J. Ital. Statist. Soc. (1997)
3, pp. 233-243

A NOTION OF COHERENT PREVISION FOR ARBITRARY RANDOM QUANTITIES

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Summary

In this paper the notion of coherent prevision given by de Finetti for bounded random quantities is extended to arbitrary ones. It is shown that the main properties of de Finetti's prevision are preserved by the extended notion and some of his conjectures on previsions of unbounded random quantities are proved. Finally, a representation theorem for finite previsions in terms of Riemann-Stieltjes integral is given for random quantities defined on a common partition of the certain event and an ensuing possible interpretation for modelling real situations is discussed.

Keywords and phrases: Coherent prevision; Finite additivity; Unbounded random quantities; Riemann-Stieltjes integral; Integral representation.

1. Introduction

A preliminary probability evaluation on a convenient set of events is required in order to calculate the expectation or, more generally, other moments of a random quantity (r.q.). It often happens in many practical problems that some moments are estimated by methods which do not require a previous assignment of the probability needed for their computation. Also for this reason, de Finetti (1970) replaces, for bounded r.q.'s, the notion of expectation with that of coherent prevision, whose evaluation can be given directly, i.e. it does not necessarily arise from a probability distribution. He is also concerned with previsions of unbounded r.q.'s, but without precisely defining them and basing his arguments on an intuitive extension of the properties of coherent prevision for bounded r.q.'s.

In a rencent paper, P. Berti *et al.* (1994) proposed a definition of coherent prevision for arbitrary r.q.'s extending that of de Finetti, but requiring the introduction of the notion of probability.

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Work performed under the auspicies of the MURST gran «Statistical models: Probabilistic foundations and procedures for inference and decisions».

In this paper we propose a definition very similar to de Finetti's which allows a direct evaluation of previsions, referred this time to arbitrary r.q.'s. Our prevision satisfies the properties of internality, linearity, monotonicity and extendibility, as does the prevision of bounded r.q.'s. Moreover it satisfies the same structure properties, with some modification for convexity (Section 3).

In Section 4, the above mentioned results on previsions for unbounded r.q.'s already conjectured by de Finetti are rigorously proved in our framework. Such results essentially concern the comparison between the notion of prevision and that of expectation, the latter meant as a Riemann-Stieltjes integral.

Provided that a probability is given on the set of events logically dependent on a partition of the certain event, we discuss in Section 5 an integral representation theorem for the previsions of the r.q.'s defined on the same partition and whose expectation is finite, showing that expectation is the evaluation which lets us give finite prevision to the largest set of these r.q.'s. This result has a remarkable interest for applications, because it explains to some extent (see 5.3) why choosing expectation as prevision is preferable to other possible choices.

In Section 2 we recall the notion of coherent prevision for bounded r.q.'s and introduce some useful notation.

2. Preliminaries

In describing and evaluating uncertainty we follow the approach proposed by de Finetti. According to de Finetti, events are defined by propositions of logic in a state of information – a proposition of logic describing the data of the problem –. Two propositions define the same event in a state of information if the subject is able to deduce from the data (the state of information) that they have the same truth value (see de Finetti (1938) p. 1 and Crisma (1996) for a more rigorous approach).

As far as r.q.'s are concerned, de Finetti considers them as *well determined* numbers, generally unknown because of lack of information. In more formal terms, we describe here r.q.'s as usual, by means of real-valued maps defined on a partition $\wp$ of the certain event Ω, assuming that two maps $x_1 : \wp_1 \to \mathbf{R}, x_2 : \wp_2 \to \mathbf{R}$ define the same r.q. (are equivalent) if and only if we have $x_1(\omega_1) = x_2(\omega_2)$ for each possible event $\omega_1 \wedge \omega_2$ belonging to the product partition $\wp_1 \wedge \wp_2$. This assumption assures that both maps identify the same well determined number whatever is the true event in $\wp_1 \wedge \wp_2$ (for further results on the topic see Crisma (1990)). It is worth remarking that the r.q.'s of any set can be described by maps defined on a common domain: the product partition of the individual partitions related to the given r.q.'s.

As for the evaluation of *bounded* r.q.'s, de Finetti (1970) p. 106 gives the following definition of coherent prevision:

2.1. *Definition*

Let $\mathcal{C}$ be a set of bounded r.q.'s. A real-valued map P defined on $\mathcal{C}$ is said to be a *coherent prevision* if and only if for each $X_1, \ldots, X_n \in \mathcal{C}$, $c_1, \ldots, c_n \in \mathbf{R}$, the following inequalities hold:

$$\inf \sum_{i=1}^{n} c_i X_i \leq \sum_{i=1}^{n} c_i P(X_i) \leq \sup \sum_{i=1}^{n} c_i X_i .$$

The definition may be interpreted in a betting scheme where a *betting-system* on $X_1, \ldots, X_n$ with coefficients $c_1, \ldots, c_n$ gives rise to $\sum_{i=1}^{n} c_i X_i$ as a *pay-off* and $\sum_{i=1}^{n} c_i P(X_i)$ as a *price*.

The notion of coherent probability is included in this definition by identifying the probability of an event E with the prevision of its indicator $|E|$. We shall use in the sequel the same symbol to denote both a set of events and the set of their corresponding indicators. In particular, the symbol $\mathcal{A}_L(\wp)$ will denote either the algebra of the logically dependent events in a partition $\wp$ or the set of their indicators. By identifying events with subsets of $\wp$, $\mathcal{A}_L(\wp)$ can be also viewed as the power set of $\wp$.

For the properties of coherent prevision see for instance Regazzini (1983), Holzer (1985).

3. Coherent previsions of unbounded random quantities

The definition of coherent prevision 2.1 might also be applied to sets of arbitrary r.q.'s, being in this way all previsions real-valued, even when a r.q. is unbounded. However, as shown in the next example this implies that important properties of previsions, such as monotonicity and extendibility do not hold.

Example. Let N be a r.q. taking values 0, 1, 2, ... If the asymptotic distribution is introduced, then the bounded r.q. $X_h = N|N \leq h| + h|N > h|$ satisfies the conditions $X_h \leq N$, $P(X_h) = h$ (since $P(N > h) = 1$) for each h. If $P(N) < +\infty$, then there exists h such that $P(X_h) > P(N)$ and the monotonicity property does not hold. In order to maintain it the evaluation $P(N) = +\infty$ must be allowed, and also $P(-N) = -\infty$ to preserve the linearity property. □

Hence it is necessary to include $+\infty$ and $-\infty$ in the set of prevision values, that becomes $\bar{\mathbf{R}} = \mathbf{R} \cup \{-\infty, +\infty\}$. Order and operations are extended from $\mathbf{R}$ to $\bar{\mathbf{R}}$ in the usual way. Recall that any operation on $\bar{\mathbf{R}}$ is *well defined* when it cannot be reduced to the forms $(\pm\infty)$-$(\pm\infty)$.

According to these arguments, we extend Definition 2.1 in the following way:

3.1. *Definition*

Let $\mathcal{C}$ be a set or r.q.'s. A map P: $\mathcal{C} \to \bar{\mathbf{R}}$ is said to be an *extended coherent prevision* (briefly a *coherent prevision* or simply a *prevision*) if and only if for each $X_1, \ldots, X_n \in \mathcal{C}$, $c_1, \ldots, c_n \in \mathbf{R}$ such that $\sum_{i=1}^{n} c_i P(X_i)$ is well defined, the following inequalities hold:

$$\inf \sum_{i=1}^{n} c_i X_i \leq \sum_{i=1}^{n} c_i P(X_i) \leq \sup \sum_{i=1}^{n} c_i X_i .$$

The next theorems state that, in this more general setting, the main properties of the coherent prevision for bounded random quantities are preserved. The proofs can easily be obtained by suitably adapting the arguments for the bounded case (see Crisma *et al.* (1997)).

3.2. *Theorem*

Let X, Y be r.q.'s, P a coherent prevision. Then

(i) $\inf X \leq P(X) \leq \sup X$ (internality),

(ii) $P(X + Y) = P(X) + P(Y)$, whenever $P(X) + P(Y)$ is well defined, $P(sX) = sP(X)$, for each $s \in \mathbf{R}$ (linearity),

(iii) $P(X) \geq P(Y)$, whenever $X \geq Y$ (monotonicity).

Note that if $\mathcal{C}$ is a singleton, then P is a coherent prevision on $\mathcal{C}$ if and only if the internality property holds. Moreover, if the r.q.'s in $\mathcal{C}$ are all bounded, Definitions 3.1 and 2.1 are equivalent. Observe for this that the internality property implies that the prevision of each bounded r.q. is finite.

As in the case of bounded r.q.'s, there is a characterisation for a coherent prevision on $\mathcal{C}$, if $\mathcal{C}$ is a linear space.

3.3. *Theorem*

Let $\mathcal{C}$ be a linear space of r.q.'s. Then P: $\mathcal{C} \to \bar{\mathbf{R}}$ is a coherent prevision if and only if the properties of internality and linearity hold.

If $\mathcal{C}$ is a set of bounded r.q.'s, the set of all coherent previsions on $\mathcal{C}$ is convex and closed under limit operations. The latter property holds also for sets of unbounded r.q.'s. Convexity fails, instead, for the plain reason that not all mixtures are well defined operations. Nevertheless, some subsets of previsions on $\mathcal{C}$ are convex, as stated in the next theorem.

3.4. *Theorem*

Let $\mathcal{C}$ be a set of r.q.'s.

(i) If $(P_k)_{k\in N}$ is a sequence of coherent previons on $\mathcal{C}$ such that $\lim_{k\to+\infty} P_k(X)$ exists for each $X \in \mathcal{C}$, then $P = \lim_{k\to+\infty} P_k$ is a coherent prevision on $\mathcal{C}$.

(ii) If P_1, P_2 are coherent previsions on $\mathcal{C}$ such that $P_1 + P_2$ is well defined on $\mathcal{C}$ and $\lambda_1, \lambda_2 \geq 0$ such that $\lambda_1 + \lambda_2 = 1$, then the map $P = \lambda_1 P_1 + \lambda_2 P_2$ is a coherent prevision on $\mathcal{C}$.

The fundamental extension theorem holds also for the extended previsions. The proof is close to that given by de Finetti in the case of coherent probability (de Finetti (1972b), pp. 78-79), with the inductive step as given in Holzer (1985).

3.5. *Extension Theorem*

Let $\mathcal{C} \subset \mathcal{C}'$ be sets of r.q.'s, P a coherent prevision on $\mathcal{C}$. Then there exists a coherent prevision P' on $\mathcal{C}'$ such that $P'|_{\mathcal{C}} = P$.

4. Prevision and expectation

It is well known that if the distribution function F_X of a bounded r.q. X is given, then the prevision of X is uniquely determined by its *expectation* $E(X) = \int_{-\infty}^{+\infty} x\, dF_X(x)$ (see for instance Gigante (1994), Regazzini (1983))[1]. This result does not hold any longer if X is unbounded, in the sense that the choice of a coherent prevision for X is not generally uniquely determined by the knowledge

1. In the sequel the symbol $\int_a^b g(x)dF(x)$ will always indicate a Riemann-Stieltjes integral, possibly in a generalized sense.

2. Given a distribution function F, the two limits $\lim_{x\to+\infty}(1 - F(x)) = 1 - F(+\infty)$, $\lim_{x\to-\infty} F(x) = F(-\infty)$ are called *adherent probability at* $+\infty$, *at* $-\infty$ respectively.

of its distribution function. In this section we study the relation between the prevision of a r.q. X (as defined in 3.1) and the corresponding distribution function, when it is given. We prove that when a r.q. X is one-side unbounded and a positive adherent probability at infinity[2] exists, the prevision of X must be $+\infty$ ($-\infty$) if X is upper (lower) unbounded. Otherwise, there exists the expectation of X, finite or infinite, and it is a lower (upper) bound for its coherent prevision, when X is upper (lower) unbounded. Moreover, if X is two-side unbounded, any $p \in \bar{\mathbf{R}}$ can be chosen as its prevision, whatever the distribution function of X is. These results have already been stated by de Finetti (1970) (see p. 155 and p. 289), but by means of intuitive arguments.

4.1. *Theorem*

Let X be a r.q. such that $\inf X > -\infty$, $\sup X = +\infty$ and $F_X(x) = P(X \le x)$ its distribution function. Then the following statements hold:

(i) If $F_X(+\infty) < 1$, then $P(X) = +\infty$ is the only coherent extension of P to $\{X\}$.

(ii) If $F_X(+\infty) = 1$, then $P(X)$ is a coherent extension of P to $\{X\}$ if and only if

$$P(X) \ge \int_a^{+\infty} x\, dF_X(x), \quad \text{for} \quad a < \inf X. \tag{1}$$

Remark. If a coherent prevision P on $\mathcal{C}$ allows a unique extension to $\mathcal{C}'$, in order to study the constraints imposed by P to the extensions on a set $\mathcal{C}''$, one can obviously consider P as defined on $\mathcal{C} \cup \mathcal{C}'$. We use this technique in the proof of the theorem since we consider bounded r.q.'s functions of a given r.q. X, whose distribution function, and hence the previsions, are determined from the distribution function of X.

Proof. Preliminarily, we observe that if $x > \max\{0, \inf X\}$ and $Y_x = X|X \le x| + x|X > x|$, then Y_x is bounded and $Y_x \le X$. Therefore, the prevision $P(X)$ of X must be such that $P(X) \ge P(Y_x) = E(Y_x)$ and hence, if $a < \inf X$, we have

$$P(X) \ge \int_a^x u\, dF_X(u) + x\big(1 - F_X(x)\big) \ge \int_a^x u\, dF_X(u) \tag{2}$$

where $\int_a^x u\, dF_X(u)$, $x > 0$, is an increasing function of x.

Now, in case (i), we have $\lim_{x \to +\infty} x\big(1 - F_X(x)\big) = +\infty$ and thus the thesis follows from the first inequality of (2).

In case (ii), since by (2) $P(X) \geq \int_a^x u\, dF_X(u)$ for any $x > \max\{0, \inf X\}$, as $x \to +\infty$ we obtain $P(X) \geq \int_a^{+\infty} x\, dF_X(x)$, i.e. (1) is a necessary condition. Condition (1) is also sufficient if $\int_a^{+\infty} x\, dF_X(x) = +\infty$. Its sufficiency must still be proved if $\int_a^{+\infty} x\, dF_X(x) < +\infty$.

Let for this $P(X) \geq \int_a^{+\infty} x\, dF_X(x)$ and consider a betting system whose pay-off and price are

$$Y = X + \sum_{h=1}^{n} c_h |X \leq x_h|, \quad P(X) + \sum_{h=1}^{n} c_h F_X(x_h),$$

where c_h, x_h, $h = 1, \ldots, n$, are arbitrary real numbers. Since $\sum_{h=1}^{n} c_h |X \leq x_h|$ is bounded, we have $\sup Y = \sup X = +\infty \geq P(X) + \sum_{h=1}^{n} c_h F_X(x_h)$. To complete the proof, we still have to prove that $\inf Y \leq P(X) + \sum_{h=1}^{n} c_h F_X(x_h)$. Let x be a value of X such that $x > max\{0, x_1, \ldots, x_n\}$ and $Z_x = X|X \leq x| + x|X > x| + \sum_{h=1}^{n} c_h |X \leq x_h|$. Then $Z_x \leq Y$ and the range of Z_x is a subset of the range of Y. Therefore we have $\inf Z_x = \inf Y$. Moreover, since Z_x is bounded, $P(Z_x) = E(Z_x)$. Then for $a < \inf X$ we obtain

$$\inf Y = \inf Z_x \leq P(Z_x) = \int_a^x u\, dF_X(u) + x(1 - F_X(x)) + \sum_{h=1}^{n} c_h F_X(x_h).$$

Observing that

$$\int_a^x u\, dF_x(u) + x(1 - F_X(x)) = \int_a^x u\, dF_X(u) + \int_x^{+\infty} x\, dF_X(u) \leq \int_x^{+\infty} u\, dF_X(u),$$

we get $\inf Y \leq \int_a^{+\infty} x\, dF_X(x) + \sum_{h=1}^{n} c_h F_X(x_h) \leq P(X) + \sum_{h=1}^{n} c_h F_X(x_h)$, which completes the proof. □

Symmetrically, this result can be easily extended to lower unbounded r.q.'s. In fact the following theorem holds:

4.2. *Theorem*

Let X be a r.q. such that $\inf X = -\infty$, $\sup X < +\infty$, $F_X(x) = P(X \leq x)$ its distribution function. Then the following statements hold:

(i) If $F_X(-\infty) > 0$, then $P(X) = -\infty$ is the only coherent extension of P to $\{X\}$.

(ii) If $F_X(-\infty) = 0$, then $P(X)$ is a coherent extension of P to $\{X\}$ if and only if

$$P(X) \leq \int_{-\infty}^{b} x \, dF_X(x), \quad \text{for } b \geq \sup X.$$

The next theorem states that if a r.q. is two-side unbounded, then any element of $\bar{\mathbf{R}}$ is a coherent prevision for it, whatever is its distribution function.

4.3. *Theorem*

Let X be a r.q. such that $\inf X = -\infty$, $\sup X = +\infty$, $F_X(x) = P(X \leq x)$ its distribution function. Then any $P(X) = p \in \bar{\mathbf{R}}$ is a coherent extension of P to $\{X\}$.

5. A representation theorem for finite previsions

Consider the set $\mathcal{X}$ of all the r.q.'s defined on a given partition $\wp$ of Ω, and suppose a coherent probability (prevision) on $\mathcal{A}_L(\wp)$ is assigned, thus determining the distribution function of each r.q. of $\mathcal{X}$. Let $\mathcal{X}_E$ be the subset of $\mathcal{X}$ of the r.q.'s whose expectation is finite. The following theorem states that whenever the prevision is forced to be finite for each r.q. in $\mathcal{X}_E$, expectation is the only prevision on $\mathcal{X}_E$ coherent with that given on $\mathcal{A}_L(\wp)$.

5.1. *Theorem*

Let $\mathcal{X}$ be the set of all the r.q.'s defined on a partition $\wp$, π a probability on $\mathcal{A}_L(\wp)$, $\mathcal{X}_E = \left\{X \in \mathcal{X}\colon F_X(+\infty) - F_X(-\infty) = 1, \left|\int_{-\infty}^{+\infty} x \, dF_X(x)\right| < +\infty\right\}$, P a coherent extension of π to $\mathcal{X}_E$ such that $|P(X)| < +\infty$. Then $P(X) = \int_{-\infty}^{+\infty} x \, dF_X(x)$, for any $X \in \mathcal{X}_E$.

The theorem is analogous to Theorem (2.14) given by Berti *et al.* (1994) with respect to their definition of coherent prevision. It can be obtained as a particular case of Theorem (2.13) of the above quoted paper, on noting that $\mathcal{X}_E$ is the set of D-integrable functions with respect to $(\wp, \mathcal{A}_L(\wp), \pi)$ and that for $X \in \mathcal{X}_E$ the D-integral of X coincides with the value of the Riemann-Stieltjes integral (for the notion of D-integral see Bhaskara Rao and Bhaskara Rao, 1983). However a direct proof of the result, which does not require the introduction of the notion of D-integral, is given in Crisma *et al.* (1997).

Moreover, we observe that the result is related to Corollary 3 by Dubins (1977), from which one can deduce that the conditions of Theorem 5.1 are equivalent to requiring that the extension of the prevision, from the set of all bounded r.q.'s $\mathcal{X}_B$ to $\mathcal{X}_E$, be regular, that is

$$P(X) = \sup\{P(Z), Z \in \mathcal{X}_B, Z \leq X\}.$$

5.2. *Remark*

Giving a probability π on $\mathcal{A}_L(\wp)$, and therefore a distribution function for each $X \in \mathcal{X}$, by 4.1, 4.2, there is only one extension of π to the set of all one-side unbounded r.q.'s having either positive adherent probability at infinity or non finite Riemann-Stieltjes integral. According to the theorem proved above, if we force the prevision to be finite for each r.q. in $\mathcal{X}_E$, then the extension of π to this set is still unique. On noting that in this hypothesis the extension to X^+ and X^- is unique for any $X \in \mathcal{X}$, by the linearity property, the prevision of X is unique as well, provided that $P(X^+)$, $P(X^-)$ are not both $+\infty$. Setting $\mathcal{X}_I = \{X \in \mathcal{X}: P(X^+) = P(X^-) = +\infty\}$, we can finally state that, if we require the prevision on $\mathcal{X}_E$ to be finite, then π has a unique extension $\mathcal{X}$-$\mathcal{X}_I$. □

5.3. *Remark*

In real situations the r.q.'s one has to consider are usually bounded. Nevertheless, for mathematical convenience it is often useful to introduce models in which some bounded r.q.'s are replaced with convenient unbounded ones. Since bounded r.q.'s have finite previsions, it is clearly desirable that these unbounded r.q.'s obey the same condition. Recalling 5.2, the r.q.'s which can be given finite prevision are those of $\mathcal{X}_E$ and possibly of $\mathcal{X}_I$. Nevertheless, the r.q.'s of $\mathcal{X}_I$ may hardly represent real situations, since their distribution functions place too much probability near both $+\infty$ and $-\infty$. Hence, the r.q.'s to model real problems should be preferably chosen in $\mathcal{X}_E$.

Theorem 5.1 states that if one wants to give finite prevision to each r.q. in $\mathcal{X}_E$, then the only admissible choice is the expectation. In other words, any coherent prevision, which does not coincide with the expectation for some r.q. in $\mathcal{X}_E$, can not be finite for all r.q.'s of this set. This means that, in order to allow previsions to be finite for as many r.q.'s as possible, the choice of expectation as prevision on $\mathcal{X}_E$ is the only one that assures this.

We finally observe that even when a probability is only partially assigned on $\mathcal{A}_L(\wp)$, but the distribution function of a r.q. X is given and the corresponding expectation is finite, if we wish to keep the possibility of giving finite prevision to the largest set of r.q.'s, the expectation of X must still be chosen as its prevision. In fact, in any extension of π to $\mathcal{A}_L(\wp)$ any other choice reduces, as seen above, the set of r.q.'s whose prevision can be given finite values.

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