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Delay of Vehicle Motion in Traffic Dynamics

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We demonstrate that in the Optimal Velocity Model (OVM) delay times of vehicles coming from the dynamical equation of motion of OVM explain the order of delay times observed in actual traffic flows *without introducing explicit delay times.* This implies that the explicit delay time is not important in contrast to the traditional car following models, in which the explicit delay time was thought to be essential to explain realistic traffic flow. Effective delay times in various cases are estimated: the case of a leader vehicle and its follower, a queue of vehicles controlled by traffic lights and the many-vehicle case of highway traffic flow. The remarkable result is that in most of the situation for which we can make a reasonable definition of an effective delay time, the obtained delay time is of order one second. This agrees with the observed data very well.

Key words: traffic dynamics, delay time, response, vehicle motion, numerical simulation

1. Introduction

In traffic theory various types of models were proposed; among which the car-following model [4, 8, 10] is the most popular one. However none of the models succeeded in explaining the most remarkable fact that traffic flow has two phases; one is free flow with low car-density and high average speed and the other is congested flow with high car-density and low average speed.

Recently, we proposed a new model which explains the existence of those two phases [1]. We call this the 'Optimal Velocity Model' (OVM).* The equation of motion of the OVM is given by

$$\ddot{x}_n(t) = a\{V(\Delta x_n(t)) - \dot{x}_n(t)\} \qquad n = 1, 2 \cdots N, \tag{1}$$

where the notations are; car number n, time t, position of n-th car x_n and its headway $\Delta x_n = x_{n-1}(t) - x_n(t)$. a is a constant parameter which we shall call

* By the term "Optimal Velocity" we mean a desired speed which a driver is trained to keep. It was used in the previous papers and is cited by papers not only on the traffic flow problems but also on other physical phenomena such as granular motion as well.

the sensitivity. The essential difference from traditional car-following models is the introduction of an optimal velocity $V(\Delta x)$ of a vehicle, whose value is a function of headway distance. A driver reacts according to the difference between the vehicle's velocity and the optimal velocity $V(\Delta x)$ and controls its velocity by accelerating (or decelerating) his vehicle proportional to this velocity difference. The dynamical equation of OVM has two different kinds of solutions. One is a homogeneous flow solution and the other is a congested flow solution which consists of two distinct regions; high density regions and low density regions. In OVM, if the density of vehicles is above some critical value the traffic congestion occurs spontaneously, which can be understood as a sort of phase transition from a homogeneous flow state to a congested flow state [1, 2]. Also OVM is successful in reproducing the characteristic features of observed traffic flow data (See Section 2).

On the other hand, the equation of motion for traditional car-following models with delay time τ of the driver's response [5] is given by

$$\ddot{x}_n(t+\tau) = \lambda\{\dot{x}_{n-1}(t) - \dot{x}_n(t)\}. \tag{2}$$

For the case of $\tau = 0$, Eq.(2) becomes a first-order differential equation and the homogeneous flow solution of the equation is always locally stable. In this model the delay time τ plays a crucial role for the local instability. When the leading vehicle changes its velocity, the velocity change of each successive vehicle becomes larger only when $\lambda\tau > \pi/2$. In the traditional viewpoint, the origin of the delay time τ has been thought to be a *physiological delay of response*. In fact, it is well-known that the motion of a vehicle accompanies some delay time in response to the motion of its preceding vehicle. Phenomenologically the delay time τ was usually taken to be of order one second.

Here we should make clear the notion of "delay time". Note that there are two different types of delay times from the phenomenological point of view. The first type is a time lag with which the driver begins an action after being conscious of a stimulus. It may also take a finite time for a vehicle to change its velocity after the operation of the driver. We define such physiological and mechanical time lag as the "delay time of response". The second type is the "*observed delay time of vehicle motion*". Suppose a vehicle changes its velocity, we then observe the velocity change of the following vehicle after some delay time, *i.e.*

$$v_n(t) \simeq v_{n+1}(t-T), \tag{3}$$

then we can define the observed delay time of motion as T. Of course in the general case, it is difficult to find a good definition of the delay time of motion of two successive vehicles. Only in the restricted case in which two vehicles behave quite similarly, can we define the delay time of vehicle motion quite definitely.

From the theoretical point of view, there are also two types of "delay time" to be distinguished. One is the delay time τ explicitly introduced as a parameter in the equation of motion (see Eq.(2)), which we call the "*explicit delay time*" in this paper. This may correspond to the delay time of response of the above discussion.

Table I. Data of delay times observed in Chuo Motorway.

	accelerating case	decelerating case
congested flow	1.4 sec	0.9 sec
free flow	3.3 sec	2.7 sec

The other is the "*effective delay time*" emerging as a result of the dynamics of traffic flow, which is a quite different notion from the explicit delay time. This may correspond to the observed delay time of vehicle motion stated above. This effective delay time of vehicle motion includes not only the contribution from the explicit delay time but also that from a purely dynamical origin. Therefore these two "delay times" should be treated as different quantities in the analysis and should be observed separately.

In this paper, we discuss the effective delay time of motion in OVM with no explicit delay time, and examine whether or not effective delay time (see Section 3 for its exact meaning) is comparable with the observed delay time. After making a brief review of the phenomenological consequences of OVM (Section 2), we define in Section 3 the effective delay time of vehicle motion in OVM in terms of the vehicle motions of a leader and its follower and make an analytical study using a linear approximation. Then we carry out numerical simulations to obtain the effective delay time in several cases: the effective delay times of vehicle motions controlled by traffic lights (Section 4) and those in uniform traffic flow and in congested flow (Section 5). Discussions and further prospects are given in Section 6.

2. Phenomenological Consequences of OVM

We first make a quick review of our previous work which shows how our OVM reproduces the freeway traffic flow data.

As the most appropriate data for our purpose, we use the data of a car-following experiment on the Chuo Motorway [7, 9]. They obtained data points on the velocity-clearance plane (Figure 1), where the clearance S is defined as the headway subtracted by the vehicle length, l_c ($\Delta x - l_c = S$).[†]

From Figure 1, we fix the following form of the Optimal Velocity Function (OVF) [3];

$$\begin{aligned} V(\Delta x) &= 16.8\,[\tanh 0.0860\,(\Delta x - 25) + 0.913] \quad &(\text{for } \Delta x > 7\text{m}) \\ &= 0 &(\text{for } \Delta x < 7\text{m}) \end{aligned} \tag{4}$$

whose parameters are determined from the Chuo Motorway car-following experi-

[†] The term "clearance" usually refers to the yellow interval of a traffic signal. Here we use the term "clearance" following to the original graph in the paper [7]. It means the "gap" or the headway distance subtracted by the vehicle length.

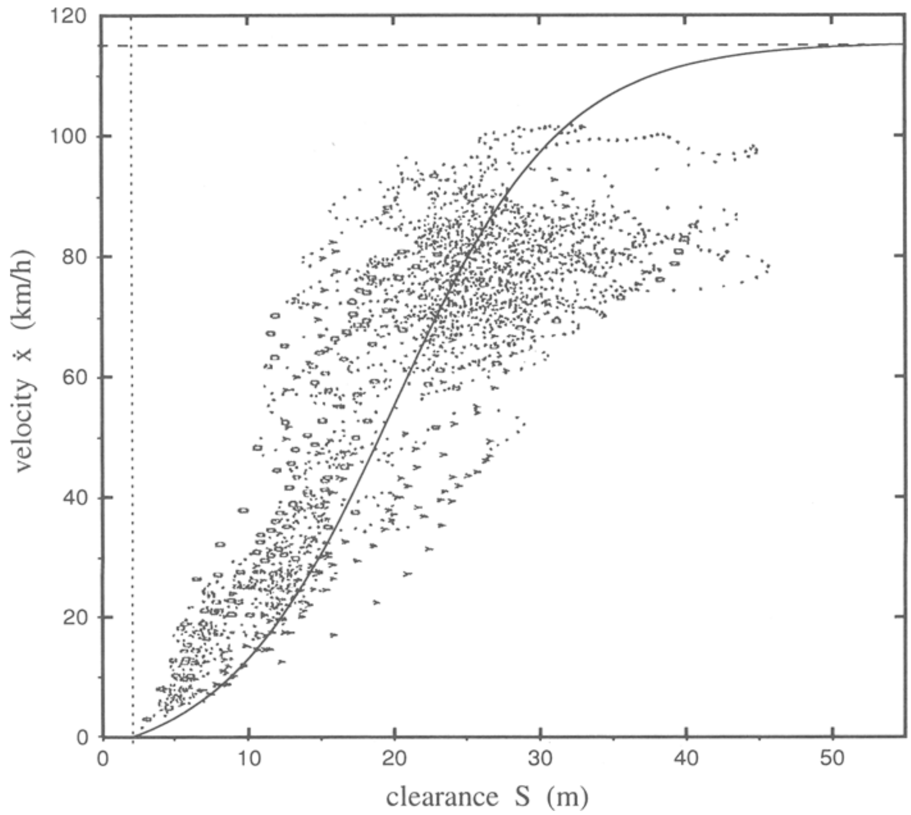


Fig. 1. Velocity-clearance data from a car-following experiment on the Chuo Motorway. Solid curve is the determined Optimal Velocity Function V (Eq.(4)).

ment [7, 9]: the inflection point is $(\Delta x, \dot{x}) = (25\text{ m}, 55\text{ km/h})$, the maximal velocity is $V_{\text{max}} = 115$ km/h and the minimal headway is $\Delta x_{\text{min}} = 7.0$ m, which includes the length of the vehicle (5 m) which was used in the experiment.

Figure 2 shows accumulated observed data of the Q-k graph taken by the Hanshin Expressway Public Corporation [11], from which we find an apparent discontinuity at approximately 25% occupancy. It indicates the existence of a critical density which separates free and congested flows. The result of our OVM simulation is shown in Figure 2 by the diamond marks together with the observed data. In the congested flow, the series of diamond marks are very dense and may look like a thick solid line. Our result agrees quite well with the observation, and reproduce the cusp-type behavior around the critical density which separates the free and congested flows.

Summarizing our results, we may conclude that OVM is successful in a unified description of both free and congested flows without introducing any explicit delay time.

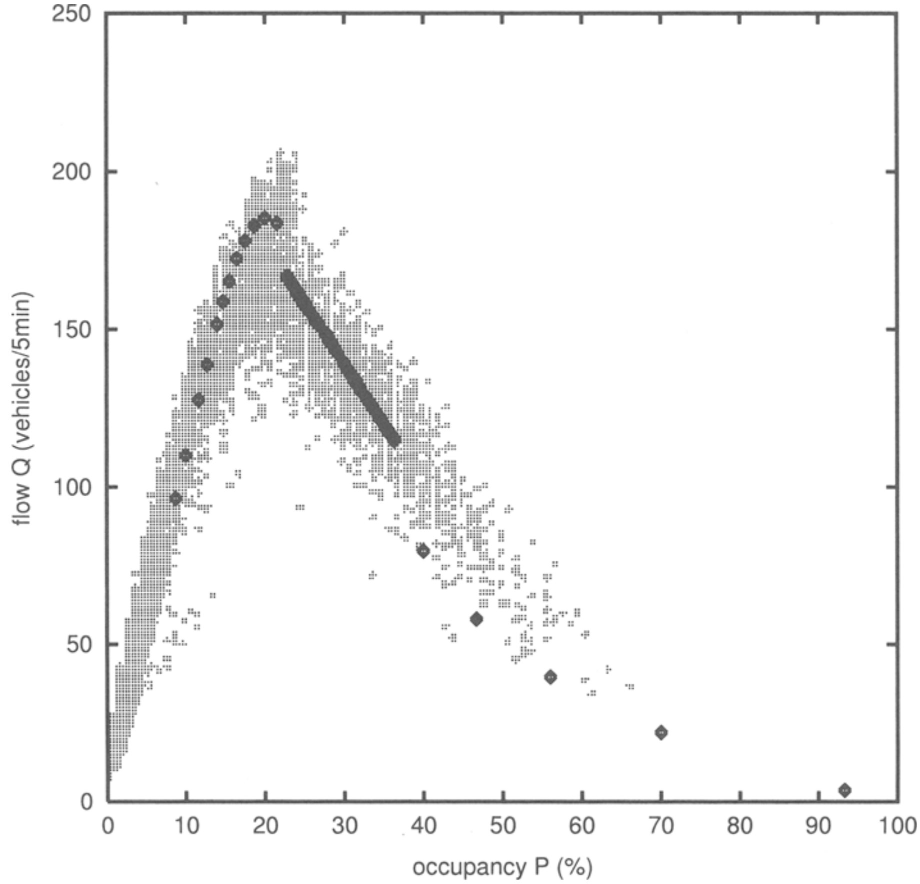


Fig. 2. Q-k diagram on the flow-occupancy plane. The result of our simulations with $a = 2.0\ \mathrm{s}^{-1}$ is shown by the diamond marks together with the observed data in Hanshin Expressway.

3. Delay Time for the Leader and Follower Case

First let us make a definition of the effective delay time of vehicle motion. Consider a pair of vehicles, a leader and its follower. This pair of vehicles may be either two separate vehicles or any successive pair of vehicles in a queue of vehicles on a highways or a queue waiting to start from a traffic light.

When a leader moves with its velocity $v(t)$, and its follower replicates the motion of the leader with some delay time T (then the follower's velocity is given by $v(t - T)$), we can define effective delay time of vehicle-motion as T. It must be remarked that *we do not define the effective delay time as the delay of motion of the follower with respect to its position but by its velocity replication.*

Let the positions of a leader and its follower be $y(t)$ and $x(t)$. In this case Eq.(1) is written as

$$\ddot{x}(t) = a\{V(y(t) - x(t)) - \dot{x}(t)\}, \tag{5}$$

which has a following solution (uniform motion)

$$y_0(t) = V(b)t + b, \quad x_0(t) = V(b)t, \tag{6}$$

where b is the headway and $V(b)$ is a constant velocity. To investigate the response of the follower vehicle to the leader vehicle, we introduce a small perturbation $\lambda(t)$ and its response $\xi(t)$:

$$y(t) = y_0(t) + \lambda(t), \quad x(t) = x_0(t) + \xi(t). \tag{7}$$

Inserting Eq.(7) into Eq.(5) and taking a linear approximation, we get

$$\ddot{\xi}(t) + a\dot{\xi}(t) + af\xi(t) = af\lambda(t), \tag{8}$$

where $f = V'(b)$. This is just equivalent to the well-known equation of motion for forced oscillation with a damping term caused by friction.

In order to find a solution, we first write $\lambda(t)$ by a Fourier expansion

$$\lambda(t) = \int \tilde{\lambda}(\omega)e^{i\omega t}d\omega. \tag{9}$$

For the Fourier component $\lambda_0 e^{i\omega t}$, the solution of Eq.(8) is given by

$$\xi(t) = \frac{\lambda_0}{1 + i\omega/f - \omega^2/af}\, e^{i\omega t}. \tag{10}$$

This is rewritten as

$$\xi(t) = |\eta|\lambda_0 e^{i\omega(t-T)}, \tag{11}$$

where

$$|\eta|^2 = \frac{|\xi|^2}{\lambda_0^2} = \frac{(af)^2}{(af - \omega^2)^2 + (a\omega)^2}, \tag{12}$$

$$T = \frac{1}{\omega}\tan^{-1}\frac{a\omega}{af - \omega^2}. \tag{13}$$

When $f < a/2$, the amplitude $|\eta|$ is a monotonically damped function of ω. On the other hand, when $f > a/2$, $|\eta|$ takes its maximum at $\omega = \omega_0$;

$$\omega_0^2 = a(f - a/2), \tag{14}$$

so we call this ω_0 the "enhanced mode" (see Figure 4(a)). Note that $f = a/2$ is the critical point for the instability condition of homogeneous flow (see the previous paper [1]). Eq.(14) shows that the non-zero enhanced mode ω_0 exists so far as the instability condition $f > a/2$ is satisfied.

Let us examine some characteristic cases. For $f < a/2$ where low frequency modes dominate $|\omega| \ll a,\ f$, we have

$$|\eta| \sim 1, \quad T \sim \frac{1}{f}. \tag{15}$$

In this case the response $\xi(t)$ to the perturbation $\lambda(t)$ becomes

$$\xi(t) = \int \tilde{\lambda}(\omega) e^{i\omega(t-T)} d\omega = \lambda(t-T), \tag{16}$$

which leads

$$\dot{x}(t) = V(b) + \dot{\xi}(t) = V(b) + \dot{\lambda}(t-T) = \dot{y}(t-T). \tag{17}$$

Thus for a sufficiently slow perturbation, the effective delay time of vehicle motion is expressed as Eq.(15); T is approximately the inverse of the derivative of the OVF at a corresponding headway.

(a)

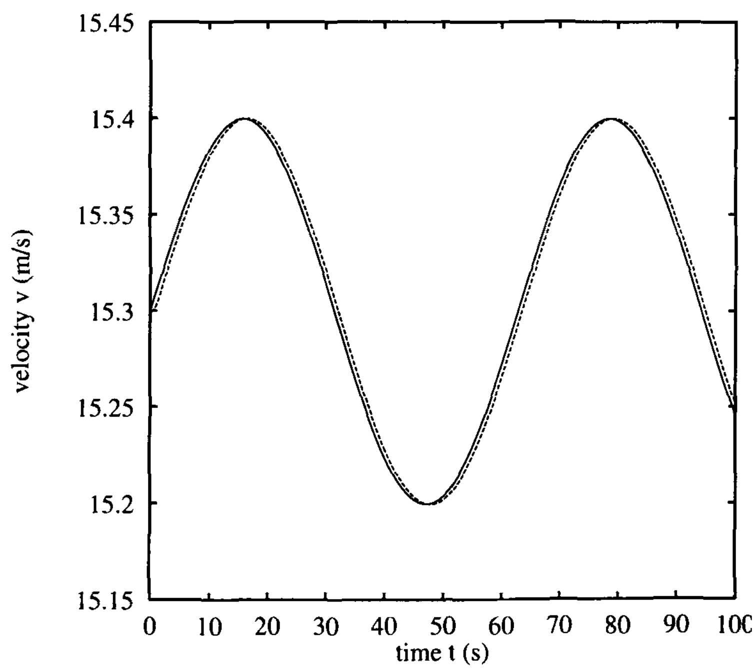
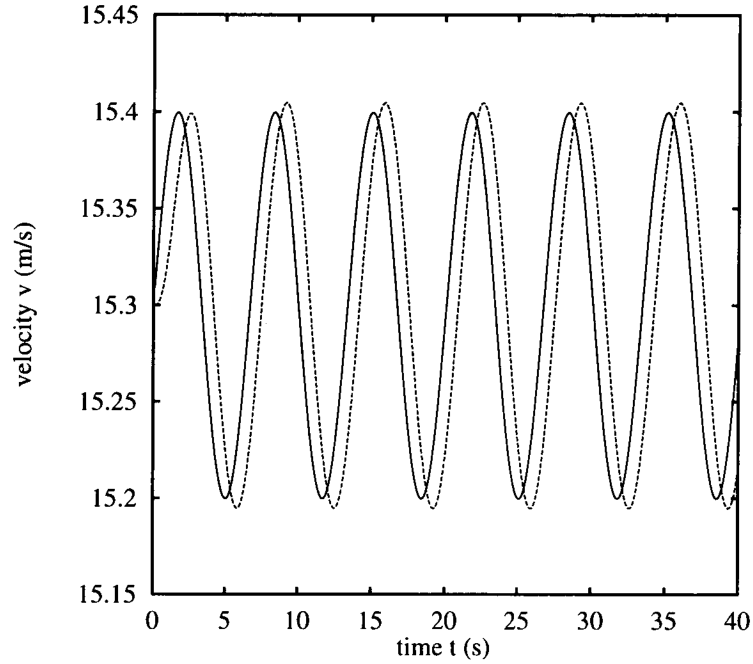


Fig. 3 (a)

(b)



(c)

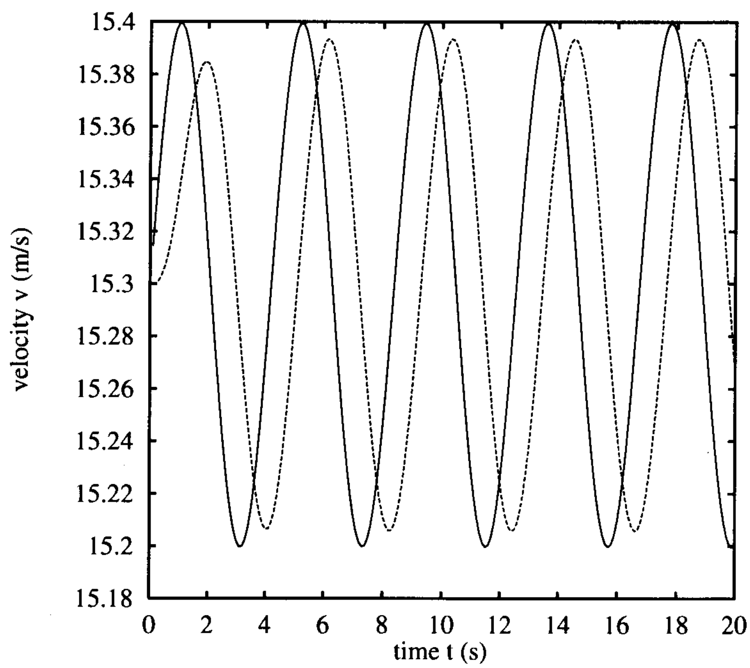
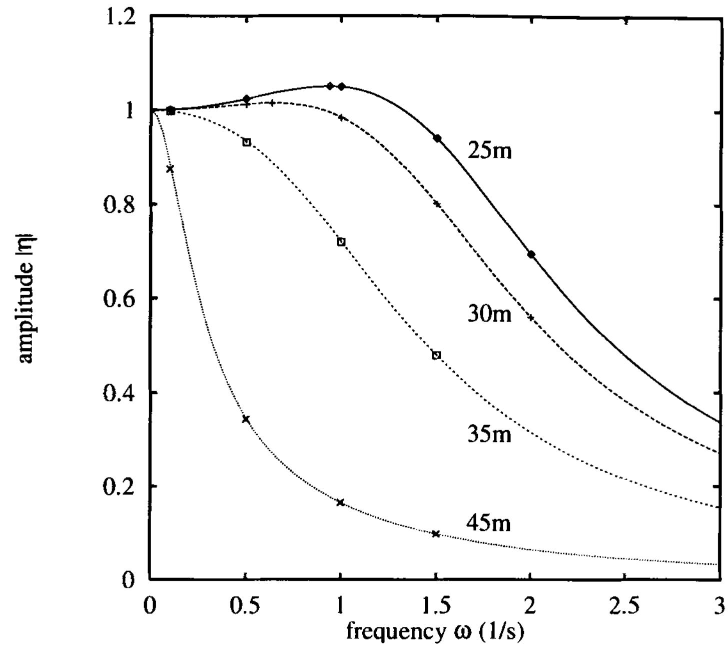


Fig. 3. Numerical results for the motion of leader and follower, where Δx =25 m and $a = 2.0\ \mathrm{s}^{-1}$. Frequencies of leader's motion are (a) $\omega = 0.1\ \mathrm{s}^{-1}$, (b) $\omega = \omega_0$ and (c) $\omega = 1.5\ \mathrm{s}^{-1}$.

(a)



(b)

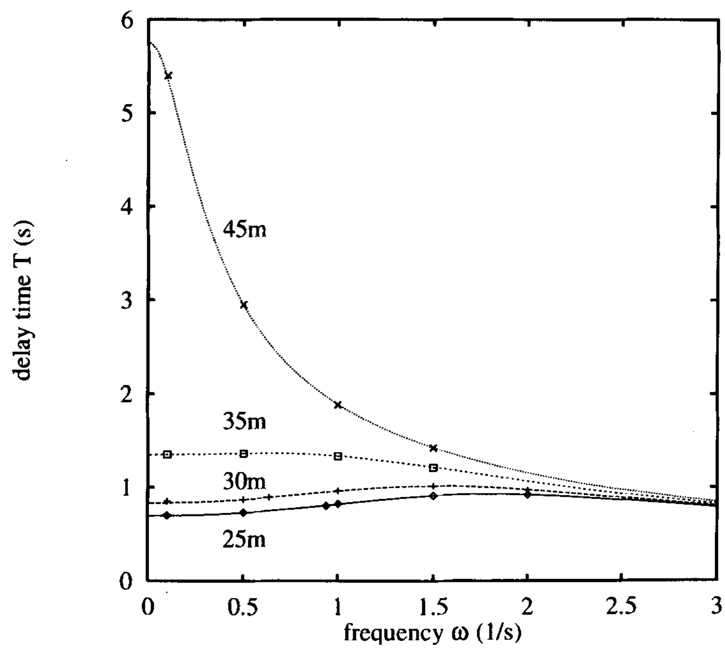


Fig. 4. Each curve shows the behavior of (a) $|\eta|$ (Eq.(12)) and (b) T (Eq.(13)) for $b = 25, 30, 35, 45$ m with $a = 2.0$ s^{-1}. Plotted marks on the curves show numerical results.

In the other case, $f > a/2$, the amplitude $|\eta|$ takes a maximum value at $\omega = \omega_0$. Then we have

$$T_{\text{enhanced}} = \frac{1}{\omega_0} \tan^{-1} \frac{2\omega_0}{a}, \tag{18}$$

which indicates that the effective delay time T for this enhanced mode depends on the sensitivity a, in contrast to the previous case Eq.(15). One can easily confirm that T tends to $1/f$ when a is close to its critical value $2f$. We should remark that an exact replication indicated in Eq.(17) is not always realized because $|\eta|$ is not equal to 1 in Eq.(11). (See also Figures 3(a)–(c) and the discussions below Eq.(19).)

Now let us see the results of numerical simulations and compare them with those of our analytical consideration. Here we use Eq.(4) as the OVF. In numerical simulations, we prepare a pair of vehicles with their unperturbed motions of Eq.(6) and with headway b. If the leader changes its motion by $\lambda(t)$ then the function $\xi(t)$ can be obtained by the numerical simulation from which the effective delay time can be read off. We choose $\lambda(t)$ as follows

$$y(t) = V(b)t + b - \lambda_0 \cos \omega t, \quad \dot{y}(t) = V(b) + v_0 \sin \omega t \quad \text{for } t \geq 0 \quad (v_0 = \lambda_0 \omega), \tag{19}$$

where v_0 is taken as 0.1 m/s.

As illustrations, we show the behaviors of $\lambda(t)$ and its response $\xi(t)$ for $a = 2.0$ s^{-1}, $b = 25$ m (therefore $\omega_0 = 0.938$ s^{-1}). Figures 3(a)–(c) are the cases for $\omega = 0.1,\ 0.938,\ 1.5$ s^{-1} respectively. To find the values T, we first rescale $\dot{\xi}(t)$ and then translate it so as to coincide with the curve $\dot{\lambda}(t)$ in Figures 3(a)–(c). The value of $|\eta|$ is this scale factor. The results for $|\eta|$ and T are shown in Figures 4(a) and (b). For reference, we also show the numerical results by making full simulations for $b = 25,\ 30,\ 35,\ 45$ m.

4. Vehicle Motions Controlled by Traffic Lights

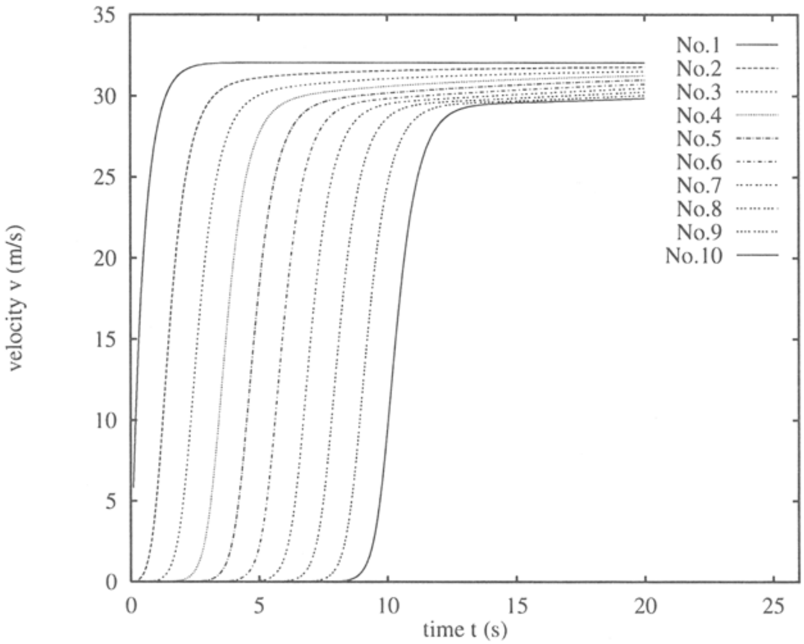
The delay time of vehicle motion is clearly recognized in motions of a series of vehicles controlled by traffic lights. Consider the situation in which every vehicle waits until a red light changes to green. The initial conditions are as follows;

$$\begin{aligned} &\Delta x_1(0) = \infty, && \dot{x}_1(0) = 0 \\ &\Delta x_n(0) = 7(\text{m}), && \dot{x}_n(0) = 0 \qquad (n = 2, 3, \ldots) \end{aligned} \tag{20}$$

When the light changes to green, the top vehicle will first start to accelerate, followed by the succeeding vehicles according to the equations of motion. Note that this situation is quite different from the previous case in which we can treat the system within a linear approximation.

In applying OVM to this case, we use the same OVF as Eq.(4), since we have no appropriate data of behavior of vehicles inside cities. Though this function does

(a)



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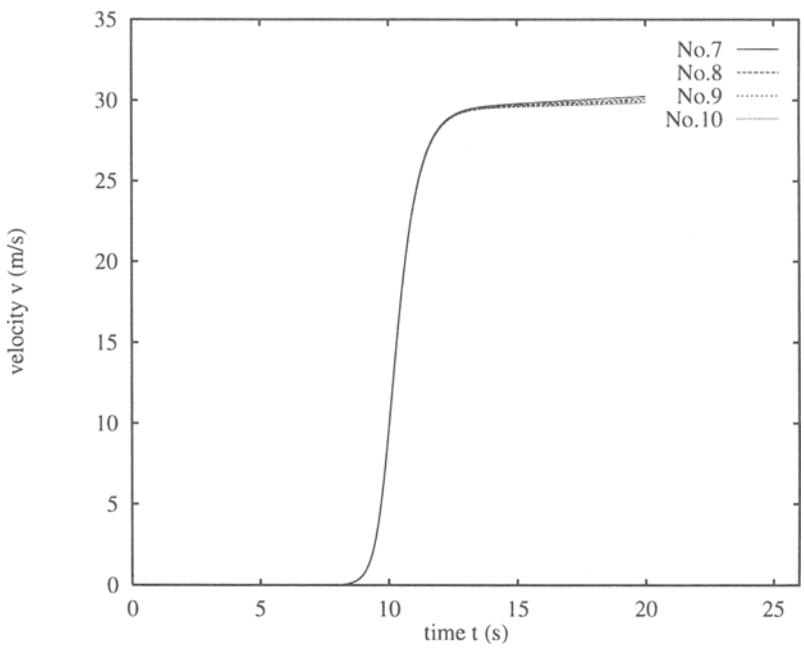
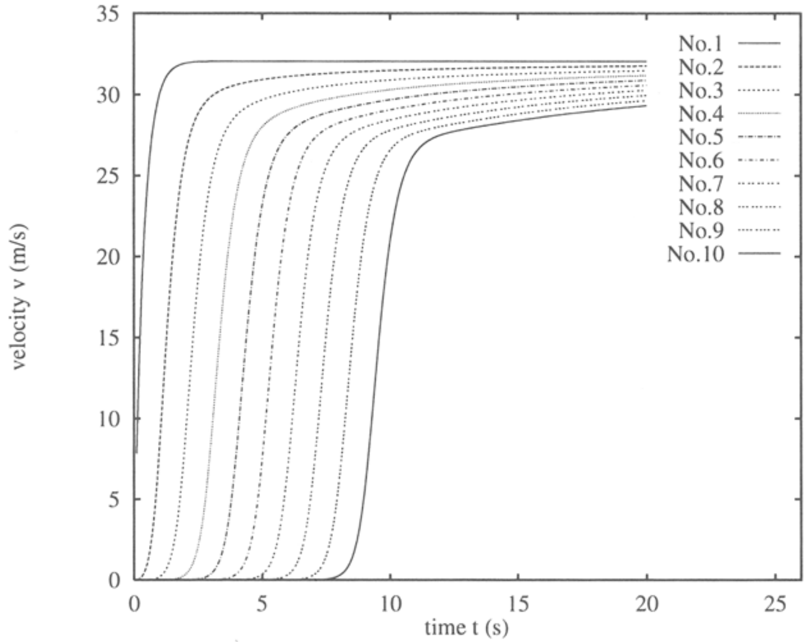


Fig. 5. (a) Simulated behavior of velocities of first ten vehicles under traffic lights with $a = 2.0\ \mathrm{s}^{-1}$. (b) Figure of shifted curves $\dot{x}_n(t-(n-10)T)$ with $T = 1.10$ s.

(a)



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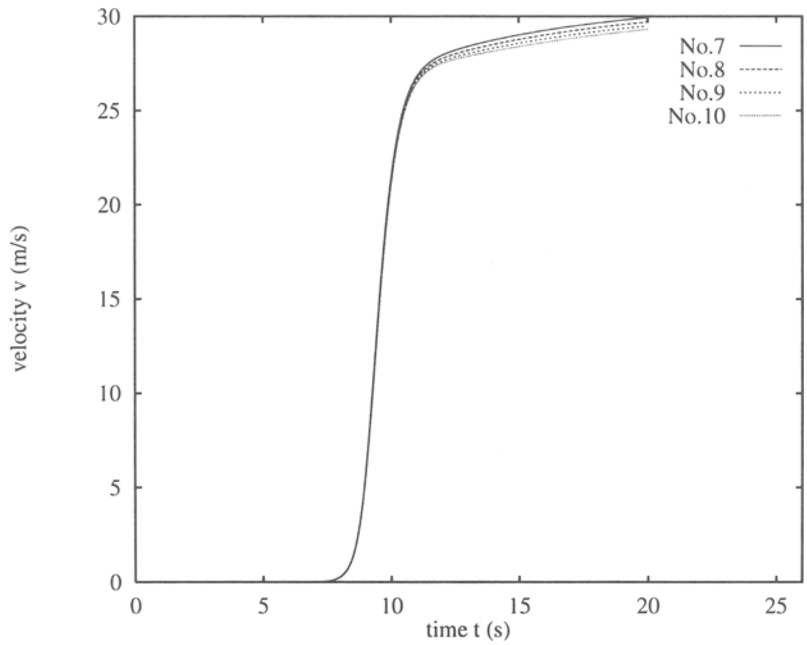

Fig. 6. (a) Simulated behavior of velocities of first ten vehicles under traffic lights with $a = 2.8\ \mathrm{s}^{-1}$. (b) Figure of shifted curves $\dot{x}_n(t - (n - 10)T)$ with $T = 1.03$ s.

not exactly correspond to the actual traffic situation, it is enough to estimate the order of the delay time of vehicle motions controlled by the the traffic lights. If we use the actual OVF, its maximum velocity becomes smaller than that of the highway data and the delay time becomes larger.

We perform the numerical simulation for the cases $a = 2.0,\ 2.8\ \mathrm{s}^{-1}$ and obtain the time dependence of the velocity $\dot{x}_n(t)$ of n-th vehicles in a queue. Figures 5(a) and 6(a) show the behavior of motion for the first ten vehicles. In the figures, we see that, except for the first few vehicles, every vehicle in the queue almost replicates the behavior of its preceding one with a certain delay time. In this case, the relation (3) can be applied for n-th vehicle with a large enough number n. In this sense we may say that a replicative pattern of vehicle motion is realized "asymptotically" and only in this occasion we can define an effective delay time of motion T in just the same way as we defined in Section 3. Note that this definition of the effective delay time slightly differs from a time lag with which the successive vehicles start, for example.

The effective delay time T is read off from Figures 5(a) and 6(a) by finding a unit quantity, with which each curve is shifted in such a way that the curves converge into a single curve. The results are $T = 1.10$ s for $a = 2.0\ \mathrm{s}^{-1}$ and $T = 1.03$ s for $a = 2.8\ \mathrm{s}^{-1}$. To demonstrate this similarity of vehicle motion, we also show plots of translated data $\dot{x}_n(t-(n-10)T)$ for the 7th, 8th, 9th and 10th vehicles in Figures 5(b) and 6(b).

5. Delay Time in Highway Traffic Flow

Next we investigate the effective delay time in a simple situation where N vehicles move on a single lane circuit with circumference L. Of course we assume that road conditions are uniform along the circuit and the drivers are identical. Numerical calculations are made with the initial condition: $x_1 = b(N-1)+\Delta, x_n = b(N-n)\ (2 \le n \le N), \dot{x}_n = V(b)\ (1 \le n \le N)$. Δ in this condition means that a small perturbation is added to the first vehicle x_1, and its value equals $2m$.

In the previous papers [1], we have shown that a homogeneous flow changes into a congested flow spontaneously if the density of vehicles is greater than the critical value. The results of simulations indicate that after enough time the traffic flow on a circuit creates an alternating pattern of high and low density regions. The motion of vehicles in this flow is visualized by plotting them in the 'phase space' $(\Delta x,\ \dot{x})$. After the traffic flow becomes stationary, the trajectory of every vehicle in this 'phase space' draws a kind of limit cycle which we named a 'hysteresis loop' in Ref. [1] (see also Figure 9). Note that this situation is far from the perturbative case treated in Section 3.

Now let us estimate effective delay times for two cases under this traffic flow: (A) the first stage and (B) the final stationary-state stage.

· *Case* A

In this case the traffic flow is almost homogeneous. Let us pick up a pair of

(a)

(b)

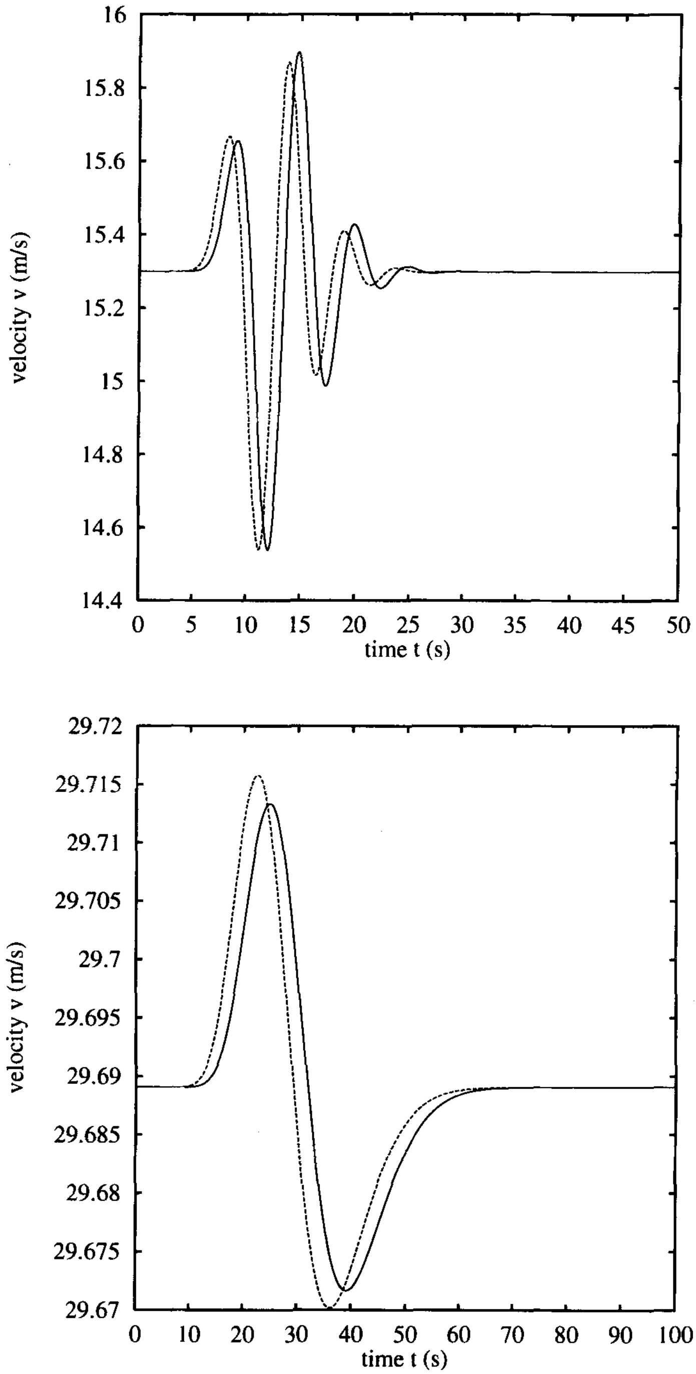

Fig. 7. Motions of 10th and 11th vehicles in the first stage of simulation with $a = 2.0\ \mathrm{s}^{-1}$: (a) Δx =25 m for 50 seconds and (b) Δx =40 m for 100 seconds.

vehicles $n = 10, 11$. A small perturbation of the first vehicle x_1 propagates backward and after several seconds that pair of vehicles change their velocities. The typical behaviors are demonstrated in Figures 7. In the same way as Section 3, the effective delay time of motion can be estimated from numerical results. The obtained values of effective delay times are shown in Table II for the case of $a = 2.0\ \mathrm{s}^{-1}$ and Table III for the case of $a = 2.8\ \mathrm{s}^{-1}$. Note that in this case many modes contribute and their total effects are obtained only from numerical simulation. As references, the reader may compare them with the effective delay times for the low frequency limit and for the enhanced mode ω_0 are also shown in Tables II, III. From these results, the low frequency limit is a good approximation and the effective delay time T is almost

Table II. Delay times for various headway with $a = 2.0\ \mathrm{s}^{-1}$. The second column indicates that traffic flow is stable $(-)$ or unstable $(+)$. The third and fourth columns show analytical results given in Section 2.

Δx (m)	$f - a/2$	$T_0 = f^{-1}$ (s)	T_{enhanced} (s)	$T_{\text{simulation}}$ (s)
10	−	2.6427	-	2.6
15	−	1.3434	-	1.35
20	+	0.8282	0.8884	0.95
25	+	0.6921	0.8017	0.85
30	+	0.8282	0.8884	0.95
35	−	1.3434	-	1.35
40	−	2.6427	-	2.6
50	−	13.101	-	13

Table III. Delay times for various headway with $a = 2.8\ \mathrm{s}^{-1}$. The second column indicates that traffic flow is stable $(-)$ or unstable $(+)$. The third and fourth columns show analytical results given in Section 2.

Δx (m)	$f - a/2$	$T_0 = f^{-1}$ (s)	T_{enhanced} (s)	$T_{\text{simulation}}$ (s)
10	−	2.6427	-	2.6
15	−	1.3434	-	1.35
20	−	0.8282	-	0.85
25	+	0.6921	0.6996	0.75
30	−	0.8282	-	0.85
35	−	1.3434	-	1.35
40	−	2.6427	-	2.6
50	−	13.101	-	13

independent of sensitivity a for stable traffic flow. For the unstable case, on the other hand, T depends explicitly on the sensitivity a.

· *Case* B

After a sufficiently long time, traffic flow forms stationary patterns of high and low density regions. Under this situation a vehicle does not change its velocity unless it encounters a boundary of high and low density regions. Let us observe the motion of a vehicle on the boundary. A vehicle which encounters the boundary changes its velocity. After some delay time, the following vehicle comes to the boundary and changes its velocity in the same way as the previous vehicle. The typical behavior of vehicles is shown in Figure 8.

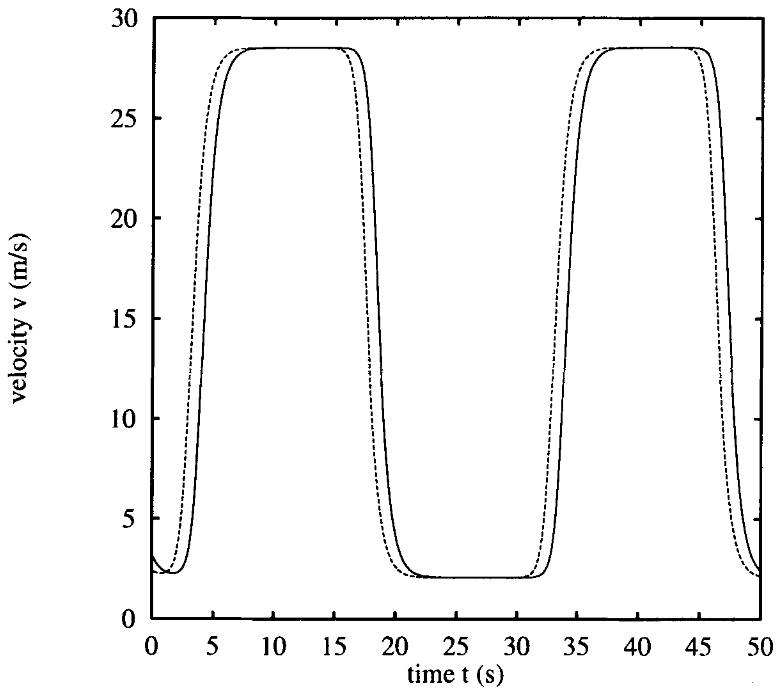

Fig. 8. Simulated motion of successive two vehicles in congested flow. Initial condition of simulation is Δx =25 m with $a = 2.0\ \mathrm{s}^{-1}$.

The effective delay time of vehicle motion in this case can be derived as follows. Consider two vehicles: one enters into a high density region from a low density region and, after a certain interval T, the next one follows. In the 'phase space' $(\Delta x,\ \dot{x})$ (Figure 9), the motion in the low density region is represented by a point $F(\Delta x_F, v_F)$, that is, vehicles are moving with velocity v_F and headway Δx_F. The time interval T is defined as the time which is needed for the next vehicle to reach the boundary and enter into a high density region. Now at the time when the first vehicle reaches the boundary of the low density region the distance between the

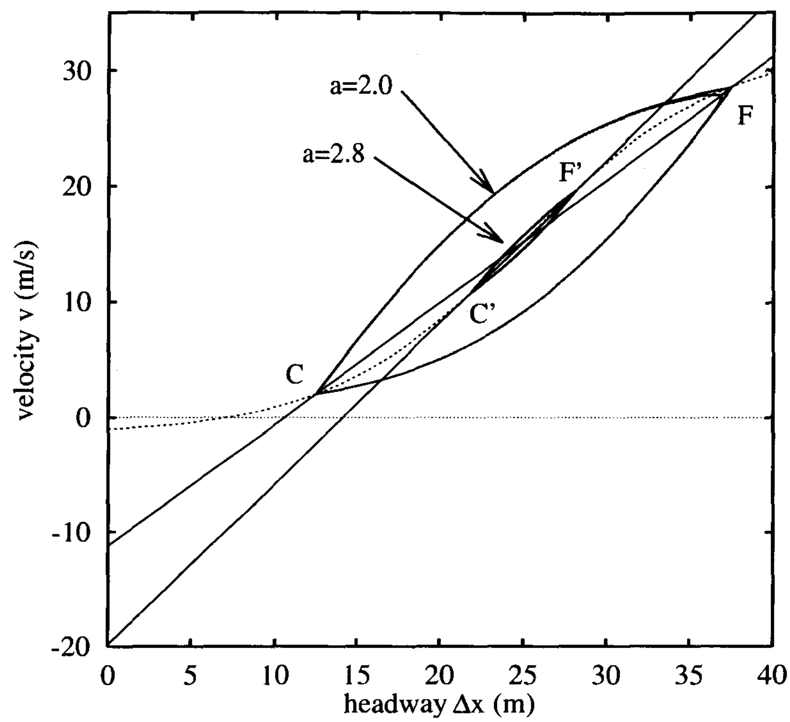

Fig. 9. The limit cycles obtained by numerical simulations for $a = 2.0\ \mathrm{s}^{-1}$ and $a = 2.8\ \mathrm{s}^{-1}$.

next vehicle and this boundary is of course Δx_F. The next vehicle moves with velocity v_F and the boundary itself also is moving backward with velocity v_B, so if the vehicle and the boundary meet after a time interval T, we can write the following relation

$$v_F T + v_B T = \Delta x_F. \tag{21}$$

A similar relation can be written at a boundary where a vehicle exits from a high density region, which corresponds to a point $C(\Delta x_C, v_C)$ in Figure 9.

$$v_C T + v_B T = \Delta x_C. \tag{22}$$

This can be confirmed if one recalls that the pattern of the flow is already stationary, and the input vehicles at a boundary of the high density region must be equal to the output at another boundary. Therefore the time interval T in Eqs.(21) and (22) must be identical, and all vehicles move in the same way with the time interval T. If we write the velocity of the first vehicle as $v(t)$, the velocity of the following vehicle is $v(t - T)$, which implies that T is just the effective delay time of vehicle motion defined in the previous section (see also Ref. [6]). From Eqs.(21) and (22), we find

$$T = \left(\frac{v_F - v_C}{\Delta x_F - \Delta x_C} \right)^{-1} . \tag{23}$$

Graphically the effective time delay T can be expressed as the slope of the line connecting F and C for $a = 2.0\ \mathrm{s}^{-1}$ and F' and C' for $a = 2.8\ \mathrm{s}^{-1}$ in Figure 9, and v_B is the intercept with the vertical axis of this line.

We have carried out numerical simulations. For sensitivity $a = 2.0\ \mathrm{s}^{-1}$, we find $C(12.51, 2.05)$ and $F(37.50, 28.55)$, yielding the effective delay of vehicle motion $T = 0.943$ s and the back velocity of the boundary $v_B = 11.2$ m/s. As for $a = 2.8\ \mathrm{s}^{-1}$, $C'(21.89, 10.92)$ and $F'(28.11, 19.68)$, yielding $T = 0.711$ s and $v_B = 19.9$ m/s.

It is interesting to find that the main contribution of the resultant delay of vehicle-motion comes from the structure of the OVM and not from the explicit delay τ.

6. Summary and Discussions

The notion of the delay time of response τ has played a significant role in the history of traffic dynamics. Indeed delays of vehicle motions are observed in many cases, in traffic light waiting queues or in highway traffic motion and the delay time of vehicle motion are usually observed to be of order one second. It has long been thought that the observed delay time must be introduced directly in the equation of motion as an explicit delay time, most of which is caused by the driver's physiological delay time and the mechanical delay of the response of the vehicles. However it is known that the physiological response time is of order 0.1 second, not of order 1 second. We should be careful that the delay time of vehicle motion comes from another origin, that is, from the equation of motion itself which we have here investigated intensively. The results are summarized as follows:

1. *The case of a leader vehicle and its follower*
   As is seen in Figure 4(b), if the headway distance is around 25 m, in which drivers are sensitive to the behavior of the motion of the preceding vehicle, we clearly recognize that the effective delay time is around 1 second, independent of the frequency of the leader's velocity-change function $\lambda(t)$. However if their headway distance is more than 40 m, the effective delay time is estimated to be larger than 1 second. For example, we obtain 6 seconds for the case $\Delta x = 45$ m and low frequency limit ($\omega \sim 0$). This is because of the structure of the Optimal Velocity Function (OVF). If the slope of the OVF is very small, drivers are insensitive to the behavior of the preceding vehicle. This can be easily understood if one considers the extreme case in which the function V is independent of Δx (and so $f = 0$). In this case, a follower never reacts to its previous vehicle and accordingly its effective delay time becomes infinite.

2. *A queue of vehicles controlled by traffic lights*
   In this case, except for the first several vehicles, most of the succeeding vehi-

cles behave almost similarly, as seen in Figures 5 and 6. From those figures, effective delay times are read off; $T = 1.10$ s for $a = 2.0$ s^{-1} and $T = 1.03$ s for $a = 2.0$ s^{-1}. Although T depends on the sensitivity adopted, the results obtained are again of order 1 second for a reasonably realistic sensitivity.

3. *Many-vehicle case of highway traffic flow*
In Figures 7(a) and (b) we show the typical behaviors of a pair of vehicles and in Table II and III the effective delay times obtained by numerical simulations are summarized with various values of its headway Δx. Again in the case of $\Delta x = 25$ m, the effective delay time is estimated to be of order 1 second. In the case where the congested flow becomes stable, $T = 0.71$ s for $a = 2.8$ s^{-1} and $T = 0.9471$ s for $a = 2.0$ s^{-1}. Since the end point $C(F)$ on the limit cycle becomes larger (smaller) as a becomes larger (see for example Figure 7 in Ref. [3]); so of course T becomes smaller for larger a (high sensitivity).

All of our results show that the effective delay time in Optimal Velocity Model (OVM) is almost enough to reproduce the order of the observed delay time. It now becomes obvious that the estimated delay time of motion arises as an effect of the dynamical equation itself without any explicit introduction of τ. This may come from the structure of the OVF itself which we have determined phenomenologically. However we believe that this remarkable fact has a more profound reason, which will be made clearer by further investigation of OVM by performing an analytical study [12].

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