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A SIMPLE EXTENSION OF THE ANALOGUE MODEL TO THE CASE OF SCALAR CURRENTS*

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An off-mass-shell continuation of the dual multiparticle amplitude is constructed in the analogue model for scalar particles. It is invariant under the *SL* (2; *R*) group, factorizable in the direct channel and the hadronic form-factor has a reasonable asymptotic behaviour; analytic properties, however, are not satisfactory. In the scaling limit the two-current amplitude shows an automatic cut-off in the transversal momentum, which is proportional to the momentum transfer (not fixed), so that an anomalous dimension appears in the model.

Introduction

SUSSKIND quite recently proposed a simple model for meson—meson interactions [1], in which the mesons are considered to be bound scalar $q\bar{q}$ pairs. During the interaction momentum is transferred only to these "valence quarks" and not to the exchanged quanta. From this assumption he was able to get the Veneziano N-point function.

NIELSEN and SUSSKIND have emphasized that in the meson—current interaction the current can be coupled directly to the exchanged quanta [2]. This assumption seems to have experimental support in high-energy lepton-hadron processes. The parton picture offers a relatively good description of these processes and one can identify the exchanged quanta with partons.

Starting from this idea the above authors constructed hadronic form-factors with convenient properties in the $q^2 \sim 0$ region. (q^2 is the mass of the scalar current.) With further assumptions they succeeded in giving a reasonable model for deep inelastic electron—proton scattering as well.

The model contains certain arbitrariness concerning the cut-off to be applied in divergences appearing in the form-factor. The choice of the cut-off influences the behaviour of the form-factor considerably.

SUSSKIND has shown that the original harmonic oscillator model is equivalent to the analogue model of NIELSEN [3]. It would be of interest to know whether this equivalence can be maintained for the current amplitudes also and whether the problem of divergences appears in the analogue model.

* Dedicated to Prof. L. JÁNOSSY on his 60th birthday.

In Section 2 the extension of the analogue model to the current amplitude is rephrased. The scalar-current form-factor for mesons will be constructed without cut-off and a two-current amplitude derived. From the factorization of the amplitude in the s-channel we can deduce excitation form-factors.

The dual and asymptotic properties of the two-current amplitude are briefly discussed in Section 3 and finally in Section 4 the problem of the connection between the scaling and duality is investigated by means of a simple analogue model.

The extension of the analogue model

In the analogue model the equivalent picture of the world-sheet of SUSSKIND is a conducting plate of the same dimensions $-\infty \geq \lambda \leq +\infty$; $0 \leq \theta \leq \pi$ and uniform conductivity σ (see Fig. 1). If the meson is free, a

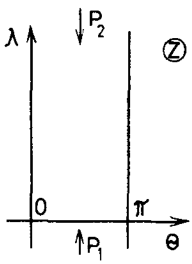

Fig. 1

four-momentum current flows in the sheet from a momentum source (p_1) at the point $\lambda = -\infty$ to source (p_2) at $\lambda = \infty$. Because of four-momentum conservation we have $p_1 = -p_2$. The current distribution ($p(\theta; \lambda)$) can be evaluated by solving the problem of the D. C. generated by the source distribution given above with the subsidiary condition $\int_0^\pi p(\Theta; \lambda) d\theta = p_1$. For uniform conductivity this gives a uniform current distribution independent of θ.

The amplitude for the interaction of an arbitrary number of particles and currents is given by NIELSEN's formula:

$$A = \frac{\int d(\text{conf.}) \exp\{-\sigma \int j^2 \, df\}}{\int d(\text{conf.})} . \tag{1}$$

Here $j(\theta; \lambda)$ describes the current distribution; $\sigma \int j^2 df$ is the total heat generation in the plate ($\delta^{(4)}(\Sigma p_i)$ is separated from A); and $\int d$ (conf.) denotes summation over all possible couplings of external momenta to the meson. We know [2] that the coupling of the on-mass-shell momenta is restricted to the boundary of the plate ($\theta = 0; \pi$), whereas the currents can couple inside the sheet, too. Two further rules must be stated for constructing the amplitude:

a) Requiring the amplitude to be 1 in the case of a free hadron, one has to subtract the heat generation in the free hadron from $\sigma\int j^2 df$, so we have

$$A = \int d(\text{conf}) \exp\{-\sigma \int (j^2 - j_0^2)\, df\} / \int d(\text{conf}). \tag{1a}$$

One can see that this "0-point" heat is

$$H_0 = \lim_{|w_1| \to \infty} 2\sigma p^2 \ln |w_1| + \text{const.}, \tag{2}$$

where $w_1 = e^{iz_1}$; $z_1 = \theta_1 + i\lambda_1$ is the position of the source associated with the ingoing momenta p_1.

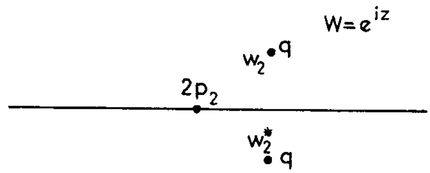

Fig. 2

b) The second remark concerns the requirement of λ-translation invariance, which is suggested by the proper-time interpretation of λ [1]. This is the partial consequence of the invariance of our amplitude under the transformation

$$w' = \frac{aw + b}{cw + d} \tag{3}$$

(a; b; c; d are real parameters with $|ab - cd| = 1$; $w = e^{iz}$, $z = \theta + i\lambda$). This property will be discussed in detail in Section 3. Because of the invariance one can fix the coupling of one of the currents at $\lambda = 0$.

With these two rules in mind let us construct the hadronic form factor (the meson—current—meson coupling).

We map the sheet into the upper half-plane by means of the function $w = e^{iz}$. In order to satisfy the boundary condition the half-plane will be reflected with respect to the real axis, without changing the signs of the sources (Fig. 2). The heat generated in the sheet can be evaluated from Ohm's law:

$$\begin{aligned}
H_{p_2 q} &= -\sigma p_2 q\, (\ln |w_q| + \ln |w_q^*|), \\
H_{qq} &= -\sigma q^2 \ln |w_q - w_q^*|, \\
H_{p_1 p_2} &= -\lim_{|w_1| \to \infty} 2\sigma p_1 p_2 \ln |w_1|, \\
H_{p_1 q} &= -\lim_{|w_1| \to \infty} \sigma p_1 q (\ln |w_1 - w_q| + \ln |w_1 - w_q^*|).
\end{aligned} \tag{4}$$

Expanding the sum $H_{p_1 p_2} + H_{p_1 q}$ in series at the point $w_1 = \infty$, one gets

$$H_{p_1 p_2} + H_{p_1 q} = -\lim_{|w_1| \to \infty} \sigma \left[2p_1(p_1+q) \ln |w_1| + \frac{1}{|w_1|} (\dots) \right]. \tag{5}$$

Taking into account four-momentum conservation, it can be seen immediately that (5) tends to (2), so $H_{p_1 p_2} + H_{p_1 q}$ does not appear in (1a), as a consequence of our first rule. By the second rule:

$$|w_q| = 1, \text{ that is } H_{p_2 q} = 0.$$

We shall now use the so-called scale invariance property of the heat generation [3], which means that the expressions given by (4) are determined only up to an additive constant. In order to ensure agreement with the NIELSEN—SUSSKIND form-factor, this constant is required to be $-\ln \pi/\lambda_0$ in H_{qq} and 0 elsewhere (λ_0 is defined in [2]).

$$H_{qq} = -\sigma q^2 \ln \left(\frac{\lambda_0}{\pi} |w_q - w_q^*| \right), \tag{4a}$$

Substituting (4a) into (1a) one gets

$$F_\theta(q^2) = \left(\frac{2\lambda_0}{\sigma} \sin \Theta \right)^{\sigma q^2}. \tag{6}$$

One can now integrate over all possible values of θ, with a normalization factor in the integrand (omitting the denominator of (1a)):

$$F(q^2) = \int_0^\pi \frac{d\Theta}{N(\Theta)} \left(\frac{2\lambda_0}{\pi} \right)^{\sigma q^2} (\sin \Theta)^{\sigma q^2}. \tag{7}$$

If we choose $N(\Theta) = \sin^2 \Theta$ and $\sigma = 1$, we get the form-factor of [2] without using any cut-off. The choice of the form of $N(\theta)$ is determined by the interaction. In [2] $N(\theta) = \sin^2 \theta$ for charge-symmetric interactions, $N(\theta) = \sin^{1/2}\theta$ for electromagnetic interactions. In the following Section it will be shown that further restrictions appear if invariance of the volume element under real Moebius transformations is required. In order to investigate the excitation form-factors the two-current amplitude (Fig. 3) will be also discussed.

According to the first rule, the contribution of the p_1-source can be omitted. The other contributions are as follows (the procedure is the same as before):

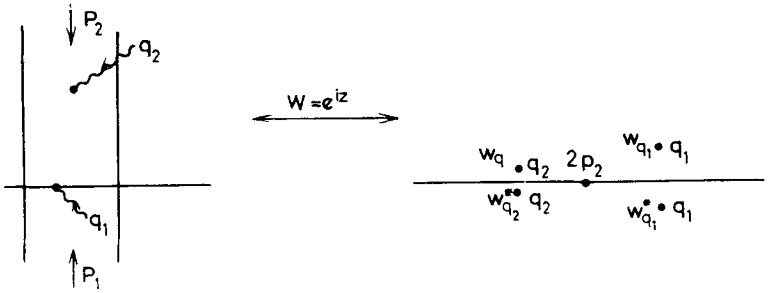

Fig. 3

$$H_{p_1 q_1} = 0 \; ; \; H_{q_1 q_1} = -\sigma q_1^2 \ln \left(\frac{\lambda_0}{\pi} |w_{q_1} - w^*_{q_1}| \right) ,$$

$$H_{p_2 q_2} = - \sigma p_2 q_1 \ln |w_{q_2}|^2 ; H_{q_2 q_2} = - \sigma q_2^2 \ln \left(\frac{\lambda_0}{\pi} |w_{q_2} - w^*_{q_2}| \right) , \tag{8}$$

$$H_{q_1 q_2} = - \sigma q_1 q_2 [\ln |w_{q_1} - w_{q_2}| + \ln |w_{q_1} - w^*_{q_2}|] ,$$

The amplitude is

$$A = \int_0^\pi \frac{d\Theta_1}{N(\Theta_1)} \int_0^\pi \frac{d\Theta_2}{N(\Theta_2)} \int_0^1 \frac{du}{u} u^{\sigma(2p_2q_2+q_2^2)} |e^{i\Theta_1} - e^{i\Theta_2} u|^{\sigma q_1 q_2} |e^{i\Theta_1} - u e^{-i\Theta_2}|^{\sigma q_1 q_2} \cdot$$
$$\cdot (\sin \Theta_1)^{\sigma q_1^2} (\sin \Theta_2)^{\sigma q_2^2} \left(\frac{2\lambda_0}{\pi} \right)^{\sigma(q_1^2 + q_2^2)} , \tag{9}$$

(the new variable $u = e^{-\lambda q_2}$ has been introduced). The same amplitude can be derived (see Appendix) by the prescriptions of [1].

Now let us study the poles of (9) in the variable s. These poles occur from the region $u \sim 0$ and their residues are proportional to polynomials in $q_1 \cdot q_2$. The leading term describes the exchange of the pole on the leading trajectory and for the $s - c = J$ ($J = 0; 1; \ldots$) pole is:

$$\mathrm{Res}_J A = \int \frac{d\Theta_1}{N(\Theta_1)} \int \frac{d\Theta_2}{N(\Theta_2)} (\sin \Theta_1)^{\sigma q_1^2} (\sin \Theta_2)^{\sigma q_2^2} \left(\frac{2\lambda_0}{\pi} \right)^{\sigma(q_1^2 + q_2^2)} \cdot$$
$$\cdot (2 q_1 q_2)^J (\cos \Theta_1 \cos \Theta_2)^J \frac{1}{J!} . \tag{10}$$

By factorization of (10) one gets the excitation form-factors shown in [2]:

$$F^J_{\mu_1 \ldots \mu_J}(q^2) = (\sqrt{2}\sigma)^J q_{\mu_1} \cdot \cdot q_{\mu_J} \int \frac{d\Theta}{N(\Theta)} (\sin \Theta)^{\sigma q^2} (\cos \Theta)^J \left(\frac{2\lambda_0}{\pi} \right)^{\sigma q_1} \frac{1}{\sqrt{J!}} . \tag{11}$$

Properties of the two-current amplitude

In this Section the properties of the amplitude (9) are briefly summarized.

Invariance of the intergrand under Sl(2; R) transformation [3].

Up to now a particular mapping of the $(\Theta; \lambda)$ sheet into the upper half-plane was used for the computation of the amplitude. In the case of an arbitrary mapping realizable by

$$w = \frac{ae^{iz}+b}{ce^{iz}+d}$$

the integrand in (1) becomes $(\sigma = 1)$

$$|w_1 - w_{q1}|^{2p_1q_1} |w_1 - w_{q_2}|^{2p_1q_2} [|w_{q_1} - w_{q_2}| \, |w^*_{q_1} - w_{q_2}|]^{q_1q_2} |w_{q_1} - w^*_{q\,1}|^{q_1^2} \cdot \\ \cdot |w_{q_2} - w^*_q|^{q_2^2} |w_2 - w_{q_1}|^{2p_2q_1} |w_2 - w_{q_2}|^{2p_2q_2} |w_2 - w_1|^{2p_2q_1}. \tag{12}$$

In order to ensure invariance of the amplitude under this change of the configuration of sources, that is under the $SL(2; R)$ transformations of the upper half-plane we have to multiply (12) by a factor $| w_2 - w_1 |^{2p_1^2}$. But from (2) it can be seen that this is precisely the factor by which (1a) differs from (1). Therefore the meaning of the formal manipulation to assure $SL(2; R)$-invariance is similar to leaving the 0-point energy in the case of harmonic systems to ensure that the results are finite ($p_1^2 = p_2^2$ was assumed).

The invariance of the volume element

$$dV_N^M = \prod_{i=1}^{N} \frac{|dw_i|}{|w_{i+1} - w_i|} \, 2^M \prod_{k=1}^{M} \frac{d^2 w_k}{|w_k - w_k^*|^2} \tag{13}$$

can be demonstrated by direct evaluation (N is the number of particles on-mass-shell, M is the number of currents). The $SL(2; R)$ being a three-parameter group we can fix the positions of the two on-shell particles and partially that of one of the currents ($| w_3 | = 1$), then one arrives at the same volume element as in (9) by putting $N = 2$, $M = 2$; $N(\Theta) = \sin^2\theta$.

Poles on the q_1^2 and q_2^2 planes

These poles arise from the region where θ_1 and θ_2 are approximately 0 or π and can be exhibited by integrating by parts at $\theta_i = 0; \pi$; they occur at $q_i^2 = 1; -1; \ldots$; if we use $N(\theta) = \sin^2\theta$. The residue of the two-fold pole at $q_1^2 = q_2^2 = 1$ coming from the region $\theta_1 = \theta_2 = 0$ or π is the dual amplitude

$$\int_0^1 du u^{-s-2} (1-u)^{-t-2}; \text{ for } N(\Theta) = \sin^2\Theta.$$

The contribution of the regions $\theta_1 = \theta_2 \pm \pi$ and $\theta_1 = 0;\pi$ can be identified with the parts of the amplitude which correspond to the $(s;u)$ and $(t; u)$ duality [1].

Poles on the s-plane

These poles arise from the $u \sim 0$ region at $s - c = J, J = 0; 1;\ldots,$ appearing independently of the actual value of θ.

Factorization in the s-channel

This problem has been investigated in Section 2. The residue contains the product of the two form-factors and a polynomial of $q_1 q_2$:

$$A = \sum_{J=1}^{\infty} \frac{F^J(q_1^2)\, F^J(q_2^2)}{J-(s-c)} (q_1 q_2)^J + \text{daughter poles}, \tag{14}$$

where

$$F^J(q^2) = (\sqrt{2})^J \int \frac{d\Theta}{N(\Theta)} (\sin\Theta)^{q_2} (\cos\Theta)^J \left(\frac{2\lambda_0}{\pi}\right)^{q^2} \frac{1}{\sqrt{\Gamma(J+1)}} .$$

Asymptotic behaviour in s

The main contribution comes from the region $u \sim 1$. The asymptotics depends strongly on the actual value of θ. If the currents couple on the edge of the interaction region, one obtains the dual part of the amplitude. But from the region $\theta_1 = \theta_2$, different rom 0; π, one would have a trajectory of a slope, which is half of that appearing in the dual part of the amplitude:

$$\lim_{s\to\infty} A_{\Theta_1=\Theta_2\neq 0;\, \pi} \sim \int du \int \frac{d\Theta}{N(\Theta)} (2p_2 q_2 + q_2^2)^{q_1 q_2} \, |1 - e^{-2i\Theta}|^{q_1 q_2} f(u). \tag{15}$$

Here $f(u)$ denotes the u-dependent part of the integrand. The part of the amplitude which comes from the region $0 < \theta_1 \neq \theta_2 < \pi$, has s-independent asymptotics.

The amplitude does not remain dual if we perform an analytic continuation from the mass-shell. It has inconvenient analytic properties in the t-channel due to the factor $|w_{q_1} - w_{q_2}|^{q_1 q_2}$. (The position of the t-channel poles depends on the actual value of q_i^2). This factor cannot be removed without fundamentally changing the physical content of the model.

It can be concluded that the amplitude has reasonable properties in the direct channel, but one is faced with having to exclude the $q_1 q_2$-poles in the crossed channel. Nevertheless the generality of the method presented here may be of interest for further investigations.

Scaling in the analogue model

NIELSEN and SUSSKIND imposed the scaling property on their model by considering the contributions of only those diagrams in which the two currents couple to the same exchanged quantum [2]. This assumption has an equivalent picture in the analogue model, which can be related directly to the parton model proposed by BJORKEN and PASCHOS [4]. If we identify $N^{-1}(\theta)$ with the density of partons [2] (this factor earlier played the role of the weighting factor in the summation of the contributions of different configurations), the longitudinal momentum carried by a parton at a given θ is $pN(\theta)/\pi$. The transversal part of the parton's momentum is neglected. Choosing

$$\sigma = \sigma_0 \delta(\Theta - \Theta q_1) \tag{16}$$

(for the q-current only), we restrict the coupling of both currents to the same parton.

The heat generation is given by

$$\begin{aligned} H_{qq} &= q^2 \int_0^\lambda d\Theta\, d\lambda' \sigma_0\, \delta(\Theta - \Theta_1) = q^2 \lambda \sigma_0, \\ H_{pq} &= \frac{2p \cdot N(\Theta_1) q}{\pi} \int_0^\lambda d\Theta\, d\lambda'\, \sigma_0\, \delta(\Theta - \Theta_1) = 2\, pq\, \lambda \sigma_0 \frac{N(\Theta_1)}{\pi}\,. \end{aligned} \tag{17}$$

From Eq. (1a) the following amplitude can be evaluated:

$$\begin{aligned} A &= \int_0^\infty \frac{d\Theta}{N(\Theta)} \int_0^\infty d\lambda \exp\left\{ - \left[q^2 + \frac{2pq}{\pi} N(\Theta) \right] \sigma_0 \lambda \right\} = \\ &= \int_0^\infty \frac{d\Theta}{N(\Theta)} \frac{1}{\sigma_0 \left(q^2 + \frac{2pq}{\pi} N(\Theta) \right)}\,. \end{aligned} \tag{18}$$

The imaginary part of (18) is

$$\frac{1}{\pi} Im\, A = \int_0^\pi \frac{d\Theta}{N(\Theta)\sigma_0} \delta\left(q^2 + \frac{2p \cdot q}{\pi} N(\Theta) \right) = \frac{1}{2\nu} \int_0^\pi \frac{d\Theta}{N(\Theta)\, \sigma_0} \delta\left(x - \frac{N(\Theta)}{\pi} \right) \tag{19}$$

and gives the scaling law of ImA. In order to have a definite scaling function, we put $N(\theta) = \pi \sin^2 \theta$

$$\frac{1}{\pi} \nu\, Im\, A = \frac{1}{\sigma_0\, \pi x^{3/2} \sqrt{1 - x^2}}\,, \tag{20}$$

where

$$x = \frac{q^2}{2\nu}\,; \quad \nu = -pq\,.$$

It is natural to try now to relate amplitude (9) in the BJORKEN limit [4] to the above picture. The aim will be to demonstrate that in the scaling limit the main contribution to the amplitude comes from the $\theta_1 = \theta_2$ region and that the amplitude shows the scaling property. We use (9) in the form given by Eq. (A3) with cut-off sums like (A4). (A3) has only technical advantages over (9); as we have seen, the results given by the two are identical. For the forward-scattering ($q_1 = -q_2$), we replace (A3) by

$$A = \int_0^\pi \frac{d\Theta_1}{N(\Theta_1)} \int_0^\pi \frac{d\Theta_2}{N(\Theta_2)} \int_0^1 dX\, X^{-s+c-1}\, e^{-q^2 \sum_k' \frac{\cos^2 k\Theta_1}{k} - q^2 \sum_k' \frac{\cos^2 k\Theta_2}{k}} \; e^{2q^2 \sum_k' \frac{\cos k\Theta_1 \cos k\Theta_2}{k} X^k} \tag{21}$$

Let us consider first the $s \to \infty$ limit only (Regge limit). The main contribution comes from the region $X \sim 1$. We introduce therefore the new variable $X = 1 - \frac{u}{s}$ and expand X^k in Newton binomial, which yields

$$A = \int \frac{d\Theta_1}{N(\Theta_1)} \int \frac{d\Theta_2}{N(\Theta_2)} \int \frac{du}{s}\, e^u\, e^{-q^2 \sum_k' \frac{1}{k} (\cos k\Theta_1 - \cos k\Theta_2)^2} \times \\ \times e^{2q^2 \sum_k' \frac{\cos k\Theta_1 \cos k\Theta_2}{k} \cdot \sum_{l=7}^{k}{}' \binom{k}{l} \left(-\frac{u}{s}\right)^l} \tag{22}$$

Let us consider next the $q^2 \to \infty$ limit. The first exponent tends to 0 if $\theta_1 \neq \theta_2$, so expanding $\cos k\, \theta_2$ about $\theta_2 = \theta_1$, retaining only the first nonvanishing term, we acquire:

$$A = \int \frac{d\Theta_1}{N(\Theta_1)} \int \frac{d\Theta_2}{N(\Theta_2)} \int \frac{du}{s}\, e^{-q^2 \sum_k^{k_0}{}' k \sin^2 k\Theta_1 (\Theta_1 - \Theta_2)^2} \cdot \\ \cdot e^u \cdot e^{\left[2q^2 \sum_k^{k_0}{}' \cos^2 k\Theta_1 - k \cos k\Theta_1 \sin k\Theta_1 (\Theta_2 - \Theta_1)/k \cdot \sum_{l=1}^{k}{}' \binom{k}{l} \left(-\frac{u}{s}\right)^l\right]}.$$

Using the definition

$$\lim_{q^2 \to \infty} e^{-q^2 \sum_k^{k_0}{}' k \sin^2 k\Theta_1 (\Theta_2 - \Theta_1)^2} \sqrt{q^2 \sum_k k \sin^2 k\Theta_1}\, \frac{1}{\sqrt{\pi}} = \delta(\Theta_2 - \Theta_1) \tag{23}$$

for the δ-function, we have

$$\lim_{\substack{q^2\to\infty \\ s\to\infty}} \int \frac{d\Theta_1}{N(\Theta_1)} \int \frac{d\Theta_2}{N(\Theta_2)} \int \frac{du}{s} \frac{\sqrt{\pi}}{\sqrt{q^2 \sum_k k \sin^2 k\,\Theta_1}} \delta(\Theta_2-\Theta_1)\, e^u \cdot$$
$$\cdot \exp\left\{2q^2 \sum_k \frac{\cos^2 k\Theta_1}{k} \sum_{l=1}^{k} \binom{k}{l} \left(-\frac{u}{s}\right)^l\right\} \tag{24}$$

Finally, we keep $q^2/s = \bar{x}$ fixed. In this case only one term remains finite in the second exponent:

$$\lim_B A = \frac{1}{s\sqrt{q^2}} \int \frac{d\Theta_1}{N^2(\Theta_1)} \int du \times$$
$$\times \sqrt{\frac{\pi}{\sum_k k \sin^2 k\Theta_1}} \cdot \exp\left\{\left(1 - 2\bar{x} \sum_k^{k_0} \cos^2 k\,\Theta_1\right) u\right\}. \tag{25}$$

Taking the u integration around $u \sim 0$:

$$\lim_B A = \frac{1}{s\sqrt{q^2}} \int \frac{d\Theta_1}{N^2(\Theta_1)} \sqrt{\frac{\pi}{\sum_k k \sin^2 k\Theta_1}} \frac{1}{1-2\bar{x}\sum_k \cos^2 k\Theta_1}. \tag{26}$$

After summing the divergent sums the θ_1-integration for the imaginary part can be made by means of the δ-functions

$$\lim_B s\sqrt{q^2}\, Im\, A\, \frac{1}{\pi} = \sqrt{\pi} \int \frac{d\Theta}{N(\Theta_1)} \frac{1}{\sqrt{\sum_k k \sin^2 k\Theta_1}}\, \delta\left(1 - 2\bar{x} \sum_k^{k_0} \cos^2 k\Theta_1\right). \tag{27}$$

The above demonstration confirms the assumption about the important region of θ-integration made in Eq. (16), but the power of ν multiplying ImA in the BJORKEN limit changes and its value is characteristic for the case of so-called anomalous dimensions, proposed in some field theoretical models of scaling. The physical basis for this change may be the different way of cutting-off the transversal momentum. In the naive analogue picture (Eq. (16)), there is no transversal part, but (as it can be seen from Eq. (23)) in the amplitude (9) the contributing region is proportional to $(\sqrt{q^2})^{-1}$.

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I would like to thank DR. I. MONTVAY for his helpful discussions and encouragement.

Appendix

The two-current amplitude is derived by applying the prescription of [1].

The contribution from a given configuration $(\Theta_1; \Theta_2)$ is

$$T_{\Theta_1;\Theta_2} = \sum_{\{n_\mu^k\}} \langle 0|T(q_1;\Theta_1)|\{n_\mu^k\}\rangle \frac{1}{s - \sum kn_\mu^k - c} \langle\{n_\mu^k\}|T(q_2;\Theta_2)|0\rangle, \tag{A1}$$

where $|0\rangle$ denotes the ground state of the meson, $|\{n_\mu^k\}\rangle$ denotes an excited state in the occupation number representation, and $T(q_1;\theta_1) = e^{iq_1 x(\Theta_1)}$ is the scalar vertex-operator with the second quantized four-position $X_\mu(\Theta;\lambda)$ given in [2]. If (A1) is rewritten in the coherent state basis and

$$T_{\Theta_1\Theta_2} = \sum_{\{n_\mu^k\}} e^{-q_1^2 \sum_k \frac{\cos^2 k\Theta_1}{k} - \sqrt{2}\sum_k \frac{\cos k\Theta_1}{\sqrt{k}}\alpha(k)\cdot q_1} \left(\prod_{\{k;\mu\}} \frac{\overleftarrow{\partial}}{\partial\alpha_\mu}\frac{\overrightarrow{\partial}}{\partial\beta_\mu}\right)^{n_\mu^k} \frac{1}{n_\mu^k!}\cdot \tag{A2}$$

$$\cdot\, e^{-q_1^2 \sum_k \frac{\cos^2 k\Theta_2}{k} + \sqrt{2}\sum_k \cos k\Theta_2 \beta(k)\cdot q_2} \left.\frac{1}{s - \sum_{\mu;k} kn_\mu^k - c}\right|_{\alpha_\mu=\beta_\mu=0}.$$

Using the identity

$$\int_0^1 dX X^{c-s-1+\sum kn_\mu^k} = \frac{1}{s - c - \sum_k kn_\mu^k}$$

we get

$$T_{\Theta_1;\Theta_2} = \int_0^1 dX \sum_{n_\mu^k} \prod_{\mu k} \frac{1}{n_\mu^k!}\left(-\frac{2}{k} q_{1\mu} q_{2\mu} \cos k\Theta_1 \cos k\Theta_2\right)^{n_\mu^k} X^{c-s-1+kn_\mu^k}\cdot$$

$$\cdot\, e^{-q_1^2 \sum_k^{\alpha} \frac{\cos^2 k\Theta_1}{k}}\, e^{-q_2^2 \sum_k \frac{\cos^2 k\Theta_2}{k}} = \tag{A3}$$

$$= \int_0^1 dX X^{c-s-1} \exp\left\{-q_1^2 \sum_k \frac{\cos^2 k\Theta_1}{k} - q_2^2 \sum_k \frac{\cos^2 k\Theta_2}{k} - \right.$$

$$\left. - 2q_1 q_2 \sum_k \cos k_1\Theta_1 \cos k\Theta_2 \frac{X^k}{k}\right\}.$$

As the exponent in the first exponential is divergent, it is necessary to introduce a cut-off. From [1] we have:

$$\sum_k^{k_0} \frac{\cos^2 k\Theta}{k} \approx -\frac{1}{2}\log\frac{2\lambda_0}{\pi} - \frac{1}{2}\log\sin\Theta. \tag{A4}$$

The third exponent can be evaluated without any cut-off:

$$\sum_k \cos k\Theta_1 \cos k\Theta_2 \frac{X^k}{k} = \frac{1}{2}\left[\ln|1 - Xe^{i(\Theta_1+\Theta_2)}| + h_1|1 - Xe^{i(\Theta_1-\Theta_2)}|\right]. \quad (A5)$$

Substituting (A4) and (A5) into (A3) $T_{\Theta_1\Theta_2}$ is obtained in its final form as:

$$T_{\Theta_2\Theta_1} = \int_0^1 dX X^{c-s-1} \left(\frac{2\lambda_0}{\pi}\right)^{q_1^2+q_2^2} (\sin\Theta_1)^{q_1^2} (\sin\Theta_2)^{q_2^2} \cdot \cdot\, 1 - Xe^{i(\Theta_1+\Theta_2)}|^{q_1q_2}\, |1 - Xe^{i(\Theta_1-\Theta_2)}|^{q_1q_2}. \quad (A6)$$

The amplitude comes from (A6) by integrating over Θ_1 and and agrees with that given by (9) for $\sigma = 1$:

$$A = \int_0^\pi \frac{d\Theta_1}{N(\Theta_1)} \int_0^\pi \frac{d\Theta_2}{N(\Theta_2)} \int_0^1 dX X^{c-s-1} \left(\frac{2\lambda_0}{\pi}\right)^{q_1^2+q_2^2} (\sin\Theta_1)^{q_1^2} (\sin\Theta_2)^{q_2^2} \cdot \cdot\, |1 - Xe^{i(\Theta_1-\Theta_2)}|^{q_1q_2}\, |1 - Xe^{i(\Theta_1+\Theta_2)}|^{q_1q_2}. \quad (A7)$$

The factorization in s is also clear from here, and the excitation form-factors are the same as in [1].

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ПРОСТОЕ РАСПРОСТРАНЕНИЕ АНАЛОГОВОЙ МОДЕЛИ НА СЛУЧАЙ СКАЛЯРНЫХ ПОТОКОВ

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Резюме

Продолжение дуальной многочастичной амплитуды вне оболочки массы было построено в аналоговой модели скалярных частиц. Оно инвариантно при $SL(2; R)$ группе, факторизируется в прямом канале, а гадронический формирующий фактор показывает обоснованное асимптотическое поведение. Все же его аналитические свойства не являются удовлетворительными. Дуальная амплитуда в скалярном пределе показывает автоматический перерыв в поперечном моменте, который пропорционален переноса момента (не является константой). Поэтому в модели возникает аномальное измерение.