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"HOW CAN WE DETERMINE THE TWO-BODY t-MATRIX?"

By

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We start with an assumed shape for the wavefunction of the ground state of the two-nucleon system. This wavefunction gives us the residue of the t-matrix at the bound-state pole and using the unitary pole approximation (UPA) provides a good extrapolation procedure for other off-shell values of the t-matrix. (E. HARMS has verified the accuracy of the UPA using the anti-bound state for the REID soft-core singlet potential.)

We now face the problem: what bound-state wavefunction should we assume? In the next several years it should be possible to determine the deuteron wavefunction experimentally, by measuring elastic electron-deuteron scattering from polarized deuterons, or equivalently by measuring the polarization of the recoil deuteron. (The wavefunction at large neutron-proton separation should also be checked by using it in the Schrödinger equation to give a local potential which should agree well with that for one pion exchange.) Angular distribution measurements provide a separation of the deuteron form factor into a charge form factor and a magnetic moment form factor. Polarization measurements separate the charge form factor into a monopole form factor G_0 and a quadrupole form factor G_2. T. BRADY has found that values of $G_2(q)$ at momentum transfers q of order $3\ F^{-1}$ are quite sensitive to the percentage of D-state (p_D) in the deuteron. At present the lack of knowledge of p_D is the main source of uncertainty in calculation of the energy of the trinucleon.

1. Introduction

We are trying to calculate the properties of the trinucleon (the three-nucleon system, in its bound state ^{3}H or ^{3}He). The Faddeev equations show us that if we limit ourselves to a non-relativistic three-nucleon problem with only two-nucleon forces, that "all" we need to know are the *off-shell* values $t(p, k; s)$ of the non-relativistic two-nucleon t-matrix. The momentum of the two-body system is initially p, and changes to k. The energy of the two-body system (in the center of mass co-ordinate system) is s, which in general is neither p^2 nor k^2. (I use units with $\hbar^2/M = 1$. Also, I am simplifying notation by considering only a central force in a state of specified angular momentum: e.g., the t-matrix for the ^1S state. I will maintain this over-simplified notation even when discussing tensor forces, for which we need 3 different functions which for the coupled $^3S_1 - {}^3D_1$ deuteron are $t_{00}(p, k; s)$, $t_{02}(p, k; s)$ and $t_{22}(p, k; s)$.)

The momenta p and k have the ranges $0 \leq p < \infty$, and $0 \leq k \leq \infty$, but we do not need accurate knowledge of t at very high momenta. If we cal-

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culate the properties of the trinucleon at energy E, we need values of s in the range $s \leq E$. In this paper I shall neglect the very interesting problems of nucleon-deuteron scattering states above the threshold for deuteron break-up. I thus limit myself to non-positive values of s.

Instead of attempting a review, I will mainly report on work of my students (past and present): S. BHATT, T. BRADY, C. BURNAP, M. FUDA, E. HARMS, L. LAROZE, A. LU, J. O'DONOGHUE, B. SIEBERT and R. STAGAT.

Two of the invited speakers here (MITRA and KHARCHENKO) pioneered the use of a separable approximation to the t-matrix. The approximation $t(p, k, s) = g(p)\, g(k)/D(s)$ reduces by one the dimensionality of the Faddeev equations, thus greatly simplifying their solution. Even with tensor forces, we need to solve only three coupled *one-dimensional* equations. I wish to emphasize that in using a separable approximation to the t-matrix, we are not asserting that the potential is really separable (LEVINGER, [1]). Of course, if the t-matrix were *exactly* separable, then so is the potential and vice-versa. (YAMAGUCHI, [2].) But as illustrated below for the REID singlet soft core potential (HARMS, [3]) a local potential can have a t-matrix that is separable *to a good approximation.*

I also wish to emphasize the present phenomenological nature of the theory of the two-nucleon interaction. If we had a good theory, comparable to that for Coulomb forces, we would be able to calculate the off-shell t-matrix from first principles, thus answering the question I ask in my title. (Of course we must turn to experiment to determine numerical values of a small number of parameters in the theory, such as the electron charge and the photon rest mass for the case of Coulomb forces.) I believe that nuclear theory is still strongly phenomenological. The one generally accepted statement on the nucleon-nucleon potential is the validity of the one-pion-exchange-potential (OPEP) at reasonably large distances. Even here there is not complete agreement as to what is the range of distance for a given accuracy for a given term in OPEP. (LOMON [4]; FESHBACH [5]). Of course, the two parameters in OPEP are determined experimentally, by performing independent experiments.

In the next Section, I outline the "conventional extrapolation procedure" which uses a local potential to extrapolate from on-shell ($s = p^2 = k^2$) to off-shell values of the t-matrix. I do not discuss other extrapolation procedures, due to AMADO [6]; BARANGER [7]); VAN DIJK [8]; FUDA [9], [10]; KOWALSKI [11]; HAFTEL [12] and others. These are based on on-shell values of the t-matrix. In Section 3, I present an answer to the preliminary question of my title by using elastic electron-deuteron scattering to determine the deuteron wavefunction, which in turn is used to determine the off-shell values of the t-matrix. The crucial experiments of measuring the polarization of the recoil deuteron seem feasible. I hope that the program of Section 3 will in the next several years materialize into a practical procedure for finding the triplet t-

matrix. In the last Section I discuss the Gordian knot of interacting problems: extrapolation procedures to find the t-matrix, mesonic effects, relativistic effects in the trinucleon, and three-body forces. I suggest a scheme for starting to untangle the knot — or does anyone have a sword to cut it?

2. Conventional extrapolation procedure

The conventional procedure starts by fitting a more or less local potential to the phase parameters for nucleon-nucleon scattering: i.e. to the on-shell values of the t-matrix. (I say "more or less local" since the potential contains Majorana exchange, and linear and quadratic spin-orbit coupling.) The potentials now in use agree with OPEP at large nucleon-nucleon separation and may include other theoretical constructs such as two-pion-exchange or one boson exchange either explicitly or implicitly. I shall pick REID's [13] soft core potential as an example of the conventional procedure.

Once we have chosen a potential to fit the on-shell values of the t-matrix, this potential specifies a prescription to find the off-shell values: namely, solve the Lippmann—Schwinger equation. We may be able to approximate these off-shell values with a separable t-matrix (or a t-matrix written as a sum of a small number of separable terms). Or we may be clever enough to solve the Faddeev equations for a local potential (MALFLIET [14], DELVES or SANDHAS at this Symposium). I will emphasize the use of a separable approximation.

It is important to distinguish between work with the spin-singlet and the spin-triplet nucleon-nucleon t-matrices. The singlet is both much simpler, since it is purely a central force; and is also much easier to measure experimentally, using proton-proton scattering. (I am neglecting the possibility of charge dependent forces, discussed by J. DABROWSKI at this Symposium.) On the other hand, the triplet involves tensor forces and involves more difficult experiments on the neutron-proton system.

We can almost neglect errors in determination of the on-shell singlet t-matrix for the ^1S state. The phase shifts are fitted by a local potential, such as REID's soft core. The off-shell t-matrices, at least for negative energy s, can be approximated with good accuracy by a separable form, using the unitary-pole-approximation, or UPA.

The UPA is designed (LOVELACE [15]; FUDA [16]) for a potential with a bound state: so let us pretend that REID's potential is some 8% stronger, giving us a bound state at an infinitesimal negative energy, $-B_s$. This bound state has a wave function $|B_s>$, and energy $-B_s = -\hbar^2\alpha_s^2/M$. It is easy to find the separable potential, with form factor $S(p)$, which would give a bound state with the same wave function and same energy: we merely take YAMAGUCHI's original paper and read it backwards. The result is that the momentum-space wavefunction $<p \mid B_s>$ is proportional to the form factor:

$$<p \mid B_s> = NS(p)/(p^2 + \alpha_s^2)\,. \tag{1}$$

HARMS and LAROZE [17] determined the wavefunction $<p \mid B_s>$ and hence the form factor, by solving the ground state for the (strengthened) REID soft core potential. HARMS' form factor is shown in Fig. 1, note how different it is from the YAMAGUCHI shape usually used: see VAN WAGENINGEN, this Symposium.

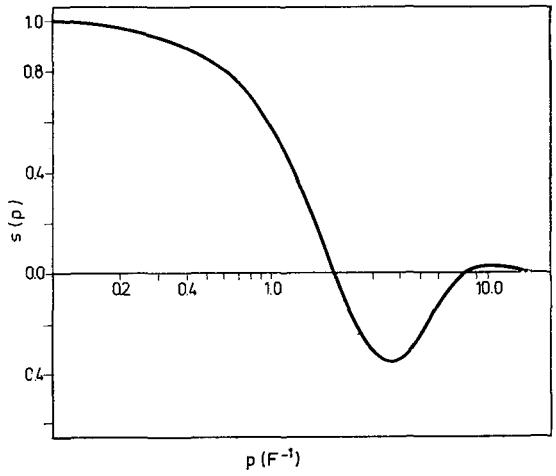

Fig. 1. Form factor $S(p)$ for UPA to REID's soft core singlet. Taken from HARMS and LAROZE [17]

C. BURNAP (unpublished) has checked HARMS' analytical form for the singlet form factor, in a direct manner. BURNAP uses Eq. (1) to determine the momentum space wavefunction (for zero binding); he then Fourier transforms to find the co-ordinate space wavefunction, which he inserts into the Schrödinger equation to find local potential. (This is the same procedure used by BURNAP et al. [18] for the triplet case.) Fig. 2 gives the ratio of HARMS' $V_H(r)$ to REID's $V_R(r)$ vs nucleon-nuclear distance r. We see that this ratio stays near 1.08 for $1 \leq r \leq 3F$.

HARMS weakens the separable potential by a factor 1.08 to find the separable t-matrix, of form (1): the denominator $D(s)$ is found in the usual way by performing an integral involving $S(p)$, and depends on the strength λ of the separable potential. Fig. 3 shows one of many comparisons between the solid line HARMS' $t_v(p, k, s)$ and the (circles) "exact" $t(p, k, s)$ found by numerical solution of the Lippmann–Schwinger equation for the local REID potential. (HARMS [3].) They agree well for reasonably large values of p, $(0 < p < 2F^{-1})$ and $-s = 1/2\hbar^2/M = 22$ MeV for the examples shown, of diagonal $(p = k)$ and off-diagonal $(p \neq k)$ elements.

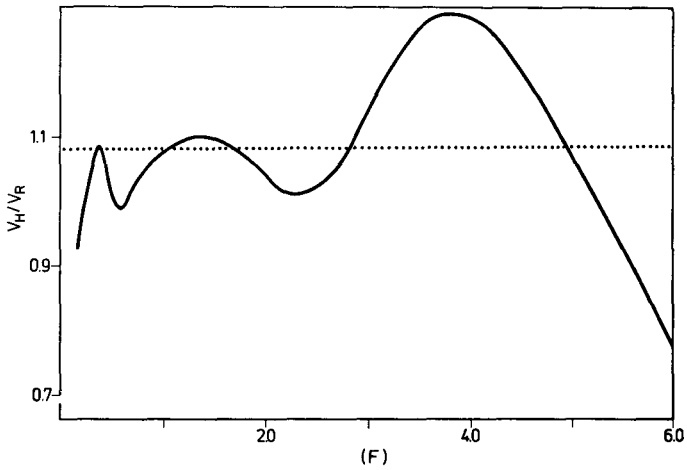

Fig. 2. Ratio $V_H(r)/V_R(r)$ vs nucleon-nucleon separation r, by BURNAP (unpublished). Here V_R is the REID soft core singlet, and V_H is the potential corresponding to HARMS' form factor (Fig. 1) for the (strengthened) REID potential

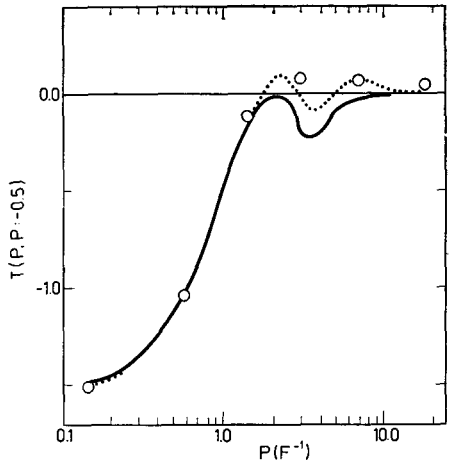

Fig. 3a. Diagonal elements of the t-matrix, for energy $s = -1/2\,\hbar^2/M$. The solid curve shows the UPA (Fig. 1), the dashed curve the UPE with 3 terms, and the circles the t-matrix for the local REID soft core potential. Taken from HARMS [3]

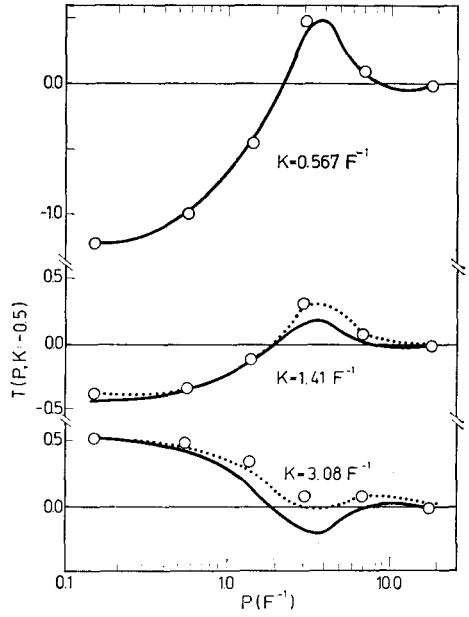

Fig. 3b. Off-diagonal elements of the t-matrix; same notation as 3a. Taken from HARMS [3]

Why does the UPA work so well? Does it work well enough for calculations of the energy and scattering length for the trinucleon? LOVELACE [15] and FUDA [16] gave a preliminary answer to the first question by using the Low equation to show that the UPA was exact in the neighborhood of the two-body pole. (The t-matrix is unitary since it is found from a hermitian potential: hence FUDA's notation, UPA.) This answer is illustrated in Fig. 4 where we

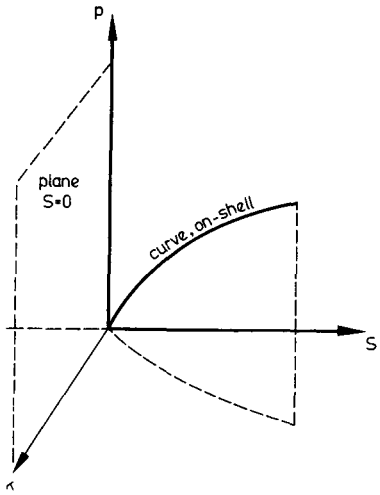

Fig. 4. Sketch of p, k, s space, where p and k are momenta and s the energy used in the t-matrix. The curve shows on-shell, $p^2 = k^2 = s$. The pk plane at an energy corresponding to the pole, shows where the UPA gives the exact residue

show p, k, s space. The on-shell t-matrix is given on the *curve* $p^2/M = k^2/M = s \geq 0$; the off-shell t-matrix at the pole is given exactly by the UPA on the *plane* $s = -B$. If we want $t(p, k, s)$ to the left of that plane, but not "terribly far" to the left, an analytic form that is exact on that plane is likely to be a good approximation. (On the other hand approximation schemes which are exact on the energy shell (FUDA [9]) would be expected to be more useful for $t(p, k, s)$ at positive energies not "terribly far" off-shell: e.g. in nucleon-nucleon bremsstrahlung.)

This general argument in favour of the UPA for negative energy does not explain why the UPA works much better for REID's potential with a soft core than for a potential which is everywhere attractive, such as the Hulthen potential treated by HARMS [3], by KOK [19] and by SANDHAS (this Symposium). The improvement due to treating a soft core potential is shown by the unitary pole expansion (UPE) introduced by HARMS [3] as a special case of the Weinberg or Hilbert—Schmidt expansion or expansion in Sturmian functions. In the UPE a potential with a soft core gives a rapidly converging series for the

t-matrix, with small terms of opposite sign due to the attractive and repulsive parts of the potential. The second and third terms nearly cancel, the higher terms are very small, and hence the first term (which is the UPA) is surprisingly accurate. No such cancellation occurs for the Hulthen potential. (The purely attractive square well treated at great length by LEVINGER [1] was unfortunately a special case since the potential is nearly separable as shown by BRAYSHAW [20].)

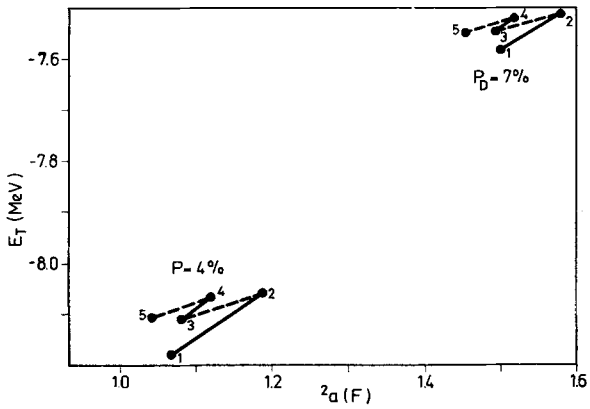

Fig. 5. Convergence of UPE for REID singlet; combined with single term central triplet (modified Hulthen) and tensor YAMAGUCHI, with parameters adjusted to 7% deuteron D-state (upper right) or 4% D-state (lower left). "1" refers to UPA

Is the UPA good enough? HARMS [3] and HARMS and LAROZE [17] have shown that it is very good for calculations of the trinucleon energy E_T and the neutron-deuteron doublet scattering length ^2a. Fig. 5 shows their results for the UPE series for the REID singlet: the energy E_T moves by *small* steps up and down as additional terms are included. The upper-right zig-zag combines the REID singlet with a central triplet of modified Hulthen form (BRADY, [21]) and YAMAGUCHI tensor, with parameters adjusted to 7% D state for the deuteron. The lower zig-zag has parameters adjusted to $p_D = 4\%$: note the change in E_T of over 1/2 MeV and corresponding change in ^2a, for the adjustment in p_D. The UPA value (for 4% D state) of −8.18 MeV is within 0.04 MeV of the estimated value for an infinite number of terms in the UPE; and the UPA result is within 1/3 MeV of experiment.

We earlier suggested (LEVINGER, [1]) that the extrapolation using a local potential and the extrapolation using the UPA were extremes (since the UPA gives a highly nonlocal potential) so that any other extrapolation procedure (e.g., AMADO [6]) should give off-shell t-matrices between those for a local and separable potential. Now we find close agreement between the local and UPA extrapolations, for a REID soft core. Will other extrapolation

procedures fall in the small region between these two? Probably not. We must fall back on Occam's razor as an argument for rejecting other extrapolation procedures.

What about the triplet form factors? Let us again base our work on Reid's soft-core fit to on-shell values of the t-matrix. E. HARMS has recently (unpublished) found an analytical form for the UPA to match REID's deuteron wave function. BHATT [22] uses this UPA together with the REID singlet UPA and calculates $E_T = -7.5$ MeV and ${}^2a = 1.6$ F. In Fig. 6 I compare these values as a circle with those for the UPA by HARMS and LAROZE [17] here

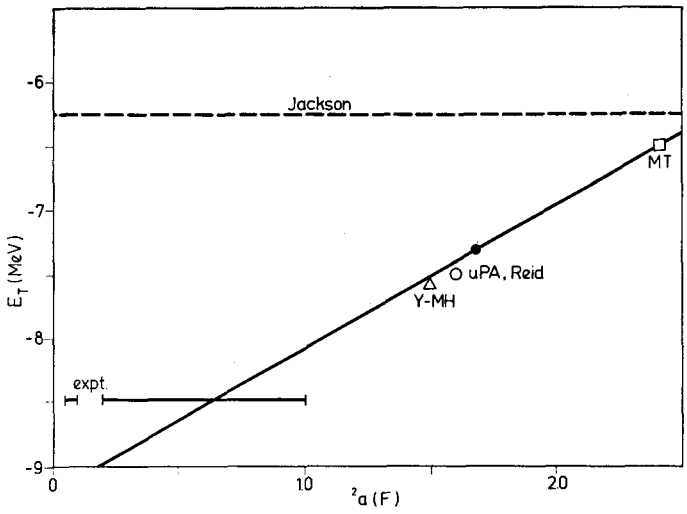

Fig. 6. Phillips plot of 4 calculations on trinucleon: the x is HARMS—LAROZE (1970), from Fig 5, for 7% D-state; the circle is the UPA for REID singlet and triplet (BHATT et al. [22]); the square is MALFLIET—TJON for REID [13]; the dashed line is JACKSON et al. (unpublished); the solid line at lower left shows experimental values

shown as an $x(p_D = 7\%)$ and also with MALFLIET's [14] calculation (a square) and JACKSON et al. (unpublished) a dashed line since ${}^2a$ is not given. DELVES' preliminary results (this Symposium) are not shown. We see good agreement between the two Rensselaer calculations: we have chosen similar values of p_D, and it seems that the change of shape of the central triplet and tensor form factors is of little significance. On the other hand, we have an MeV disagreement between the UPA calculation and the MALFLIET and JACKSON calculations using the same REID potential. (In all cases 2-body interactions in higher partial waves are neglected; they contribute about -0.2 MeV to E_T.)

I do not want to pursue this disagreement in detail here, since work is in progress to understand it better. I note that one could argue that the

agreement within an MeV provides an excellent check on extremely difficult calculations for very complicated potentials. (In what other nuclear physics calculation for finite nuclei have 3 different groups agreed to within 1/3 MeV on the binding energy per nucleon?) Still, we all aim at an order of magnitude better accuracy. Some of the 1 MeV disagreement is due to calculational errors by each group. There are also two systematic errors in the right directions to explain the discrepancy. First, the UPA sets the $^3D_1$ phase shift as identically zero. (FUDA, [16]). Second, MALFLIET neglects the D-state part of the wavefunction for the spectator nucleon (with respect to the center of mass of the two interacting nucleons.)

I conclude that the UPA is quite accurate for the singlet potential (assumed to be local); its accuracy is now being studied for specified triplet potentials.

3. The deuteron wavefunction

Current measurements of triplet phase parameters are inaccurate, particularly for the mixing parameter ε_1. (MACGREGOR, [23]). This inaccuracy leads to large differences among potentials which fit the phase parameters: e.g., REID's soft core ($p_D = 6.4\%$), or MONGAN's [24] two-term separable. ($p_D = 1\%$).

The trinucleon energy E_T is sensitive to the value chosen for p_D; see Fig. 5. (Actually, E_T depends on the strength of the central triplet, which in turn is a monotonic decreasing function of p_D.) Is there another method of determining the value of p_D?

Also, how do we know if a given potential agrees with OPEP at large r? This question is trivial for a local potential. But for YAMAGUCHI's or MONGAN's wavefunctions the answer is not obvious. BURNAP [18] found the answer for the YAMAGUCHI triplet wavefunctions $| B >$ in the same way outlined above for BURNAP's work with HARMS' UPA to the REID singlet. BURNAP found fair (but not excellent) agreement between the potential derived from $| B >$ and OPEP for YAMAGUCHI shapes with p_D or 4% or 7%, but very poor agreement for $0.78 \leq p_D \leq 2\%$. Roughly speaking, the failure for low values of p_D comes from the requirement that $| B >$ give a satisfactory value of the quadrupole moment Q. We can fit Q at low p_D by using a D-state part of the wavefunction with a very long tail, which in turn corresponds to using a very long-range potential, in disagreement with OPEP.

We propose in this Section to answer both questions (the value of p_D and the validity of OPEP at large r) at the same time, by use of experiments that directly measure the deuteron wave function $| B >$. The wave function is just about as good an observable as the on-shell values of the t-matrix, and may be at least as useful for extrapolation purposes to find the t-matrix at negative energies. (Fig. 4) It has been neglected too long!

I must first remind you that measurements on the deuteron and low energy $n-p$ scattering give three numbers: the enegy $-B$, the root-mean-square radius $< r^2 >$, and the quadrupole moment Q. (Usually one writes the scattering length a_t, or the triplet effective range, instead of $<r^2>$; but one can juggle freely among these 3 quantities using effective range theory, so I have chosen the one most useful for my discussion here.) On the other hand the central $V_C\,(r)$ and tensor $V_T\,(r)$ each has a range and a depth, so we have 4 adjustable parameters to fit only 3 data. Thus one adjustable parameter, which I choose as p_D, remains.

If we were naive, we would use the deuteron magnetic moment to give $p_D = 4\%$: perhaps it is not an accident that this naive choice together with reasonable shapes gives the fair agreement with trinucleon experiments shown in Fig. 5. Being more sophisticated, we argue that the magnetic moment is sensitive to relativistic effects, effects of meson exchange, admixture of nucleon isobars in the deuteron wavefunction, and/or non-locality in the nucleon-nucleon potential. (It is unclear to me how much double or triple counting we do if we treat each of these 4 effects separately.)

Corresponding to each of the static deuteron properties (charge, quadrupole moment and magnetic moment) we have a form factor for elastic electron-deuteron scattering. Since I am ignoring the static magnetic moment, I shall be consistent and also ignore where possible the magnetic form factor, $G_M\,(q)$. The differential cross section $d\sigma/d\Omega$ for elastic $e-d$ scattering in the laboratory system gives (GLENDENING, [25]) a form factor $G\,(q)$

$$G^2\,(q) = (d\sigma/d\sigma)/(d\sigma/d\Omega)_{\text{point}} = G_0^2 + G_2^2 + (2\tan^2 1/2\Theta + 1)G_M^2. \qquad (2)$$

Here $(d\sigma/d\Omega)_{\text{point}}$ is the calculated differential cross section for a deuteron of negligible dimensions composed of point nucleons. The form factors $G_0\,(q)$ and $G_2\,(q)$ depend on the momentum transfer q, the deuteron wavefunction, and the isoscalar nucleon electric form factor $G_{ES}\,(q)$. (See MEHROTRA [26]).

$$G_k\,(q) = 2\;G_{ES}\,(q)\;F_k\,(q)\;\;k = 0{,}2, \qquad (3)$$

$$F_0\,(q) = \int_0^\infty (u^2 + w^2)\;j_0\;(1/2qr)\,dr, \qquad (4)$$

$$F_2\,(q) = \int_0^\infty 2w\;(u - 8^{-1}\,\sqrt{2}w)\;j_2\;(1/2qr)\,dr. \qquad (5)$$

By measuring $G^2\,(q)$ at fixed q as a function of $\tan^2 1/2\Theta$ the experimentalist can use Eq. (2) to separate out $(G_0^2 + G_2^2)$ (WILSON, [27]) and thus ignore the effect of the magnetic scattering. However, the sum $(G_0^2 + G_2^2)$ turns out to be insensitive to the choice of deuteron wavefunction: we need to measure the ratio G_2/G_0. (Note that the isoscalar nucleon form factor cancels in this ratio.)

BERTOZZI of M. I. T. suggested to me several years ago that the right way to measure $G_2(q)$ was to measure the *tensor* polarization of the recoil deuteron

from elastic e, d scattering. (See PREPOST [28] for the *vector* polarization.) I believe that no such experiments have yet been made; though they appear to be of not much more than moderate difficulty with a suitable electron accelerator. The energy should be several hundred MeV, and the machine should have a good duty cycle so that e, d coincidences will eliminate background. The deuteron polarization would be measured by nuclear scattering of the deuterons.

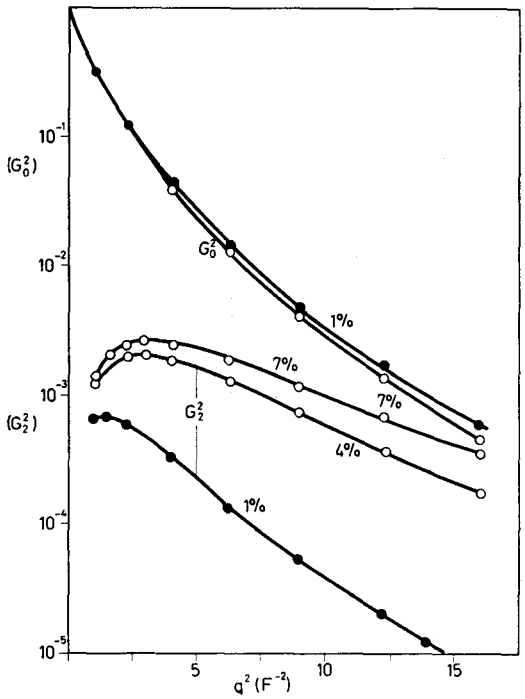

Fig. 7. Form factors for electron-deuteron scattering. The upper two curves show the squared monopole form factor, G_0^2 vs squared momentum transfer q^2, for YAMAGUCHI shapes, with deuteron D-state at 1% and 7% respectively. The lower three curves show the squared quadrupole form factor G_2^2 for 7%, 4% and 1% D-state, respectively. From BRADY [21]. See Eqs. (3) to (5) of text

I find it easier to understand the calculation of the electric part of the differential scattering cross section $(G_0^2 + G_2^2)$ due to polarized (aligned) deuterons. From time reversal invariance, this cross section is proportional to the polarization of the recoil deuteron; and BERTOZZI assures me that the recoil polarization experiment is easier. I just learned that the polarization effect has been calculated and published: GOURDIN [29], BRADY and TOMUSIAK at Saskatchewan and I independently arrived at the same answer, which I state without proof. Let M be the quantized component of deuteron angular

momentum along the direction of the momentum transfer vector q. We find that the value of M is unchanged due to electric scattering. We can then consider $(d\sigma/d\Omega)_0$, $(d\sigma/d\Omega)_1 = (d\sigma/d\Omega)_{-1}$ and $(d\sigma/d\Omega)_u = (2/3)\ (d\sigma/d\Omega)_1 + + (1/3)\ (d\sigma/d\Omega)_0$. Here the subscripts 0 and $\pm$ 1 give the value of M, and u stands for "unpolarized". Define an element of the deuteron polarization tensor P

$$P = 3\ (2)^{-1/2}[(d\sigma/d\Omega)_0 - (d\sigma/d\Omega)_1]/(d\sigma/d\Omega)_u = (2G_0G_2 + 2^{-1\ 2}G_2^2)/(G_0^2 + G_2^2). \quad (6)$$

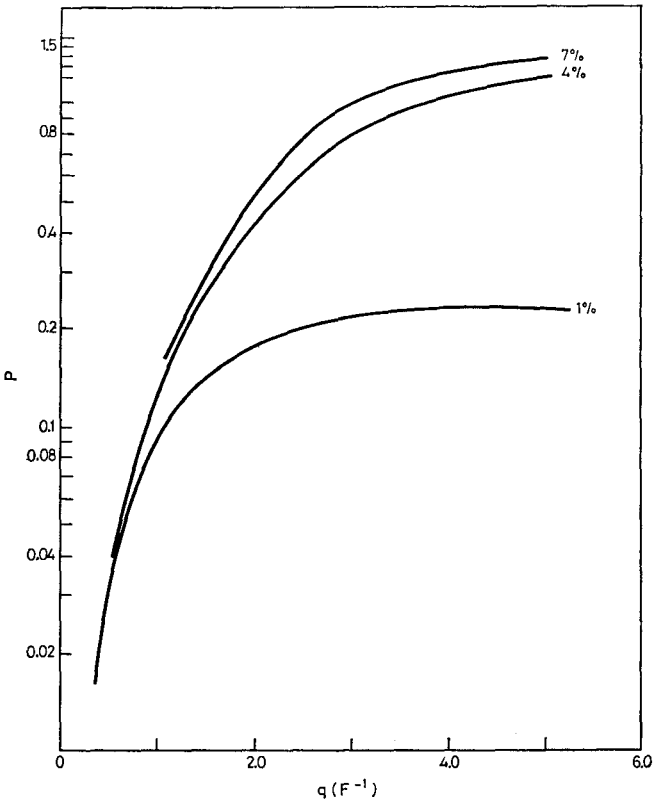

Fig. 8. Deuteron polarization P vs momentum transfer q (in F^{-1}) for YAMAGUCHI shape, with different percentage D-state. See Fig. 7 and Eq. (6). From BRADY (unpublished)

A measurement of $P\ (q)$ provides the ratio $G_2/G_0 = F_2/F_0$ as a function of momentum transfer q.

Fig. 7 shows on a semi-log scale several of BRADY's results [21] for $G_0^2\ (q)$ and $G_2^2\ (q)$ vs q^2 in F^{-2}. The curves labeled 1%, 4% or 7% are for YAMAGUCHI shapes with the designated percentage D-state. Note the large changes in G_2^2, compared to rather small ones in G_0^2. The values of P shown in Fig. 8 are very low for 1% D-state. 4% and 7% D-state give large but simi-

lar values of the polarization. Clearly a rough measurement could throw out the low values of p_D (already discredited by the disagreement with OPEP); a *precise* measurement would be needed for the region of p_D from 4 to 7%.

It is amusing to plot the form factors F_0 and F_2 in a different manner. If we plot $F_0(q^2)$ vs q^2 on a linear scale, it is well known that we get a straight line with intercept unity, and slope proportional to the mean square radius of the deuteron. So the linear approximation gives negligible additional know-

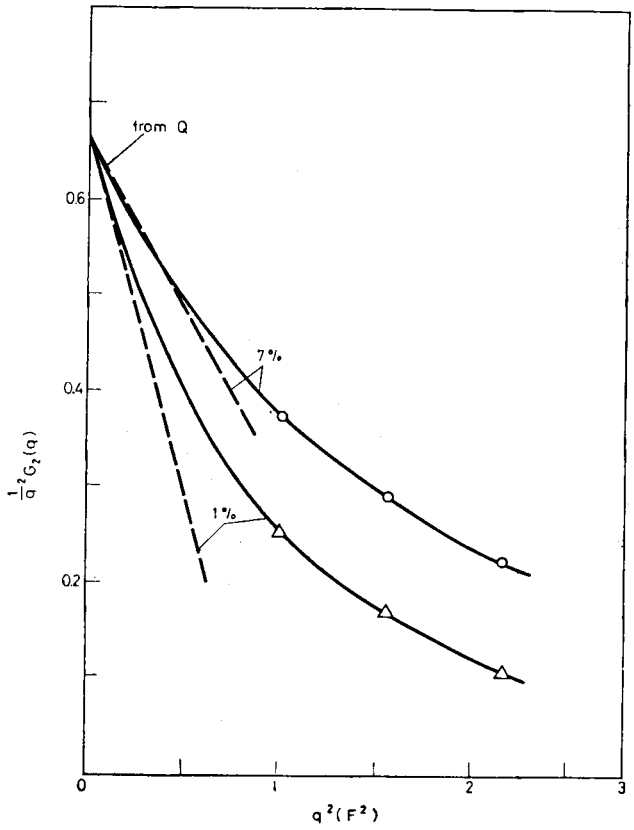

Fig. 9. Plot of G_2/q^2 vs q^2. The intercept is proportional to the quadrupole moment (same for 7% and 1% D-states); the slope is proportional to the parameter f: see Eqs. (7) and (8)

ledge of the deuteron. For $F_2(q)$ we expand the spherical Bessel function $j_2(1/2qr)$ in a power series, and keep the first two terms. Define

$$f = \int_0^\infty 2w(u - 8^{-1/2}w)\, r^4\, dr. \tag{7}$$

Then

$$F_2(q)/q^2 = Q/3\times 2^{1/2} \quad (q^2/2^3\times 3\times 5\times 7)f. \tag{8}$$

Fig. 9 illustrates the different values of the deuteron parameter f for 1% and 7% D-state: note the same intercept, proportional to quadrupole moment Q. We see that the linear approximation fails even at $1F^{-2}$. Nevertheless this new parameter f may turn out to be useful.

4. The Gordian knot

It would take too long to try to explain all the strands that are tangled together in one giant knot involving the nucleon-nucleon potential and the trinucleon problem. Let me merely enumerate some of the loose ends seen by a casual observer: i) the on-shell and near-off-shell two-body t-matrix; ii) extrapolation procedures to get farther off-shell; iii) relativistic effects in the trinucleon; iv) 3-body forces in the trinucleon; v) meson exchange and/or nucleon isobar effects in the trinucleon.

i) Some needed measurements of the on-shell t-matrix suffer from the lack of a suitable free neutron target. Experimentalists sometimes try to get around this lack by scattering nucleons using loosely bound neutrons, as in the deuteron: but this involves knowledge of the trinucleon problem. I proposed above studying the near-off-shell triplet t-matrix by studying elastic electron-deuteron scattering. (One might also study inelastic $e-d$ scattering, or the deuteron photo-effect, or neutron-proton bremsstrahlung.) This is also a three-body problem, but one of the three bodies interacts only by electromagnetic or weak forces, so the Born approximation is useful. But, as discussed above in connection with the deuteron magnetic moment, various effects not described by the two-nucleon Schrödinger equation with a local potential seem likely to enter, particularly for the magnetic interaction. Are they important for the electric interactions, $G_0(q)$ and $G_2(q)$?

ii) We need to get further off-shell for application of the trinucleon. I quoted HARMS' work above on a local singlet potential with a soft core, which showed that the UPA agreed surprisingly well with a local potential. While the UPA is likely to be quite good for any reasonable central potential, its validity must be studied further for a tensor potential.

iii) Trinucleon relativistic effects, as evaluated by JACKSON [30], are only of order 1/4 MeV, and in the right direction to improve agreement between calculations of E_T (with 4 to 7% deuteron D-state) and experiment. Still we don't have a complete relativistic theory so this loose strand is not completely disentangled.

iv) Three-body forces still seem hard to calculate. (After all, if we could calculate three-body forces accurately, we would have no trouble with calculation of two-body forces, and the messy phenomenological arguments above on the two-nucleon problem could be avoided!)

v) If we are only comparing two (linearly related) quantities E_T and ${}^2a$ with experiment, it is hard to assess the significance of agreement or disagreement. Thus a further study of trinucleon properties, by calculation of the Coulomb energy and of electron-trinucleon scattering is very useful. (TJON,[31] MALFLIET and also DELVES, this Symposium). Here though we must be wary of *new* mesonic contributions (or nucleon isobar contributions): e.g., a one-pion isovector term enters, and is almost certainly responsible for the anomalous magnetic moments of the trinucleon (i.e., outside the Schmidt lines).

My program for unravelling the knot follows. We combine nucleon-nucleon and electron-deuteron elastic scattering to find the on-shell and near-off-shell two-nucleon t-matrix. We *assume* that mesonic exchange or nucleon isobar effects are negligible for the electric form factors in $e-d$ scattering. We can later improve this assumption by using a theory which agrees with experiments on the corrections to the magnetic scattering, and using the same theory for the much smaller corrections to $G_0(q)$ and $G_2(q)$. (BLANCENBECLER, [32]). We extrapolate off-shell using the UPA, with possible corrections using the UPE for the tensor force. If we are able thus to solve the first two problems, we can use the comparison between experiment and calculation for E_T and ${}^2a$ as a check on theoretical estimates of relativistic effects and of three-body forces. Hopefully we can treat the new mesonic effects in the trinucleon as we did for the deuteron: i.e., verify a theory (KLOET and TJON, preprint) for their contribution to the magnetic scattering, and use the same theory for smaller contributions to electric scattering.

Looking still further ahead, we could use the alpha particle as an additional test of relativistic effects, three-body (and four-body) forces, and (only isoscalar) mesonic and isobar effects. Of course the calculations are much harder than for the trinucleon. (KHARCHENKO, this Symposium).

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"КАК ОПРЕДЕЛЯЕТСЯ *t*-МАТРИЦА ДВУХ ТЕЛ?"

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Резюме

Сначала предлагаем определенную форму волновой функции основного состояния системы двух ядер. Эта волновая функция дает остаток *t*-матрицы в полюсе связанного состояния, и использование приближения единичного полюса дает хорошую экстраполяцию для значений *t*-матрицы в не энергетической поверхности. Сейчас возникает проблема, какую волнову функцию связанного состояния мы должны подобрать? В течение следующих лет становится возможным экспериментальным путем определить волновую функцию дейтерия путем измерения упругого рассеяния электрона и действия в поляризованном дейтерии или аналогичным путем измерить поляризацию отраженного дейтерия.