PRODUCTION OF HEAVY UNSTABLE PARTICLES IN EXTREMELY ENERGETIC NUCLEON-NUCLEON COLLISIONS

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The problem of multiple production of heavy unstable particles is analyzed. Possible reaction schemes are established on the basis of Gell-Mann's theory. In order to apply the Fermi-Landau thermodynamical model to the production of heavy unstable particles, their interaction with pions is investigated. Some characteristic quantities of the two-proton system are calculated by making use of the aforesaid model. The results seem not to contradict experimental results.

1. Introduction

In recent years several publications have been appearing on the production of heavy unstable particles in extremely energetic nuclear collisions. Although the statistics of experimental data is rather poor, and there are already some theoretical papers on the subject[1] we hope it may not be superfluous to carry out some investigations concerning the production mechanism of these particles.

Choosing a simple model, we shall deal with central, totally inelastic collisions of two nucleons at very high energies. (For the justification of this model, see the work of FEINBERG and CHERNAVSKI [1]). We assume further, that the nucleon has a pionic proper field, the linear dimensions of which are $\sim 1/\mu$ (μ stands for the mass of pions ; in the course of the present paper we put $\hbar = c = 1$) and similarly a K-mesonic proper field with linear dimensions $\sim 1/m$ (m = mass of K-mesons). During a collision the two nucleons are assumed to form an intermediate state with highly excited proper fields. The latter ones give up their energy by emission of several quanta of the pionic, nucleonic and K-mesonic fields respectively. If the lifetime of the "intermediate state" is sufficiently long, we are allowed to assume that in the proper fields a statistical equilibrium exists and we may try to calculate the number of different particles produced by methods of statistical physics. This idea of FERMI [2][2] was modified by LANDAU [3], [4], who states that during the first

[1] We mention only the early work of HABER-SCHAIM, YEIVIN and YEKUTIELI, Phys, Rev., **94.** 184 (1954) applying Fermi's theory and BELENKI's papers, quoted in ref [4].

[2] In FERMI's and LANDAU's papers only the pionic proper field of the nucleon is dealt with. The above-mentioned model seems to be a natural extension of the former. We do not introduce, however, hyperonic proper fields ; we shall regard for sake of simplicity hyperons as "composite particles" — e. g. according to the model of GYÖRGYI [5]. This assumption does not affect seriously our later considerations.

stage of its decay, the intermediate state consists of a continuously varying-number of particles, since, because of strong interactions the number of particles is not a "good quantum number". Only at a later stage, when the particles have got out of the influence of each other, may we speak of a fixed number of particles and apply formulae of statistical mechanics. LANDAU defines a "critical temperature" of the system at which we can consider the particles as free.

According to LANDAU's calculations, the latter temperature for pions and nucleons is given by

$$T_c^{(\pi)} = a \cdot \mu$$

in energy-units, where a is a numerical factor near to unity.

Now the following problems arise :

a) Do multiple processes play a role in the production of K-mesons and hyperons, and if so, which are the reaction equations.

b) If we want to describe the production of the heavy unstable particles by means of the Fermi-Landau model, what is the critical temperature for the latter.

In connection with *a)* we remark, that if multipe production of heavy unstable particles takes place at all, then conservation laws allow the following reactions :

$$N + N \rightarrow \left\{ \begin{array}{l} N + N \\ \Lambda + N + K \\ \Sigma + N + K \\ \Xi + N + 2K \\ \Xi + \Lambda + 3K \\ \Lambda + \Sigma + 2K \\ \Xi + \Sigma + 3K \\ 2\Lambda + 2K \\ 2\Sigma + 2K \\ 2\Xi + 4K \end{array} \right. + \left\{ \begin{array}{l} \cdots K + \bar{K} \\ \cdots N + \bar{N} \\ \cdots \Lambda + \bar{\Lambda} \\ \quad \cdots\cdot \end{array} \right. + \left\{ \begin{array}{l} \cdots \Xi + \bar{N} + 2K \\ \cdots \Xi + \bar{\Lambda} + K \\ \cdots \Xi + \bar{Z} + K + \cdots \pi \\ (+\, ch \cdot conj.) \end{array} \right.$$

Concerning problem *b)* we expect that either the interaction of K-mesons and hyperons with pions and nucleons is weak in relation to that of pions and nucleons or it is nearly of the same strength. In the first case the critical temperature would be $T_c^{(K)} \sim m$, while in the second one we expect that

$$T_c^{(K)} \sim T_c^{(\pi)}$$

In sections 2 and 3 in order to answer problem *b)* we deal with the π, π) and (K, π) interaction ; while in the fourth and fifth sections we calculate different observable quantities on the basis of the Fermi-Landau-model and try to compare them with experimental results.

2. The (π, π) interaction

The (π, π) interaction seems to be an experimentally observable fact. Although we have no direct experiment yet, the analysis of the reactions [6]

$$\pi + N \rightarrow \pi + \pi + N \tag{2,1}$$

carried out by ITO and MINAMI [7] gives indirect evidence for the existence of such an interaction.

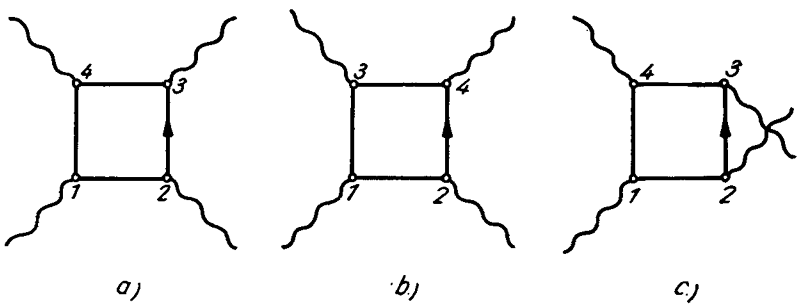

Fig. 1

From a theoretical point of view, this interaction is similar to the photon-photon scattering, *scil*, it also is a vacuum effect and is described by the same types of Feynman-diagrams. Photon-photon scattering has been investigated theoretically in detail by KARPLUS and NEUMANN [8], [9]. We want to give here a rough estimation of the (π, π)-cross section for high energies, using $ps(ps)$ meson theory and lowest order perturbation approximation. We neglect isobaric variables, and consider a neutral pion field only. In such a rough calculation this simplification seems to be justified. The basic interaction diagrams, describing the process are given in Fig. 1.

To each of these diagrams corresponds another, equivalent one, with the nucleon loop in the opposite direction.

In order to estimate the behavior of the cross section, we shall calculate the S-matrix element corresponding to diagram *a)* only, since interference terms will not affect seriously the order of magnitude.

In the center-of-mass system and momentum representation we obtain apart from numerical factors

$$S^{(4)} = G^4 \frac{\delta\left(\sum_{i=1}^{4} k_i\right)}{\omega^2} \int d^4p \, Sp \left\{ \gamma_5 \frac{i\gamma p - M}{p^2 + M^2} \gamma_5 \frac{i\gamma(p-k) - M}{(p-k)^2 + M^2} \cdot \right.$$
$$\left. \cdot \gamma_5 \frac{i\gamma(p - k_2 - k_3) - M}{(p - k_2 - k_3)^2 + M^2} \cdot \gamma_5 \frac{i\gamma(p + k_1) - M}{(p + k_1)^2 + M^2} \right\},$$

where M stands for the nucleon mass, k_i is the momentum four-vector of the i-th pion (choosing the signs in such a way as if all the pions were outgoing ones) G is the dimensionless coupling constant between the pion and nucleon field ; $\gamma p \equiv p_\mu \gamma^\mu$ and $\omega^2 = k^2 + \mu^2$. We perform the integration in (2, 1) using FEYNMAN's method [10] . Having carried through the transformations, quoted above, we find, that there is occurring a logarithmically divergent integral, namely

$$\int \frac{(p^2)^2 \, d^4p}{(p^2 + a^2)^4} \,. \tag{2,2}$$

(After having taken the trace, we can neglect odd powers of p since they give zero in a symmetrical integration over the angles in the p-space ; cf. FEYNMAN, *loc. cit.* or JAUCH's and ROHRLICH's Quantum Electrodynamics [11].) We separate now the physically meaningless divergence in a well-known manner.

Using the identity

$$\frac{1}{\alpha^n} - \frac{1}{\beta^n} = - \int_0^1 \frac{n(\alpha - \beta)\,dz}{[(\alpha - \beta)z + \beta]^{n+1}} \tag{2,3}$$

we find

$$\int \frac{(p^2)^2 \, d^4p}{(p^2 + a^2)^4} = \int \frac{(p^2)^2 \, d^4p}{(p^2 + M^2)^4} - 4 \int_0^1 du \int d^4p \frac{(a^2 - M^2)(p^2)^2}{[p^2 + M^2 \cdot u + a^2(1-u)]^5} \tag{2,4}$$

The second integral on the right side is already convergent. The first one is divergent, but can be removed *e. g.* by the formalistic regularization method of PAULI and VILLARS [12]. We have chosen the latter method for sake of simplicity ; we do not attribute to it any physical interpretation, but regard it as an invariant cut-off procedure only.

Calculation from here on runs along the same lines as the one by KARPLUS and NEUMANN, *loc. cit.* We calculate the asymptotic expression for high energies of the forward-scattering amplitude. The total cross section is then obtained making use of the well known "optical theorem" :

$$\sigma(\omega) = \frac{4\pi}{\omega} Im\,(a(\omega)), \tag{2,5}$$

where $a(\omega)$ is the forward-scattering amplitude.

The calculation yields, apart from some numerical factors

$$\sigma_{\pi\pi}(\omega) \sim \frac{G^4}{M^2} \frac{M^2}{\omega^2} \left(\ln \frac{\omega}{M}\right)^2 \tag{2,6}$$

The form of this exprρssion agrees with that of the cross section for high-energy light quanta, and was obtained first by ACHIESER (see KARPLUS, NEUMANN, *loc. cit.*)

3. The (K, π) interaction

We turn now to the investigation of the interaction between K-mesons and pions, assuming an "elementary" interaction between (K, N) and (π, N) fields only.

We neglect again isobaric variables, and treat a scalar K-meson field with scalar coupling. The interaction-operator between the K-meson and nucleon field then will be of the form

$$W = g/m\,(\bar{\psi}\,\psi)\,(\bar{x}\,x)\,, \tag{3,1}$$

where ψ is the field operator of the nucleon and x that of the K-meson field, g is the dimensionless interaction constant. Both ψ and x are isospinors of the first kind.[1]

The basic graph in lowest, non vanishing order for an interaction is shown in Fig. 2. The corresponding S matrix element, — again apart from numerical factors — reads :

$$s \sim \frac{\delta(\Sigma p_i)\, G_g^2}{\sqrt{\omega_1\,\omega_2\,\omega_3\,\omega_4}} \cdot \frac{1}{4m} \int Sp\, \gamma_5 \frac{i\gamma(k+p_4)-M}{(k+p_4)^2+M^2}\, \gamma_5 \frac{i\gamma(k+p_4-p_1)-M}{(k+p_4-p_1)^2+M^2} \cdot \cdot \frac{i\gamma k - M}{k^2+M^2}\, d^4 k \tag{3,2}$$

Taking the trace under the integral, one can transform the denominator according to the standard method, indicated in the preceding section, and separate a logaritmically divergent term

$$\int \frac{k^2\, d^4 k}{(k^2+M^2)^3}$$

which can be removed by the Pauli-VILLARS regularization procedure.

All further calculations are carried out in the same manner, as has been shown in the preceding section. They give for the high-energy limit of the $(K\ \pi)$ cross section :

[1]. An interaction Lagrangian of this form has been proposed by GYÖRGYI, loc. cit.

$$\sigma_{K\pi} \sim G^2 g \frac{M}{m} \frac{1}{\omega^2}. \tag{3,3}$$

Compared with the $(\pi\,\pi)$ cross section, and taking some reasonable value for the coupling constants [17], we see, that $\sigma_{K\pi}$ is comparable with $\sigma_{\pi\pi}$ within a factor of the order of unity. (In this comparison we have taken into account some numerical factors, not written down explicitly in (2, 6) and (3, 31).

The consequence of this for the following thermodynamical calculation is, that the critical temperature for K-mesons will be nearly the same as for pions. Since the critical temperature varies rather slowly with the cross section [4], we shall choose both temperatures for sake of simplicity, exactly equal.

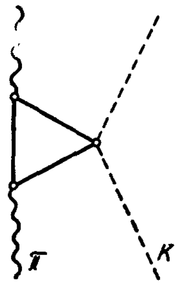

Fig. 2

We want to call attention once again to the very rough character of our calculations : we have used the high-energy limit of the cross sections, calculated by peturbation theory in the case of moderate energies and rather strongly coupled fields. Indeed, there are some indications [13], that $(\pi\,\pi)$ cross section may have a different energy dependence than obtained by us. We hope, however, that the *ratio* of these cross sections is of the correct order of magnitude, and the qualitative conclusions drawn from the calculation are not false.

4. Thermodynamical model of particle production

According to the programme outlined in sec. 1 we are going to calculate some observable quantities of the two-nucleon system, on the basis of the Fermi-Landau-model. We shall follow — as far as possible — the notations of [4].

From secs. 2 and 3 we conclude that at high energies — where thermodynamical approximation has any meaning at all — K-mesons and — possibly — hyperons interact strongly with pion and nucleon fields. Therefore, we choose in the Fermi-Landau-model

$$T_c^{(\pi)} = T_c^{(K)} = \mu\,. \tag{4,1}$$

(The value of the constant factor before μ is obtained by means of a graphical estimation, based on the results of [4]).

The density of particles of type i is given by

$$n_i = w_i \int_0^\infty \frac{4\pi p^2 dp/(2\pi)^3}{\exp(\sqrt{p^2 + m_i^2/Tc}) \mp 1}, \tag{4,2}$$

where w_i is a weight factor arising from summations over spin, isobaric spin etc. m_i is the mass of the particle ; the sign in the denominator depends on the statistics followed by the particle.

The latter formula can be written in the more convenient form

$$n_i = w_i/2\pi^2 T_c^3 F^{\mp}(\xi_i),$$

$$\xi_i = m_i/T_c; \quad F^{\mp}(z) = z^3 \int_0^\infty \frac{x^2 dx}{\exp(z\sqrt{1+x^2}) \mp 1}. \tag{4,3}$$

($F^{\mp}(z)$ and similar functions are tabulated in reference [4].) Conservation laws can be taken into account by means of the multiplicator method. If there is one integral of motion, then $F^{\mp}(z)$ should be replaced by

$$F^{\mp}(z, y) = z^3 \int_0^\infty \frac{x^2 dx}{\exp(z\sqrt{1+x^2} - y) \mp 1}, \tag{4,4}$$

where y is the corresponding multiplicator. (5, 4) can be generalized for the case of several integrals of motion in a straightforward manner. For small values of y

$$F^{\mp}(\xi, y) \approx e^y F^{\mp}(\xi). \tag{4,4a}$$

The multiplicators can be determined from the equations expressing the conservation of the quantity in question. These can be written in the following form. If i, j are two kinds of particles, and the *numbers* of particles are denoted by capital letters, so, if $T_c^{(i)} = T_c^{(j)}$, then $N_i/N_j = n_i/n_j$.

Further, if the conserved quantity is ω, its value for particle i is ω_i, then the conservation equation reads

$$\omega_{N_1} + \omega_{N_2}/N_i = \sum_k' n_k \omega_k/n_i. \tag{4,5}$$

(N_1, N_2 stands for the two colliding nucleons.)

We consider as an example the collision of two protons. Denote by y, z, u the multiplicators, corresponding to N, T_3, U respectively. (It can be shown, that the neglect of the conservation of T introduces a very small error.)

Regarding the N, T_3, U-values of the different particles and (4, 4a), the conservation-equations are found to be,

$$(N): n_{\pi_0}/N_{\pi_0} = n_N^0\, sh\,(y+z+u) + n_N^0\, sh\,(y-z+u) + n_\Lambda^0\, sh\,y + \\ + n_\Xi^0\, sh\,(y+z-u) + n_\Xi^0\, sh\,(y-z-u) + n_\Sigma^0\, sh\,(y+2z) + \\ + n_\Sigma^0\, sh\,y + n_\Sigma^0\, sh\,(y-2z);$$

$$(2T_3): n_{\pi_0}/N_{\pi_0} = n_N^0\, sh\,(y+z+u) - n_N^0\, sh\,(y-z+u) + n_K^0\, sh\,(z-u) + \\ + n_K^0\, sh\,(z+u) + n_\Xi^0\, sh\,(y+z-u) - n_\Xi^0\, sh\,(y-z-u) + \\ + 2n_\pi^0\, sh\,2z + 2n_\Sigma^0\, sh\,(y+2z) - 2n_\Sigma^0\, sh\,(y-2z);$$

$$(u): n_{\pi^0}/N_{\pi^0} = n_N^0\, sh\,(y+z+u) + n_N^0\, sh\,(y-z+u) + \\ + n_k^0\, sh\,(z+u) - n_k^0\, sh\,(z-u) - n_\Xi^0\, sh\,(y+z-u) - \\ - n_\Xi^0\, sh\,(y-z-u).$$

Here we have chosen neutral pions as "reference-particles" and $n_i^0 = w_i/2\pi^2 \cdot T^3 F^{\mp}(\xi_i)$.

The approximate roots of the system (4, 6) are :[1]

$$y = 7{,}667/N_{\pi^0}, \quad z = 0{,}2131/N_{\pi^0}, \quad u = 0{,}8407/N_{\pi^0}.$$

We see, that for very high energies, as a rough estimation, we may put :

$$n_i \approx n_i^0. \tag{4,7}$$

As another important quantity, the average energy which is carried away by a definite kind of particles is calculated. The energy-density is given by

$$\varepsilon_i = T^4 w_i/2\pi^2 \Phi^{\mp}(\xi_i),$$

$$\Phi^{\mp}(\xi_i) = \xi_i^3 \int_0^\infty \frac{x^2\sqrt{1+x^2}\,dx}{\exp(\xi_i\sqrt{1+x^2}) \mp 1} \tag{4,8}$$

and a corresponding function, if some conservation laws are taken into account. Similarly, as in (4, 4*a*) and (4, 7), we may put

$$\Phi^{\mp}(z,y) \approx e^y \Phi^{\mp}(z). \tag{4,9}$$

The result of the calculation, outlined here, without making use of the approximations (4, 7), (4, 9), is plotted in Table I. Columns 2—3 give the density, resp.

[1] The relative error of the approximation decreases with energy as $(N_{\pi^0})^2$. Since the most important role is played by the conservation of N, our further considerations apply — at least qualitatively — to (N, P) and (P, P) collisions as well.

energy density of the particle, indicated in column 1, divided by a common factor. Column 4 gives the average energy of one particle in the c. m. system, according to the approximate expression $\langle E \text{ c. m.}\rangle \approx \varepsilon/n$. Column 5 gives the fraction of the total available energy, carried away by the particles in question ; while the last column gives the relative number of emited particles. In columns 5 and 6 the approximation (4, 7) and the similarity $\varepsilon_i \sim \varepsilon_i^0$ is used.

Table I

1	2		3	4	5	6
Particle	$n\,(T^3/_2\,\pi^2)^{-1}$		$\varepsilon\,(T^4/_2\,\pi^2)^{-1}$ *	$\langle$ Ec. m. $\rangle$ [Be V]	$\varepsilon/\sum_j \varepsilon_j$ **	$n/\sum_j n_j$
π^0 $\pi+$ $\pi-$	1 exp $(0{,}462/N_{\pi^0})$ exp $(-0{,}462/N_{\pi^0})$	. 1,78	. 5,90	1,12	0,23	0,19
K^0 $\bar{K}^0$ $\bar{K}^+$ K^-	exp $(0{,}628/N_{\pi^0})$ exp $(-0{,}628/N_{\pi^0})$ exp $(1{,}054/N_{\pi^0})$ exp $(-1{,}054/N_{\pi^0})$	. 0,478	. 2,39	1,71	0,062	0,078
N^0 $\bar{N}^0$ $P+$ $\bar{P}^-$	exp $(8{,}29/N_{\pi^0})$ exp $(-8{,}29/N_{\pi^0})$ exp $(8{,}72/N_{\pi^0})$ exp $(-8{,}72/N_{\pi 9})$	. 0,0714	. 0,606	2,88	0,009	0,020
Λ^0 $\bar{\Lambda}^0$	exp $(767/N_{\pi^0})$ exp $(-7{,}67/N_{\pi^0})$	. 0,023	. 0,23	3,34	0,003	0,074
$\Sigma+$ Σ^0 $\Sigma-$ $\bar{\Sigma}+$ $\bar{\Sigma}^0$ $\bar{\Sigma}-$	exp $(8{,}09/N_{\pi^0})$ exp $(7{,}67/N_{\pi^0})$ exp $(7{,}24/N_{\pi^0})$ exp $(-7{,}24/N_{\pi^0})$ exp $(-7{,}67/N_{\pi^0})$ exp $(-8{,}09/N_{\pi^0})$	. 0,015	. 0,15	3,40	0,002	0,0048
Ξ^0 Ξ^0 $\Xi-$ $\Xi+$	exp $(7{,}04/N_{\pi^0})$ exp $(-7{,}04/N_{\pi^0})$ exp $(6{,}61/N_{\pi^0})$ exp $(-6{,}61/N_{\pi^0})$	. 0,007	. 0,076	3,69	0,0009	0,0024

* The figures of column 3 have to be multiplied by the same exponential factor as the corresponding figures in column 2.

** Columns 5 and 6 are calculated in the high-energy limit : $N_{\pi^0} \sim \infty$

The number of a definite kind of particles is obtained by multiplying its density with the interaction volume.

The latter is expressed by $N_t/\sum_j n_j$, where N_t is the total number of emitted particles. Thermodynamical calculations show (cf. [2] [4]), that, N_t is proportional to the fourth root of the primary energy in the L-system :

$$N_t \approx 2\,(E_p/M)^{1/4}\,.$$

So the average number of particle i becomes

$$N_i \approx 2n_i/\sum_j n_j\,(E_p/M)^{1/4} \tag{4,10}$$

and in particular

$$N_{\pi^0} \approx 0{,}38\,(E_p/M)^{1/4}\,. \tag{4,11}$$

Consider an example. Let be, $E_p = 10^5$ BeV, then our calculations give the results $N_t \approx 35$; we quote the average numbers of some "interesting" particles :

$$N_{K^0} = 2{,}3\,;\quad N_{\bar{K}^0} = 1{,}9\,;\quad N_{K^+} = 2{,}4\,;\quad N_{\bar{K}^-} = 1{,}7\,;\quad N_{\Lambda^0} = 0{,}31\,;$$
$$N_{\Sigma^+} \approx N_{\Sigma^0} \approx N_{\Sigma^-} \approx 0{,}2\,;\quad N_{\Xi^0} \approx N_{\Xi^{--}} \approx 0{,}07\,.$$

(The latter figure indicates, that the correction to the counted number of neutral pions due to the γ-decay of Ξ°-cf. BRISBOUT et al.'s work, quoted later — is very small.) One observes further that the number nucleon-antinucleon pairs is rather *small.*

Similarly, the number of antihyperons is considerably smaller than that of their charge-conjugate pairs.

The fraction of energy, carried away by heavy particles (heavy mesons, nucleons, hyperons) is about 50%.

5. Comparison with experimental results

Now we try to compare our results with the experiments ; we do not intend to give a full account of experiments and discuss the experimental procedure, but we aim rather at obtaining some information about the power and limitations of our model.

A measurable quantity is the ratio of neutral pions to charged shower particles.

Measurements have been carried out by several authors (of [14]—[19]).

Some of their results are summarized in Table II.

We see that the R-values agree within the — rather large — statistical errors, and further that R is approximately constant in a large range of pri-

Table II

Ratio R of neutral pions to charged shower particles injets

Author	$R = N_{\pi^0}/N_s$
Daniel et al. [14]	0,33 $\pm$0,07
Mulvey [15]	0,25 $\pm$0,1
Naugle et al. [16]	0,44 $\pm$0,14
Kaplon et al. [17]	0,46 $\pm$0,09
Lal et al. [18]	0,40 $\pm$0,08
Brisbout et al. [19]	0,383$\pm$0,044
Weighted average	0,375$\pm$0,029

mary energy ($50 \leq E_p \leq 3000$ BeV/nucleon). The possible variation of R is obscured by statistical uncertainties.

From R we obtain the ratio of the non-pionic charged shower particles to charged pions, if we assume that $N_{\pi^0}/N_{\pi^\pm} = 0{,}5$, which is a very good approximation.

The above-mentioned ratio from the weighted average of Table II is

$$\bar{f} = \overline{\left(\frac{N_s - N_{\pi^\pm}}{N_{\pi^\pm}} \right)} = 0{,}33 \pm 0{,}10 \, .$$

Our calculated result is (summing over charged particles, which are not pions)

$$f_{th} = 0{,}33 \, .$$ [1]

Concerning the ratio of charged to neutral heavy unstable particles, we quote the works of Leighton and Trilling [20] and of Giacconi et al. [21], who have carried out measurements with cloud chamber and *lead* absorber. According to their measurements this ratio is approximately

$$\frac{N_u^\pm}{N_u^0} = 0{,}21 \pm 0{,}03 \, ,$$

while Table I yields

$$\frac{N_u^\pm}{N_u^0} = 0{,}9$$

in significant contradiction with the experimental result. We mention, however, that the quoted measurements were carried out with Pb target nuclei, so, the possibility of explaining the discrepancy between theory and experiment by secondary effects, does not seem excluded.

[1] Our model predicts a variation for f of about 10% in the energy-range 50 BeV $< E_p < \infty$; this — if it really exists — is completely obscured by uncertainties of present experiments.

9. Discussion and conclusion

We want to stress once again the main assumptions, made in this paper :

a) To consider collisions to be central collisions only.

b) To regard interaction between heavy unstable particles and pions resp. nucleons as strong.

c) To consider it possible to apply thermodynamic calculations. We believe, that the validity of hypothesis *a)* is rather doubtful, but as long as a correct field theory is not constructed, we cannot consider non-central collisions.

Hypothesis *c)* is partly dependent on *b)* ; we do not want to discuss its validity in detail, but refer the reader to the works of FERMI and the LANDAU-school, already quoted.

Looking at Table I and (4, 10), one observes, that the multiplicity of heavy unstable particles is not too high ; therefore, the application of thermodynamics may be a very rough approximation.

Bearing the aforesaid comments in mind, it might appear strange, that this model could give an — at least qualitative — agreement with experiment.

We believe this in great part to be due to the uncertainties of the experiments. We only mention here, that even the estimation of the *primary energy* of a jet from the angular distribution is a procedure of rather doubtful accuracy. Further, we do not have sufficient information as to the mass spectrum of shower particles. The above-mentioned experiments give only a ratio of non-pions to pions or similar "composite" ratios.

We try to point out some features of our model, which it will perhaps be possible to test experimentally.

a) The variation of the ratio of the numbers of different particles with primary energy might be investigated. In our model this depends apart from the exponential factor arising from conservation equations — on the variation of interaction cross sections in the "cloud" around the nucleon. (Variation of T_{c}.).

b) The fraction of energy, carried away by heavy particles, decreases — although rather slowly — with primary energy. Its variation becomes faster at primary energies of the order of ten or hundred BeV. In this region, however, our model breaks down, and we cannot make quantitative predictions. (In the region of only a few BeV-s, one can easily understand the increase of the big fraction of energy, carried away by nucleons : there the *elastic* part of the cross section becomes large, and the energy is carried away by the *original* nucleons.)

Concerning the problems mentioned in the introduction, we cannot assert definitely, that *multiple* production of heavy unstable particles really does take place, but this uncertainty might be due to the fact, that at energies available at present, their multiplicity is rather low; so, we might expect, that extension of experiments to the region of still higher energies, will yield a definite answer.

A simple model, assuming excited self-fields around the colliding nucleons is not in qualitative contradiction with present experimental data.[1]

We hope to have the opportunity of returning to some of the problems concerning this subject.

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[1] After this work had been finished, the author got acquintanted with a paper, reporting of the multiple emission of heavy unstable particles, from stars in photographic emulsion. (TSAI-CHÜ, MORAND, Phys. Rev., **104,** 1943, 1956). The reported event is not a jet. The authors assume a reaction

$$N + N \rightarrow \Sigma^+ + \Sigma^- + K^- + 3\,\vartheta^\circ + \pi^+$$

or

$$\pi^- + N \rightarrow \Sigma^+ + \Sigma^- + K^- + \vartheta^\circ + N$$

leading to the emission of heavy unstable particles. A further question remains, however namely, whether the quoted events may not be explained by two separate reactions, each of them being a "simple" associated production.

Recently, DEBENEDETTI et al (Nuovo Cim., **4,** 1142, 1956) reported the value of R (see p. 27) to be 0,426 $\pm$ 0,06, measured on a single event. The author is indebted to Dr. TSAI-CHÜ for information about their event and the Turin-group for having sent him a reprint.

РОЖДЕНИЕ ТЯЖЕЛЫХ НЕСТАБИЛЬНЫХ ЧАСТИЦ В СОУДАРЕНИЯХ НУКЛЕОНОВ БОЛЬШОЙ ЭНЕРГИИ

Г. ДОМОКОШ

Резюме

Рассматривается проблема множественного рождения тяжелых нестабильных частиц. Устанавливаются возможные схемы реакций на основе теории Гелл—Манн. Исследуется взаимодействие тяжелых нестабильных частиц с π—мезонами, на основании которого образование тяжелых нестабильных частиц рассматривается применением термодинамической модели Ферми—Ландау. С помощью этой модели вычисляются некоторые характеристические величины системы, состоящей из двух протонов. Результаты повидимому не противоречат данным опытов.