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MARTENSITIC FORMATION AND INTERNAL FRICTION

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Coherent phonons which start martensitic formation are the solutions of the Duffing equation. By using this idea the frictional force was included in this undamped nonlinear equation. Numerical results were obtained for Au–Cu–Zn (30 at% Cu, 47 at% Zn) and In–Tl (21 at% Tl) single crystal. Dependence of vibrational amplitude of the phonons on frequency of driving force was researched for various temperatures in the austenitic range. It was found that damping effects change that strongly as the martensitic transformation temperature, M_s, is approached. The proposed model in this study can explain the jump phenomenon in the response curves more clearly and realistically when damping is included.

1. Introduction

Nonlinear differential equation systems have become increasingly important in metal physics. Martensitic transformation is associated with certain thermodynamical features relating to phonons: This transformation occurs with the velocity of sound, pre-transformation vibrational mode softening, etc. Therefore, in recent years several phonon models relevant to martensitic transformation have been presented [1]. Zhang has stated that the coherent phonons will be able to start martensitic transformation [2]. In his model the researcher has neglected the frictional effects which have a retarding role against leaving of interface from austenite. In this paper pre-martensitic internal friction effects, contrary to Zhang's investigation were included in the nonlinear differential equation of atomic motion. The main result of this study is that the damping parameter affects considerably the vibrational amplitude of the phonons during the martensitic transformation. The new model improved in this paper can explain the characteristics of the transformation and reverse transformation and describes rather well hysteresis phenomena associated with the transformation.

2. Forced oscillations

2.1 Undamped equation of atomic motion

According to the theory of coherent phonon starting martensitic transformation, a phenomenon starting transformation depends on cooperative atomic movement in a potential well of coherent phonon waves propagated in austenite phases.

Therefore, it is supposed that there are various regions including the lattice displacement waves above the martensitic starting temperature, M_s. The amplitude of the lattice displacement wave is enhanced enough to go beyond the critical amplitude atoms crowd through the energy barriers into the positions determined by the crystal lattice of the low-temperature equilibrium phase and these dynamical displacements are frozen out, resulting in the martensitic structure. The equation describing the motion of the atoms can be derived from a potential $\varphi(r)$ as martensitic transformation is characterized by cooperative atomic movements. By using the expression

$$F = -d\varphi(r)/dr \tag{1}$$

it is obtained as

$$m\ddot{r} + kr + \gamma r^3 = 0, \tag{2}$$

where

$$\varphi(r)/m = \mu r^2/2 + \beta r^4/4,$$

which is called coherent phonon potential, m is the total mass of atoms, k and γ are constants. The frequency of the soft-mode of the transformation is

$$w_0^2 = \mu = C_2(T - T_c) = k/m, \tag{3}$$

where T_c is martensitic start temperature and $\beta = \gamma/m$. Since the amplitude of atoms is enhanced by the coherent phonon waves the motion of atoms must also be under the influence of an external force $F_0 \cos wt$. This driving force represents the stress caused by the vibrating atoms driven by the pre-transformation lattice displacement waves which crowd into the space between the neighbouring atoms and cause the deformation of crystal lattice in the transformed region. By considering this force the equation describing the motion of atoms termed Duffing equation can be represented by

$$d^2r/dt^2 + \mu r + \beta r^3 = (F_0/m) \cos wt. \tag{4}$$

The solution of this equation is given [3]

$$\begin{aligned} r &= r_0 \cos wt, \\ w^2 &= w_0^2 + 3\beta r_0^2/4 - f/r_0, \end{aligned} \tag{5}$$

where $f = F_0/m$. Zhang used this equation to study the amplitude of atomic displacements during martensite transformation.

2.2 Damped equation of atomic motion

During the martensitic phase transformation it is known that the interface of austenite–martensite moves with the velocity of sound. When the interface is influenced from the austenite, this effect appears as frictional force. Therefore, the frictional force which is proportional to the velocity of the interface should be added to the equation describing the motion of the atoms so that the potential used is much more reasonable. In this study this force acting on such a system executing small oscillations of phonons may be written $-\alpha\dot{r}$, where α is a positive coefficient and the minus sign indicates that the force acts in the direction opposite to that of the velocity. Adding this force on the right-hand side of the Eq. (4) we obtain

$$d^2r/dt^2 + 2\lambda dr/dt + \mu r + \beta r^3 = f \cos wt, \tag{6}$$

where $2\lambda = \alpha/m$. By using the Van der Pol method [4–6], the solution of this equation is found as

$$\begin{aligned} r &= r_0 \cos wt, \\ w^2 &= w_0^2 + 3\beta r_0^2/4 \pm [(f/r_0)^2 - (2w_0\lambda)^2]^{1/2} \end{aligned} \tag{7}$$

or

$$\nu^2 = \nu_0^2 + 3\beta r_0^2/16\pi^2 \pm [(F_0/4\pi^2 m r_0)^2 - (\nu_0\lambda/\pi)^2]^{1/2}.$$

The coefficient β which depends strikingly on the properties of the material was taken as a positive constant in the equations mentioned. Therefore, the solutions already obtained above were found for the hard spring case. In the soft spring case this constant is negative [7].

3. Results and discussion

It is interesting to compare the results obtained in this paper with those for the response curve which is suggested by Zhang. For this purpose the amplitude of oscillation displaying atomic displacements during the martensitic transformation is plotted against the frequency of the driving force for damping parameter, λ, and a given amplitude of that force. The values $C_2 = 48\times10^6\,\mathrm{jm^{-3}K^{-1}}$, $\beta = 6\times10^{11}\,\mathrm{jm^{-3}}$, $m = 95.1 \times 10^{-3}$ kg for a gram mole of the alloy Au-30 at% Cu-47 at% Zn and $C_2 = 2.5 \times 10^6\,\mathrm{jm^{-3}K^{-1}}$, $\beta = 1.04 \times 10^{12}\,\mathrm{jm^{-3}}$, $m = 133.6 \times 10^{-3}$ kg for a gram of the alloy In-21 at% Tl were used in computing amplitude-frequency response curves [8–10].

The effect of the damping parameter λ, which represents the friction of the interface between austenite and martensite, is shown in Fig. 1 when all other parameters are held constant. The peak amplitude of the response increases as λ decreases and its corresponding frequency decreases. But at $\nu = \nu_0$ (resonance), the amplitude of the response does not decrease appreciably as λ increases. The curves are nested and bent to the left (since $\beta < 0$).

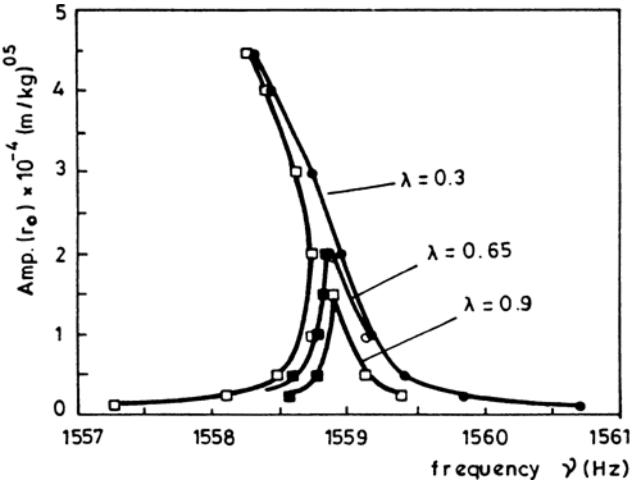

Fig. 1. Effect of the damping parameter λ on the response. Driving amplitude $F_0 = 0.25$, $T = 210$ K

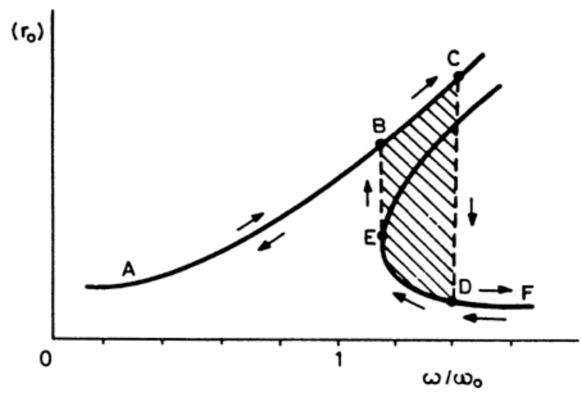

Fig. 2. The phase transformation jump phenomenon described by the Duffing equation. This curve was taken from reference [2]. Here $\lambda = 0$, $\beta > 0$

The response curve for $\lambda = 0$ was discussed by Zhang's paper (Fig. 2). From this Figure, it is not understood that the amplitude jumps definitely to which values of the driving frequency. But the response curves with damping have several fixed peak values.

Figure 3 shows the effect of the driving amplitude F_0 on the response. The amplitude of response increases as F_0 increases and the curves are spaced out in all regions of the driving frequency.

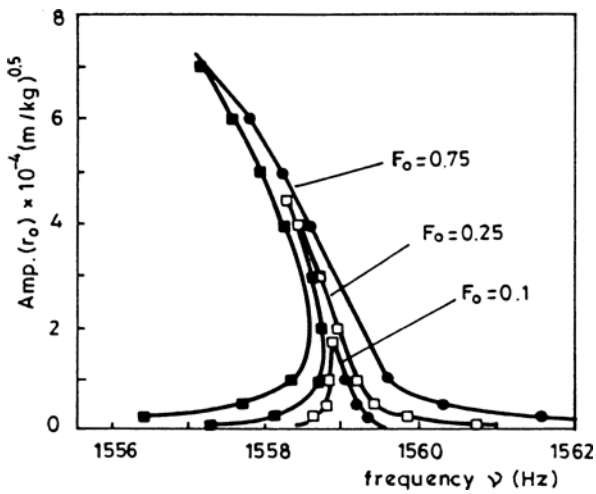

Fig. 3. Effect of driving amplitude F_0 on the response curve for $\lambda = 0.3$, $T = 210$ K

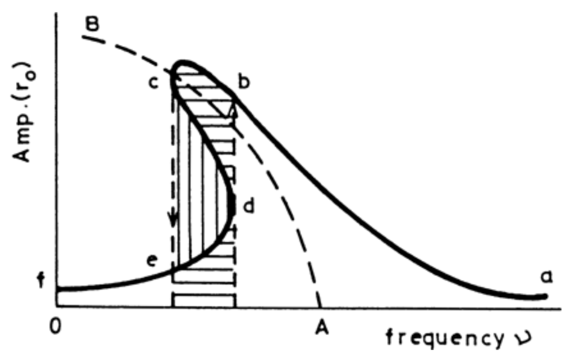

Fig. 4. Amplitude response curve in the case of a damped soft spring Duffing equation ($\beta < 0$)

The most characteristic feature in Eq. (7) is a jump in the response when the driving amplitude is held constant and the frequency is slowly varied through the response region. In the case of the soft spring system the amplitude response curve will be as shown in Fig. 4. The path *cd* is unstable and there is a sudden fall in the response from *c* to *e* when the frequency of the driving force is decreased whereas there is a corresponding jump in response from *d* to *b* on increasing the driving force. Consequently, the location of the peak response will depend upon direction when slowly sweeping the driving frequency, i.e. whether it is upward or downward. The hatched region in Fig. 4 shows clearly the transformation hysteresis loop. Both reverse transformation and transformation hysteresis are striking features of martensitic transformation [11].

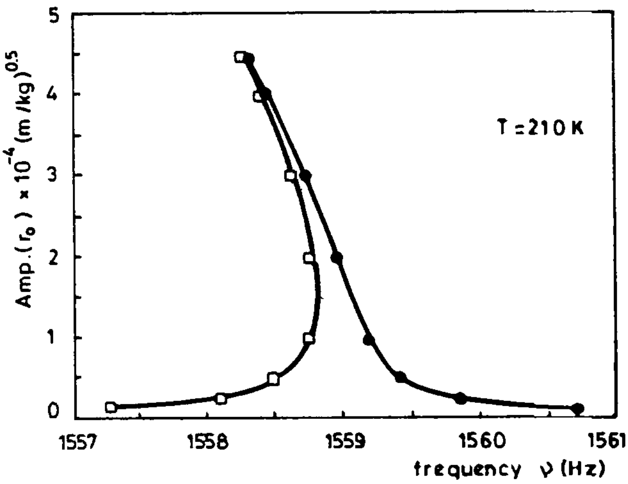

Fig. 5. Amplitude response curve of Au-30 at% Cu, 47 at% Zn alloy for $F_0 = 0.25$ and $\lambda = 0.3$

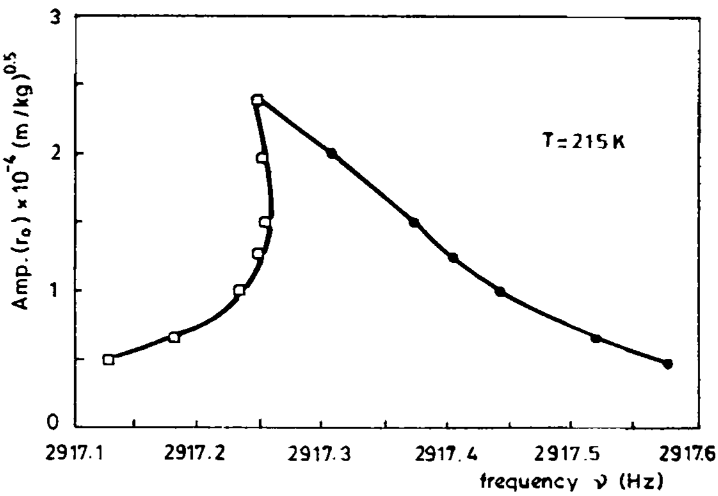

Fig. 6. Amplitude response curve of Au-30 at% Cu, 47 at% Zn alloy for $F_0 = 0.25$ and $\lambda = 0.3$

Figures 5 through 8 demonstrate the nonlinear character of the resonance curves of Au-30 at% Cu-47 at% Zn alloy (M_s = 208 K). It can be seen from these

Figures that the nonlinearity effects increase and the response character changes systematically as the martensitic transformation is approached.

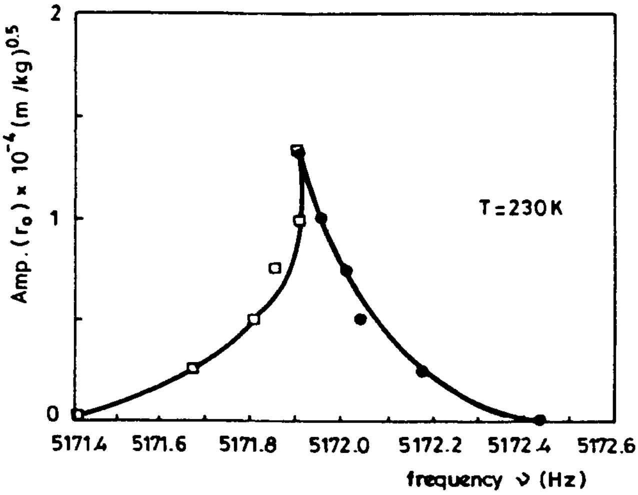

Fig. 7. Amplitude response curve of Au-30 at% Cu, 47 at% Zn alloy for $F_0 = 0.25$ and $\lambda = 0.3$

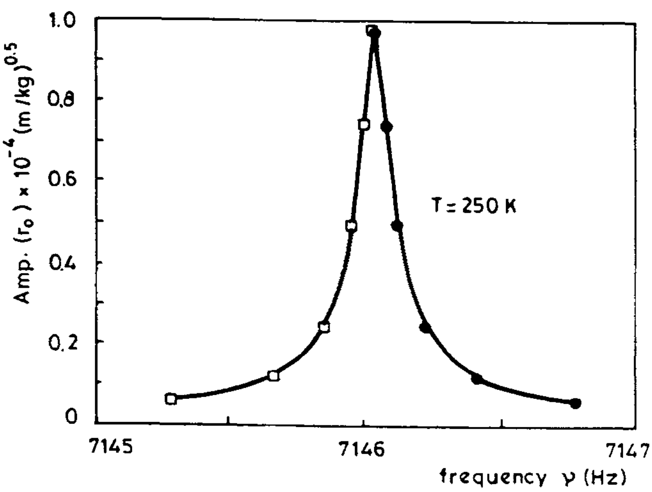

Fig. 8. Amplitude response curve of Au-30 at% Cu, 47 at% Zn alloy for $F_0 = 0.25$ and $\lambda = 0.3$

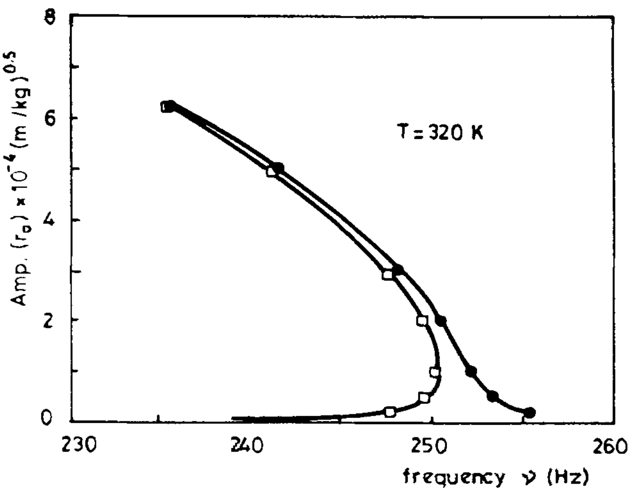

Fig. 9. Amplitude response curve of In-21 at% Tl alloy for $F_0 = 0.25$ and $\lambda = 0.3$

Figure 9 demonstrates the nonlinear character of the response curve for an In-21 at% Tl alloy (M_s = 314 K).

Consequently, the maximum amplitude of the resonance curve is governed by the value of the damping parameter and a similarity between the shape of the curves in this paper and those of reference [12] can be readily seen.

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