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EFFECTS OF MASS TRANSFER ON STEADY HYDROMAGNETIC FREE CONVECTIVE FLOW OF AN INCOMPRESSIBLE VISCOUS FLUID PAST AN INFINITE VERTICAL POROUS WALL

By

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An analysis of the mass transfer effects on the hydromagnetic free — convective flow of an electrically conducting, incompressible viscous fluid, past an infinite, non-conducting, porous, vertical wall with constant suction, has been carried out, in presence of a transverse magnetic field. The induced magnetic field is taken into consideration and the terms representing the viscous dissipative heat and the Joule heating are included in the energy equation. Approximate solutions to coupled non-linear equations governing the flow are obtained, when the magnetic Prandtl number is unity and the magnetic parameter $M < 1$. Expressions are given for the velocity, the induced magnetic field, the temperature, the skin friction, the electric current density and the rate of heat transfer in terms of the Nusselt number. The variations of the above quantities are presented graphically, and the paper is concluded with a quantitative discussion.

1. Introduction

It is known that flows arising from differences in concentration or material constitution alone and in conjunction with temperature differences have great significance not only for their own interest but also for the applications to geophysics, aeronautics and engineering. There are many interesting aspect of such flows, so in recent years analytical solutions to such problems of flow have been presented by many authors. SPARROW et al [4] have presented an analytical study of the effects of buoyancy in a binary boundary layer into which a foreign gas is injected through a porous surface. SOUNDALGEKAR [3] has studied the effects of mass transfer on steady free convective flow of a dissipative, incompressible fluid past an infinite vertical porous wall, with constant suction. Recently, HALDAVNEKAR and SOUNDALGEKAR [1] have carried out an analysis of the mass transfer effects on the steady free convective flow of an incompressible electrically conducting, viscous fluid past an infinite porous plate with constant suction and transverse magnetic field. In this study the magnetic Reynolds number of the flow is taken to be small enough so that the induced magnetic field can be neglected. Also the viscous dissipative heat in the equation of energy is assumed to be negligible as compared to Joule dissipative heat.

Hence, in the present analysis we study the effects of the mass-transfer on the steady free convective flow, of an electrically conducting, incompressible, viscous fluid, past an infinite vertical non-conducting porous wall with constant suction, in the presence of a uniform transverse magnetic field. The induced magnetic field is not assumed negligible and the terms, which represent the viscous dissipative heat and the Joule dissipative heat remain in the equation of energy. Approximate solutions to a coupled non-linear system of equations governing the flow are derived when the magnetic Prandtl number is unity, and expressions are obtained for the velocity field, the induced magnetic field, the temperature field, the skin friction, the rate of heat transfer, in terms of the Nusselt number and for the electrical current density. Finally, all the above quantities are shown graphically, followed by a discussion.

2. Mathematical analysis

We assume as the coordinate origin 0, an arbitrary point on an infinite vertical porous wall, which is taken to be an electrical insulator. The x'-axis is chosen along the vertical wall in the upward direction and the y'-axis is chosen normal to it. The electrostatic system of units has been used throughout, and we assume that, in the present analysis, all the physical variables are function of the space coordinate y only. Also the applied magnetic field is uniform and perpendicular to the wall, so that in the region considered, $H = H(H_x, H_y, 0)$. Under these assumptions, the steady free convective flow on an electrically conducting, viscous incompressible fluid is governed by the following set of equations

$$v' \frac{\partial u'}{\partial y'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta(T'_w - T'_\infty) + g\beta^*(C' - C'_\infty) + \frac{\mu_0}{\varrho} + H_y \frac{\partial H_x}{\partial y'}, \tag{1}$$

$$v' \frac{\partial H_x}{\partial y'} = H_y \frac{\partial u'}{\partial y'} + \frac{1}{\sigma\mu_0} \frac{\partial^2 H_x}{\partial y'^2}, \tag{2}$$

$$v' \frac{\partial T'}{\partial y'} = \frac{k}{\varrho c_p} \frac{\partial^2 T'}{\partial y'^2} + \frac{\nu}{c_p} \left(\frac{\partial u'}{\partial y'}\right)^2 + \frac{1}{\sigma\varrho c_p} \left(\frac{\partial H_x}{\partial y'}\right)^2, \tag{3}$$

$$v' \frac{\partial C'}{\partial y'} = D \frac{\partial^2 C'}{\partial y'^2}, \tag{4}$$

$$\frac{\partial v'}{\partial y'} = 0, \tag{5}$$

where all the above physical quantities have their usual meaning, except C' which is known as the species concentration, D is the molecular diffusivity and

β^* is the volume coefficient of expansion with concentration. The second and the third terms on the right hand side of Eq. (3) signify, respectively, the heat generated by friction (or viscous dissipative heat) and the Joule heating.

The boundary conditions for the velocity field, for the temperature field and for the species concentration are:

$$\begin{aligned} y &= 0; \quad u' = 0, \quad T' = T'_w, \quad C' = C'_w, \\ y &\to \infty: \quad u' \to 0, \quad T' \to T'_\infty \quad C' \to C'_\infty . \end{aligned} \tag{6}$$

The appropriate boundary conditions on H_x are (for detailed discussion see PANDE [2]):

$$\begin{aligned} y &= 0: \quad H_x = 0, \ H_y = H_0, \\ y &\to \infty: \quad H_x \to 0, \quad H_y \to H_0. \end{aligned} \tag{7}$$

From Maxwell's equations the components of electrical current density are given by

$$j_x = 0, \quad j_y = 0$$

and

$$j_z = -\left(\frac{\partial H_x}{\partial y'}\right); \tag{8}$$

and the divergence equation for the magnetic field gives

$$H_y = \text{constant} = H_0, \tag{9}$$

where H_0 is the externally applied transverse magnetic field.

Integration of (5) gives

$$v' = -v_0 , \tag{10}$$

where v_0 is the constant suction velocity. The negative sign in (10) indicated that the suction velocity is directed towards the wall.

We now define the following non-dimensional parameters:

$$u = \frac{u'}{v_0}, \quad y = \frac{y' v_0}{\nu}, \quad \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty},$$

$$C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \quad H = \left(\frac{\mu_0}{\varrho}\right)^{1/2} \frac{H_x}{v_0},$$

$$G_r = \frac{\nu g \beta (T'_w - T'_\infty)}{v_0^3} \text{ (the Grashof number)},$$

$$G_c = \frac{\nu g \beta^* (C'_w - C'_\infty)}{v_0^3} \text{ (the modified Grashof number)},$$

$$P_m = \sigma \nu \mu_0 \qquad \text{(the magnetic Prandtl number)},$$

$$P = \frac{\varrho \nu c_p}{k} \qquad \text{(the Prandtl number)},$$

$$E = \frac{v_0^2}{c_p(T'_w - T'_\infty)} \qquad \text{(the Eckert number)},$$

$$S_c = \frac{\nu}{D} \qquad \text{(the Schmidt number)},$$

$$M = \left(\frac{\mu_0}{\varrho}\right)^{1/2} \frac{H_0}{v_0} \qquad \text{(the magnetic field parameter)}. \tag{11}$$

With the help of Eqs. (9) and (10) and of the non-dimensional quantities (11) the Eqs. (1), (2), (3) and (4) reduce to:

$$\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} = -G_r \theta - G_c C - M \frac{\partial H}{\partial y}, \tag{12}$$

$$\frac{1}{p_m} \frac{\partial^2 H}{\partial y^2} + \frac{\partial H}{\partial y} + M \frac{\partial u}{\partial y} = 0, \tag{13}$$

$$\frac{\partial^2 \theta}{\partial y^2} + P \frac{\partial \theta}{\partial y} = PE \left(\frac{\partial u}{\partial y}\right)^2 - \frac{PE}{P_m} \left(\frac{\partial H}{\partial y}\right)^2, \tag{14}$$

$$\frac{\partial^2 C}{\partial y^2} + S_c \frac{\partial C}{\partial y} = 0, \tag{15}$$

and the boundary conditions (6) and (7) in the non-dimensional form become:

$$\begin{aligned} y = 0: \quad & u = 0, \quad \theta = 1, \quad C = 1, \quad H = 0, \\ y \to 0: \quad & u \to 0, \quad \theta \to 0, \quad C \to 0, \quad H \to 0. \end{aligned} \tag{16}$$

Eqs. (12)—(15) are coupled non-linear differential equations and to solve we follow the power series solution method. As the fluid is incompressible and the suction velocity is small the Eckert number E is also small ($\ll 1$). Hence, we expand u, H, θ and C in powers of E and neglect terms of order E^2 and higher.

Thus, we have

$$\begin{aligned} u &= u_0 + Eu_1, \\ \theta &= \theta_0 + E\theta_1, \\ H &= H_0 + EH_1, \\ C &= C_0 + EC_1. \end{aligned} \tag{17}$$

On substituting (17) into Eqs. (12)—(15), equating the coefficient of E and neglecting terms in E^2 and higher order, we get

$$u_0'' + u_0' = -G_r\theta_0 - G_c C_0 - MH_0', \tag{18}$$

$$u_1'' + u_1' = -G_r\theta_1 - G_c C_1 - MH_1', \tag{19}$$

$$\frac{1}{P_m} H_0'' + H_0' + Mu_0' = 0, \tag{20}$$

$$\frac{1}{P_m} H_1'' + H_1' + Mu_1' = 0, \tag{21}$$

$$\theta_0'' + P'\theta = 0, \tag{22}$$

$$\theta_1'' + P\theta_1' = Pu_0'^2 - \frac{P}{P_m} H_0'^2, \tag{23}$$

$$C_0'' + S_c C_0' = 0, \tag{24}$$

$$C_1'' + S_c C_1' = 0, \tag{25}$$

where the dashes indicate derivatives with respect to y.

The corresponding boundary conditions are:

$$\begin{aligned} &y = 0: u_0 = 0,\ u_1 = 0,\ \theta_0 = 1,\ \theta_1 = 0,\ C_0 = 1,\ C_1 = 0,\ H_0 = 0,\ H_1 = 0, \\ &y \to 0: u_0 \to 0,\ u_1 \to 0,\ \theta_0 \to 0,\ \theta_1 \to 0,\ C_0 \to 0,\ C_1 \to 0,\ H_0 \to 0, H_1 \to 0. \end{aligned} \tag{26}$$

Solving Eqs. (18)—(25) under the boundary conditions (26), when the magnetic Prandtl number $P_m = 1$ and substituting the solutions obtained in (17) we have

$$\begin{aligned} u(y) = {} & A_1(e^{-\alpha y} - e^{-Py}) + A_2(e^{-\alpha y} - e^{-Scy}) + A_3(e^{-\beta y} - e^{-Py}) + \\ & + A_4(e^{-\beta y} - e^{-Scy}) + \frac{E}{2}(\Gamma_7 e^{-\alpha y} + \Delta_8 e^{-\beta y} - (B_6 + \Gamma_0) e^{-Py} + \\ & + (B_7 + \Gamma_8) e^{-2\alpha y} + (B_8 + \Gamma_9) e^{-2Py} - (B_9 + \Delta_1) e^{-(\alpha+P)y} + \\ & + (\Gamma_1 + \Delta_2) e^{-2\,Scy} - (\Gamma_2 + \Delta_3) e^{-(\alpha+S_c)y} + (\Gamma_3 + \Delta_4) e^{-2\beta y} - \\ & - (\Gamma_4 + \Delta_5) e^{-(\beta+P)y} - (\Gamma_5 + \Delta_6) e^{-(\beta+S_c)y} + (\Gamma_6 + \Delta_7) e^{-(P+S_c)y}), \end{aligned} \tag{27}$$

$$\begin{aligned} H(y) = {} & A_1(e^{-\alpha y} - e^{-Py}) + A_2(e^{-\alpha y} - e^{-Scy}) - A_3(e^{-\beta y} - e^{-Py}) - \\ & - A_4(e^{-\beta y} - e^{-Scy}) + \frac{E}{2}(\Gamma_7 e^{-\alpha y} - \Delta_8 e^{-\beta y} + (\Gamma_0 - B_6) e^{-Py} + \\ & + (B_7 - \Gamma_8 e^{-2\alpha y} + (B_8 - \Gamma_9) e^{-2Py} + (\Delta_1 - B_9) e^{-(\alpha+P)} + \\ & + (\Gamma_1 - \Delta_2) e^{-2Scy} + (\Delta_3 - \Gamma_2) e^{-(\alpha+S_c)y} + (\Gamma_3 - \Delta_4) e^{-2\beta y} + \\ & + (\Delta_5 - \Gamma_4) e^{-(\beta+P)y} + (\Delta_6 - \Gamma_5) e^{-(\beta+S_c)y} + (\Gamma_6 - \Delta_7) e^{-(P+S_c)y}), \end{aligned} \tag{28}$$

$$\theta(y) = e^{-Py} + E(B_5 e^{-Py} - A_5 e^{-2\alpha y} - A_6 e^{-2Py} + A_7 e^{-(\alpha-P)y} - A_8 e^{-2Scy} + A_9 e^{-(\alpha+S_c)y} - B_1 e^{-2\beta y} + B_2 e^{-(\beta+P)y} + B_3 e^{-(\beta+S_c)y} - B_4 e^{-(P+S_c)y}) \quad (29)$$

and

$$C = e^{-Scy}, \quad (30)$$

where

$$a = 1 + M,\ \beta = 1 - M,\ A_1 = \frac{G_r}{2P(P-\alpha)},\ A_2 = \frac{G_c}{2S_c(S_c-\alpha)},$$

$$A_3 = \frac{G_r}{2P(P-\beta)},\ A_4 = \frac{G_c}{2S_c(S_c-\beta)},\ A_5 = \frac{\alpha P(A_1+A_2)^2}{2\alpha - P},$$

$$A_6 = P(A_1^2 + A_3^2),\ A_7 = \frac{4A_1P^2(A_1+A_2)}{(\alpha+P)},\ A_8 = \frac{S_cP(A_1^2+A_4^2)}{2S_c - P},$$

$$A_9 = \frac{4\alpha A_2 S_c P(A_1+A_2)}{(\alpha+S_c)(\alpha+S_c-P)},\ B_1 = \frac{\beta P(A_3+A_4)^2}{2\beta - P},$$

$$B_2 = \frac{4P^2A_3(A_3+A_4)}{\beta+P},\quad B_3 = \frac{4\beta PS_cA_4(A_3+A_4)}{(\beta+S_c)(\beta+S_c-P)},$$

$$B_4 = \frac{4P^2(A_1A_2+A_3A_4)}{P+S_c},\ B_5 = A_5+A_6-A_7+A_8-A_9+B_1-B_2-B_3+B_4$$

$$B_6 = \frac{B_5G_r}{P(P-\alpha)},\ B_7 = \frac{G_rA_5}{2\alpha^2},\ B_8 = \frac{G_rA_6}{2P(2P-\alpha)},$$

$$B_9 = \frac{G_rA_7}{P(P+\alpha)},\ \Gamma_1 = \frac{G_rA_8}{2S_c(2S_c-\alpha)},\ \Gamma_2 = \frac{G_rA_9}{S_c(S_c+\alpha)},$$

$$\Gamma_3 = \frac{G_rB_1}{2\beta(2\beta-\alpha)},\ \Gamma_4 = \frac{G_rB_2}{(\beta+P)(\beta+P-\alpha)},\ \Gamma_5 = \frac{G_rB_3}{(\beta+S_c)(\beta+S_c-\alpha)},$$

$$\Gamma_6 = \frac{G_rB_4}{(P+S_c)(P+S_c-\alpha)},\ \Gamma_7 = B_6-B_7-B_8+B_9-\Gamma_1+\Gamma_2-\Gamma_3+\Gamma_4+\Gamma_5-\Gamma_6,$$

$$\Gamma_0 = \frac{B_5G_r}{P(P-\beta)},\ \Gamma_8 = \frac{A_5G_r}{2\alpha(2\alpha-\beta)},\ \Gamma_9 = \frac{A_6G_r}{2P(2P-\beta)},$$

$$\Delta_1 = \frac{A_7G_r}{(\alpha+P)(\alpha+P-\beta)},\ \Delta_2 = \frac{A_8G_r}{2S_c(2S_c-\beta)},\ \Delta_3 = \frac{A_9G_r}{(\alpha+S_c)(\alpha+S_c-\beta)},$$

$$\Delta_4 = \frac{B_1G_r}{2\beta^2},\ \Delta_5 = \frac{B_2G_r}{\beta(\beta+P)},\ \Delta_6 = \frac{B_3G_r}{S_c(S_c+\beta)}, \quad (31)$$

$$\Delta_7 = \frac{B_4G_r}{(P+S_c)(P+S_c-\beta)}\ \Delta_8 = \Gamma_0-\Gamma_8-\Gamma_9+\Delta_1-\Delta_2+\Delta_3-\Delta_4+\Delta_5+\Delta_6-\Delta_7.$$

Using the expressions (27), (28) and (29) the skin friction τ, the electric current density, Z, and the rate of heat transfer, expressed in terms of the Nusselt number Nu, in the nondimensional terms, are given respectively, by

$$\tau = \frac{\tau_w \nu}{v_0^2} = \left(\frac{\partial u}{\partial y}\right)_{y=0}$$
$$= A_1(P-\alpha) + A_2(S_c - \alpha) + A_3(P-\beta) + A_4(S_c-\beta) +$$
$$+ \frac{E}{2}(-\alpha\Gamma_7 - \beta\Delta_8 + P(B_6+\Gamma_0) - 2\alpha(B_7+\Gamma_8) -$$
$$- 2P(B_8+\Gamma_9) + (\alpha+P)(B_9+\Delta_1) - 2S_c(\Gamma_1+\Delta_2) +$$
$$+ (\alpha+S_c)(\Gamma_2+\Delta_3) - 2\beta(\Gamma_3+\Delta_4) + (\beta+P)(\Gamma_4+\Delta_5) +$$
$$+ (\beta+S_c)(\Gamma_5+\Delta_6) - (P+S_c)(\Gamma_6+\Delta_7)), \tag{32}$$

$$Z = \frac{j_z \nu}{v_0^2}\left(\frac{\mu_0}{\varrho}\right)^{1/2} = -\left(\frac{\partial H}{\partial y}\right) =$$
$$= -A_1(Pe^{-Py} - \alpha e^{-\alpha y}) - A_2(S_c e^{-S_c y} - \alpha e^{-\alpha y}) +$$
$$+ A_3(Pe^{-Py} - \beta e^{-\beta y}) + A_4(S_c e^{-S_c y} - \beta e^{-\beta y}) -$$
$$- \frac{E}{2}(\beta\Delta_8 e^{-\beta y} - \alpha\Gamma_7 e^{-\alpha y} - P(\Gamma_0 - B_6)\, e^{-Py} -$$
$$- 2\alpha(B_7-\Gamma_8)e^{-2\alpha y} - 2P(B_8-\Gamma_9)\, e^{-2Py} -$$
$$- (\alpha+P)(\Delta_1-B_9)\, e^{-(\alpha+P)y} - 2S_c(\Gamma_1-\Delta_2)\, e^{-2S_c y} -$$
$$- (\alpha+S_c)(\Delta_3-\Gamma_2)\, e^{-(\alpha+S_c)y} - 2\beta(\Gamma_3-\Delta_4)\, e^{-2\beta y} -$$
$$- (\beta+P)(\Delta_5-\Gamma_4)\, e^{-(\beta+P)y} - (\beta+S_c)(\Delta_6-\Gamma_5)\, e^{-(\beta+S_c)y} -$$
$$- (P+S_c)(\Gamma_6-\Delta_7)\, e^{-(P+S_c)y}) \tag{33}$$

and

$$Nu = \frac{q'\nu}{k(T'_w - T'_\infty)\, v_0} = -\left(\frac{\partial\theta}{\partial y}\right)_{y=0} = -P + E(P(-B_2 + B_4 -$$
$$- B_5 + 2A_6 - A_7) + \alpha(2A_5 - A_7 - A_9) + \beta(2B_1 - B_2 - B_3) +$$
$$+ S_c(2A_8 - A_9 - B_3 + B_4)). \tag{34}$$

3. Discussion

This paper is concerned with the study of the effects of mass transfer on the hydromagnetic free-convection flow past an infinite vertical porous wall with constant suction. The results are displayed in Figs. (1)—(8), respectively, for the dimensionless forms of the velocity, the induced magnetic fields, the skin friction, the electric current density and the Nusselt number. The variations of the temperature field are given in Table I for different values of the magnetic parameter M. In order to be realistic, the values of the Schmidt number S_c, are chosen to be 0.22, 0.60 and 0.75 which correspond to hydrogen, water-vapour and oxygen, respectively, at approximately 25 °C and 1 atmosphere, when for the Prandtl number P we get the value $P = 0.71$, corresponding in the air. The values of all the other parameters are chosen arbitrarily.

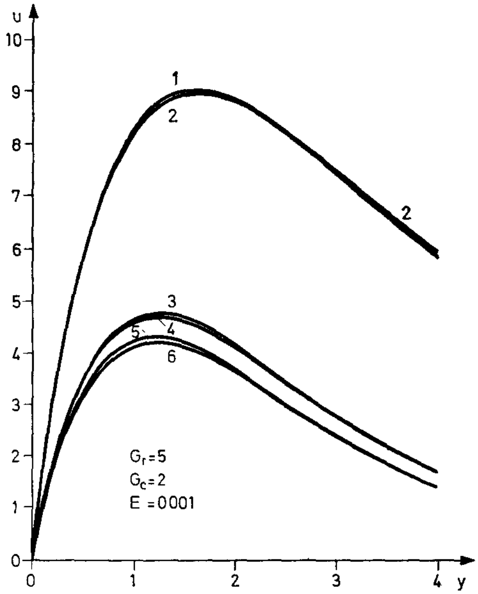

Fig. 1. The velocity profiles u for $P = 0.71$

The variation of the velocity field for different values of S_c and M are shown in Fig. 1. From this Figure we see that the velocity is greater in the case of the hydrogen ($S_c = 0.22$) than in the case of the oxygen ($S_c = 0.75$). Also we remark that as magnetic parameter M increases the velocity decreases for all the values of the Schmidt number S_c, which quantitatively agrees with the expectations since the magnetic field exerts a retarding force on the flow. In Fig. 2 the velocity profiles are shown for constant G_c and M and for different values of S_c, G_r, and E. It is known that the Eckert number E may be interpreted as the addition of heat due to viscous dissipation while the Grashof number G_r as the addition of heat due to free-convection currents. Thus the case when $(T'_w - T'_\infty) > 0$ or $G > 0$ with $E > 0$ corresponds to the external cooling of the wall, while the case when $(T'_w - T'_\infty) < 0$ or $G < 0$ with $E < 0$ corresponds to the external heating of the wall. From Fig. 2 we observe that, in the case $G < 0$ with $E < 0$, for large values of Schmidt number S_c, the velocity is negative and decreases as E increases. Thus the

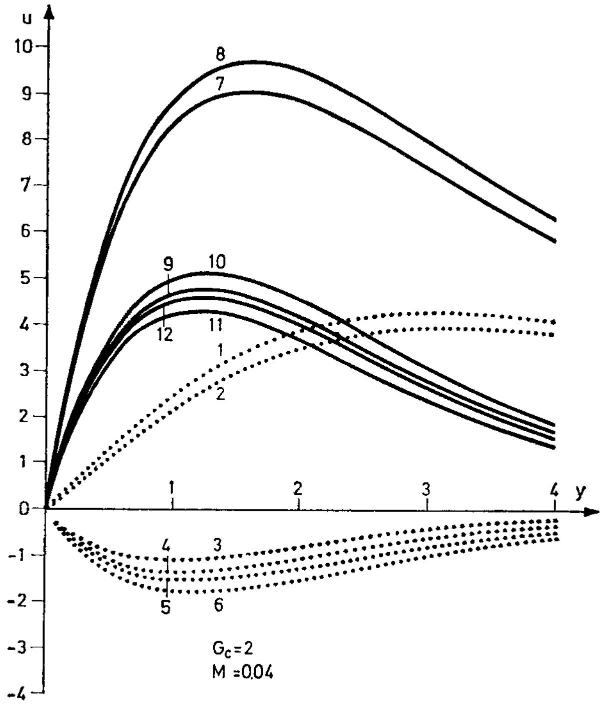

Fig. 2. The velocity profiles u for $P = 0.71$

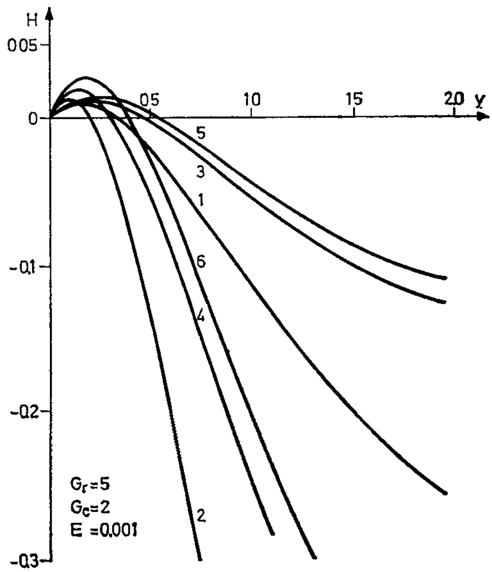

Fig. 3. The induced magnetic field H for $P = 0.71$

velocity profile is of reversed type in the case of the water vapour ($S_c = 0.60$) and oxygen ($S_c = 0.75$). Finally from this Figure we see that, in the case $G > 0$ with $E > 0$, the velocity is positive and increases as E increases for all values of S_c.

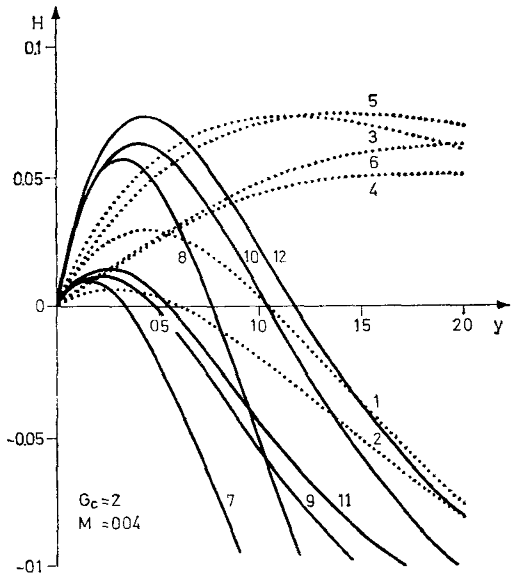

Fig. 4. **The induced magnetic field H for $P = 0.71$**

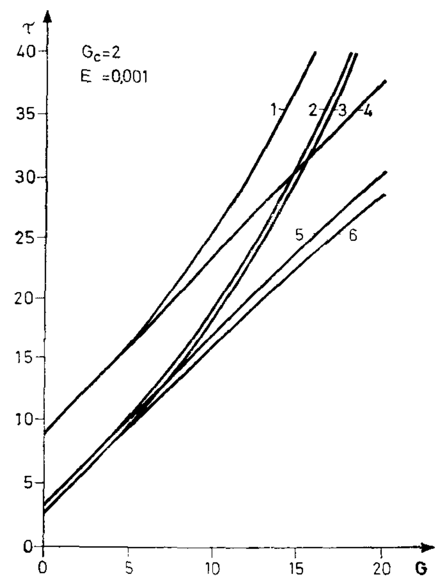

Fig. 5. **The variations of the skin friction τ for $P = 0.71$**

The variations of the induced magnetic field H are shown in Fig. 3 for different values of S_c and M. From this Figure we see that the induced magnetic field gets positive values near to the wall, while far from the wall it gets negative values, and this means that there is a reverse of the induced magnetic

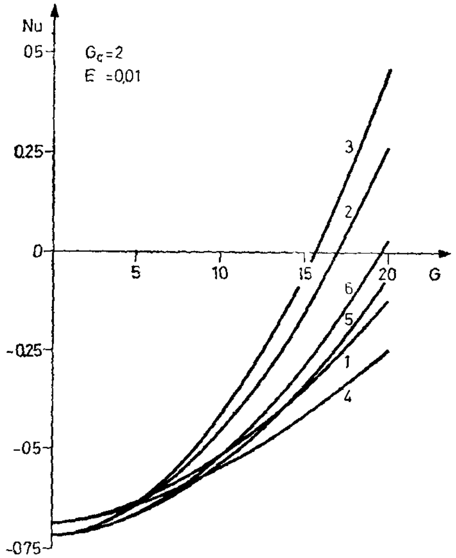

Fig. 6. The variations of the Nusselt number Nu for $P = 0.71$

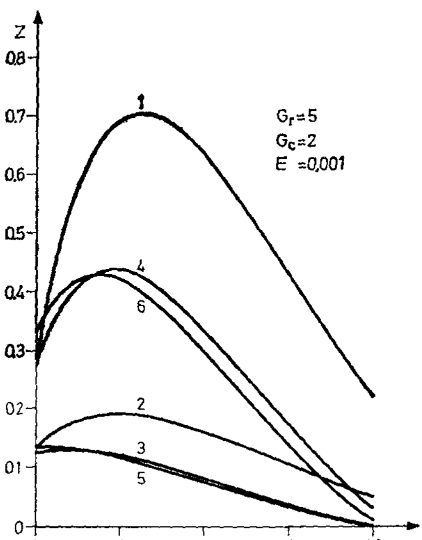

Fig. 7. The electric current density Z for $P = 0.71$

field. Also we remark that as M increases the induced magnetic field also increases for all values of the Schmidt number. In Fig. 4 the variations of induced magnetic field H are shown with y for different values of S_c and E. We observe that, in the case $G < 0$ with $E < 0$, the induced magnetic field

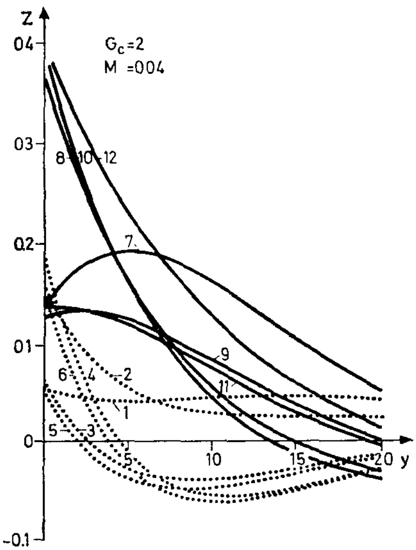

Fig. 8. The electric current density Z for $P = 0.71$

decreases as the Eckert number E increases, while in the case $G > 0$ with $E > 0$ as E increases the induced magnetic field also increases for all values of Schmidt number S_c.

The numerical values of temperature θ, calculated from expression (29) are given in Table I. We remark that as magnetic parameter M increases, the temperature decreases for all values of S_c. Also, from this Table we see that the temperature increases as E increases.

The skin friction τ is plotted against G in Fig. 5 for different values of M and S_c. We see that, for all values of Schmidt number S_c, as M increases the skin friction decreases. Thus the presence of the magnetic field helps in reducing the frictional drag on the wall.

The Fig. 6 displays the variation of the Nusselt number which represent the local dimensionless coefficient of heat transfer. We see that an increase in the strength of the magnetic field causes the Nusselt number to decrease.

The variation of the electric current density Z is shown in Figs. 7 and 8. From the Fig. 7 we see that as magnetic parameter M increases the

Table I

The variation of the temperature θ profiles for $P = 0.71$

	$G_r = 5$		$G_c = 2$		$E = 0.001$	
	$S_c = 0.22$		$S_c = 0.60$		$S_c = 0.75$	
y	$M = 0.04$	$M = 0.15$	$M = 0.04$	$M = 0.15$	$M = 0.04$	$M = 0.15$
0.0	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
0.2	0.769644	0.767742	0.768657	0.763513	0.769126	0.763703
0.4	0.587249	0.584201	0.587289	0.580236	0.587218	0.580162
0.6	0.447354	0.443656	0.447592	0.440307	0.446990	0.439841
0.8	0.341264	0.337269	0.340715	0.334003	0.339324	0.333193
1.0	0.260985	0.256952	0.259137	0.253331	0.257399	0.252294

	$G_r = -5$		$G_c = 2$		$M = 0.04$	
	$S_c = 0.22$		$S_c = 0.60$		$S_c = 0.75$	
y	$E = -0.001$	$E = -0.003$	$E = -0.001$	$E = -0.003$	$E = -0.001$	$E = -0.003$
0.0	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
0.2	0.747046	0.735604	0.740685	0.716521	0.739263	0.712257
0.4	0.557748	0.539929	0.550220	0.517345	0.549234	0.514386
0.6	0.416179	0.395414	0.409617	0.375730	0.409417	0.375128
0.8	0.310326	0.288775	0.305472	0.274214	0.305904	0.275510
1.0	0.231076	0.210099	0.228137	0.200984	0.228948	0.203416

	$G_r = 5$		$G_c = 2$		$M = 0.14$	
	$S_c = 0.22$		$S_c = 0.60$		$S_c = 0.75$	
y	$E = 0.001$	$E = 0.003$	$E = 0.001$	$E = 0.003$	$E = 0.001$	$E = 0.003$
0.0	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
0.2	0.769644	0.803400	0.768657	0.800437	0.769126	0.801846
0.4	0.587249	0.628433	0.587289	0.628553	0.587218	0.628338
0.6	0.447354	0.488940	0.447593	0.489658	0.446790	0.487247
0.8	0.341264	0.381591	0.340715	0.379942	0.339324	0.375771
1.0	0.260985	0.299528	0.259137	0.293984	0.257399	0.288769

electric current also increases for all values of S_c. Finally from Fig. 8 we see that the electric current density increases as the Eckert number E increases.

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