Acta Phys. Hung. B 20/1–2 (2004) 101–104

QUANTUM
ELECTRONICS

Deconstructing Wigner's Density Matrix Concerning the Mind–Body Question

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Received 30 July 2003

Abstract. In honor of the centennial of Eugene Wigner's birth, a possible interpretation is given of the density matrix appearing in his classic paper, "Remarks on the Mind–Body Question".

Keywords: quantum mechanics, quantum measurement, decoherence, mind–body question, Wigner's friend
PACS: 03.65.-w, 03.65.Ta, 03.65Yz

1. Introduction

At the *International Colloquium on Group Theoretical Methods in Physics* at Austin in 1978, Eugene Wigner received the first Wigner Medal. Following the conference, I wrote him a brief letter congratulating him on his award, and also taking the opportunity to ask him two questions that had been bothering me for some time concerning his famous "Remarks on the Mind–Body Question" [1]. My questions were 1) What nonlinear generalization of quantum mechanics do you propose? and 2) What does the parameter δ, appearing in your density matrix,

$$\begin{bmatrix} |\alpha|^2 & \alpha\beta^* \cos\delta \\ \alpha^*\beta\cos\delta & |\beta|^2 \end{bmatrix}, \tag{1}$$

have to do with consciousness? I was particularly interested in the second question. Wigner graciously replied, thanking me for the congratulation, and responding to my two questions as follows: "Question 1. I am more inclined to modify my earlier ideas on non-linearity and will propose a linear equation for the density matrix. However, this leads to a non-deterministic theory. I do not understand your Question 2. What is δ? Where do I speak about this δ?"

Wigner did not answer my second question, and unfortunately I did not follow up with a return letter. However, twenty years later, Peter Landsberg invited

1589-9535/04/ $ 20.00

me to write a review article on quantum computing [2], and while writing the section on Schrödinger cat states and quantum decoherence, I recognized the obvious correspondence between Wigner's density matrix, Eq. (1), and the reduced density matrix describing a qubit interacting with its environment, thereby providing insight regarding the parameter δ. Quite simply, it can be argued that

$$\delta = \cos^{-1} \langle \chi_2 | \chi_1 \rangle \,, \tag{2}$$

where $|\chi_1\rangle$ and $|\chi_2\rangle$ are the states of Wigner's friend (including the friends measuring device and any relevant environment) which are correlated with the two states, $|\psi_1\rangle$ and $|\psi_2\rangle$, of some object measured by Wigner subsequent to an earlier measurement by his friend. The states $|\chi_1\rangle$ and $|\chi_2\rangle$ are the states of Wigner's friend that correspond to the friend measuring the object to be in states $|\psi_1\rangle$ and $|\psi_2\rangle$, respectively. Evidently, $|\chi_1\rangle$ and $|\chi_2\rangle$ are both collective states, parameterized in terms of collective coordinates [3] representing an enormous number of degrees of freedom for such a complex many-body system. (A greatly expanded version of the present work appears in *Found. Phys. Lett.* **15** (2002) 287).

2. Deconstructing Wigner's Density Matrix

To see, in some didactic detail how Eq. (2) arises, one can proceed as follows. (All of the following is at least implicit in other works such as [3].) Consider an object having only two possible orthonormal states $|\psi_1\rangle$ and $|\psi_2\rangle$ in superposition and interacting with Wigner's friend while the friend measures the object. The state vector of the system consisting of the object and Wigner's friend at time t is

$$|\psi(t)\rangle = \alpha\, |\psi_1\rangle\, |\chi_1\rangle + \beta\, |\psi_2\rangle\, |\chi_2\rangle \,, \tag{3}$$

in which the two states of the object $|\psi_1\rangle$ and $|\psi_2\rangle$, through unitary evolution, have become entangled with the corresponding normalized states, $|\chi_1\rangle$ and $|\chi_2\rangle$, respectively, of Wigner's friend. The states of Wigner's friend are normalized, but are not a priori orthogonal. The density operator of the total system, consisting of the object and Wigner's friend, becomes

$$\begin{aligned} \rho(t) = {} & |\alpha|^2\, |\psi_1\rangle\, |\chi_1\rangle\, \langle\psi_1|\, \langle\chi_1| + \alpha\beta^*\, |\psi_1\rangle\, |\chi_1\rangle\, \langle\psi_2|\, \langle\chi_2| \\ & + \alpha^*\beta\, |\psi_2\rangle\, |\chi_2\rangle\, \langle\psi_1|\, \langle\chi_1| + |\beta|^2\, |\psi_2\rangle\, |\chi_2\rangle\, \langle\psi_2|\, \langle\chi_2| \,. \end{aligned} \tag{4}$$

If Wigner is interested only in the state of the object, and not the state of his friend, he need only know the reduced density matrix of the object, with the correlated states of his friend traced out. For this purpose, it is convenient to choose, for a complete set of basis vectors in the correlated Hilbert space of Wigner's friend, the state $|\chi_1\rangle$ and the state $|\bar{\chi}_1\rangle$, orthonormal to $|\chi_1\rangle$. The reduced density operator of the object $\rho_o(t)$ is then given by

$$\rho_o(t) = \mathrm{Tr}_f\, \rho(t) = \langle \chi_1 | \rho(t) | \chi_1 \rangle + \langle \bar{\chi}_1\, | \rho(t) |\, \bar{\chi}_1 \rangle \,, \tag{5}$$

where $\mathrm{Tr_f}$ denotes the trace over the basis of the states of Wigner's friend that are correlated with the object. Substituting Eq. (4) in Eq. (5), using the state-orthonormality relations and the fact that $|\chi_1\rangle$ and $|\bar{\chi}_1\rangle$, orthonormal to $|\chi_1\rangle$, form a complete set of basis vectors, one obtains the reduced density operator:

$$\begin{aligned}\rho_o(t) = |\alpha|^2 |\psi_1\rangle \langle\psi_1| + \alpha\beta^* \langle\chi_2|\chi_1\rangle |\psi_1\rangle \langle\psi_2| \\ + \alpha^*\beta \langle\chi_1|\chi_2\rangle |\psi_2\rangle \langle\psi_1| + |\beta|^2 |\psi_2\rangle \langle\psi_2| .\end{aligned} \tag{6}$$

The corresponding reduced density matrix, defined by

$$[\rho_o(t)] = \begin{bmatrix} \langle\psi_1|\rho_o(t)|\psi_1\rangle & \langle\psi_1|\rho_o(t)|\psi_2\rangle \\ \langle\psi_2|\rho_o(t)|\psi_1\rangle & \langle\psi_2|\rho_o(t)|\psi_2\rangle \end{bmatrix}, \tag{7}$$

then becomes

$$[\rho_o(t)] = \begin{bmatrix} |\alpha|^2 & \alpha\beta^* \langle\chi_2|\chi_1\rangle \\ \alpha^*\beta \langle\chi_1|\chi_2\rangle & |\beta|^2 \end{bmatrix}. \tag{8}$$

Finally, comparing Eq. (8) with Eq. (1), and, without loss of generality, letting $\langle\chi_1|\chi_2\rangle$ be real, one obtains Eq. (2).

Thus, a possible interpretation of Wigner's parameter δ is that it is the inverse cosine of the overlap between the states of Wigner's friend that become correlated with the states of the object. If the interaction between Wigner's friend and the object includes the friend's measurement of the object, then the states $|\chi_1\rangle$ and $|\chi_2\rangle$ are the states of the friend corresponding to the friend's having measured the object to be in the states $|\psi_1\rangle$ and $|\psi_2\rangle$, respectively. Because Wigner's friend (including the friend's measuring device and environment) is an extremely complex many body system, the interaction with the object will result in near-instantaneous decoherence [3] of the state of the object, and the overlap between the states $|\chi_1\rangle$ and $|\chi_2\rangle$ of Wigner's friend will extremely rapidly become negligible. One then has extremely rapid orthogonalization of the states of Wigner's friend, and extremely rapid vanishing of the off-diagonal components of the reduced density matrix of the object, namely,

$$\cos\delta = \langle\chi_2|\chi_1\rangle \to 0 . \tag{9}$$

In this case, Eq. (8) becomes

$$[\rho_o(t)] \to \begin{bmatrix} |\alpha|^2 & 0 \\ 0 & |\beta|^2 \end{bmatrix}. \tag{10}$$

Because of the complexity of the states $|\chi_1\rangle$ and $|\chi_2\rangle$ of Wigner's friend, the reduced density matrix, Eq. (8), would decohere to the diagonal form, Eq. (10), so rapidly that the coherences (represented by the off-diagonal terms) would not be observable. As a result of the decoherence, the resulting reduced density matrix becomes effectively a statistical mixture, and the paradoxical features of the problem of Wigner's friend largely evaporate. 'For all practical purposes', there is simply a probability $|\alpha|^2$ that Wigner will measure the object to be in state $|\psi_1\rangle$, and a probability $|\beta|^2$ that he will measure it to be in state $|\psi_2\rangle$, with no mysterious quantum interferences. It is significant to realize that if Wigner's friend were replaced by any complex measuring automaton (conscious or not), the same type of dynamical evolution described above would evidently apply.

Acknowledgements

This work was supported by the U.S. Army Research Laboratory and the Defense Advanced Research Projects Agency. The author wishes to thank Prof. József Janszky for inviting him to the Advisory Board for the *Centennial Conference in Commemoration of Eugene Paul Wigner* in Pécs, Hungary, 8–12 July, 2002, and thereby motivating this work.

Note

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