.\frac{\partial v}{\partial y}\right|_b = \mu_w T_b, \tag{13a}$$

$$-\lambda_s \left.\frac{\partial T}{\partial y}\right|_b = -\frac{3}{8}T_b\sqrt{3T_b}\left\{\frac{7}{2}(1+e_w)\mu_w^2-(1-e_w)\right\}, \qquad (13b)$$

where $\nu_s=\tau_p T/(2+\nu\tau_p)$, $\lambda_s=\frac{15}{4}\tau_p T/(3+\zeta\tau_p)$, the subscript "b" refers to the wall; e_w = 0.7 and μ_w = 0.23 are the coefficients of restitution and sliding friction for a particle colliding with the wall, respectively[21]. Concentration boundary condition is specified with the experimental data at the nearest measured point adjacent to the wall, and zero gradient conditions are set for $\langle v_x \rangle$ and T at the free water surface.

Einstein et al.[17] conducted three sets of experiments in an open-channel, corresponding to sand sizes of 0.274, 0.94, and 1.3 mm in diameter, respectively, to explore the diffusion characteristics of sediments in a region under the influence of the wall (0.03< y/H <0.4).

Figure 1 illustrates profiles of the predicted mass-weighted mixture velocity u_m, where u^* is the shear velocity, $\eta = y/H$ is dimensionless flow depth, H is flow depth, and u_m is defined as

$$u_m=\frac{\rho_s C\langle v_x\rangle+\rho_f(1-C)\langle u_x\rangle}{\rho_s C+\rho_f(1-C)}. \qquad (14)$$

In the low-concentration flows considered, the prediction of the kinetic model agrees well with the experimental data through using the log-wake law for fluid mean velocity. Fig. 2 presents the predicted concentration profiles. For the fine (d_p=0.274 mm), medium (d_p=0.94 mm) and large (d_p=1.3 mm) sediments, the predictions are also in good agreement with the measured, respectively.

In eq. (9), the lift force and the gradient in particle vertical velocity fluctuation affect the characteristics of sediment vertical distribution. If the sum of them exceeds the effective gravity, $\partial C/\partial y>0$, i.e. the profile of pattern I appears.

Figure 3 shows the relative importance of $F_L-\partial\langle v_y'^2\rangle/\partial y$ against the effective gravity above the reference level. In the region of 0.2< η <0.4, the ratio of

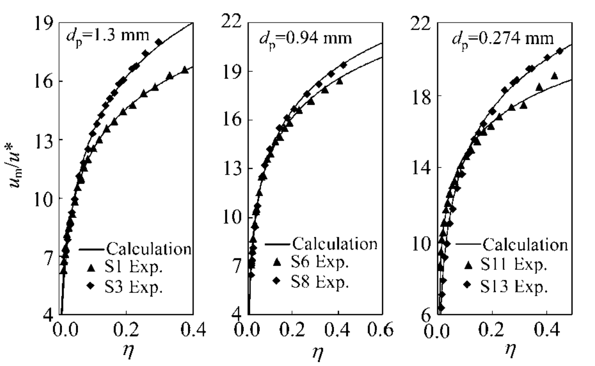

Fig. 1. Profiles of calculated versus measured mixture velocity u_m under the flow conditions of Einstein et al.[17].

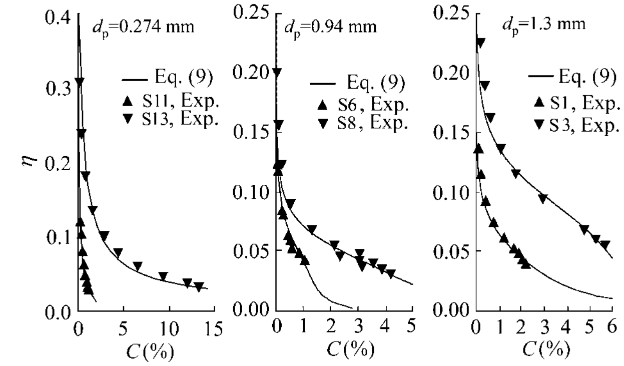

Fig. 2. Profiles of sediment concentration C under the flow conditions of Einstein et al.[17].

$F_L-\partial\langle v_y'^2\rangle/\partial y$ to the effective gravity does not exceed 2%. For 0.03< η <0.2, however, the ratio becomes distinctly large in magnitude, especially for the medium and coarse sediments. For the fine sediments, $F_L-\partial\langle v_y'^2\rangle/\partial y<0$ occurs below $\eta=0.1$, while for the larger sediments $F_L-\partial\langle v_y'^2\rangle/\partial y \geqslant 0$ holds. As a result, $F_L-\partial\langle v_y'^2\rangle/\partial y$ enhances the gravitational settling of the fine sediments in this region, but still reduces that of medium and large sediments. Moreover, the calculation shows that F_L is negligible compared with $-\partial\langle v_y'^2\rangle/\partial y$ above the reference level, which agrees with the previous argument that shear lift force only plays an important role close to the wall where strong velocity gradient exists[22]. It implies that particle-wall interactions change the vertical velocity fluctuation of the medium and large sediments far from that of the fluid in the near wall region. Accordingly, $F_L-\partial\langle v_y'^2\rangle/\partial y$ need be accounted for describing sediment diffusion below η = 0.2 in the flows considered.

The Stokes number St, a parameter describing sedi-

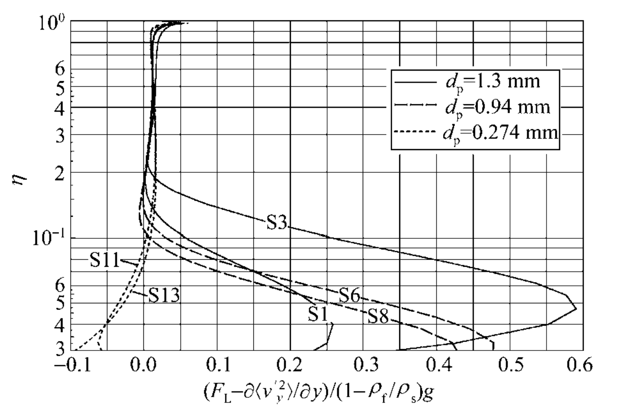

Fig. 3. Profiles of $F_L-\partial\langle v_y'^2\rangle/\partial y$ to the effective gravity ratio under the flow conditions of Einstein et al.[17].

ment inertia, affects the magnitude of sediment diffusion coefficient directly (eq. (12)). Because $\langle v_y'^2 \rangle \to \beta \langle u_y'^2 \rangle$ and T_{L_p} approach to the Lagrangian integral time-scale of fluid particles T_L as $St \ll 1$, ε_{yy} is close to $T_L \langle u_y'^2 \rangle = \nu_f^t$, which suggests that sediment particles are so fine that they follow fluid turbulent fluctuations adequately. However, for sediments with finite inertia, $\varepsilon_{yy} = \nu_f^t$ does not hold due to $\langle v_y'^2 \rangle \neq \beta \langle u_y'^2 \rangle$ and the difference between T_{L_p} and T_L. Fig. 4 demonstrates the profiles of sediment diffusion coefficient ε_{yy} in a dimensionless form. For the fine sediments, ε_{yy} is slightly greater than ν_f^t for η <0.1. As a comparison, $\varepsilon_{yy} > \nu_f^t$ becomes distinct below η=0.2 for the medium and large sediments. In addition, in the measured region of 0.03 < η < 0.4, ε_{yy}/ν_f^t increases with the decrease in η, which demonstrates the effects of particle-wall interactions. It can be noted that the larger size sediments corresponds to the larger ε_{yy}/ν_f^t, which agrees with the general understanding of ε_{yy}/ν_f^t, increasing with the increment of particle velocity ω[1,2].

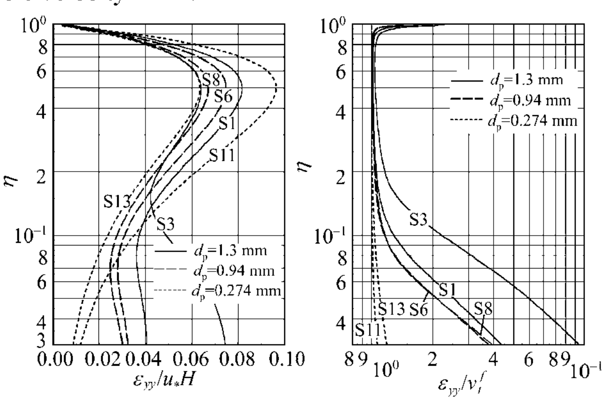

Fig. 4. Profiles of ε_{yy} under the flow conditions of Einstein et al.[17].

In Fig. 5, the profiles of the Stokes number St are presented. From this figure, St of fine sediments is far less than that of the medium and large sediments; St of the largest sediments is greater than that of smaller sediments. In the region of 0.03<η<0.4, St increases with the decrease in η. Considering the corresponding profiles of ε_{yy} in Fig. 4, it can be inferred that ε_{yy}/ν_f^t tends to increase with St increasing. In contrast, the diffusion coefficient proposed by Czernuszenko[6], involving an additional drift diffusion coefficient, is

$$\frac{\varepsilon_{yy}}{\nu_f^t} = 1 + \frac{\pi d_p}{2T_L\sqrt{\langle u_y'^2 \rangle}}\sqrt{\frac{\langle v_y'^2 \rangle}{\langle u_y'^2 \rangle}}. \quad (15)$$

With the consideration of $\langle v_y'^2 \rangle / \langle u_y'^2 \rangle \approx \beta = (1+St)^{-1}$, for fine sediments, eq. (15) predicts a decreasing ε_{yy}/ν_f^t with the increase in St as d_p and flow conditions keep unchanged. It contradicts with the present results and the statement of ε_{yy}/ν_f^t increasing with the increase in ω[2].

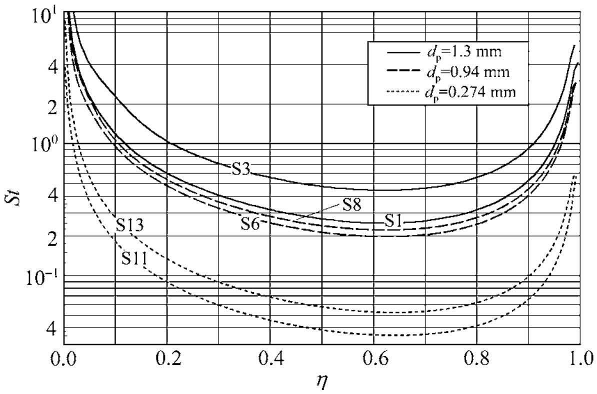

Fig. 5. Profiles of St under the flow conditions of Einstein et al.[17].

4 Conclusions

A closed kinetic model was employed to analyze the vertical diffusion of natural sands with different sizes in an open-channel flow. It can be drawn that:

(1) the diffusion coefficient ε_{yy} of the medium and large sediments distinctly exceeds the fluid eddy viscosity ν_f^t especially in the near wall region, while ε_{yy} of fine sediments is approximately equal to ν_f^t. In the measured region of 0.03 < η < 0.4, ε_{yy}/ν_f^t increases with the decrease in η.

(2) $F_L - \partial \langle v_y'^2 \rangle / \partial y$ changes sediment gravitational settling remarkably below y/H = 0.2, and need to be accounted for describing sediment diffusion in this region.

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