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THEORY OF WEAK INTERACTION WITHOUT DIVERGENCIES

By

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Based on the existence of a cosmic field suggested by [1, 2] theory of a weak interaction without divergencies has been set up. The current-current theory of Gell-Mann and Feynman is applied in combination with the theory of nonpolynomial interaction.

It is proved that all terms of perturbation theory series are convergent. Some interesting terms of second order are considered in detail.

1

It is commonly known that the current-current weak interaction theory of Gell-Mann and Feynman encountered serious difficulties connected with the ultraviolet infinities. Although this theory is unrenormalizable, the first order amplitudes of perturbation theory series are in good agreement with the experimental data. Therefore it is possible that this theory really reflects reality. In a series of papers [3–5] attempts are made to consider the weak interaction theory based on the existence of hypothetical intermediate bosons.

The recent developments of the interaction theory with the nonpolynominal structure of interaction Lagrangians raised for the first time by EFIMOV [6] and FRADKIN [7] revealed a new direction in constructing the quantized field theory without ultraviolet infinities [8–10] and, in particular, led to the construction of the nonpolynomial weak interaction theory [11, 12]. LANE and CHODOS [11] proposed the current-current theory, in which some second order processes are convergent. Their theory is based on the nonpolynomial interaction of currents with the hypothetical charged scalar particles. In our opinion, this theory contains the following defects: the charged scalar bosons as well as their interaction with current are introduced artificially, and the theory is not totally free of divergencies; only some second order amplitudes of perturbation theory series converge, for example, the νe scattering of second order has the finite matrix element while the self-energy of electron diverges logarithmically.

The weak interaction theory proposed by FIVEL and MITTER [12] encountered also some serious difficulties as it was analysed by LANE and CHODOS.

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In the present paper we make an attempt to outline the weak interaction theory, in which the above mentioned difficulties are overcome.

In paragraph 2 it is shown that the first order matrix elements of perturbation series in our theory coincide with those of the ordinary weak interaction theory. However, in the present theory the second order as well as nth order amplitudes are convergent. The small distance behaviour of amplitudes is given.

Paragraph 3 concerns the νe scattering of second order approximation with respect to the weak coupling constant G. The expression for the amplitude of the first order in the very small coupling constant 1 is also obtained.

Finally, in paragraph 4 the self-energy of electron is considered. The cutoff has the form

$$\Lambda_{cutoff} = \frac{G}{l^2} .$$

In paragraph 5 we present conclusion and discussions.

2

Let us remember the main points of previous papers [1-3]. It has been proved that the existence of cosmic field is characterized uniquely by the following space-time:

$$ds^2 = e^{-2l\varkappa(x)} (dt^2 - d\vec{r}^{\,2}) , \tag{2.1}$$

where l is the new constant having the dimension of length and $\chi(\varkappa)$-cosmic field.

Under the conformal transformations the metric (2.1) is invariant and the χ-field transforms according to the law:

$$\chi(x) \to \chi'(x') = \chi(x) + l^{-1} \ln \left| \det \frac{\partial x'}{\partial x} \right| . \tag{2.2}$$

Due to TAUB [13] the metric (2.1) is the only one possessing the conformal group as its motion group. In this paper let us confine ourselves to considering only the weak interaction of leptons proposed by Gell-Mann and Feynman.

It is well known that the interaction Lagrangian reads

$$\mathfrak{L}_w = \frac{G}{\sqrt{2}} \; : J^\mu (x) J^+_\mu (x) : \, , \; J_\mu = j^{(h)}_\mu + j^{(l)}_\mu$$

in which

$$j^{(l)}_\sigma = i\bar{e}\gamma_\sigma (1 + \gamma_5) \nu_e + i\bar{\mu}\gamma_\sigma (1 + \gamma_5) \nu_\mu$$

is the lepton current and $j^{(h)}_\mu$ the usual weak current of hadrons in the form of Cabibbo.

In the presence of a cosmic field, Lagrangian takes the form

$$\mathfrak{L}_w \to L = \frac{G}{\sqrt{2}} : J^\mu(\alpha)\, J_\mu^+(x) :: e^{-6l\chi(x)} : .$$

Using (2.2) we can satisfactorily construct the weak interaction theory.

We first notice that the matrix elements for any first order process corresponding to (2.2) and not involving the emission or absorption of cosmons χ are the same as in the case when $1 = 0$. Indeed, suppose that $|f>$ contains n cosmons

$$|f\rangle = |f'\rangle \oplus |n\chi\rangle$$

and $|i>$ does not contain cosmons. Then we have the following expression for the first order amplitude

$$\begin{aligned}\langle f|L_w|i\rangle &= \langle f'|\mathfrak{L}_w|i\rangle \langle n\chi| : e^{-6l\chi(x)}: |0\rangle \\ &= (-6l)^n \langle f'|\mathfrak{L}_w|i\rangle\, e^{iPx} ,\end{aligned}$$

where P = total momentum of cosmons. In the case when $n = 0$, our assumption is proved.

As it is mentioned in [1,2], for usual weak interaction processes we have to consider only the processes without external cosmons. In other words, the final and initial states contain no cosmons.

Now we consider the second order amplitudes given by

$$\begin{aligned} T_{if} &= \int d^4x \langle f|T[L_w(x)L_w(0)]|i\rangle = \\ &= \int d^4x \langle f|T[\mathfrak{L}_w(x)\mathfrak{L}_w(0)]|i\rangle \langle o|T: e^{-6l\chi(x)}|:: e^{-6l\chi(o)}|0\rangle = \\ &= \int d^4x \langle f|T[\mathfrak{L}_w(x)\mathfrak{L}_w(0)]|i\rangle \exp[36\, l^2 D_0(x)]. \end{aligned} \tag{2.3}$$

It is known that in the spirit of the nonpolynominal interaction theory all momenta in the states $|i>$ and $|f>$ are assumed first to lie in the Euclidean region, then the integrations over Minkowski space-time are reduced to the integration over Euclidean space of four dimensions. After all the necessary integrations have been performed we continue analytically these amplitudes to the physical region of momenta. The content of the above procedure is called Euclidicity postulate. As a consequence of this postulate the space-time interval $x^2 = \vec{r}^2 - x_0^2$ is converted to the Euclidean length $r^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2 \geq 0$.

It is known that in the Euclidean metric the propagator has the form

$$D_0^c(x) = -\frac{1}{4\pi^2}\frac{1}{r^2}$$

and, then, the superpropagator for cosmic field takes the form

$$G(x) = \exp\,[36\, l^2 D_0^c\,(x)] = \exp\left(-\frac{\lambda^2}{r^2}\right),$$

where

$$\lambda = \frac{3l}{\pi}\ .$$

It is convenient to present $G(x)$ in the following form

$$G(x) = \frac{i}{2}\int_c dz\,\Gamma(-z)\left(\frac{\lambda}{r}\right)^{2z}$$

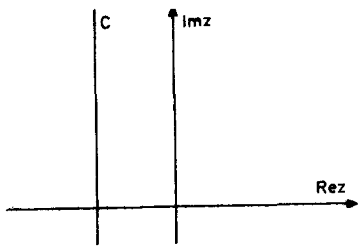

Fig. 1

in which the path C is shown in Fig. 1. Let us now perform the integration (2.3 over an angle. We obtain

$$T_{if} = \frac{i}{2}\int_c dz\,\Gamma(-z)\int_0^{+\infty} dr r^3 F_{if}\,(r)\left(\frac{\lambda}{r}\right)^{2z} \tag{2.4}$$

where

$$F_{if} = \int d^3\Omega\,(x)\,\langle f\,|\,T[\mathfrak{L}_w\,(x)\,\mathfrak{L}_w\,(0)]\,|\,i\rangle\ .$$

It is clear that the integral (2.4) converges at a small distance for $F_{if}(r)$ behaves like

$$F_{if}(r) \sim \frac{A}{r^n} \qquad \text{as } r \to 0\,,$$

where n is a certain integer.

In the case when $\lambda \to 0$, the integral (2.4) would be divergent for $n \geq 4$. It is easily seen that the small distance behaviour of T_{if} has the form

$$T_{if} \sim \begin{cases} \dfrac{\pi}{2}\,\Gamma(4-n)\,(\lambda)^{4-n} & \text{for } n \neq 4, \\ c\,\ln\dfrac{1}{\lambda} & \text{for } n = 4. \end{cases} \tag{2.5}$$

Thus λ plays the role of inverse of cutoff. The convergence of second order amplitudes has been entirely proved. Now, let us consider an arbitrary nth-order amplitude

$$T_{if}^{(n)} = \frac{i^n}{n!}\int dx_1 \ldots dx_n \langle f \mid T\,[L_w(x_1) \ldots L_w(x_n)] \mid i\rangle = \tag{2.6}$$

$$= \frac{i^n}{n!}\int dx_1 \ldots dx_n \langle f \mid T\left[\prod_{k=1}^{n} \mathfrak{L}_w(x)\right] \Big| i\rangle \times \langle 0 \mid T\left[\prod_{j=1}^{n} : \exp\,(-\bar{l}\chi(x_j)):\right] 0\rangle\,,$$

where $\bar{l} = 61$. In order to calculate the n-point superpropagator, we make use of the HORI's formula [14]:

$$F^{(n)} = T\left[\prod_{j=1}^{n} : \exp\,(-\bar{l}\chi(x)):\right] =$$

$$= :\exp\left[\frac{1}{2}\sum_{i\neq j}^{n} D_0^c(x_i - x_j)\frac{\delta^2}{\delta\chi(x_i)\delta\chi(x_j)}\right]\exp\left\{-\bar{l}\sum_{k=1}^{n}\chi(x_k)\right\}: =$$

$$= \exp\left[\frac{1}{2}\bar{l}^2\sum_{i\neq j}^{n} D_0^c(x_i - x_j)\right] : \exp\left\{-\bar{l}\sum_{k=1}^{n}\chi(x_k)\right\}\,.$$

Therefore

$$G(x_1, \ldots, x_n) = \langle 0 \mid T\left\{\prod_{j=1}^{n} : \exp\,\{-\bar{l}\chi(x)\}:\right\}\Big| 0\rangle = \exp\left\{\frac{\bar{l}^2}{2}\sum_{i\neq j}^{n} D_0^c(x_i - x_j)\right\}. \tag{2.7}$$

Finally the amplitude $T_{if}^{(n)}$ takes the form

$$T_{if}^{(n)} = \frac{i^n}{n!}\int dx_1 \ldots dx_n\, F_{if}^{(n)}(x_1, \ldots, x_n) \times \exp\left\{\frac{\bar{l}^2}{2}\sum_{i\neq j}^{n} D_0^c(x_i - x_j)\right\}. \tag{2.8}$$

It is easily seen that if $F_{if}^{(n)}$ behaves like

$$F_{if}^{(n)} \sim \frac{A}{x_{ik}^n} \qquad \text{as } x_{ik} \to 0\,,$$

where $x_{ik}^2 = (x_i - x_k)^2$ and n is an arbitrary natural number, then the integral (2.8) would be convergent.

Hence the convergence of the proposed weak interaction theory has been proved. In the next paragraph we shall consider some interesting second order processes.

3

In this paragraph the second order amplitude for scattering

$$\nu_\mu + e^- \to \nu_\mu + e^-$$

is considered. The Feynman graph describing this process is given in Fig. 2.

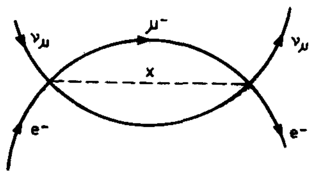

Fig. 2

Its matrix element reads

$$\langle \nu_\mu (p_2) e^-(q_2) | S^{(2)} | \nu_\mu (p_1) e^-(q_1) \rangle =$$

$$= i \int dx \langle \nu_\mu (p_2) e^-(q_2) | T [\mathfrak{L}_w (x) \mathfrak{L}_w (0)] | \nu_\mu (p_1) e^-(q_1) \rangle e^{l^{-2} D_0^c(x)}.$$

Applying the Euclidicity postulate we have

$$\langle \nu_\mu (p_2) e^-(q_2) | S^{(2)} | \nu_\mu (p_1) e^-(q_1) \rangle = \left(\frac{G}{\sqrt{2}} \right)^2 M^{\varrho\sigma} F_{\varrho\sigma} (p),$$

where

$$M^{\varrho\sigma} = \bar{u}_e (q_2) \gamma^\lambda \gamma^\varrho \gamma^\mu (1 - \gamma_5) u_e (q_1) \bar{u}_\nu (p_2) \gamma_\lambda \gamma^\sigma \gamma_\mu (1 - \gamma_5) u_\nu (p_1)$$

and

$$F_{\varrho\sigma} = i^3 \int d^4 x e^{ipx} \frac{x_\varrho x_\sigma}{r^2} \Delta_1 (r; m_\mu) \Delta_1 (r; 0) \exp \{ l^{-2} D_0^c (x) \},$$

here r is the Euclidean length and

$$\Delta_1 (r; m) = - m K_2(mr)/4\pi^2 r$$

$$\to - \frac{1}{2\pi^2 r^3} \qquad \text{for } m \to 0 .$$

First we suppose that the momentum $p = q_1 + q_2$ belongs to the Euclidean space. Then $F_{\varrho\sigma}(p)$ can be written in the following form

$$F_{\varrho\sigma} = g_{\varrho\sigma} F_1(q^2) + p_\varrho p_\sigma F_2(q^2),$$

where $q^2 = -p^2 > 0$ and

$$F_1(q^2) = -\frac{1}{q}\frac{d}{dq}F(q^2), \qquad F_2(q^2) = \left(\frac{1}{q}\frac{d}{dq}\right)^2 F(q^2),$$

$$F(q^2) = \frac{4\pi^2}{q}\int_0^{+\infty} drJ_1(qr)\,\Delta_1(r;m)\,\Delta_1(r;0)e^{-\frac{\lambda^2}{r^2}}, \tag{3.1}$$

$$F_1(q^2) = \frac{4\pi^2}{q^2}\int_0^{+\infty} drrJ_2(qr)\,\Delta_1(r;m)\,\Delta_1(r;0)e^{-\frac{\lambda^2}{r^2}}, \tag{3.2}$$

$$F_2(q^2) = \frac{4\pi^2}{q^3}\int_0^{+\infty} drr^2J_3(qr)\,\Delta_1(r;m)\,\Delta_1(r;0)e^{-\frac{\lambda^2}{r^2}}. \tag{3.3}$$

The above expression is different from that obtained in the LANE—CHODOS' theory. The integral for Eq. (3.1) in our case is clearly convergent as $r \to 0$.

Our main task is to calculate explicitely the form factor $F_{1,2}(q^2)$ given by (3.2) and (3.3) in the Euclidean momentum space and then to continue them analytically into the physical region.

To do this, it is convenient to represent $F_{1,2}(q^2)$ in the following form

$$F_1(q^2) = \frac{4\pi^2q^2}{mi}\int_c dz\Gamma(-z)\,\lambda^{2z}\int_0^{+\infty} drr^{-2z-3}J_2(qr)\,K_2(mr),$$

$$F_2(q^2) = \frac{mi}{4\pi^2q^3}\int_c dz\Gamma(-z)\,\lambda^{2z}\int_0^{+\infty} drr^{-2z-2}J_3(qr)K_2(mr).$$

Applying the standard formulae [15] we have

$$\int_0^{+\infty} drr^{-2-z3}J_2(qr)\,K_2(mr) = \frac{q^2}{2^5}\left(\frac{m}{2}\right)^{2z}\Gamma(1-z)\,\Gamma(-1-z)\times$$

$$\times\,{}_2F_1\left((1-z,-1-z;3;-\frac{q^2}{m^2}\right),$$

$$\int_0^{+\infty} drr^{-2z-2}J_3(qr)\,K_2(mr) = \frac{q^3}{2^3\Gamma(4)m^2}\left(\frac{m}{2^2}\right)^{2z}\Gamma(2-z)\,\Gamma(-z)\times$$

$$\times\,{}_2F_1\left(2-z_1-z;4;-\frac{q^2}{m^2}\right).$$

Therefore

$$F_1(q^2) =$$
$$= \frac{mi}{2^5(2\pi)^2}\int_c dz\Gamma(-z)\,\Gamma(-1-z)\,\Gamma(1-z)\left(\frac{\lambda m}{2}\right)^{2z} {}_2F_1\left(1-z, -1-z; 3; -\frac{q^2}{m^2}\right) \tag{3.4}$$

$$F_2(q^2) =$$
$$= \frac{im^{-1}}{3!\,2^3(2\pi^2)^2}\int_c dz\Gamma(-z)^2\,\Gamma(2-z)\left(\frac{\lambda m}{2}\right)^{2z} {}_2F_1\left(2-z, -z; 4; -\frac{q^2}{m^2}\right). \tag{3.5}$$

The expressions (3.4) and (3.5) for the form factors $F_{1,2}(q^2)$ allow us to perform the analytical continuation into the physical region of momentum.

$$F_1(p^2) =$$
$$= \frac{mi}{2^5(2\pi)^2}\int_c dz\Gamma(-z)\,\Gamma(-1-z)\,\Gamma(1-z)\left(\frac{\lambda m}{2}\right)^{2z} {}_2F_1\left(1-z, -1-z; 3; \frac{m^2}{p^2}\right), \tag{3.6}$$

$$F_2(p^2) = \frac{im^{-1}}{3!\,2^3(2\pi)^2}\int_c dz(\Gamma(-z))^2\,\Gamma(2-z)\left(\frac{\lambda m}{2}\right)^{2z} {}_2F_1\left(2-z, -z; 4; \frac{p^2}{m^2}\right). \tag{3.7}$$

The singularities of $F_1(p^2)$ occur at the points $z = -1,0,1,2,\ldots$. At the point $z = -1$ we have a simple pole, at $z = 0$ a dipole, at $z = 1,2,3,\ldots$ we have tripoles. The pole at $z = -1$ is the leading singularity of $F_1(p^2)$; because it contains the factor $\left(\frac{\lambda m}{2}\right)^{2z}$. Similarly, singularities at $z = 0,1$ are dipoles of $F_2(p^2)$ and the leading singularity occurs at $z = 0$; the remaining singularities at $z = 2,3,4,\ldots$ are all tripoles.

Now we evaluate the form factors $F_{1,2}(p^2)$ in the lowest order of λ. It is easily found that

$$F_1(p^2) \approx \frac{mi}{2^5(2\pi)^2}\cdot 2\pi i\,(\mathrm{Res}|_{z=-1} + \mathrm{Res}|_{z=0})$$

$$\mathrm{Res}|_{z=-1} = \left(\frac{\lambda m}{2}\right)^{-2}$$

In order to find $\mathrm{Res}|_{z=0}$ we first assume $\left|\frac{p^2}{m^2}\right| < 1$, then we have the following expression for ${}_2F_1\,(1-z,\ -1-z;\ 3;\ p^2/m^2)$:

$$
{}_2F_1\,(1-z,\ -1-z;3;x) =
$$

$$
1-\frac{(1-z)(1+z)}{3.1\,!}\,x-\frac{(1-z)(1-z+1)(1+z)(1-z-1)}{3(3+1)2\,!}\,x^2+\dots \tag{3.8}
$$

for $|x|<1$.
The residue at $z=0$ reads

$$
\mathrm{Res}|_{z=0} =
$$

$$
=\frac{\partial}{\partial z}\left[z^2\Gamma(-z)\,\Gamma(-1-z)\,\Gamma(1-z)\left(\frac{\lambda m}{2}\right)^{2z}{}_2F_1\left(1-z,\ -1-z;\ 3;\ \frac{p^2}{m^2}\right)\right]_{z=0}
$$

or

$$
\mathrm{Res}|_{z=0}=C\left(1-\frac{u}{3}\right)+\ln\frac{\lambda m}{2}\left(1-\frac{u}{3}\right)+1-\frac{u}{3}-
$$

$$
-\frac{u}{3}^{-2}(u^3-3u^2+3u-1)\ln\frac{1}{1-u}-\frac{1}{18\,u}(11u^2-15u+6)\,.
$$

Thus we obtain

$$
F_1(p^2)\approx\frac{m}{2^5\,2\,\pi}\left\{\frac{1}{3\,u^2}(u^3-3\,u^2+3\,u-1)\ln\frac{1}{1-u}+\right.
$$

$$
\left.+\frac{1}{18\,u}(11\,u^2-15\,u+6)-\left(1-\frac{u}{3}\right)\left(\ln\frac{\lambda m}{2}+C+1\right)+\left(\frac{\lambda m}{2}\right)^{-2}\right\},
$$

where $u=p^2/m^2$ and C is an Euler constant. We can now continue analytically the expression obtained for $F_1(p^2)$ into the $p^2>m^2$ region. Notice that in the p^2 plane there is a cut from $p^2=m^2$ to infinity.

Now let us evaluate $F_2(p^2)$ confining ourselves to the leading singularity at $z=0$.

$$
F_2(p^2)\approx\frac{im^{-1}}{3\,!\,2^3\,(2\,\pi)^2}\,2\,\pi i\ \mathrm{Res}|_{z=0}\ ,
$$

where

$$
\mathrm{Res}|_{z=0}=\frac{\partial}{\partial z}\left\{z^2(\Gamma(-z))^2\,\Gamma(2-z)\left(\frac{\lambda m}{2}\right)^{2z}{}_2F_1\left(2-z,\ -z;4;\frac{p^2}{m^2}\right)\right\}\Bigg|_{z=0}=
$$

$$
=2\ln\frac{\lambda m}{2}+4\ln(1-u)\frac{u^3-3\,u+2}{u^3}+4\,\frac{1-u}{u^2}-\frac{11}{3}\ .
$$

We obtain

$$F_2(p^2) \approx$$

$$\approx -\frac{1}{3\,!\,2^3\,2\,\pi m}\left(2\ln\frac{\lambda m}{2} + 4\ln(1-u)\frac{n^3 - 3u + 2}{u^3} + 4\frac{1-u}{u^2} - \frac{11}{3}\right).$$

The above expression for $F_2(p^2)$ allows us to continue analytically to the $p^2 > m^2$ region.

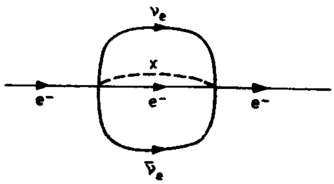

Fig. 3

Finally, the expression for $F_{\sigma\varrho}(p^2)$ of the lowest order is as follows

$$F_{\varrho\sigma}(p^2) \approx \frac{m}{2^3\,2\,\pi}\left\{\frac{1}{4}g_{\varrho\sigma}\left[\ln\frac{1}{1-u}\,\frac{u^3 - 3u^2 + 3u - 1}{3u^2} + \right.\right.$$

$$\left. + \frac{11u^2 - 15u + 6}{18u} - \left(1 - \frac{u}{3}\right)\left(\ln\frac{\lambda m}{2} + C + 1\right) + \left(\frac{\lambda m}{2}\right)^{-2}\right] -$$

$$\left. - \frac{1}{6m^2}p_\varrho p_\sigma\left[2\ln\frac{\lambda m}{2} + 4\ln(1-u)\frac{u^3 - 3u + 2}{u^2} + 4\frac{1-u}{u^2} - \frac{11}{3}\right]\right\}.$$

At high energies, i.e. for $|p^2| \gg m^2$, the asymptotic behaviour of $F_{\varrho\sigma}(p^2)$ reads

$$F_{\varrho\sigma}(p^2) \approx -\frac{m}{16\,\pi}\frac{p^2}{m^2}\ln\frac{p^2}{m^2}\left(\frac{1}{12}g_{\varrho\sigma} + \frac{2}{3m^2}p_\varrho p_\sigma\right)$$

which does not depend on the parameter λ.

4

In this paragraph we consider the lepton self-energy taking into account the contribution of weak interaction; the corresponding graph is shown in Fig. 3.

We have

$$\bar{u}_e(p)\,\Sigma(p)\,u_e(p) = \frac{i}{2}\int dx\,\langle e^-(p)|\,T\,[\mathfrak{L}_{ee}(x)\,\mathfrak{L}_{ee}(o)]\,|e^-(p)\rangle\exp\,[l^2 D_0^c(x)]\,.$$

The expression for $\Sigma(p)$ can be represented in the following form

$$\Sigma(p) = \hat{p}\sigma(p^2)(1-\gamma_5)$$

where $\sigma(p^2)$ is given by

$$\sigma(p^2) = \frac{128\,\pi^2 G^2}{p^2}\int_0^{+\infty} drr^2 I_2(pr)\,[\Delta_1(r;o)]^2\,\Delta_1(r;m_e)e^{-\frac{\lambda^2}{r^2}}\,.$$

This integral is clearly convergent at a small distance. It is quadratically divergent when $\lambda \to 0$. Now we calculate this integral by means of the above used method. The expression for $\sigma(p^2)$ is rewritten as follows

$$\sigma(p^2) = -\frac{4\,iG^2m}{\pi^4p^2}\int_c dz\Gamma(-z)\,\lambda^{2z}\int_0^{+\infty} drr^{-2z-5}\,I_2(pr)K_2(mr)\,.$$

Owing to the standard formula [14] we have

$$\int_0^{+\infty} drr^{-2z-5}\,I_2(pr)\,K_2(mr) =$$

$$= \frac{\Gamma(3)}{p^2}\,2^{-2z-5}\,m^{2z+2}\,\Gamma(-z)\,\Gamma(-2-z)\,{}_2F_1\left(-z,\,-2-z;\,3;\frac{p^2}{m^2}\right).$$

After substituting it in the expression for $\sigma(p^2)$ we obtain

$$\sigma(p^2) = i\,\frac{G^2m^4}{(2\pi)^4}\int_c dz\,(\Gamma(-z))^2\,\Gamma(-2-z)\left(\frac{\lambda m}{2}\right)^{2z}{}_2F_1\left(-z,\,-2-z;\,3;\frac{p^2}{m^2}\right).$$

The singularities of the integrand occur at $z = -2,\ -1,0,1,2\ldots$. The simple pole occurs at $z = -2,\ -1$ and the remaining singularities are tripoles. The leading singularity is, of course, $z = -2$.

The approximate expression for $\sigma(p^2)$ reads

$$\sigma(p^2) \approx \frac{G^2m^4}{(2\pi)^3}\left\{\left(\frac{\lambda m}{2}\right)^{-4} + \left(\frac{\lambda m}{2}\right)^{-2}\left(1-\frac{1}{3}\,\frac{p^2}{m^2}\right)\right\}.$$

In order to find the mass correction for electron we expand $\Sigma(p)$ in power series of $\hat{p}-m$:

$$\Sigma(p) = \Sigma_0(m^2) + \Sigma_1(m^2)(\hat{p}-m) + \ldots$$

from where we have

$$\delta m = m\sigma(m^2)$$

or in the first approximation

$$\frac{\delta m}{m} \approx \frac{2}{\pi^3} \left|\frac{G}{\lambda^2}\right| .$$

The parameter G/λ^2 plays here the role of a cutoff.

5

It is known that in quantum electrodynamics all divergencies could be suppressed by some redefinition of mass and coupling constant, i.e. by introducing a finite number of counter-terms. However, in the current-current weak interaction theory this is not the case. In fact, each order of perturbation series introduces a new kind of divergence. For example, the nth order is $\Lambda^{2(n-1)}$ divergent, where Λ is a cutoff. Therefore, one would need an infinite number of renormalization constants to make the theory convergent.

In our case, we introduce only one new parameter 1 to make the theory finite. This parameter characterizes a new interaction, namely the interaction of the matter with the cosmic field.

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