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# Acoustic Diffraction from a Slit in an Absorbing Sheet

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The diffraction of an acoustic wave, by a slit in an infinite absorbing plane in the presence of still air and moving fluid, is investigated. The problem is solved using integral transforms, the Wiener-Hopf technique and asymptotic methods. It is found that the diffracted field is the sum of fields produced by two edges of the planes formed by the slit and an interaction field.

*Key words*: Wiener-Hopf technique, integral transforms, diffraction theory, asymptotic methods, kernel factorization

## Introduction

In recent years, noise has become a serious issue of environmental protection. Noise abatement has, therefore, attracted the attention of many scientists. An effective method of noise reduction is to use barriers in heavily built up areas [1, 2]. An ideal barrier should be such that it is a good attenuator of sound and economical at the same time. Such barriers have an absorbing lining on the surfaces and satisfy absorbing boundary conditions. In case of noise radiated from aero engines and for noise inside wind tunnels, it is necessary to discuss acoustic diffraction in the presence of a moving fluid. The absorbing half plane was first considered by Rawlins [3, 4]. He discussed the diffraction of an acoustic wave from a half plane satisfying absorbing boundary conditions and determined how effectively the sound radiation is reduced by an absorbing lining in the presence of a fluid flow. However, no attempt has been made so far to discuss the acoustic scattering from a slit in an infinite absorbing plane. This situation arises when there is a finite opening in an infinite barrier intercepting the line of sight from the noise source to receiver.

The aim of this paper is to calculate the diffracted fields due to a plane wave and line source incidences in an infinite absorbing plane in still air and in the presence of a moving fluid. It is found that the two edges of the planes give rise to two diffracted fields (one from each edge) and an interaction field (double diffraction of the two edges). The field due to a slit in an infinite *rigid* barrier can be recovered as a special case taking the absorption parameter to be zero.

## 1. Plane Wave Incidence

Let $(x, y)$ define a system of cartesian coordinates with origin $O$. The absorbing planes are at the positions $x < x_1$, $x > x_2$ and a slit (aperture) is at $x_1 \leq x \leq x_2$ as shown in the Fig. 1. The absorbing planes are assumed to be of infinitesimal

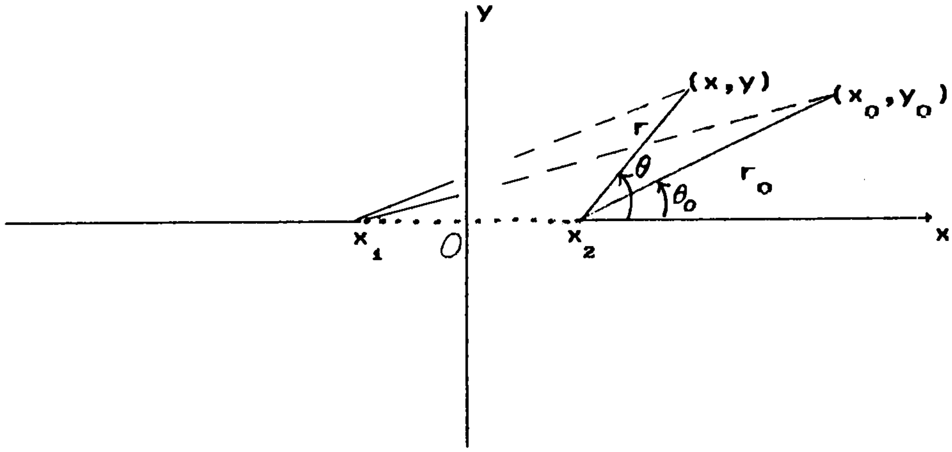


Fig. 1.

thickness and satisfying the absorbing boundary conditions $p - u_n z = 0$ [5] on both sides of the surfaces. Here, $p$ is the pressure, $u_n$ is the normal derivative of the perturbation velocity and $z$ is the acoustic impedance of the planes. The system responds to an incident plane wave $\psi_i$ given by

$$\psi_i = \exp(-\mathrm{i}kx\cos\vartheta_0 - \mathrm{i}ky\sin\vartheta_0),$$

where $\vartheta_0$ is the angle measured from the $x$-axis. The time dependence is assumed to be of harmonic nature $e^{-\mathrm{i}\omega t}$ ($\omega$ is low angular frequency), with the free space wave number of the form

$$k = \omega/c = k_1 + \mathrm{i}k_2, \tag{1}$$

where $c$ is the speed of sound. In Eq.(1), $k$ has a small positive imaginary part which has been introduced to ensure the convergence (regularity) of the Fourier transform integrals defined subsequently (Eq.(10a)). On suppressing the time harmonic factor, the wave equation satisfied by the total velocity potential $\psi$ is given by

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + k^2\psi = 0. \tag{2}$$

On the absorbing plates, we have the boundary conditions

$$\left(\frac{\partial}{\partial y} \pm \mathrm{i}k\beta\right)\psi(x, 0^{\pm}) = 0, \quad \begin{cases} x < x_1 \\ x > x_2 \end{cases} \tag{3}$$

where $\beta$ $(= \rho_0 c/z$, $\rho_0$ is the density of the undisturbed stream) is the small absorbing parameter. We remark that $\beta = 0$ corresponds to the rigid barrier and $\beta = \infty$ corresponds to the pressure release barrier. The velocity potential $\psi$ and its derivative $\frac{\partial\psi}{\partial y}$ are continuous on the slit, i.e.,

$$\left.\begin{aligned}&\psi(x,0^+)=\psi(x,0^-),\\&\frac{\partial}{\partial y}\psi(x,0^+)=\frac{\partial}{\partial y}\psi(x,0^-).\end{aligned}\right\}\;x_1\le x\le x_2 \tag{4}$$

In addition, we insist that $\psi$ represents an outward travelling wave as $r = (x^2+y^2)^{1/2} \longrightarrow \infty$ and satisfies the edge conditions [6]

$$\left.\begin{aligned}&\psi(x,0)=O(1),\\&\frac{\partial\psi}{\partial y}(x,0)=O(x^{-1/2}).\end{aligned}\right\}\quad \text{as}\quad x\longrightarrow x_1^+ \quad\text{and}\quad x_2^-. \tag{5}$$

It is appropriate to split the total field $\psi$ as

$$\psi=\begin{cases}\psi_i+\psi_r+\phi, & y\ge 0,\\ \phi, & y\le 0,\end{cases} \tag{6}$$

where

$$\psi_r=\exp(-\mathrm{i}kx\cos\vartheta_0+\mathrm{i}ky\sin\vartheta_0),$$

and $\phi$ is the diffracted field.

The boundary value problem can now be reformulated in terms of the diffracted field $\phi$ through Eq.(6) as

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+k^2\phi=0, \tag{7}$$

subject to the boundary conditions

$$\left.\begin{aligned}&\frac{\partial}{\partial y}\phi(x,0^+)+\mathrm{i}k\beta\phi(x,0^+)+2\mathrm{i}k\beta e^{-\mathrm{i}kx\cos\vartheta_0}=0,\\&\frac{\partial}{\partial y}\phi(x,0^-)-\mathrm{i}k\beta\phi(x,0^-)=0,\end{aligned}\right\}\;\begin{cases}x<x_1\\x>x_2\end{cases} \tag{8}$$

$$\left.\begin{aligned}&\phi(x,0^+)-\phi(x,0^-)=-2e^{-\mathrm{i}kx\cos\vartheta_0},\\&\frac{\partial}{\partial y}\phi(x,0^+)=\frac{\partial}{\partial y}\phi(x,0^-).\end{aligned}\right\}\;x_1\le x\le x_2 \tag{9}$$

## 2. Solution of the Problem

We define the Fourier transform pair by

$$\begin{aligned}\overline{\phi}(\alpha,y)&=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\phi(x,y)e^{\mathrm{i}\alpha x}dx,\\ \phi(x,y)&=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\overline{\phi}(\alpha,y)e^{-\mathrm{i}\alpha x}d\alpha,\end{aligned} \tag{10a}$$

where $\alpha$ is a complex variable. In order to accommodate three-part boundary conditions on $y=0$, we split $\overline{\phi}(\alpha,y)$ as

$$\overline{\phi}(\alpha,y)=\overline{\phi}_+(\alpha,y)e^{\mathrm{i}\alpha x_2}+\overline{\phi}_-(\alpha,y)e^{\mathrm{i}\alpha x_1}+\overline{\phi}_1(\alpha,y), \tag{10b}$$

where

$$\begin{aligned}\overline{\phi}_+(\alpha,y) &= \frac{1}{\sqrt{2\pi}}\int_{x_2}^{\infty}\phi(x,y)e^{\mathrm{i}\alpha(x-x_2)}dx,\\ \overline{\phi}_1(\alpha,y) &= \frac{1}{\sqrt{2\pi}}\int_{x_1}^{x_2}\phi(x,y)e^{\mathrm{i}\alpha x}dx,\\ \overline{\phi}_-(\alpha,y) &= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x_1}\phi(x,y)e^{\mathrm{i}\alpha(x-x_1)}dx.\end{aligned}$$

In Eq.(10b), $\overline{\phi}_+(\alpha,y)$ is regular for $\operatorname{Im}\alpha>-\operatorname{Im}k$, $\overline{\phi}_-(\alpha,y)$ is regular for $\operatorname{Im}\alpha<\operatorname{Im}k$ and $\overline{\phi}_1(\alpha,y)$ is an integral function. For this we recall that $k$ is complex and $\phi$ represents an outward travelling wave of the form $|\phi|<\exp(-k_2|x|)$ as $|x|\longrightarrow\infty$ for any fixed $y$. Taking Fourier transform of Eq.(7), we obtain

$$\frac{d^2}{dy^2}\overline{\phi}(\alpha,y)-\gamma^2\overline{\phi}(\alpha,y)=0, \tag{11}$$

where $\gamma=\sqrt{(\alpha^2-k^2)}$ and the $\alpha$-plane is cut such that $\operatorname{Re}\gamma>0$. The solution of Eq.(11) satisfying the radiation condition is given by

$$\overline{\phi}(\alpha,y)=\begin{cases}A_1(\alpha)e^{-\gamma y}, & y\geq 0,\\ A_2(\alpha)e^{\gamma y}, & y\leq 0.\end{cases} \tag{12}$$

Transforming boundary conditions (8) and (9), we get

$$\overline{\phi}'_-(\alpha,0^+)+\mathrm{i}k\beta\overline{\phi}_-(\alpha,0^+)+\frac{2k\beta e^{-\mathrm{i}k\cos\vartheta_0 x_1}}{\sqrt{2\pi}(\alpha-k\cos\vartheta_0)}=0, \tag{13a}$$

$$\overline{\phi}'_-(\alpha,0^-)-\mathrm{i}k\beta\overline{\phi}_-(\alpha,0^-)=0, \tag{13b}$$

$$\overline{\phi}'_+(\alpha,0^+)+\mathrm{i}k\beta\overline{\phi}_+(\alpha,0^+)-\frac{2k\beta e^{-\mathrm{i}k\cos\vartheta_0 x_2}}{\sqrt{2\pi}(\alpha-k\cos\vartheta_0)}=0, \tag{14a}$$

$$\overline{\phi}'_+(\alpha,0^-)-\mathrm{i}k\beta\overline{\phi}_+(\alpha,0^-)=0, \tag{14b}$$

$$\overline{\phi}_1(\alpha,0^+)-\overline{\phi}_1(\alpha,0^-)=2\mathrm{i}G(\alpha), \tag{15a}$$

$$\overline{\phi}'_1(\alpha,0^+)=\overline{\phi}'_1(\alpha,0^-), \tag{15b}$$

where

$$G(\alpha)=\frac{1}{\sqrt{2\pi}(\alpha-k\cos\vartheta_0)}\left\{e^{\mathrm{i}(\alpha-k\cos\vartheta_0)x_2}-e^{\mathrm{i}(\alpha-k\cos\vartheta_0)x_1}\right\}. \tag{15c}$$

Using the boundary conditions (13)–(15) in Eq.(12) and eliminating $\overline{\phi}'_+$ and $\overline{\phi}'_-$, we get

$$e^{\mathrm{i}\alpha x_2}\overline{\chi}_+(\alpha,0)+\frac{\overline{\phi}'_1(\alpha,0)}{(\gamma-\mathrm{i}k\beta)}+e^{\mathrm{i}\alpha x_1}\overline{\chi}_-(\alpha,0)=-\mathrm{i}G(\alpha), \tag{16}$$

where

$$\begin{aligned}\overline{\phi}_+(\alpha,0^+)-\overline{\phi}_+(\alpha,0^-)&=2\overline{\chi}_+(\alpha,0),\\ \overline{\phi}_-(\alpha,0^+)-\overline{\phi}_-(\alpha,0^-)&=2\overline{\chi}_-(\alpha,0).\end{aligned}$$

Eq.(16) is the standard Wiener-Hopf functional equation. For the solution of this equation, we make the following factorizations:

$$\gamma=K_+(\alpha)K_-(\alpha)=(\alpha+k)^{1/2}(\alpha-k)^{1/2}, \tag{17}$$

and

$$\left(1-\frac{\mathrm{i}k\beta}{\gamma}\right)=L_+(\alpha)L_-(\alpha)=L(\alpha), \tag{18}$$

where $L_+(\alpha)$ and $K_+(\alpha)$ are regular for $\mathrm{Im}\,\alpha>-\mathrm{Im}\,k$ and $L_-(\alpha)$ and $K_-(\alpha)$ are regular for $\mathrm{Im}\,\alpha<\mathrm{Im}\,k$. The factorization (18) has been discussed by Noble [7] and is given by

$$L_\pm(\alpha)=1-\frac{\mathrm{i}\beta}{\pi}[(\alpha/k)^2-1]^{-1/2}\cos^{-1}(\pm\alpha/k). \tag{19}$$

Thus, using Eqs.(17) and (18) in Eq.(16), we obtain

$$e^{\mathrm{i}\alpha x_2}\overline{\chi}_+(\alpha,0)+\frac{\overline{\phi}'_1(\alpha,0)}{S_+(\alpha)S_-(\alpha)}+e^{\mathrm{i}\alpha x_1}\overline{\chi}_-(\alpha,0)=-\mathrm{i}G(\alpha), \tag{20}$$

where $S_+(\alpha)$ $[=K_+(\alpha)L_+(\alpha)]$ is regular for $\mathrm{Im}\,\alpha>-\mathrm{Im}\,k$ and $S_-(\alpha)$ $[=K_-(\alpha)L_-(\alpha)]$ is regular for $\mathrm{Im}\,\alpha<\mathrm{Im}\,k$.

With the help of Eqs.(10b), (12) and (13a–15c), the unknown functions $A_1(\alpha)$ and $A_2(\alpha)$ are given by

$$\begin{aligned}2A_1(\alpha)&=e^{\mathrm{i}\alpha x_2}\left(\overline{\phi}_+(\alpha,0^+)-\overline{\phi}_+(\alpha,0^-)\right)\\&\quad+e^{\mathrm{i}\alpha x_1}\left(\overline{\phi}_-(\alpha,0^+)-\overline{\phi}_-(\alpha,0^-)\right)+2\mathrm{i}G(\alpha)\\&\quad+\frac{\mathrm{i}k\beta}{\gamma}\Big\{e^{\mathrm{i}\alpha x_2}\left(\overline{\phi}_+(\alpha,0^+)+\overline{\phi}_+(\alpha,0^-)\right)\\&\quad+e^{\mathrm{i}\alpha x_1}\left(\overline{\phi}_-(\alpha,0^+)+\overline{\phi}_-(\alpha,0^-)\right)+2\mathrm{i}G(\alpha)\Big\}.\end{aligned} \tag{21a}$$

$$-2A_2(\alpha)=e^{\mathrm{i}\alpha x_2}\left(\overline{\phi}_+(\alpha,0^+)-\overline{\phi}_+(\alpha,0^-)\right)$$

$$
\begin{aligned}
&+ e^{\mathrm{i}\alpha x_1}\left(\overline{\phi}_-(\alpha,0^+) - \overline{\phi}_-(\alpha,0^-)\right) + 2\mathrm{i}G(\alpha) \\
&- \frac{\mathrm{i}k\beta}{\gamma}\Big\{e^{\mathrm{i}\alpha x_2}\left(\overline{\phi}_+(\alpha,0^+) + \overline{\phi}_+(\alpha,0^-)\right) \\
&+ e^{\mathrm{i}\alpha x_1}\left(\overline{\phi}_-(\alpha,0^+) + \overline{\phi}_-(\alpha,0^-)\right) + 2\mathrm{i}G(\alpha)\Big\}. \qquad (21\mathrm{b})
\end{aligned}
$$

We assert that $k\beta/\gamma$ is very very small provided $|\alpha/k|$ is not too near to 1. This assertion can be justified under the assumptions of small absorbing parameter $\beta$ and low frequency of the acoustic wave. Thus, using this approximation, Eqs.(19) and (21a, b) yield

$$
L_\pm(\alpha) \sim 1 \pm \mathrm{i}\alpha\beta/\gamma, \qquad (22\mathrm{a})
$$

$$
\begin{aligned}
2A_1(\alpha) = -2A_2(\alpha) &= e^{\mathrm{i}\alpha x_2}\left(\overline{\phi}_+(\alpha,0^+) - \overline{\phi}_+(\alpha,0^-)\right) \\
&+ e^{\mathrm{i}\alpha x_1}\left(\overline{\phi}_-(\alpha,0^+) - \overline{\phi}_-(\alpha,0^-)\right) + 2\mathrm{i}G(\alpha). \qquad (22\mathrm{b})
\end{aligned}
$$

Note that in writing Eqs.(22a, b) we have retained the terms of the order $O(\beta/\gamma)$ and neglected the terms of $O(k\beta/\gamma)$.

Now, multiplying Eq.(20) by $S_+(\alpha)e^{-\mathrm{i}\alpha x_2}$ and using the general decomposition theorem [7] (§1.3, p.13) we obtain

$$
\begin{aligned}
S_+(\alpha)\overline{\chi}_+(\alpha,0) &+ \frac{\mathrm{i}e^{-\mathrm{i}k\cos\vartheta_0 x_2}}{\sqrt{2\pi}(\alpha - k\cos\vartheta_0)}\left(S_+(\alpha) - S_+(k\cos\vartheta_0)\right) + U_+(\alpha) + V_+(\alpha) \\
&= \frac{-\mathrm{i}e^{-\mathrm{i}k\cos\vartheta_0 x_2}S_+(k\cos\vartheta_0)}{\sqrt{2\pi}(\alpha - k\cos\vartheta_0)} \\
&\qquad - \frac{e^{\mathrm{i}\alpha x_2}\overline{\phi}_1'(\alpha,0)}{S_-(\alpha)} - U_-(\alpha) - V_-(\alpha), \qquad (23)
\end{aligned}
$$

where

$$
S_+(\alpha)\overline{\chi}_-(\alpha,0)e^{-\mathrm{i}\alpha(x_2-x_1)} = U(\alpha) = U_+(\alpha) + U_-(\alpha),
$$

$$
\frac{-\mathrm{i}e^{-\mathrm{i}\alpha(x_2-x_1)-\mathrm{i}k\cos\vartheta_0 x_1}S_+(\alpha)}{\sqrt{2\pi}(\alpha - k\cos\vartheta_0)} = V(\alpha) = V_+(\alpha) + V_-(\alpha),
$$

and

$$
\begin{aligned}
U_+(\alpha) &= \frac{1}{2\pi\mathrm{i}}\int_{-\infty}^{\infty}\frac{U(\xi)}{(\xi-\alpha)}d\xi, \quad \operatorname{Im}\alpha > 0, \\
U_-(\alpha) &= -\frac{1}{2\pi\mathrm{i}}\int_{-\infty}^{\infty}\frac{U(\xi)}{(\xi-\alpha)}d\xi, \quad \operatorname{Im}\alpha < 0, \\
V_+(\alpha) &= \frac{1}{2\pi\mathrm{i}}\int_{-\infty}^{\infty}\frac{V(\xi)}{(\xi-\alpha)}d\xi, \quad \operatorname{Im}\alpha > 0, \\
V_-(\alpha) &= -\frac{1}{2\pi\mathrm{i}}\int_{-\infty}^{\infty}\frac{V(\xi)}{(\xi-\alpha)}d\xi, \quad \operatorname{Im}\alpha < 0.
\end{aligned}
$$

Now, multiplying Eq.(20) by $S_-(\alpha)e^{-\mathrm{i}\alpha x_1}$, we get

$$\begin{aligned} S_-(\alpha)\overline{\chi}_-(\alpha,0) - \frac{\mathrm{i}e^{-\mathrm{i}k\cos\vartheta_0 x_1}S_-(\alpha)}{\sqrt{2\pi}(\alpha - k\cos\vartheta_0)} + R_-(\alpha) - Q_-(\alpha) \\ = -\frac{e^{-\mathrm{i}\alpha x_1}\overline{\phi}_1'(\alpha,0)}{S_+(\alpha)} - R_+(\alpha) + Q_+(\alpha), \end{aligned} \tag{24}$$

where

$$S_-(\alpha)\overline{\chi}_+(\alpha,0)e^{\mathrm{i}\alpha(x_2-x_1)} = R(\alpha) = R_+(\alpha) + R_-(\alpha),$$

$$\frac{-\mathrm{i}e^{\mathrm{i}\alpha(x_2-x_1)-\mathrm{i}k\cos\vartheta_0 x_2}S_-(\alpha)}{\sqrt{2\pi}(\alpha - k\cos\vartheta_0)} = Q(\alpha) = Q_+(\alpha) + Q_-(\alpha),$$

and

$$\begin{aligned} R_+(\alpha) &= \frac{1}{2\pi\mathrm{i}}\int_{-\infty}^{\infty}\frac{R(\xi)}{(\xi-\alpha)}d\xi, \quad \mathrm{Im}\,\alpha > 0, \\ R_-(\alpha) &= -\frac{1}{2\pi\mathrm{i}}\int_{-\infty}^{\infty}\frac{R(\xi)}{(\xi-\alpha)}d\xi, \quad \mathrm{Im}\,\alpha < 0, \\ Q_+(\alpha) &= \frac{1}{2\pi\mathrm{i}}\int_{-\infty}^{\infty}\frac{Q(\xi)}{(\xi-\alpha)}d\xi, \quad \mathrm{Im}\,\alpha > 0, \\ Q_-(\alpha) &= -\frac{1}{2\pi\mathrm{i}}\int_{-\infty}^{\infty}\frac{Q(\xi)}{(\xi-\alpha)}d\xi, \quad \mathrm{Im}\,\alpha < 0. \end{aligned}$$

Let $f_1(\alpha)$ define a function equal to both sides of Eq.(23). Since the left hand side of Eq.(23) is regular for $\mathrm{Im}\,\alpha > -\mathrm{Im}\,k$ and the right hand side is regular for $\mathrm{Im}\,\alpha < \mathrm{Im}(k\cos\vartheta_0)$ respectively, therefore, by analytic continuation, the definition of $f_1(\alpha)$ can be extended throughout the complex $\alpha$ plane. The form of $f_1(\alpha)$ is ascertained by examining the asymptotic behaviour of the terms in Eq.(23) as $|\alpha| \longrightarrow \infty$. From Eq.(19), we note that $|L_\pm(\alpha)| \sim O(1)$ as $|\alpha| \longrightarrow \infty$ and with the help of the edge conditions, we find that $\overline{\chi}_+(\alpha)$ and $\overline{\chi}_-(\alpha)$ must be at least of $O(|\alpha|^{-1/2})$ as $|\alpha| \longrightarrow \infty$. Using extended form of Liouville's theorem, it can be seen from Eq.(23) that $f_1(\alpha) \sim O(|\alpha|^{-1/2})$ and therefore, the polynomial representing $f_1(\alpha)$ can only be a constant equal to zero. Hence, from Eq.(23), we obtain

$$\begin{aligned} S_+(\alpha)\overline{\chi}_+^*(\alpha,0) + \frac{1}{2\pi\mathrm{i}}\int_{-\infty+\mathrm{i}c}^{\infty+\mathrm{i}c}\frac{S_+(\xi)\overline{\chi}_-^*(\xi,0)e^{-\mathrm{i}\xi(x_2-x_1)}}{(\xi-\alpha)}d\xi \\ - \frac{\mathrm{i}e^{-\mathrm{i}k\cos\vartheta_0 x_2}S_+(k\cos\vartheta_0)}{\sqrt{2\pi}(\alpha - k\cos\vartheta_0)} = 0, \end{aligned} \tag{25}$$

where

$$\overline{\chi}_+(\alpha,0) + \frac{\mathrm{i}e^{-\mathrm{i}k\cos\vartheta_0 x_2}}{\sqrt{2\pi}(\alpha - k\cos\vartheta_0)} = \overline{\chi}_+^*(\alpha,0),$$

$$\overline{\chi}_-(\alpha,0) - \frac{\mathrm{i}e^{-\mathrm{i}k\cos\vartheta_0 x_1}}{\sqrt{2\pi}(\alpha - k\cos\vartheta_0)} = \overline{\chi}^*_-(\alpha,0).$$

Similarly, from Eq.(24), we have

$$S_-(\alpha)\overline{\chi}^*_-(\alpha,0) - \frac{1}{2\pi\mathrm{i}}\int_{-\infty+\mathrm{i}d}^{\infty+\mathrm{i}d} \frac{S_-(\xi)\overline{\chi}^*_+(\xi,0)e^{\mathrm{i}\xi(x_2-x_1)}}{(\xi-\alpha)}d\xi = 0. \tag{26}$$

The unknown functions $\overline{\chi}_+(\alpha,0)$ and $\overline{\chi}_-(\alpha,0)$ appearing in Eqs.(25) and (26) have been determined by using the procedure discussed by Noble [7] and are given by

$$\begin{aligned} 2\overline{\chi}_+(\alpha,0) &= \overline{\phi}_+(\alpha,0^+) - \overline{\phi}_+(\alpha,0^-) \\ &= -\frac{2\mathrm{i}}{\sqrt{2\pi}S_+(\alpha)}\left(G_1(\alpha) + C_1(k)T(\alpha)\right), \end{aligned} \tag{27}$$

$$\begin{aligned} 2\overline{\chi}_-(\alpha,0) &= \overline{\phi}_-(\alpha,0^+) - \overline{\phi}_-(\alpha,0^-) \\ &= -\frac{2\mathrm{i}}{\sqrt{2\pi}S_-(\alpha)}\left(G_2(-\alpha) + C_2(k)T(-\alpha)\right), \end{aligned} \tag{28}$$

where

$$C_1(k) = \frac{1}{S_+(k)}\left(1 - \frac{T^2(k)}{S_+^2(k)}\right)^{-1}\left\{G_2(k) + \frac{G_1(k)T(k)}{S_+(k)}\right\},$$

$$C_2(k) = \frac{1}{S_+(k)}\left(1 - \frac{T^2(k)}{S_+^2(k)}\right)^{-1}\left\{G_1(k) + \frac{G_2(k)T(k)}{S_+(k)}\right\},$$

$$G_1(\alpha) = P_1(\alpha)e^{-\mathrm{i}k\cos\vartheta_0 x_2} - R_1(\alpha)e^{-\mathrm{i}k\cos\vartheta_0 x_1}, \tag{29a}$$

$$G_2(\alpha) = P_2(\alpha)e^{-\mathrm{i}k\cos\vartheta_0 x_1} - R_2(\alpha)e^{-\mathrm{i}k\cos\vartheta_0 x_2}, \tag{29b}$$

$$P_{1,2}(\alpha) = \frac{S_+(\alpha) - S_\pm(k\cos\vartheta_0)}{(\alpha \mp k\cos\vartheta_0)},$$

$$R_{1,2}(\alpha) = \frac{E_0\left(W_0[-\mathrm{i}(k \pm k\cos\vartheta_0)(x_2-x_1)] - W_0[-\mathrm{i}(k+\alpha)(x_2-x_1)]\right)}{2\pi\mathrm{i}(a \mp k\cos\vartheta_0)},$$

$$T(\alpha) = \frac{1}{2\pi\mathrm{i}}E_0W_0[-\mathrm{i}(k+\alpha)(x_2-x_1)],$$

$$E_0 = 2e^{\mathrm{i}\pi/2}\frac{e^{\mathrm{i}k(x_2-x_1)}}{(x_2-x_1)^{1/2}},$$

$$W_0(z) = \Gamma(3/2)e^{z/2}(z)^{-1/4}W_{-3/4,1/4}(z),$$

($W_{i,j}$ is a Whittaker function and $z = -\mathrm{i}(k+\alpha)(x_2-x_1)$). Substitution of Eqs.(27) and (28) in Eq.(22b) yields

$$A_1(\alpha) = -\frac{\mathrm{i}}{\sqrt{2\pi}}\left\{\frac{e^{\mathrm{i}\alpha x_2}}{S_+(\alpha)}\left(G_1(\alpha) + C_1(k)T(\alpha)\right)\right.$$

$$+\frac{e^{\mathrm{i}\alpha x_1}}{S_-(\alpha)}\left(G_2(-\alpha)+C_2(k)T(-\alpha)\right)\Bigg\}+\mathrm{i}G(\alpha). \tag{30}$$

Now, substituting the value of $A_1(\alpha)$ in Eq.(12) and using the approximations (29a, b), the field $\phi(x,y)$ can be written as

$$\phi=\phi^{\mathrm{sep}}(x,y)+\phi^{\mathrm{int}}(x,y),$$

where

$$\begin{aligned}\phi^{\mathrm{sep}}(x,y) &= \frac{\mathrm{i}}{2\pi}\int_{-\infty}^{\infty}\left\{\frac{K_+(k\cos\vartheta_0)L_+(k\cos\vartheta_0)e^{\mathrm{i}(\alpha-k\cos\vartheta_0)x_2}}{K_+(\alpha)L_+(\alpha)(\alpha-k\cos\vartheta_0)}\right.\\ &\qquad\left.-\frac{K_-(k\cos\vartheta_0)L_-(k\cos\vartheta_0)e^{\mathrm{i}(\alpha-k\cos\vartheta_0)x_1}}{K_-(\alpha)L_-(\alpha)(\alpha-k\cos\vartheta_0)}\right\}\\ &\quad\times e^{-\mathrm{i}\alpha x-\gamma y}d\alpha, \end{aligned}\tag{31}$$

$$\begin{aligned}\phi^{\mathrm{int}}(x,y) &= \frac{\mathrm{i}}{2\pi}\int_{-\infty}^{\infty}\left\{\left(R_1(\alpha)e^{\mathrm{i}k\cos\vartheta_0 x_1}-C_1(k)T(\alpha)\right)\frac{e^{\mathrm{i}\alpha x_2}}{K_+(\alpha)L_+(\alpha)}\right.\\ &\qquad\left.+\left(R_2(-\alpha)e^{-\mathrm{i}k\cos\vartheta_0 x_2}-C_2(k)T(-\alpha)\right)\frac{e^{\mathrm{i}\alpha x_1}}{K_-(\alpha)L_-(\alpha)}\right\}\\ &\quad\times e^{-\mathrm{i}\alpha x-\gamma y}d\alpha. \end{aligned}\tag{32}$$

Here, $\phi^{\mathrm{sep}}(x,y)$ represents the field diffracted by the edges at $x=x_2$ and $x=x_1$, and $\phi^{\mathrm{int}}(x,y)$ gives the interaction of one edge upon the other. The integrals appearing in Eqs.(31) and (32) can be evaluated asymptotically by using the steepest descent method. For that, we put $x=r\cos\vartheta$, $y=r\sin\vartheta$ and deform the contour by the transformation $\alpha=-k\cos(\vartheta+\mathrm{i}\nu)$ $(0<\vartheta<\pi,-\infty<\nu<\infty)$. Hence for large $kr$,

$$\phi^{\mathrm{sep}}(x,y) = \frac{\mathrm{i}\sin\vartheta}{\sqrt{2\pi kr}}\mathcal{F}_1(-k\cos\vartheta)e^{\mathrm{i}(kr-\pi/4)}, \tag{33}$$

$$\phi^{\mathrm{int}}(x,y) = \frac{\mathrm{i}k\sin\vartheta}{\sqrt{2\pi kr}}\mathcal{F}_2(-k\cos\vartheta)e^{\mathrm{i}(kr-\pi/4)}, \tag{34}$$

where

$$\begin{aligned}\mathcal{F}_1(-k\cos\vartheta) &= \left\{\frac{K_+(k\cos\vartheta_0)L_+(k\cos\vartheta_0)e^{-\mathrm{i}k(\cos\vartheta+\cos\vartheta_0)x_2}}{K_+(-k\cos\vartheta)L_+(-k\cos\vartheta)(\cos\vartheta+\cos\vartheta_0)}\right.\\ &\qquad\left.-\frac{K_-(k\cos\vartheta_0)L_-(k\cos\vartheta_0)e^{-\mathrm{i}k(\cos\vartheta+\cos\vartheta_0)x_1}}{K_-(-k\cos\vartheta)L_-(-k\cos\vartheta)(\cos\vartheta+\cos\vartheta_0)}\right\}, \end{aligned}\tag{35}$$

$$\begin{aligned}\mathcal{F}_2(-k\cos\vartheta) &= -\left(R_1(-k\cos\vartheta)e^{-\mathrm{i}k\cos\vartheta_0 x_1}-C_1(k)T(-k\cos\vartheta)\right)\\ &\qquad\times\frac{e^{-\mathrm{i}k\cos\vartheta x_2}}{K_+(-k\cos\vartheta)L_+(-k\cos\vartheta)}\end{aligned}$$

$$- \big(R_2(k\cos\vartheta)e^{-\mathrm{i}k\cos\vartheta_0 x_2} - C_2(k)T(k\cos\vartheta)\big) \times \frac{e^{-\mathrm{i}k\cos\vartheta x_1}}{K_-(-k\cos\vartheta)L_-(-k\cos\vartheta)}. \tag{36}$$

In Eqs.(35) and (36), $L_\pm(\,.\,)$ are given by Eqs.(22a).

## 3. Line Source Incidence

In this section, we consider the diffraction of an acoustic wave due to a line source from the slit. We consider the line source to be located at the position $(x_0, y_0)$ and the inhomogeneous wave equation satisfied by the total velocity potential $\Psi$ takes the form

$$\frac{\partial^2\Psi}{\partial x^2} + \frac{\partial^2\Psi}{\partial y^2} + k^2\Psi = \delta(x - x_0)\delta(y - y_0), \tag{37}$$

subject to the boundary conditions

$$\left(\frac{\partial}{\partial y} \pm \mathrm{i}k\beta\right)\Psi(x, 0^\pm) = 0, \quad \begin{cases} x < x_1 \\ x > x_2 \end{cases} \tag{38}$$

$$\left.\begin{aligned} &\Psi(x,0^+) = \Psi(x,0^-), \\ &\frac{\partial}{\partial y}\Psi(x,0^+) = \frac{\partial}{\partial y}\Psi(x,0^-). \end{aligned}\right\} \quad \Big\{ x_1 \le x \le x_2 \tag{39}$$

The total velocity potential $\Psi$ may be expressed as

$$\Psi = \Psi_0 + \Psi_d, \tag{40}$$

where $\Psi_0$ is the incident wave corresponding to the source term and $\Psi_d$ is the solution of the homogeneous wave equation that corresponds to the diffracted potential. The solution of the inhomogeneous wave equation can be written in a straight forward manner as

$$\begin{aligned} \Psi_0 &= \frac{1}{4\pi\mathrm{i}} \int_{-\infty}^{\infty} \frac{e^{-\mathrm{i}\alpha(x-x_0)+\mathrm{i}(k^2-\alpha^2)^{1/2}|y-y_0|}}{(k^2-\alpha^2)^{1/2}} d\alpha, \\ &= -\frac{1}{4\mathrm{i}} H_0^{(1)}(k[(x-x_0)^2 + (y-y_0)^2]^{1/2}). \end{aligned} \tag{41}$$

The diffracted field $\Psi_d$ is obtained by using the procedure in Section 1 and is given by

$$\Psi_d(x,y) = \Psi_d^{\text{sep}}(x,y) + \Psi_d^{\text{int}}(x,y),$$

where

$$\Psi_d^{\text{sep}}(x,y) = \mathrm{i}\frac{\sin\vartheta}{4\pi k\sqrt{rr_0}}\mathcal{F}_1(-k\cos\vartheta)e^{\mathrm{i}k(r+r_0)}, \tag{42}$$

$$\Psi_d^{\text{int}}(x,y) = \mathrm{i}\frac{\sin\vartheta}{4\pi\sqrt{rr_0}}\mathcal{F}_2(-k\cos\vartheta)e^{\mathrm{i}k(r+r_0)}. \tag{43}$$

In Eqs.(42) and (43)

$$r = (x^2+y^2)^{1/2}, \quad r_0 = (x_0^2+y_0^2)^{1/2},$$

and $\mathcal{F}_1(-k\cos\vartheta)$ and $\mathcal{F}_2(-k\cos\vartheta)$ are given by Eqs.(34) and (35) respectively.

## 4. The Effects of Convection

In this section, we make an assessment of the effects to be expected if the sound is propagating in a moving fluid. We consider a small amplitude sound wave on a main stream moving with velocity $U$ parallel to the $x$-axis and discuss the diffraction of a line source from the slit in a moving fluid. The perturbation velocity $u$ of the irrotational sound wave can be written in terms of the velocity potential $\eta$, as $u = \operatorname{grad}\eta$. The resulting pressure in the sound field is then given by

$$P = -\rho_0\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\eta, \tag{44}$$

where $\rho_0$ is the density in the undisturbed stream. Then our problem becomes one of solving the following convective wave equation

$$\left\{(1-M^2)\frac{\partial^2}{\partial x^2} + 2\mathrm{i}kM\frac{\partial}{\partial x} + \frac{\partial^2}{\partial y^2} + k^2\right\}\eta(x,y) = \delta(x-x_0)\delta(y-y_0), \tag{45}$$

subject to the boundary conditions

$$\left(\frac{\partial}{\partial y} \mp \beta M\frac{\partial}{\partial x} \pm \mathrm{i}k\beta\right)\eta(x,0^\pm) = 0, \quad \begin{cases} x < x_1 \\ x > x_2 \end{cases} \tag{46}$$

$$\left.\begin{aligned} &\eta(x,0^+) = \eta(x,0^-), \\ &\frac{\partial}{\partial y}\eta(x,0^+) = \frac{\partial}{\partial y}\eta(x,0^-), \end{aligned}\right\} \quad \{x_1 \le x \le x_2 \tag{47}$$

where $M = \frac{U}{c}$ is the Mach number. We assume that the flow is subsonic i.e. $|M| < 1$ and make the following substitutions

$$\begin{aligned} &x = (1-M^2)^{1/2}X, \quad x_0 = (1-M^2)^{1/2}X_0, \quad y_0 = Y_0, \quad y = Y, \\ &x_1 = (1-M^2)^{1/2}X_1, \quad x_2 = (1-M^2)^{1/2}X_2, \\ &k = (1-M^2)^{1/2}K, \quad \beta = (1-M^2)^{1/2}B, \\ &\eta(x,y) = \varphi(X,Y)e^{-\mathrm{i}KMX}. \end{aligned}$$

Using these substitutions in Eqs.(45)–(47), we get

$$\left(\frac{\partial^2}{\partial X^2}+\frac{\partial^2}{\partial Y^2}+K^2\right)\varphi=\frac{e^{-\mathrm{i}KMX_0}}{(1-M^2)^{1/2}}\delta(X-X_0)\delta(Y-Y_0), \tag{48}$$

$$\left(\frac{\partial}{\partial Y}\mp MB\frac{\partial}{\partial X}\pm \mathrm{i}KB\right)\varphi(X,0^{\pm})=0,\quad \begin{cases}X<X_1\\ X>X_2\end{cases} \tag{49}$$

$$\begin{aligned}&\varphi(X,0^+)=\varphi(X,0^-),\\ &\frac{\partial}{\partial Y}\varphi(X,0^+)=\frac{\partial}{\partial Y}\varphi(X,0^-).\end{aligned}\quad \left\{X_1\le X\le X_2\right. \tag{50}$$

As before, the Wiener-Hopf functional equation in case of the moving fluid is found to be of the form

$$\begin{aligned}e^{\mathrm{i}\alpha X_2}\overline{\zeta}_+(\alpha,0)&+\frac{\overline{\varphi}_1'(\alpha,0)}{\widetilde{\gamma}\widetilde{L}(\alpha)}+e^{\mathrm{i}\alpha X_1}\overline{\zeta}_-(\alpha,0)\\ &=\frac{e^{\mathrm{i}KMX_0}e^{\mathrm{i}(KR_0-\pi/4)}}{2(1-M^2)^{1/2}\sqrt{2\pi KR_0}}G(\alpha),\end{aligned} \tag{51}$$

where

$$\begin{aligned}&\overline{\varphi}_+(\alpha,0^+)-\overline{\varphi}_+(\alpha,0^-)=2\overline{\zeta}_+(\alpha,0),\\ &\overline{\varphi}_-(\alpha,0^+)-\overline{\varphi}_-(\alpha,0^-)=2\overline{\zeta}_-(\alpha,0),\\ &R_0^2=X_0^2+Y_0^2,\quad \widetilde{\gamma}=(\alpha^2-K^2)^{1/2}=\widetilde{K}_+(\alpha)\widetilde{K}_-(\alpha),\\ &\widetilde{L}(\alpha)=[1-\mathrm{i}B(M\alpha+K)/\widetilde{\gamma}].\end{aligned}$$

In order to solve the Wiener-Hopf equation (51), we need to factorize the kernel function $\widetilde{L}(\alpha)$ as

$$\widetilde{L}(\alpha)=\widetilde{L}_+(\alpha)\widetilde{L}_-(\alpha), \tag{52}$$

where $\widetilde{L}_+(\alpha)$ is regular for $\operatorname{Im}\alpha>-\operatorname{Im}K$, and $\widetilde{L}_-(\alpha)$ is regular for $\operatorname{Im}\alpha<\operatorname{Im}K$. The factorization (52) has been obtained in Appendix A. It is important to note that

$$|\widetilde{L}_\pm(\alpha)|\sim O(|\alpha|^{\mp\delta}), |\overline{\zeta}_\pm(\alpha)|\sim O(|\alpha|^{-\varepsilon}),\quad \text{as } |\alpha|\longrightarrow\infty \text{ [3]},$$

where

$$\delta=\frac{1}{2\pi}\arg\left(\frac{1-\mathrm{i}MB}{1+\mathrm{i}MB}\right),\quad \varepsilon=\frac{1}{2}-\delta.$$

Now, following the same method of solution as in Section 1, the diffracted field $\eta$ can be written as

$$\eta(x,y)=\eta^{\mathrm{sep}}(x,y)+\eta^{\mathrm{int}}(x,y), \tag{53}$$

where

$$\eta^{\rm sep}(x,y) = {\rm i}\frac{e^{-{\rm i}KM(X-X_0)}\sin\vartheta}{(1-M^2)^{1/2}4\pi K\sqrt{RR_0}}\widetilde{\mathcal{F}}_1(-K\cos\vartheta)e^{{\rm i}K(R+R_0)}, \tag{54}$$

$$\eta^{\rm int}(x,y) = {\rm i}\frac{e^{-{\rm i}KM(X-X_0)}\sin\vartheta}{(1-M^2)^{1/2}4\pi\sqrt{RR_0}}\widetilde{\mathcal{F}}_2(-K\cos\vartheta)e^{{\rm i}K(R+R_0)}. \tag{55}$$

In Eqs.(54) and (55)

$$R = (X^2+Y^2)^{1/2},$$

$$\widetilde{\mathcal{F}}_1(-K\cos\vartheta) = \left\{\frac{\widetilde{K}_+(K\cos\vartheta_0)\widetilde{L}_+(K\cos\vartheta_0)e^{-{\rm i}K(\cos\vartheta+\cos\vartheta_0)X_2}}{\widetilde{K}_+(-K\cos\vartheta)\widetilde{L}_+(-K\cos\vartheta)(\cos\vartheta+\cos\vartheta_0)} - \frac{\widetilde{K}_-(K\cos\vartheta_0)\widetilde{L}_-(K\cos\vartheta_0)e^{-{\rm i}K(\cos\vartheta+\cos\vartheta_0)X_1}}{\widetilde{K}_-(-K\cos\vartheta)\widetilde{L}_-(-K\cos\vartheta)(\cos\vartheta+\cos\vartheta_0)}\right\},$$

$$\begin{aligned}\widetilde{\mathcal{F}}_2(-K\cos\vartheta) = &-\left(R_1(-K\cos\vartheta)e^{-{\rm i}K\cos\vartheta_0 x_1} - C_1(K)T(-K\cos\vartheta)\right)\\ &\times\frac{e^{-{\rm i}K\cos\vartheta X_2}}{\widetilde{K}_+(-K\cos\vartheta)\widetilde{L}_+(-K\cos\vartheta)}\\ &-\left(R_2(K\cos\vartheta)e^{-{\rm i}K\cos\vartheta_0 X_2} - C_2(K)T(K\cos\vartheta)\right)\\ &\times\frac{e^{-{\rm i}K\cos\vartheta X_1}}{\widetilde{K}_-(-K\cos\vartheta)\widetilde{L}_-(-K\cos\vartheta)}.\end{aligned}$$

From Eq.(53), we observe that as a result of fluid motion the field is increased by the factor $(1-M^2)^{-1/2}$ in comparison to still fluid. Also, the field is independent of the direction of the flow since the fluid velocity $U$ appears as $|U|^2$ in the factor $(1-M^2)$. These results also take care of acoustic diffraction from a slit in an infinite *rigid* barrier in a moving fluid which can be obtained by putting $\beta = 0$ in Eq.(53).

## APPENDIX A

The function $\widetilde{L}(\alpha)$ is given by

$$\widetilde{L}(\alpha) = \left(1 + \frac{B(M\alpha+K)}{(K^2-\alpha^2)^{1/2}}\right),$$

The factorization of the function $[1+B(K-M\alpha)/(K^2-\alpha^2)^{1/2}]$ has been discussed by Rawlins [3]. The same procedure can be adopted for $\widetilde{L}(\alpha)$. Thus, employing the technique of Rawlins and omitting the details of calculations, the function $\widetilde{L}(\alpha)$ may be factorised as

$$\widetilde{L}(\alpha) = \widetilde{L}_+(\alpha)\widetilde{L}_-(\alpha), \tag{A1}$$

where

$$\widetilde{L}_\pm(\alpha) = \widetilde{L}_\pm(0) \exp \int \lambda_\pm(\alpha) d\alpha, \tag{A2}$$

and

$$\widetilde{L}_+(0) = \widetilde{L}_-(0) = \sqrt{1+B}.$$

In Eq.(A2)

$$\lambda_+(\alpha) = -\frac{1}{2(\alpha+K)} + \frac{BK}{(1+B^2M^2)\pi} \times \left( \frac{(M-\alpha_1)F(\alpha,K\alpha_1)}{(\alpha_1-\alpha_2)} - \frac{(M-\alpha_2)F(\alpha,K\alpha_2)}{(\alpha_1-\alpha_2)} \right),$$

$$\lambda_-(-\alpha)\,|_{M=-M} = -\lambda_+(\alpha),$$

$$F(\alpha,\alpha_0) = \frac{1}{(\alpha-\alpha_0)}\left(f(\alpha) - f(\alpha_0)\right),$$

$$f(P) = \int_K^{\infty K} \frac{dt}{(t+P)(t^2-K^2)^{1/2}} = \frac{\cos^{-1}(P/K)}{(K^2-P^2)^{1/2}},$$

$$\alpha_{1,2} = \frac{1}{(1+B^2M^2)}\left(-MB^2 \pm (1-B^2+M^2B^2)^{1/2}\right). \tag{A3}$$

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