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TOP MESON AND PROPERTIES OF TOPONIUM

By

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We discuss the properties of states containing the hypothetical top quark with mass $m_t \geq 20$ GeV in the framework of a flavour independent potential model.

1. Introduction

Recently the spectacular results of the CLEO and CUSB [1] experimental groups at the CESR machine have confirmed the idea that the T system is a bound state of a pair of heavy nonrelativistic quarks b and $\bar{b}$. This has compelled us to anticipate that the next hypothetical particle ζ is a bound state of the much heavier quark t and antiquark $\bar{t}$ where the hypothetical top quark t [2] is the weak isospin partner of the b-quark. The present experimental lower limit on its mass is 17 GeV, but we will extend our studies up to the mass of 20 GeV.

There have been various fits of the charmonium and bottomonium spectra with various potentials like those containing the effect of one gluon exchange [3] in the short range part, and like $V = A + Br^{\alpha}$ pointing towards a value of α which is close to zero and positive. The advantage of that kind of fit is that it is simple and one has simple properties of scaling with respect to the quark mass. The other advantage is that it allows to appreciate the statements according to which the comparison of the $b\bar{b}$ and $c\bar{c}$ spectra gives evidence for a short range part of the potential behaving as predicted by QCD, i.e. $V \sim \left(\frac{-8\pi}{27}\right) r^{-1} \left|\log \frac{r}{r_0}\right|^{-1}$. In this study we use the WKB fit for the spectra of the next heavy particle called the top meson, if it exists at all because for all potentials usually considered in the quarkonium problem, the WKB approximation is excellent (with an error of a few percent for s-waves).

The paper will be organised as follows: in Section 2 we will discuss the $T^* - T$ mass splittings and predict the masses of the pseudoscalar and vector mesons; in Section 3 the WKB fit of the top meson spectrum is studied; in Section 4 meson decay constants will be discussed in the context of a nonrelativistic potential model.

2. $t\bar{t}$ spectroscopy and mass splitting

We notice that mesons $(Q\bar{q})$, where Q is a heavy quark and q is the light quark u or d behave to some extent like a hydrogen atom (a heavy proton and a light electron). If the proton is replaced by a deuteron the binding energy of the electron is not appreciably changed. Similarly, in the first approximation the binding energy of $\bar{q}$ in $(Q\bar{q})$ will not depend on the flavours of Q if Q is heavy enough. Hence

$$M(Q\bar{u}) = M_Q + C. \tag{1}$$

A QQ state will be stable if

$$\begin{aligned} M(Q\bar{Q}) &= 2M + E < 2M(Q\bar{Q}) \\ &= 2M_Q + C\,, \end{aligned} \tag{2}$$

i.e.

$$E\,(\text{binding energy}) < 2C\,,$$

where C is given by the potential model [4].

Using Eq. (1) and the quark masses as $m_t = 20$ GeV, $m_b = 4.86$ GeV, $m_c = 1.45$ GeV; $m_s = 0.54$ GeV and $m_u \sim m_d = 0.33$ GeV we have

$$\begin{aligned} M(T_u) &= 20.42\ \text{GeV}\,, \\ M(T_s) &= 20.69\ \text{GeV}\,, \\ M(T_c) &= 21.645\ \text{GeV}\,, \\ M(T_b) &= 24.95\ \text{GeV}\,. \end{aligned} \tag{3}$$

Adopting the QCD inspired picture in which the spin—spin forces of the constituents are proportional to their colour magnetic moments, we find that the mass splitting between pseudoscalar states T and the vector states T^* decreases rapidly with increasing quark masses. We write

$$M(T_q^*) - M(T_q) \sim \frac{m_c}{m_t}\,\frac{m_s}{m_q} \times \left[M(F^{*0}) - M(F^0)\right]. \tag{4}$$

Using the values of quark masses given above we obtain the mass splittings of at most a few MeV. This suggests that T^* cannot decay strongly into $T + \pi$. The results obtained from Eq. (4) are

$$\begin{aligned} M(T_u^*) - M(T_u) &= 10.85\ \text{MeV}\,, \\ M(T_s^*) - M(T_s) &= 6.6\ \text{MeV}\,, \\ M(T_c^*) - M(T_c) &= 2.31\ \text{MeV}\,, \\ M(T_b^*) - M(T_b) &= 0.78\ \text{MeV}\,. \end{aligned} \tag{5}$$

3. WKB fit of the spectra

For a power potential the WKB approximation has the advantage of giving a particularly transparent expression for the energy levels. It has also the advantage of being relatively accurate. We write for the $l = 0$ n state [5]

$$M(n, m_q) = 2m_q + A + C_\alpha B^{1-\alpha/(\alpha+2)} m_q^{-\alpha/(\alpha+2)} (n - 1/4)^{2\alpha/(\alpha+2)}, \tag{6}$$

with $V = A + Br^\alpha$, $\alpha > 0$ and C_α depends on α. In order to find α, we write in terms of the ratio of the quark masses

$$\frac{(E_{2s} - E_{1s})_{t\bar{t}}}{(E_{2s} - E_{1s})_{b\bar{b}}} = \left(\frac{m_b}{m_t}\right)^{\alpha/(\alpha+2)}, \tag{7}$$

This gives $(E_{2s} - E_{1s})_{t\bar{t}} = 0.517$ GeV. Similarly $(E_{3s} - E_{1s})_{t\bar{t}} = 0.83$ GeV.

A naive application the of WKB approximation gives for the ζ system [6]

$$\frac{E_{3s} - E_{1s}}{E_{2s} - E_{1s}} \frac{(11)^{2\alpha/(\alpha+2)} - (3)^{2\alpha/(\alpha+2)}}{(7)^{2\alpha/(\alpha+2)} - (3)^{2\alpha/(\alpha+2)}} .$$

In this way we get $\alpha_{\mathrm{WKB},\zeta} = 0.17$.

Thus, we predict

$$E_{4s} - E_{1s} = (E_{2s} - E_{1s}) \times \frac{(15)^{2\alpha/(\alpha+2)} - (3)^{2\alpha/(\alpha+2)}}{(7)^{2\alpha/(\alpha+2)} - (2)^{2\alpha/(\alpha+2)}} = 1.05 \text{ GeV} .$$

We now argue that the spin-dependent effect can be removed by considering the combination

$$3/4\, M(\text{triplet}) + 1/4\, M\ (\text{singlet})$$

or

$$M = M_{\text{triplet}} - 1/4\, (M_{\text{triplet}} - M_{\text{singlet}})\,;$$

we further assume

$$M_{\text{triplet}} - M_{\text{singlet}} = C \times \frac{|\psi_n(0)|^2}{m_Q^2} .$$

The value of C is adjusted to the $\zeta - n_t$ mass difference and use WKB for $\psi_n(0)$, then one gets

$$M(\text{triplet} - \text{singlet}, n, m_q) =$$

$$= 0.67\,\text{MeV} \left(\frac{m_Q}{m_t}\right)^{-\frac{1-2\alpha}{2+\alpha}} \times \left(\frac{4n-1}{3}\right)^{\frac{2\alpha-2}{\alpha+2}} . \tag{8}$$

Here we take $\alpha = 0.17$ and from this equation the energy difference obtained is nearly the same as obtained above.

4. Decay constants

For a nonrelativistic system

$$f^2({}^3S_1(\bar{\mu}\mu) \rightarrow e^+e^-) = |\varphi^{(0)}|^2 \otimes 4 \otimes M_{\mu\bar{\mu}}^{-1} = \\ = \frac{2\, m_\mu\, \Gamma_{e^+e^-}}{2\pi\,\alpha^2}, \tag{9}$$

where $\varphi(0)$ is the Schrödinger wave function at the origin. For charmonium and bottomonium we find

$$\frac{f_v^2(T \rightarrow e^+e^-)}{M_T} = \frac{f_v^2(J/\psi \rightarrow e^+e^-)}{M_\psi} \tag{10}$$

and extend (10) to systems containing top quarks. We thus write the decay constant as [4]

$$F_v \sim \frac{30\sqrt{\mathrm{MeV}}}{\sqrt{m_t + m_q}\left(\frac{1}{m_t} + \frac{1}{m_q}\right)}. \tag{11}$$

This relation is also valid for pseudoscalar mesons if one of the constituents is sufficiently heavy to make the spin-spin effects negligible. From Eq. (11) the decay constants of vector T-mesons are

$$f(T_u) = 65 \ \mathrm{MeV}; \qquad f(T_s) = 105 \ \mathrm{MeV};$$

$$f(T_c) = 263 \ \mathrm{MeV}; \qquad M(T_b) = 705 \ \mathrm{MeV}$$

and

$$f(t\bar{t}) = 1.4 \ \mathrm{GeV}.$$

5. Conclusion

When (or if) toponium or top production is observed, novel features will appear. For $m_t \gtrsim 20$ GeV weak effects will become sufficiently strong to determine not only the decays of the pseudoscalar mesons, but also those of the vector mesons. We have studied the various properties of these hypothetical mesons.

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Appendix

Mass spectrum of toponium family as obtained from Eq. (6)

Label	*Energy* (GeV)
$\zeta(t\bar{t})$	39.86
ζ'	40.377
ζ''	40.69
ζ'''	40.91
ζ''''	41.115