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Learning mathematics from worked-out examples: Analyzing and fostering self-explanations

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Recent research has shown that learning from worked-out examples is of major importance for initial skill acquisition in well-structured domains such as mathematics. However, only those learners who actively process the presented examples profit noticeably from this learning mode. Specifically, the learning outcomes depend on how well the learners explain the solution steps presented in the examples to themselves ("self-explanation effect"). In a series of studies on learning mathematics from examples, learners' spontaneous self-explanations and instructional means used to encourage self-explanations were investigated. In this research, the following main findings were obtained. Most learners were rather passive with respect to their spontaneous self-explanations. Among the active and successful learners, two subgroups employing different self-explanation styles could be identified. With regard to the instructional means used to induce effective example processing, it turned out that to employ "learning by teaching" in order to stimulate explanation activities was of very limited use. Attempts to directly train for or elicit certain types of self-explanations were more successful. However, even in the latter case, self-explanations had inherent deficits (e.g., proneness to errors). Thus, we sought to design learning arrangements that try to integrate self-explanations with well-timed and well-adapted instructional explanations (e.g., from tutors) in order to enhance students' problem-solving skills.

In this article, an overview is given of the research the author and his colleagues have conducted on the significance of self-explanations in mathematics learning from worked-out examples. The emphasis of this work is on interventional studies in which we tried to enhance the quality of learners' self-explanations and, as a consequence, to improve learning outcomes. In the first section, the significance of worked-out examples is briefly outlined. In the second, the relation between individual differences in self-explanations and learning outcomes is described. The third and the fourth sections are devoted to the discussion of experiments in which instructional techniques for fostering self-explanations were investigated. Although, the interventional studies were in part successful, the learners' self-explanations were far from

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optimal. Possible further improvements of instructional means are discussed in the final section. In line with the emphasis of this Special Issue, the metacognitive aspects of self-explanations are emphasized throughout this article.

Worked-out examples as a source of learning

Recent research has shown that learning from worked-out examples is of major importance for initial skill acquisition in well-structured domains such as mathematics (Reimann, 1997; VanLehn, 1996). It is not only a learning mode preferred by novices, but also an effective one. Zhu and Simon (1987) found that their carefully designed and sequenced mathematical examples were sufficient to induce skill acquisition and abstract problem representations without providing explicit instruction. Studies performed by Sweller and his colleagues (e.g., Sweller & Cooper, 1985) showed that learning from worked-out examples can be more effective than learning by problem solving. This finding is explained by the argument that problem solving requires so much working memory capacity that it interferes with learning in the sense of schema acquisition; that given this load, too few resources are left for the induction of abstract and generalizable problem-solving schemata (cf. Sweller, 1994). With regard to metacognition, it can be argued that effective metacognitive control is also impeded by problem-solving tasks when they "absorb" the learners' cognitive capacity.

Besides the capacity arguments, there is probably another important advantage of worked-out examples in comparison to problem solving – an advantage that is strongly related to metacognition. When learners are told to solve problems, their primary goal is, of course, to solve problems and not necessarily to learn. The learners adopt a performance orientation. In contrast, when confronted with worked-out examples, there is no demand to perform. The only tasks confronted by the learners is to understand and to learn. Thus, worked-out examples foster intentional learning (Bereiter & Scardamalia, 1989), or in other words, a learning orientation (see also the distinction between performance and learning orientation by Dweck & Leggett, 1988).

Although worked-out examples have significant advantages, their employment as a learning methodology does not, of course, guarantee effective learning. The extent to which learners profit from the study of examples strongly depends on how well they explain the solutions of the examples to themselves (Chi, Bassok, Lewis, Reimann, & Glaser, 1989). In the next section, important research on individual differences in the quality of these self-explanations is described.

Individual differences in self-explanations

The classical study of Chi et al.

In what has become a classical study, Chi et al. (1989) have shown that the extent to which learners profited from the study of worked-out examples depended on how well they explained the rationale of the presented solutions to themselves. This was called the "self-explanation effect". Specifically, the successful learners as compared to those who were less successful could be characterized as follows: (a) The successful learners devoted more time to the study of the worked-out examples; (b) they elaborated more frequently on the application conditions and goals of operators; (c) they related more frequently operators to domain principles (principle-based explanations); (d) they explicated more comprehension problems; Chi et al. (1989) interpreted this finding as indicating that the successful learners actually noticed when they had comprehension problems whereas less successful learners frequently had "illusions of understanding". Pirolli and Recker (1994) were able to replicate these results.

A problem that became evident in the study of Chi et al., was, however, that the successful and the unsuccessful learners differed with respect to both quantitative (learning time) and qualitative (quality of the self-explanations) aspects. Thus, it was not clear to what extent qualitative self-explanation differences were responsible for individual differences in learning outcomes. This limitation was avoided in a study by Renkl (1997b), which will be described in the next section.

Individual differences in self-explanations when learning mathematics

Renkl (1997b) fixed the learning time for each individual so that the pure impact of qualitative differences in self-explanation activities could be isolated. The participants, who were first-year students of education, studied worked-out examples from the domain of probability calculation. The solutions of the examples were presented on a computer monitor in a step-by-step manner. The learners could determine their speed of processing the examples: They were to move to the next solution step, or if the example was fully presented, to the next example by a mouse click. The learning time was fixed to 25 minutes for each individual. The self-explanations were assessed by the thinking-aloud method (Ericsson & Simon, 1993). Learning outcomes were measured by test problems of different transfer distance to the examples presented for learning (for details see Renkl, 1997b).

The following main results were obtained. It was found that the quality of self-explanations was significantly related to learning outcomes even when learning time was kept constant. Specifically, the successful and the unsuccessful learners differed with respect to the following main points: (a) The successful learners frequently assigned meaning to operators by identifying the underlying domain principle (*principle-based explanations*; e.g., "It gets multiplied, because the events are independent of each other"; this statement referred to the meaning of the multiplication rule). (b) They frequently assigned meaning to operators by identifying the (sub-)goals achieved by these operators (*explication of goal-operator combinations*; e.g., "Through this multiplication we get the probability of tiles with color and form faults"). (c) They tended to anticipate the next solution step instead of looking it up (*anticipative reasoning*; e.g., "Then the probability of tiles with color and form faults is 1/50" [before reading this probability]). (d) The less successful learners explicated a greater number of comprehension problems, that is, they had more metacognitive awareness of their own learning difficulties (*metacognitive monitoring*; e.g., "Now I don't understand it any more"). This latter finding diverged from the results of Chi et al. (1989). Probably in contrast to the learners in the investigation conducted by Chi et al. (1989), the learners in Renkl's study very often could not resolve their comprehension impasses as informal observations indicated. The latter learners would have needed external support. This helps to explain the negative relation between self-diagnosed comprehension problems and learning outcomes.

In addition, Renkl (1997b) found that the successful learners frequently did not provide all of the types of self-explanations that were positively related to learning outcomes. A cluster analysis showed that there were two types of successful learners. *Principle-based explainers* concentrated their self-explanation efforts on the assignment of meaning to operators, both by principle-based explanations and by explicating goal-operator combinations. They did not frequently anticipate solution steps. This was extensively done, however, by the *anticipative reasoners*, who refrained from many principle-based explanations and from the frequent explication of goal-operator combinations. In sum, there were two ways utilized in successful learning.

Besides these two types of successful learners, there were two groups of unsuccessful ones: passive and superficial explainers (for details see Renkl, 1997b). The *passive explainers'* poor learning outcomes could be explained by the very low level of self-explanation activity. *Superficial explainers*, on the other hand, assigned relatively little time to each worked-out example. Although they were moderately successful they explicated few comprehension problems. With respect to their deficient metacognitive awareness of their learning difficulties, the superficial explainers resembled the less successful learners described by Chi et al. (1989).

It is important to note that most learners belonged to the unsuccessful groups. The behavior of these unsuccessful learners revealed that learning from worked-out examples is connected with two drawbacks that are closely related to metacognition: (a) At least some learners have experienced mainly feelings of understanding when studying examples. Subsequent problem-solving performance reveals, however, that such feelings of understanding are frequently illusionary. Worked-out examples may result in "illusions of understanding" because they do not require the learners to do something that is followed by intrinsic feedback (i.e., obvious failure to accomplish a task) or extrinsic feedback (i.e., right-wrong information from an external source such as a tutor). (b) Many learners do not seem to have metaknowledge of how to learn from worked-out examples or, at least, they do not use this knowledge (cf. Renkl, Mandl, & Gruber, 1996). These learners glance over the worked-out examples without elaborating them; they obviously believe that this will lead to learning. They do not conceptualize the demand to learn as a non-trivial problem what would be necessary for effective intentional learning (Bereiter & Scardamalia, 1989).

Given these deficits, it is important to search for instructional interventions in order to foster self-explanation activities and, as a consequence, to enhance learning results. Some researchers have already performed experiments in which self-explanations were successfully fostered. These studies did not, however, concentrate on learning from worked-out examples (Bielaczyc, Pirolli, & Brown, 1995: text and examples; Chi, DeLeeuw, Chiu, & LaVancher, 1994: text; Neuman & Schwarz, 1998: problem solving).

In the next two sections, interventional studies performed by the author and his colleagues are described. Figure 1 presents an overview of this research. The investigations can be differentiated with respect to the interventional approach that was adopted (cf. Friedrich & Mandl, 1992). In *indirect interventions,* the objective was not to directly train for or elicit self-explanations, but to set incentives so that the learners give up their (in most cases) passive or superficial mode of processing the presented examples. Specifically, "learning by teaching" was employed in order to stimulate explanation activities. In *direct interventions,* self-explanations were directly trained for or elicited. In particular, each of the two self-explanation styles that proved to be effective in Renkl (1997b) was induced in a separate study (see Figure 1). In the next section, the effects of indirect interventions are described.

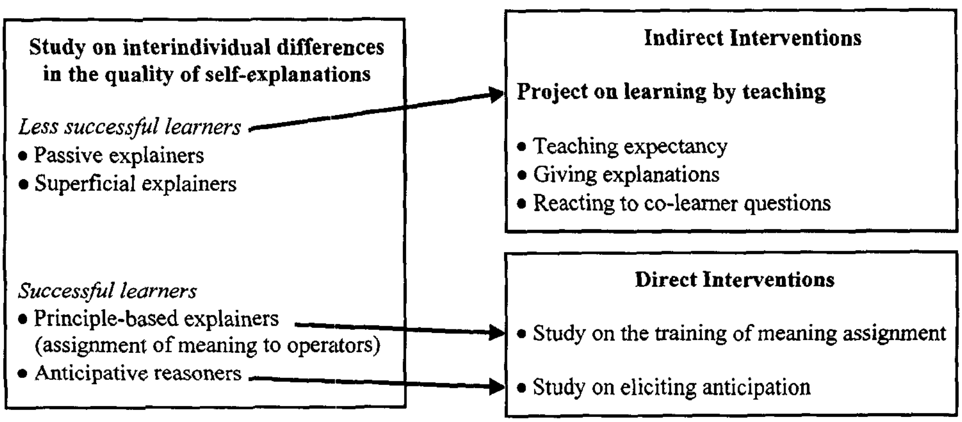

Figure 1. An overview of the research program

Indirect interventions

In a series of experiments, Renkl (1995, 1996, 1997a, 1997c, 1997d, 1998) investigated the extent to which learning from worked-out examples can be fostered by assigning the role of a "teacher" or explainer to learners (*learning by teaching*). The main idea was that if most learners do not spontaneously generate elaborated self-explanations, it might be helpful to put them into the role of an explainer for somebody else. This should motivate them and in some sense even urge them to increased explanation activities. Many studies have shown that giving explanations for co-learners can foster knowledge acquisition (for an overview, see Webb, 1991). That is, explanations for others are in some sense also explanations for oneself (i.e., self-explanations).

Specifically, the effects of three central components of learning by teaching on explanation activities and learning outcomes were investigated. These components were (a) teaching expectancy, (b) generation of explanations, and (c) reacting to co-learner questions. In this project, the same domain, the same kind of participants (first-year students of education), and to a large extent the same materials and instruments were employed as in Renkl's study on individual differences in self-explanations (for details see Renkl, 1997d).

In a first experiment on *teaching expectancy*, Renkl (1995, 1997d) analyzed the extent to which learners' self-explanation activities during the study of examples and the resulting learning outcomes were fostered by creating the expectation that they would later have to explain the solution rationale of similar examples to a co-learner (experimental group). The participants in a control group expected that they would later have to solve similar problems. The learning outcomes were assessed after the individual learning phase; no actual explaining took place. Surprisingly, the teaching expectancy did not significantly foster self-explanations. In some respect, it was even detrimental because it increased stress and reduced intrinsic motivation. As a result, the teaching expectancy did not foster learning outcomes. A pure teaching expectancy was obviously not sufficient to effectively foster explanation activities and learning. In consequence, it was sought to determine the extent to which having learners actually explain the solution rationale of worked examples might prove a more promising intervention.

Renkl (1996, 1997a, 1997d) analyzed the effects of *generating explanations*. Specifically, he investigated the extent to which learners profit from learning by teaching when they actually have to explain the solution rationale of worked-out examples to a co-learner. For this purpose, yoked pairs were formed. After a preparatory individual learning phase, one partner (experimental group) explained the solution rationale of examples to the other partner (control group). It turned out that the demand to explain actually increased explanation activities as compared to the amount of spontaneous (self-)explanations. This did not, however, result in increased learning outcomes. On the contrary, the listeners outperformed the explainers. In addition, as was the case in the previous study (Renkl, 1995), the learning-by-teaching component induced substantial stress.

Renkl (1997c, 1997d, 1998) analyzed the effects of *co-learner questions*. He sought to determine to what extent learning by explaining is beneficial when the explainers are stimulated by sophisticated co-learner questions. In this experiment, the participants explained the solution rationale of examples to a putative co-learner (confederate). In the experimental group, the confederate posed semi-standardized "what-if" questions, asking how a problem solution must be modified if a problem's concluding question phrase were to be changed. In the control group, the putative co-learner was more or less totally passive. It was found that the co-learner questions fostered merely one type of explanations (i.e., situation elaborations), whereas all other types were reduced. As a consequence, the learning outcomes of intrinsically motivated learners were impeded because the co-learner questions hampered their sophisticated spontaneous explanation activities which tended, for example, to include many principle-based explanations. The learning outcomes of learners with low intrinsic motivation, whose spontaneous explanation activities were very poor, were fostered. The co-learner questions raised the quality of their explanation activities at least to a medium level. In addition, it turned out that those learners with a high level of prior knowledge were the learners who principally profited from the co-learner questions because they were able to generate correct elaborated explanations in response.

On the whole, learning by teaching resulted in poor results. Additional analyses suggested that there were two main reasons for this failure. First, the learners were not acquainted with the role of an explainer (i.e., tutor) and, second, the learning materials were rather difficult. Both factors contributed to the fact that the learners were overwhelmed and stressed by the dual task of teaching and learning.

To sum up, the results of this project were important because they qualify the enthusiastic judgments about learning-by-teaching frequently found in the literature. With respect to the goal to foster (self-)explanations and, as a consequence, learning outcomes, the attempt to employ learning-by-teaching failed for the most part. In the next section, the question of whether direct interventions were more successful in fostering learning mathematics from worked-out examples will be discussed.

Direct interventions

The findings of Renkl (1997b) suggest that there are two ways of successful learning from worked-out examples: (a) via the assignment of meaning to operators by principle-based explanations and by the explication of goal-operator combinations; (b) via the anticipation of solution steps. In each of the following studies, one of these two successful ways was experimentally analyzed.

Experiment on fostering the assignment of meaning to operators

Renkl, Stark, Gruber, and Mandl (1998) addressed the finding of Renkl (1997b) that some effective learners frequently assign meaning to operators, both by principle-based explanations and by explicating goal-operator combinations. They tested experimentally the extent to which an elicitation procedure for fostering the explication of goal-operator combinations and principle-based explanations enhances the acquisition of skills in the computation of compound interest and real interest[1] (in addition, the effects of example variability were investigated; this is not, however, relevant in this context). In an experimental group, the learners were informed about the importance of self-explanations. In other words, they were provided with metacognitive knowledge about self-explanations. Then a model on how to self-explain was presented to the participants in the experimental group. Subsequently, they self-explained a worked-out example, coached by the experimenter. Finally, the participants independently learned from worked-out examples, just as the members of the control group had done. The latter merely received a thinking-aloud training instead of a self-explanation training. The learning outcomes were measured by near-transfer problems (in comparison to the examples presented for learning: same underlying structure, different surface features) and far-transfer problems (changed structure and changed surface features).

The following main findings were obtained. The elicitation procedure had a very strong effect on self-explanation activities (effect size of about two standard deviations; Renkl, Stark, Gruber, & Mandl, in press). As a consequence, learning outcomes were enhanced. The effect sizes with respect to near- and far-transfer performance corresponded to about half a standard deviation. In the case of near-transfer, the positive effect of the elicitation procedure was primarily caused by the learners with low prior topic knowledge (aptitude-treatment interaction). It was only those persons who substantially profited from this instructional support (Renkl et al., 1998).

Despite the encouraging results of this experiment, the effects of the elicitation procedure were not totally satisfying. Analyses of the learners' verbal protocols revealed that there were three major problems: (1) Although the elicitation procedure was successful in increasing the number of self-explanation elements, the quality and correctness of the self-explanations were far from optimal in many cases. (2) Some learners processed the examples rather passively and superficially, although they were supported by the elicitation procedure. Thus, in some cases, the instructional intervention was not successful. (3) Some learners had substantial comprehension problems, irrespective of whether they were supported by the elicitation procedure or not. Hence, the difficulty of weak learners already found in Renkl (1997b) re-appeared.

These problems demonstrate the necessity of searching for further instructional methods in the effort to optimize learning from worked-out examples.

Experiment on eliciting anticipative reasoning

In his dissertation project, Stark (1998) took up the finding of Renkl (1997b) that one group of successful learners concentrated their efforts on the anticipation of solution steps. He investigated the extent to which the insertion of "blanks" that, in a certain sense, forced the learners to determine (anticipate) the next solution step on their own fostered learning. It was assumed that elements of problem solving were integrated into learning from examples by forcing the learners to anticipate (Renkl, 1997b; Stark, 1998). Hence, the learners could gain metacognitive knowledge about the extent to which they were already able to solve problems. This should reduce the frequently found "illusions of understanding" (cf. Chi et al., 1989; Pirolli & Recker, 1994). In addition, the learners began to do what they were ultimately expected to do, namely to solve problems (i.e., to generate solution steps).

In his experiment, Stark (1998) employed the worked-out examples, the instruments, and the materials of Renkl (1997b) in slightly modified versions. The examples were presented successively, that is, in a step-by-step procedure. Half of the participants studied incomplete examples (experimental group), the other half learned from complete examples (control group). In the experimental group, part of the example solutions presented were replaced by "question marks". The learners were to name what was missing. After doing that or at least making the attempt, the complete solution step was presented so that there was feedback on the correctness of the learners' anticipation.

The following main results were obtained. The employment of incomplete examples fostered performance on problems with the same structure, but changed surface features (defined as near-transfer) and on problems with changed structure, but similar surface features (medium-transfer). The effect sizes corresponded to about one standard deviation. The learners who studied incomplete examples also performed better on far-transfer problems (changed structure and surface features). This difference did not, however, reach the 5%-level of significance (effect size: about half a standard deviation). Aptitude-treatment interactions were not found in this study. That means that the effects of incomplete examples were independent of the learners' cognitive prerequisites.

Protocol analyses of the self-explanations in both groups showed that incomplete examples significantly fostered the quality of self-explanations. Nevertheless, the self-explanations of the learners studying incomplete examples were far from optimal. The problems of unresolved comprehension impasses and of the occurrence of incorrect self-explanations were again found, just as in the studies described above. Hence, a lot could and should be done to further improve the quality and correctness of the learners' self-explanations.

Conclusions from the studies on direct interventions

The instructional means that were chosen in order to foster learning from worked-out examples were successful with respect to the enhancement of learning outcomes. However, even when instructional interventions were employed, the quality of self-explanations was far from satisfying. These limitations are probably inherent in learning arrangements in which learners are totally dependent on their self-explanation activities. In order to further improve learning from worked-out examples, it is reasonable to look for fruitful possibilities that would combine self-explanation activities with explanations from others such as tutors or teachers (in the following shortly: instructional explanations).

Outlook: Combining self-explanations and instructional explanations

Up to now, we have focused on self-explanations and attempts to improve them. Explanations from more knowledgeable persons (e.g., teachers or tutors) were not considered, although they dominate traditional forms of instruction. The major reason for this was that empirical results indicate that instructional explanations are very often rather ineffective and inferior to self-explanations (e.g., Brown & Kane, 1988). Instructional explanations as compared to self-explanations have, at least, three main disadvantages that help to explain these findings (see Table 1): (a) *Non-adaptation to the learner's prior knowledge.* Instructional explanations are very often not adapted to the prior knowledge of the individual learner with the result that the learner cannot understand it. Self-explanations, in contrast, are constructed out of the learner's prior knowledge; hence, they are necessarily adapted. (b) *Timing.* Some research findings suggest that a learner profits from instructional explanations only when she/he can integrate them into an on-going activity such as problem solving or reasoning about something (cf. Neber, 1995; Webb, 1992). Whereas self-explanations are an integral part of on-going learner activity, it is not a trivial task to assure appropriate timing for instructional explanations. (c) *Generation effect.* Many studies on human memory have shown that self-generated information is better remembered than presented information. Thus, self-explanations should be better remembered than instructional explanations (see also Lovett, 1992).

Table 1

Advantages and disadvantages of self-explanations and instructional explanations

	Self-explanations	Instructional explanations
Adaptation to prior knowledge	YES	Uncertain
Timing	FAVORABLE	Uncertain
Generation effect	YES	No
Correctness	Uncertain	GIVEN
Resolution of comprehension problems	Difficult	MEDIUM
Comprehension monitoring	Unfavorable	MEDIUM

Note. Capital letters: More favorable features of self-explanations or instructional explanations respectively.

The findings of the studies reported above show, however, that relying only on self-explanations has serious implications in three aspects (see Table 1): (a) *Correctness.* Self-explanations are often only partially correct or even incorrect. This can lead to the construction of incorrect knowledge that, in the worst case, can severely impede further learning. Instructional explanations, by contrast, are in the large majority of cases correct. (b) *The issue of solving comprehension problems.* When confronted with new contents, learners frequently have comprehension impasses that they cannot resolve on their own. External help is sometimes necessary to overcome problems in understanding. (c) *A need for comprehension monitoring.* Learners have the "metacognitive problem" that they frequently have the illusion of understanding when explaining the solutions of worked-out examples to themselves (see above). As a consequence, they do not try to further deepen their understanding, although this would be necessary for effective learning. Instructional explanations can show the learners, at least in some cases, that they do not yet have a sufficient understanding.

With respect to these points, instructional explanations can be extremely helpful. They can effectively support the learners' knowledge-construction activities. Thus, a challenging task for further research on learning from worked-out examples is to find ways to combine self-explanations and instructional explanations in a way that combines their respective advantages. Such an arrangement should satisfy the following criteria:

(1) *Provide for as much self-explanations as possible, as much instructional explanation as necessary.* It is desirable that learners acquire the metacognitive competence to effectively learn from worked-out examples in a self-regulated way. Hence, the learners should rely on self-explanations as much as possible. Instructional explanations should only be provided when the learners are not able to understand the learning content on their own.

(2) *Provide feedback.* The learning arrangement should be designed in a way that will substantially reduce the learners' illusions of understanding frequently found in these studies. For this purpose, some intrinsic or extrinsic feedback should be available. As explained above, the employment of incomplete examples is one possibility for heightening the learners' metacognitive awareness of their comprehension problems. The provision of instructional explanations is, of course, another instructional means that can fulfill a feedback function.

Three additional criteria for improving effectiveness should be considered when providing instructional explanations:

(3) *Timing.* Instructional explanations should be presented on learners' demand. This should assure that the instructional explanations are appropriately timed and are actually used in the on-going knowledge-construction activities of the learners.

(4) *Adaptation to prior knowledge.* Of course, it is extremely important that the explanations are formulated in a way that will make them accessible to the learners. Another important point is that instructional explanations should not be "over-extensive" and tell the learners things that they already know or that they do not need to know in the immediate instance. Hence, instructional explanations should be as parsimonious as possible. Only when a lack of prior knowledge makes it necessary should more extensive explanations be provided.

(5) *Focus on principles.* With respect to the content of instructional explanations, it is argued that their focus should be on the underlying principles of the respective content (sub-)domain. Among others, this claim is supported by the significance of principle-based explanations when studying worked-out examples (e.g., Chi et al., 1989; Renkl, 1997b). Furthermore, as Alexander (1997) argues, learning progress from a novice stage to the stage of competence is generally characterized by the development of a principle-based understanding. In addition, a principle-based understanding is especially important for mathematics learning (Hiebert, 1986; Renkl & Helmke, 1992; Steiner & Stoecklin, 1997) if the acquired knowledge should be flexibly transferable and is not, as it is very often the case, inert.

The author is presently preparing studies on learning arrangements in which the objective is to combine the advantages of self-explanations and instructional explanations as much as possible when learning from worked-out examples. This effort is in accord with findings from research on mathematics learning, and even on learning in general, that emerged in recent years (e.g., Renkl et al., 1998; Stark, Graf, Renkl, Gruber & Mandl, 1995). In order to come to a deep understanding that allows for the transfer of acquired skills, learners have to actively construct their own understanding. This can be fostered in learning arrangements that provide the opportunity for self-regulated learning. However, learners are often overtaxed when they have to guide their learning on their own. Thus, they need instructional support. The instructional principle emerging from these arguments is that effective learning arrangements should provide for self-regulated learning and simultaneously support the learners in their corresponding efforts ("supported self-regulated learning"). Learning arrangements that try to combine self-explanations and instructional explanations in the way outlined above are in accord with the principle of supported self-regulated learning: The learners should *self*-explain as much as possible and simultaneously be supported by instructional explanations.

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Notes

1 Real interest refers to the net return in relation to the invested capital (real interest may differ from nominal interest for many reasons). Compound interest includes the accumulated interest as well as that computed on interested already distributed.

Les recherches récentes montrent que l'apprentissage à partir d'exemples est d'une grande utilité dans l'acquisition initiale de la maîtrise de domaines bien structurés comme les mathématiques. Cependant, seuls les apprenants qui traitent activement les exemples proposés tirent un profit notable de cette forme d'apprentissage. Plus précisément, les résultats de l'apprentissage dépendent du soin avec lequel les apprenants expliquentpour eux-mêmes les étapes de résolution présentées dans l'exemple ("effet d'auto-explication"). Dans une série d'études sur l'apprentissage des mathématiques à partir d'exemples, on a étudié l'effet d'auto-explications spontanées ainsi que l'effet d'incitations à utiliser de telles auto-explications. Dans cette recherche les principaux résultats constatéssont les suivants. La plupart des apprenants sont relativement inactifs dans la production d'auto-explications. Parmi ceux qui s'y livrent activement et avec succès, deux sous-groupes purent être identifiés par leur style d'auto-explication. En ce qui concerne les moyens pédagogiques mis en oeuvre pour induire un traitement efficace des exemples, il apparaît que l'enseignement magistral destiné à stimuler les activités d'explication *donne peu de résultats. Plus heureuses sont les tentatives de faire produire certaines formes d'auto-explication ou d'entraîner les apprenants à les utiliser. Cela dit, même dans ce dernier cas, les auto-explications présentent des faiblesses (e.g. risque d'erreurs). Il faut donc parvenir à associer les auto-explications et des explications pédagogiques bien adaptées et bien planifiées, si l'on veut améliorer les capacités de résolution de problème.*

Key words: Learning, Mathematics, self-explanations, Training, Worked-out examples.

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Current theme of research:

Learning from worked-out examples; learning by teaching; cognitive processes.

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