TOMONAGA'S INTERMEDIATE COUPLING THEORY USING CONFIGURATION SPACE METHODS

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With the intermediate coupling theory — using the configuration space methods of the quantum theory of fields — we determine the state vector characterizing the real nucleon. We carry out our calculations for the case of interaction of the nucleon field described by the Dirac equation and the scalar, resp. pseudoscalar meson field. Pair creation is completely disregarded. Remaining within the frameworks of the configuration space method the recoil of the nucleon is considered. With the aid of the state vector we also calculate the mean value of some characteristic physical quantities. The use of the configuration space method — particularly in connection with the computation of local physical quantities — makes possible to form a very clear picture about the real nucleon.

Introduction

For the quantum theoretical treatment of the interacting fields the covariant perturbation method proved to be very succesful in quantum electrodynamics but it cannot be applied in case of strongly coupled fields. The results calculated with its aid do not agree with the experimental results owing to the bad convergence. Recently it has been becoming more and more obvious, that the renormalization method which can be unambiguously formulated with the aid of the S-matrix is not satisfactory, as after the renormalization physically inadmissable results occure. It is for this reason, that consideration of methods, other than the theory of the S-matrix, is of considerable importance.

In the following Tomonaga's intermediate coupling theory [1]—[21] is dealt with in the case of a nucleon field, described by the Dirac equation, being in interaction with the scalar resp. symmetrical pseudoscalar meson field. The state vectors characterizing the real nucleons are determined in an adequate approximation. Our calculations are based on configuration space methods, and throughout the interaction picture is made use of.

The four-momentum of interacting fields is

$$P_\mu[\sigma] = P_\mu^0 - \frac{1}{c} \int_\sigma H(x)\, d\,\sigma_\mu(x)\,, \tag{1}$$

where P_μ^0 is the sum of the four-momenta of the individual interacting fields and thus the operator of the infinetesimal displacement for the interaction picture operators.

According to (1) the energy-momentum eigenvalue equation is

$$P_\mu [\sigma] \,|\, \sigma > = \Pi_\mu \,|\, \sigma > , \tag{2}$$

where in case of a neutral scalar coupling

$$H(x) = g : \bar{\psi}(x)\, \psi(x)\, \Phi(x) : . \tag{3}$$

In case of the symmetric pseudoscalar field pseudoscalar coupling according to the DYSON-FOLDY theorem [22], [23]

$$H(x) = \frac{i g}{2 \varkappa} : \left(\bar{\psi} \gamma_\mu \gamma_5 \sqrt{2}\, \tau_- \psi\, \partial_\mu \Phi + \bar{\psi} \gamma_\mu \gamma_5 \sqrt{2}\, \tau_+ \psi\, \partial_\mu \Phi^* + \bar{\psi} \gamma_\mu \gamma_5\, \tau_3 \psi\, \partial_\mu \Phi_3 \right) : +$$
$$+ \lambda \frac{g^2}{2 M c^2} : \bar{\psi} \psi \left(2\, \Phi \Phi^* + \Phi_3^2 \right) : , \tag{4}$$

where δ-like interaction terms were neglected, λ serves for the pair suppression suggested by BRÜCKNER and others [24], [25], according to BRÜCKNER its most probable value is 0,2. Here : : denotes, as is usual, a normal product. It is known that the pseudoscalar coupling is preferred as against pseudovector coupling owing to its renormalizability. Recently the possibility of the renormalization of pseudovector coupling was also suggested [26], [27]. Thus the substitution $\lambda = 0$ is justified too.

The state vector of the field according to the configuration space method [28], [29] applied here (detailed literature in the latter) in case of a nucleon and neutral scalar meson field is

$$|\sigma> = \sum_{n\, n'\, m} (i)^{n+n'} \left(\frac{i}{\hbar c} \right)^m \int_\sigma \cdots \int_\sigma \overline{| \, x^1, \ldots \; x^n ; \; x'^1 \; \ldots \; x'^{n'}} ; y^1 \ldots y^m >$$
$$\Pi \gamma_{\nu i}^{(i)} d\, \sigma_{\nu i} (x^i)\, \Pi \gamma_{\mu j}^{(j)} d\, \sigma_{\mu j} (x'^j) \,.\, \prod^m d_{\varrho k} (y^k)\, d\, \sigma_{\varrho k} (y^k)$$
$$< x^1, \ldots x^n ; x'^1, \ldots x'^{n'} ; y^1, \ldots y^m \,|\, \sigma > , \tag{5}$$

in case of a nucleon and symmetrical pseudoscalar meson field

$$|\sigma> = \sum_{n, n', m^+, m^-, m^3} (i)^{n+n'} \left(\frac{i}{\hbar c} \right)^{m^+ + m^- + m^3} \int_\sigma \cdots \int_\sigma | \, x^1, \ldots x^n ; \; x'^1, \ldots x'^{n'} ;$$
$$\xi^1, \ldots \xi^{m^+} ; \eta^1, \ldots \eta^{m^-} ; \zeta^1, \ldots \zeta^{m^3} >$$
$$\prod^n \gamma_{\nu i}^{(i)} d\, \sigma_{\nu i} \prod^{n'} \gamma_{\mu j} d\, \sigma_{\mu j} \prod^{m^+} d_{\varrho i}\, d\, \sigma_{\varrho i} \prod^{m^-} d_{\varrho k}\, d\, \sigma_{\varrho k} \prod^{m^3} d_{\,\varrho e}\, d\, \sigma_{\,\varrho e} \tag{6}$$
$$< x^1, \ldots x^n ; x'^1, \ldots , x'^{n'} ; \xi^1, \ldots , \xi^{m+} ; \eta^1, \ldots , \eta^{m^-} ; \zeta^1, \ldots \zeta^{m^3} \,|\, \sigma > \cdot$$

Here

$$d_\nu = \left(\frac{\overleftarrow{\partial}}{\partial x_\nu} - \frac{\overrightarrow{\partial}}{\partial x_\nu} \right).$$

The state vectors and through them the amplitudes are defined by

$$\overline{| x^1, \ldots x^n ; x'^1, \ldots x'^{n'}} ; y^1, \ldots y^m > = (n!\, n'!\, m!)^{-\frac{1}{2}}\, \bar{\psi}^{(+)} (x^1) \ldots \bar{\psi}^{(+)} (x^n),$$
$$\bar{\psi}'^{(+)} (x'^1) \ldots \bar{\psi}^{(+)} (x'^{n'})\, \Phi^{(-)} (y^1) \ldots \Phi^{(-)} (y^m) \,|\, 0 > . \qquad (7)$$

resp.

$$| x^1, \ldots x^n ; x'^1 \ldots x'^{n'} ; \xi^1, \ldots \xi^{m+} ; \eta^1, \ldots \eta^{m-} ; \zeta^1, \ldots \zeta^{m^3} > =$$
$$= (n!\, n'!\, m^+!\, m^-!\, m^3!)^{-\frac{1}{2}} . \bar{\psi}^{(+)} (x^1), \ldots \bar{\psi}'^{(+)} (x'^1) \ldots \Phi^{(+)*} (\xi^1) \ldots$$
$$\ldots \Phi^{(-)} (\eta^1) \ldots \Phi_3^{(-)} (\zeta^m) \,|\, 0 > .$$

resp.

$$\psi^{(+)} (x) \,|\, 0 > = \psi'^{(+)} (x) \,|\, 0 > = \Phi^{(+)} (x) \,|\, 0 > = 0 , \quad < 0 \,|\, 0 > = 1 \qquad (8)$$

where $| 0 >$ is the vacuum state:

$$\psi^{(+)}(x) \,|\, 0 > = \psi'^{(+)}(x) \,|\, 0 > = \Phi_3^{(+)}(x) \,|\, 0 > = \Phi^{(-)*}(x) \,|\, 0 > = 0 , < 0 \,|\, 0 > = 1 ,$$

Solving the eigenvalue equation (2) just means the determination of all the amplitudes $< x^1, \ldots | \sigma >$ occurring in (5) resp. (6). In the following weshall determine these amplitudes in a suitable approximation. With the help of these amplitudes we may — since they have a direct probability meaning in the coordinate space and at the same time determine also the number of mesons — form a clear picture about a real nucleon. Our picture can be completed by determining the mean value of some physical quantities characterizing the system. From this point of view the local quantities are of particular interest. Thus in the environment of the real nucleon, the mean values of the meson potential, the electric charge density, as well as the energy density will be determined.

The intermediate coupling theory

The intermediate coupling theory is the variational method in the quantum theory of fields. According to this instead of the exact solution of the eigenvalue equation (2) only the mean value of $- ic < \sigma \,|\, P_4 [\sigma] \,|\, \sigma >$ is minimized, satisfying the condition $< \sigma \,|\, \sigma > = 1$, with the help of suitable trial functions. In this paper pair creation is disregarded throughout ; thus in (5) resp. (6) — and everywhere, where this may occur in the course of the calculations — zero is written for all amplitudes $< x^1, \ldots; x'^1, \ldots | \sigma >$ containing at least one antinucleon.

In the following we examine such states in which a specified number of real nucleons (say A) is contained. Then owing to the neglect of pair creation in the state vectors only such amplitudes can occur which characterize (apart from possible mesons) exactly A bare nucleons. We carry out our calculations first for the case the nucleon and the scalar field.

1. *The nucleon and the neutral scalar meson field*

Our aim is now to determine the state vector describing A real nucleons and to calculate for this state the values of some characteristic physical quantities. Neglecting pair creation, from (1), (3) and (5) using the formulae (7), (8) as well as the properties of the functions S and Δ occurring in the commutation relations we obtain :

$$< \sigma | P_\mu [\sigma] | \sigma > = \sum_n i^A \left(\frac{i}{\hbar c}\right)^n \int_\sigma \ldots \int_\sigma < \sigma | \overline{x^1, \ldots x^A}; y^1, \ldots y^n >$$

$$\prod^A \gamma_{\nu i} \, d\, \sigma_{\nu i} (x^i) \prod^n d_{\mu j} d\, \sigma_{\mu j} (y^j) \, .$$

$$\cdot \frac{\hbar}{i} \left\{ \sum^A \frac{\partial}{\partial x^i_\mu} + \sum^n \frac{\partial}{\partial y^j_\mu} \right\} < x^1, \ldots x^A ; y^1, \ldots y^n | \sigma > -$$

$$- g \frac{A\, (i)^{A-1}}{c} \sum_n (n+1)^{1/2} \int_\sigma d\, \sigma_\mu (x) \left(\frac{i}{\hbar c}\right)^n \int_\sigma \ldots \int_\sigma < \sigma | \overline{x, x^2, \ldots x^A}; y^1, \ldots y^n > .$$

$$\prod_{i=2}^A \gamma^{(i)}_{\nu i} \, d\, \sigma_{\nu i} (x^i) \prod^n d_{\mu j} d\, \sigma_{\mu j} (y^j) < x, x^2, \ldots x^A ; x, y^1, \ldots y^n | \sigma > -$$

$$- g \frac{A\, (i)^{A-1}}{c} \sum_n n^{1/2} \int_\sigma d\, \sigma_\mu (x) \left(\frac{i}{\hbar c}\right)^{n-1} \int_\sigma \ldots \int_\sigma < \sigma | \overline{x, x^2, \ldots x^A} ; x, y^2, \ldots y^n >$$

$$\prod_{i=2}^A \gamma^{(i)}_{\nu i} \, d\, \sigma_{\nu i} (x^i) \prod_{j=2}^n d_{\mu j} d\, \sigma_{\mu j} (y^j) < x, x^2, \ldots x^A ; y^2, \ldots y^n | \sigma > . \qquad (9)$$

Similarly

$$< \sigma | M | \sigma > = \sum_n n\, (i)^A \int_\sigma \ldots \int_\sigma \left(\frac{i}{\hbar c}\right)^n \int_\sigma \ldots \int_\sigma < \sigma | \overline{x^1, \ldots x^A} ; y^1, \ldots y^n >$$

$$\prod^A \gamma^{(i)}_{\nu i} \, d\, \sigma_{\nu i} (x^i) \cdot \prod^n d_{\mu j} d\, \sigma_{\mu j} (y^j) < x^1, \ldots x^A ; y^1, \ldots y^n | \sigma > , \qquad (10)$$

where $M = \frac{i}{\hbar c} \int_n \Phi^{(-)} d_\nu \Phi^{(+)} d\sigma_\nu$ is the operator of the meson number. Similarly

$$< \sigma | \Phi(x) | \sigma > = \sum_n{}' (i)^A \int_\sigma \dots \int_\sigma \left(\frac{i}{\hbar c}\right)^n \int_\sigma \dots \int_\sigma (n+1)^{1/2} < \sigma | \overline{x^1, \dots x^A}; x, y^1, \dots y^n >$$
$$\cdot \prod^A \gamma_\nu^{(i)} d\sigma_{\nu i}(x^i) \prod^n d_{\mu j} d\sigma_{\mu j}(y^j) < x^1, \dots x^A; y^1, \dots y^n | \sigma > +$$
$$+ \sum_n{}' (i)^A \int_\sigma \dots \int_\sigma \left(\frac{i}{\hbar c}\right)^{n-1} \int_\sigma \dots \int_\sigma n^{1/2} < \sigma | \overline{x^1, \dots x^A}; y^2, \dots y^n >$$
$$\prod^A \gamma_{\nu i}^{(i)} d\sigma_{\nu i}(x^i) \prod_{j=2}^n d_{\mu j} d\sigma_{\mu j}(y^j) < x^1, \dots x^A; x, y^2, \dots y^n | \sigma > . \tag{11}$$

The normalization condition is now

$$< \sigma | \sigma > = 1 = \sum_n{}' (i)^A \int_\sigma \dots \int_\sigma \left(\frac{i}{\hbar c}\right)^n \int_\sigma \dots \int_\sigma < \sigma | \overline{x^1, \dots x^A}; y^1, \dots y^n >$$
$$\prod^A \gamma_{\nu i}^{(i)} d\sigma_{\nu i}(x^i) \prod^n d_{\mu j} d\sigma_{\mu j}(y^j) < x^1, \dots x^A; y^1, \dots y^n | \sigma > . \tag{12}$$

In the above expression σ is not affected by the differentiation. Later when going over to the plane $\sigma \to t =$ const., the substitutions

$$d_4 \to 2(-\Delta + \mu^2)^{1/2}$$
$$-i\hbar \partial_4 \to i\hbar \gamma_4 \gamma_i \partial_i + i\hbar \gamma_4 \varkappa$$
$$-i\hbar \partial_4 \to i\hbar(-\Delta + \mu^2)^{1/2} \tag{13}$$

may therefore be introduced to advantage. According to (7) and (8), because ψ and Φ are operators in the interaction picture, these are permissible.

a) *The intermediate coupling theory without considering the nucleon recoil*

Disregarding the pair creation our equations are yet exact. In the following a single real nucleon is investigated and we assume

$$< x; y^1, \dots y^n | \sigma > = C_n(\sigma) \varphi(x) \prod^n f(y^i) . \tag{14}$$

The functions φ and f may also depend on σ, which can be chosen arbitrarily, however, we do not especially denote this dependence. Substituting (14) into (9) we obtain

$$< \sigma | P_\mu[\sigma] | \sigma > = \sum_n{}' C_n^*[\sigma] C_n[\sigma] \{p_\mu + n k_\mu\} +$$
$$+ \sum_n{}' (n+1)^{1/2} C_n^*[\sigma] C_{n+1}[\sigma] \alpha_\mu +$$
$$+ \sum_n{}' n^{1/2} C_n^*[\sigma] C_{n-1}[\sigma] \beta_\mu , \tag{15}$$

where

$$p_\mu = i \int_\sigma \bar{\varphi}\, \gamma_\nu (-i\hbar)\, \partial_\mu \varphi\, d\sigma_\nu\,, \qquad i \int_\sigma \bar{\varphi}\, \gamma_\nu\, \varphi\, d\, \sigma_\nu = 1$$

$$k_\mu = \frac{i}{\hbar c} \int_\sigma f^* d_\nu (-i\hbar)\, \partial_\mu f d_\nu\,, \quad \frac{i}{\hbar c} \int_\sigma f^* d_\nu f d\, \sigma_\nu = 1$$

$$a_\mu = -\frac{g}{c} \int_\sigma d\, \sigma_\mu \bar{\varphi}\, \varphi f, \qquad \beta_\mu = -\frac{g}{c} \int_\sigma d\, \sigma_\mu \bar{\varphi}\, \varphi f^*\,. \tag{16}$$

The normalization condition $< \sigma \mid \sigma > = 1$ is fulfilled if

$$\sum C_n^* C_n = 1\,. \tag{17}$$

We now go over to the plane $t =$ const. and change denotation, so that x, $y \ldots$ etc. are now the vectors of the threedimensional space; $dx = dx_1 dx_2 dx_3$. Besides, the mean values of operators at an arbitrarily chosen time will be denoted instead of by $< t \mid \Omega \mid t >$ frequently by $<\Omega>$. Neither will the dependence on time following from the transition $\sigma \to t$ in (14) be denoted.

Taking into account the condition (17) and varying $-\, i c < P_4 >$ with respect to C_n we obtain

$$C_n \{E + n\, \varepsilon - W\} + C_{n+1} (n+1)^{1/2} a + C_{n-1} n^{1/2} a^* = 0\,, \tag{18}$$

where

$$E = \frac{c}{i} p_4 = \int \bar{\varphi}\, (\hbar c\, \gamma_i \partial_i + M c^2)\, \varphi\, d\, x\,,$$

$$\varepsilon = \frac{c}{i} k_4 = 2 \int f^* (-\Delta + \mu^2) f d\, y\,,$$

$$a = \frac{c}{i} a_4 = g \int \bar{\varphi}\, \varphi f d\, x\,. \tag{19}$$

Transforming equation (18) according to the method of GLAUBER and LUTTINGER [3] to the problem of the harmonical oscillator, it can be easily solved. The solution is

$$W^{(\nu)} = E + \nu\, \varepsilon - \frac{a\, a^*}{\varepsilon}\,, \tag{20}$$

$$C_n^{(\nu)} = e^{-\frac{a a^*}{2\varepsilon^2}} \sum_{l=0}^{\nu} \frac{(\nu!)^{1/2} (n!)^{1/2} (-1)^{n-\nu+l}}{l!\, (\nu - l)!\, (n - \nu + l)!} \left(\frac{a^*}{\varepsilon} \right)^{n-\nu+l}\,. \tag{21}$$

With the aid of the solution thus obtained the mean value of the meson number operator (10) can be calculated in the ν-th state. From (14), (16) and (21) we obtain

$$\langle M \rangle^{(\nu)} = \frac{a a^*}{\varepsilon^2} + \nu . \tag{22}$$

Now we determine the meson amplitude in the ground state ($\nu = 0$). For this we vary $W^{(0)}$ with respect to f. The condition (16) need not be taken into consideration when varying, because f is determined by (20) except for an indefinite constant. Varying (20) we obtain

$$\bar{\varphi} \varphi - \frac{\int \bar{\varphi} \varphi f^* \, d x}{\int f^* (-\Delta + \mu^2) f \, d x} (-\Delta + \mu^2) f = 0 , \tag{23}$$

the solution of which is

$$f(x) = a (-\Delta + \mu^2)^{-1} \, \bar{\varphi}(x) \, \varphi(x) . \tag{24}$$

From here we can already calculate the mean value of the meson potential in the ground state. Substituting into (11) and (16) the quantities determined above, we finally obtain

$$\langle \Phi(x) \rangle = -g (-\Delta + \mu^2)^{-1} \, \bar{\varphi}(x) \, \varphi(x) \tag{25}$$

as expected. Using (24) the mean value of the meson number operator in the ground state according to (22) is

$$\langle M \rangle = \frac{g^2}{2 \hbar c} \int \bar{\varphi} \varphi (-\Delta + \mu^2)^{-\frac{3}{2}} \, \bar{\varphi} \varphi \, d x . \tag{26}$$

The energy of the fields from (20) and (24) is thus

$$W^{(0)} = E - \frac{g^2}{2} \int \bar{\varphi} \varphi (-\Delta + \mu^2)^{-1} \, \bar{\varphi} \varphi \, d x . \tag{27}$$

Finally minimizing this with respect to φ and taking into account the normalization condition (16) referring to φ we obtain that the energy of the fields is minimal, assuming

$$(\hbar c \gamma_4 \gamma_i \partial_i + \gamma_4 M c^2 - g^2 \gamma_4 (-\Delta + \mu^2)^{-1} \, \bar{\varphi} \varphi) \varphi = \lambda \varphi . \tag{28}$$

According to (7) here and further on for similar equations only solutions giving positive frequency are to be taken into consideration.

As regards the interpretation of the above formulae the following idea is due to G. HEBER [12], [13], [20], [21]. It seems to be clear at once from (27) — at least qualitatively — that for some suitable g, W has a minimum in case of a concentrated φ packet. This has of course to be determined from (28). Thus the following idea may be formed about the real nucleon: each real nucleon consists of a core concentrated into a small volume which is swarmed around by mesons. When the nucleons are treated in other calculations as plane waves, then these plane waves have nothing to do with the present φ — this is always concentrated into a small volume — but it describes simply the centre of mass of the φ packet.

The working out of the qualitative picture requires naturally detailed calculations. The result of such calculations does not seem very convincing. Ensurance of the nucleon concentration requires an unusually high value of g and on the other hand as was shown later by HEBER the recoil of the nucleon also counteracts concentration (see further below).

The other more common possibility is the renormalization. Indeed, the interaction Hamiltonian [3] completed by the term $-\delta Mc^2\bar{\psi}\psi$ can be easily checked to give back formulae (24)—(26) unchanged, expression (27), however, is modified

$$W^{(0)'} = E - \frac{g^2}{2} \int \bar{\varphi}\varphi(-\Delta + \mu^2)^{-1}\bar{\varphi}\varphi\, dx - \delta M c^2 \int \bar{\varphi}\varphi\, dx, \qquad (27')$$

where now in E the experimentally observed mass occurs. Assuming

$$\delta M = -\frac{g^2}{2c^2} \int \bar{\varphi}\varphi(-\Delta + \mu^2)^{-1}\bar{\varphi}\varphi\, dx \Big/ \int \bar{\varphi}\varphi\, dx \qquad (29)$$

the energy of the fields is just E, which has a minimum if φ satisfies the energy eigenvalue equation

$$(\hbar c \gamma_4 \gamma_i \partial_i + \gamma_4 M c^2)\varphi = \lambda\varphi \qquad (28')$$

containing now already the real mass.

The mass correction δM, however, depends strongly on the form of the bare nucleon, showing that our solution is not exact, the trial function (14) is too simple. Nevertheless accepting the normalization as an approximation, according to the foregoing we may form the following picture about the real nucleon in a coordinate system moving with the nucleon. According to (20) and (27) the total energy of the field is Mc^2, where M is the real mass of the nucleon. According to (26) if the state function φ of the bare nucleon is normalized to the volume V the mean value of the meson number is $g^2/2\hbar c\,\mu^3 V$. (25) gives the mean value of the meson potential as $-g/\mu^2 V$. The mean value of the energy of the mesons surrounding the bare nucleon is from (16), (19) and (24) mc^2, their momenta are zero. From (15) follows that the mean value of the momentum of the field is also zero and from (29) that the mass correction is $\delta M = -g^2/2c^2 V \mu^2$.

* * *

Our method may be applied without encountering difficulties to the case of many nucleons as well. For the amplitudes, similarly to (14) we now assume:

$$\langle x^1, \ldots x^A ; y^1, \ldots y^n \,|\, t \rangle = C_n \varphi (x^1, \ldots x^A) \prod^{n} f(y^i) . \tag{30}$$

Owing to (7) φ must be antisymmetric. Repeating our calculations with the trial function (30) instead of (24) we obtain

$$f(x) = a\,(-\Delta + \mu^2)^{-1} A\, \bar{\varphi}(x)\, \varphi(x) ,$$

$$\bar{\varphi}(x)\, \varphi(x) = \overbrace{\int \ldots \int}^{A-1} \bar{\varphi}(x, x^2, \ldots x^A)\, \gamma_4^{(2)}\, d\,x^2 \ldots \gamma_4^{(A)}\, d\,x^A\, \varphi(x, x^2, \ldots x^A) \tag{31}$$

from which the mean value of the meson number operator becomes

$$\langle M \rangle = \frac{g^2 A^2}{2 \hbar c} \int \bar{\varphi} \varphi\, (-\Delta + \mu^2)^{-\frac{3}{2}}\, \bar{\varphi} \varphi\, d\,x \tag{32}$$

and the mean value of the energy of the field

$$W^{(0)} = A\,E - \frac{g^2 A^2}{2} \int \bar{\varphi} \varphi\, (-\Delta + \mu^2)^{-1}\, \bar{\varphi} \varphi\, d\,x . \tag{33}$$

For atomic nuclei in zeroth approximation $\bar{\varphi}\varphi = 1/V = 3/4 r_0^3\, \pi\, A$. In this case W^0 is indeed proportional to A and the avarage meson number (32) becomes.

$$\langle M \rangle = \frac{g^2}{2 \hbar c} \, \frac{A^2}{V \mu^3} \approx \frac{g^2}{4 \pi \hbar c} A \quad \text{if} \quad \mu = \mu_\pi .$$

b) *The intermediate coupling theory considering the nucleon recoil*

Now the recoil of the nuclon will be considered. This may be done remaining within the framework of the method, by the modification of the trial function (14). Considering the recoil be now

$$\langle x ; y^1, \ldots y^n \,|\, t \rangle = C_n\, \varphi(x) \prod^{n} f(y^i - x) . \tag{34}$$

Taking into account the normalization conditions

$$\sum{}' C_n^* C_n = 1 , \quad \int \bar{\varphi} \gamma_4 \varphi\, d\,x = 1 , \quad \frac{2}{\hbar c} \int f^* (-\Delta + \mu^2)^{1/2} f\, d\,x = 1 \tag{35}$$

[compare equ. (16)] and calculating the mean value of the momentum of the fields from (9) we obtain

$$\langle P_i \rangle = \int \bar{\varphi} \gamma_4 (-i \hbar)\, \partial_i \varphi\, d\,x . \tag{36}$$

i.e. while according to (15) the total momentum of the field depends also on the momenta of the mesons, here the mean value of the total momentum of the field is determined only by the bare nucleon. Hence we may say that when a meson is emitted the momentum of the bare nucleon decreases to just the necessary extent (see Appendix).

Let us calculate the mean value of the energy. Using (9), (34) and (35) we obtain

$$\frac{c}{i}\langle P_4\rangle = \sum_n{}' C_n^* C_n (E + n(\varepsilon - \beta)) + \sum_n{}' C_n^* C_{n+1}(n+1)^{1/2} a + \sum_n{}' C_n^* C_{n-1} n^{1/2} a^* , \tag{37}$$

where [compare (19)]

$$\begin{aligned} E &= \int \bar{\varphi}(\hbar c \gamma_i \partial_i + M c^2)\varphi \, d x , \\ \varepsilon &= 2 \int f^* (-\Delta + \mu^2) f \, d x , \\ a &= g f(0) \int \bar{\varphi}\varphi \, d x , \\ \beta &= \hbar c \int \bar{\varphi}(x)\gamma_j \varphi(x) \, d x \int f^*(y) \partial_j f(y) \, d y = - i \hbar \mathfrak{i} \int f^* \nabla f \, d y . \end{aligned} \tag{38}$$

Repeating the calculations carried out in the first part we obtain successively

$$W^{(\nu)} = E + \nu(\varepsilon - \beta) - \frac{a a^*}{\varepsilon - \beta} , \tag{39}$$

$$C_n^{(\nu)} = e^{-\frac{a a^*}{2(\varepsilon-\beta)^2}} \sum_{l=0}^{\nu}{}' \frac{(\nu!)^{1/2}(n!)^{1/2}(-1)^{n-\nu+l}}{l!(\nu-l)!(n-\nu+l)!} \left(\frac{a^*}{\varepsilon-\beta}\right)^{n-\nu+l} , \tag{40}$$

$$\langle M \rangle^{(\nu)} = \frac{a a^*}{(\varepsilon-\beta)^2} + \nu . \tag{41}$$

Varying (39) with respect to f we obtain in the ground state

$$f(x) = a \left(-\Delta + \mu^2 + \frac{i\hbar}{2} \mathfrak{i} . \nabla \right)^{-1} \delta(x) . \tag{42}$$

In the following only nucleons at rest will be dealt with ($\mathfrak{i} = 0$). From (39) and (42) the total energy of the field is obtained as

$$W^{(0)} = E - \frac{g^2}{2} \left(\int \bar{\varphi}\varphi \, d x \right)^2 \int \delta(x)(-\Delta + \mu^2)^{-1} \delta(x) \, d x . \tag{43}$$

Completing the interaction Hamiltonian (3) for the sake of mass renormalization the second term of (43) can again be made to vanish if

$$\delta M = -\frac{g^2}{2c^2} \int \bar{\varphi} \varphi \, d x \int \delta(x) \left(-\Delta + \mu^2\right)^{-1} \delta(x) \, d x . \tag{44}$$

Because δM is yet weakly depending on the form of the bare nucleon, the solution is exact only for particles of infinite masses, in other cases it is an approximation.

The mean value of the meson potential from (11) and (42) is thus

$$\langle \Phi(x) \rangle = -g \int \bar{\varphi} \varphi \, d x \left(-\Delta + \mu^2\right)^{-1} \bar{\varphi}(x) \gamma_4 \varphi(x) \tag{45}$$

and the mean value of the meson number

$$\langle M \rangle = \frac{g^2}{2 \hbar c} \left(\int \bar{\varphi} \varphi \, d x\right)^2 \int \delta(x) \left(-\Delta + \mu^2\right)^{-\frac{3}{2}} \delta(x) \, d x . \tag{46}$$

Summarizing the results : after renormalization in case of a real nucleon in a coordinate system moving with the nucleon the total energy of the field is Mc^2 and its momentum zero. The mean value of the meson potential is $-g/\mu^2 V$ and that of the meson number infinite.

For the determination of the meson-mode belonging to the ν-th excited state of the nucleon we obtain from (39) ($i = 0$) :

$$f(x) = \left[\left(\frac{2 \varepsilon \nu}{a g} + \frac{2 a^*}{\varepsilon g}\right) \left(-\Delta + \mu^2\right) + \frac{2 \lambda \varepsilon}{\hbar c a g} \left(-\Delta + \mu^2\right)^{1/2}\right]^{-1} \delta(x) ,$$

where λ is the Lagrange factor belonging to the normalization condition referring to f. In case of strong coupling the present f agrees with (42), namely then only the second term of the bracket remains ($a \sim g$).

Let us finally examine the energy distribution in the environment of a real nucleon in the ground state. Let us thus determine the mean value of the energy density:

$$\varrho = : \frac{1}{2} \left[\nabla \Phi \nabla \Phi - \partial_4 \Phi \partial_4 \Phi + \mu^2 \Phi^2\right] + \hbar c \bar{\psi} \left(\gamma_i \partial_i + \varkappa\right) \psi + g \bar{\psi} \psi \Phi - - \delta M c^2 \bar{\psi} \psi :$$

Similarly to the preceding methods we obtain with the aid of the state vector determined before

$$\begin{aligned} \langle \varrho \rangle = {} & \frac{g^2}{2} \left(\int \bar{\varphi} \varphi \, d x\right)^2 \int \bar{\varphi}(x^1) \gamma_4 \varphi(x^1) \Big\{ \nabla \left(-\Delta + \mu^2\right)^{-1} \delta(x - x^1) \nabla \left(-\Delta + \right. \\ & \left. + \mu^2\right)^{-1} \delta(x - x^1) + \mu^2 \left(-\Delta + \mu^2\right)^{-1} \delta(x - x^1) \left(-\Delta + \mu^2\right)^{-1} \delta(x - x^1) \Big\} d x^1 + \\ & + \hbar c \bar{\varphi}(x) \left(\gamma_i \partial_i + \varkappa\right) \varphi(x) - \frac{g^2}{2} \int \bar{\varphi} \varphi \, d x \cdot \bar{\varphi}(x) \varphi(x) \left(-\Delta' + \mu^2\right)^{-1} \delta(x') \big|_{x'=0} - \\ & - \frac{g^2}{2} \int \bar{\varphi} \varphi \, d x \, \bar{\varphi}(x) \varphi(x) \left(-\Delta' + \mu^2\right)^{-1} \delta(x') \big|_{x'=0} - \delta M c^2 \bar{\varphi}(x) \varphi(x) . \end{aligned} \tag{47}$$

According to (44) the last two terms just cancel each other. It can be similarly seen, that forming $\int \varrho\, d\, x$, the first and third terms also become zero. Compensation of these terms, however, does not takes place locally, thus finally

$$\langle \varrho(x) \rangle = \hbar c \bar{\varphi}(x)(\gamma_i \partial_i + \varkappa)\varphi(x) + \frac{g^2}{2}\int \bar{\varphi}\varphi\, d\, x \Big\{ \int \bar{\varphi}\varphi\, d\, x \int \bar{\varphi}(x^1)\gamma_4 \varphi(x^1)$$

$$\left[\left(\mu + \frac{1}{|x - x^1|}\right)^2 + \mu^2\right] \cdot (-\Delta + \mu^2)^{-1}\delta(x - x^1)(-\Delta + \mu^2)^{-1}\delta(x - x^1)\, dx^1 -$$

$$- \bar{\varphi}(x)\varphi(x)(-\Delta' + \mu^2)^{-1}\delta(x')\,|_{x'=0}\Big\}\,.$$

Hence the energy density depends — with the exception of g — only on the form of the bare nucleon. If this is prescribed, then from the above equation the energy density can be determined.

Earlier BHABHA [30] carried out calculations concerning the theory of cosmic showers assuming, that the energy of the nucleon is concentrated into two regions, in an internal region of the order of a nucleon Compton wave lenght and an external meson region of the order of a meson Compton wave length in the proportion of $(1-\varepsilon)Mc^2$ resp. εMc^2. Let us examine now how much energy falls — according to our calculations — in the case of our present model into the individual regions. Let us suppose as an approximation that the bare nucleon is pointlike: $\bar{\varphi}(x)\gamma_4\varphi(x) = \delta(x)$ and $\int \bar{\varphi}\varphi dx = 1$. We then obtain a lower limit for the energy falling into the external region. Thus (m is the π meson mass, $g = 5e$, $M = 6{,}8\ m$):

$$\int_{x^2 \geq \left(\frac{\hbar}{Mc}\right)^2} < \varrho(x) > d\, x = \frac{g^2}{4\pi\hbar c}\,\frac{e^{-\frac{2m}{M}}}{2}(m + M)\, c^2 \approx \frac{g^2}{4\pi\hbar c}\, 0{,}43\, M\, c^2 =$$

$$= 0{,}08\, M\, c^2\,.$$

* * *

Let us deal now with the many-nucleon problem. As a generalization of (34) let us take the trial function in the following form:

$$\langle x^1, \ldots x^A; y^1, \ldots y^n | t > = C_n \varphi(x^1, \ldots x^A) \prod^{n} \left\{ \sum_{j=1}^{A} f(y^i - x^j) \right\}. \quad (48)$$

Here according to (7) φ is antisymmetric. The normalization condition (12) is now

$$\sum C_n^* C_n\, l(n) = 1\,,$$
$$l(n) = \int \ldots \int B(x^1, \ldots x^A)^n\, \bar{\varphi}(x^1, \ldots x^A) \prod^{A} \gamma_4^{(i)}\, d\, x^{(i)}\, \varphi(x^1, \ldots x^A)\,,$$
$$l(0) = 1\,, \quad (49)$$

where

$$B(x^1, \dots x^A) = \frac{2}{\hbar c}\Big[A \int f^*(y)(-\Delta + \mu^2)^{1/2} f(y)\, dy +$$

$$+ \sum_{\substack{i,j \\ i \neq j}} \int f^*(y - x^i)(-\Delta + \mu)^{1/2} f(y - x^j)\, dy\Big] = D + b(x^1, \dots x^A). \quad (50)$$

According to (9) the mean value of the total momentum of the field is

$$\langle P_i \rangle = \sum_n{}' C_n^* C_n \int \dots \int B(x^1, \dots x^A)^n \bar{\varphi}(x^1, \dots x^A)$$

$$\prod^A \gamma_4^{(i)} d x^{(i)} \frac{\hbar}{i} \sum^A{}' \frac{\partial}{\partial x_i} \varphi(x^1, \dots x^A). \quad (51)$$

Let us determine the mean value of the energy. From (10) we obtain

$$\frac{c}{i}\langle P_4 \rangle = \sum_n{}' C_n^* C_n \big(A E(n) + n\, \varepsilon(n)\big) + \sum_n{}' C_n^* C_{n+1} (n+1)^{1/2} a(n) +$$
$$+ \sum_n{}' C_n^* C_{n-1} n^{1/2} a^*(n-1)$$

$$E(n) = \int \dots \int B(x^1, \dots x^A)^n \varphi(x^1, \dots x^A) \prod^A \gamma_4^{(i)} d x^{(i)}$$
$$(\hbar c \gamma_4^{(1)} \gamma_i^{(1)} \partial_i + \gamma_4^{(1)} M c^2) \varphi(x^1, \dots x^A)$$

$$\varepsilon(n) = 2 \int \dots \int B(x^1, \dots x^A)^{n-1} \bar{\varphi}(x^1, \dots x^A) \prod^A \gamma_4^{(i)} d x^{(i)} \varphi(x^1, \dots x^A)$$
$$\sum_{i,j}{}' \int f^*(y - x^i)(-\Delta + \mu^2) f(y - x^j)\, dy$$

$$a(n) = A g \int \dots \int dx\, B(x, x^2, \dots x^A)^n \bar{\varphi}(x, x^2, \dots x^A) \prod_{i=2}^A \gamma_4^{(i)} dx^{(i)} \varphi(x, x^2, \dots x^A)$$

$$\cdot \left(\sum_{j=2}^A f(x - x^j) + f(0)\right), \quad (52)$$

where the terms corresponding to β of the expression (37) were omitted. Considering the normalization condition (49) and varying we obtain

$$C_n\left(A \frac{E(n)}{l(n)} + n \frac{\varepsilon(n)}{l(n)} - W\right) + C_{n+1}(n+1)^{1/2} \frac{a(n)}{l(n)} + C_{n-1} n^{1/2} \frac{a^*(n-1)}{l(n)} = 0. \quad (53)$$

Now the coefficients of the C_n-s depend yet on n. The coefficients will all be independent of n, if it is assumed that when substituting (50) into any integral of (52) b may be neglected against D. Further be f normalized: $D = 1$. Thus

$$\frac{E(n)}{l(n)} \to E = \int \ldots \int \bar{\varphi}(x^1, \ldots x^A) \prod^A \gamma_4^{(i)} d\,x^{(i)} (\hbar\, c\, \gamma_4^{(1)} \gamma_i^{(1)} \partial_i + \gamma_4^{(1)} M\, c^2)\, \varphi(x^1, \ldots x^A),$$

$$\frac{\varepsilon(n)}{l(n)} \to \varepsilon = 2 \int \ldots \int \bar{\varphi}(x^1, \ldots x^A) \prod^A \gamma_4^{(i)} d\,x^{(i)} \varphi(x^1, \ldots x^A)$$
$$\sum_{i,j}' f^*(y - x^i)\,(-\Delta + \mu^2) f(y - x^j)\, d\,y,$$

$$\frac{a(n)}{l(n)} \to a = A\, g \int \ldots \int d\,x^1\, \bar{\varphi}(x^1, \ldots x^A) \prod_{i=2}^{A} \gamma_4^{(i)} d\,x^{(i)} \varphi(x^1, \ldots x^A)$$
$$\left(\sum_{j=2}^{A} f(x^1 - x^j) + f(0)\right), \qquad \frac{a^*(n-1)}{l(n)} \to a^*. \tag{54}$$

Using these expressions the solution of equation (53) is as before

$$W^{(\nu)} = A\, E - \frac{a\, a^*}{\varepsilon} + \nu\, \varepsilon,$$
$$C_n^{(\nu)} = e^{-\frac{a a^*}{2\varepsilon^2}} \sum_{l=0}^{\nu} \frac{(\nu!)^{1/2} (n!)^{1/2} (-1)^{n-\nu+l}}{l!\,(\nu - l)!\,(n - \nu + l)!} \left(\frac{a^*}{\varepsilon}\right)^{n-\nu+l}. \tag{55}$$

f can be again determined by the variation of W. Thus the solution is for the ground state

$$f(x) = a\,(-\Delta + \mu^2)^{-1}\,\delta(x),$$

if only $\bar{\varphi}\gamma_4 \approx \bar{\varphi}$

Using this the total energy of the field becomes

$$W^{(0)} = A\, E - \frac{g^2}{2} A \int \ldots \int d\,x\, \bar{\varphi}(x, x^2, \ldots x^A) \prod_{i=2}^{A} \gamma_4^{(i)} d\,x^{(i)} \varphi(x, x^2, \ldots x^A)$$
$$\sum_{j=2}^{A} (-\Delta + \mu^2)^{-1}\, \delta(x - x^j), \tag{56}$$

here the self-energies were left out. Finally varying $W^{(0)}$ with respect to φ we obtain, that the total energy of the field is minimal, if

$$\sum_j \left[\hbar\, c\, \gamma_4^{(j)} \gamma_i^{(j)} \partial_i^{(j)} + \gamma_4^{(i)} \left(M\, c^2 - \frac{g^2}{2} (-\Delta^{(j)} + \mu^2)^{-1} \sum_{\substack{i,j \\ i \neq j}} \delta(x^j - x^i)\right)\right] \varphi(x^1, \ldots x^A) =$$
$$= \lambda\, \varphi(x^1, \ldots x^A), \tag{57}$$

where λ is the Lagrangian factor belonging to the normalization condition (49) $[l(0) = 1]$. In the equation resulting from iterating (57) as it has been shown by G. MARX and G. SZAMOSI [32], [33] relativistical repulsive and many-body forces occur which might result the saturation.

The mean value of the meson number from (11), (48), (49) and (55) is

$$\langle M \rangle = \frac{a^* a}{\varepsilon^2} = \frac{g^2 A}{2 \hbar c} \int \delta(y) (-\Delta + \mu^2)^{-3/2} \delta(y) \, dy \simeq A \langle M \rangle_{\text{one nucleon}} ,$$

hence our approximation used in this part means essentially the omission of the mesons giving rise to the interaction between the individual nucleons, against those belonging to the self-fields of the nucleons.

According to the foregoing the divergent self-energies can be approximately eliminated by mass renormalization, other quantities, however, remain divergent. For the elimination of these the usual method is the cutting-off method. This can be explained according to the considerations of part *a)* by attributing to the bare nucleon a finite extension. As against this from the calculations performed taking into consideration the recoil, it is evident that if the cut-off is to be justified, (3) cannot be of general validity.

Thus we obtain from (46) for a nucleon at rest if the cutting-off is carried out at a value $\delta\mu = \varkappa \approx 6{,}8 \; \mu$ with $g = 5e$

$$\langle M \rangle = 0{,}09 .$$

2. *The nucleon and the symmetrical pseudoscalar meson field*

In our calculations here we consider only the recoil of the nucleon, simplest calculations not taking into account the recoil can be carried out similarly. Let us first of all consider the mean value of $\langle \sigma | P_\mu [\sigma] | \sigma \rangle$.

We obtain from (1), (4) and (6) similarly to (9)

$$\langle \sigma | P_\mu [\sigma] | \sigma \rangle = i \int_\sigma \sum \left(\frac{i}{\hbar c} \right)^{n^+ + n^- + n^3} \int \cdots \int \langle \sigma | \bar{x}; \xi^1 \ldots \xi^{n^+}; \eta^1, \ldots \eta^{n^-}; \zeta^1, \ldots \zeta^{n^3} \rangle$$

$$\cdot \prod^{n^+} d_{\nu_i} \, d\sigma_{\nu i}(\xi^i) \prod^{n^-} d_{\nu i} \, d\sigma_{\nu i}(\eta^i) \prod^{n^3} d_{\nu_k} \, d\sigma_{\nu k}(\zeta^k) \, \gamma_\nu \, d\sigma_\nu(x) .$$

$$\frac{\hbar}{i} \left\{ \frac{\partial}{\partial x_\mu} + \sum^{n^+} \frac{\partial}{\partial \xi^i_\mu} + \sum^{n^-} \frac{\partial}{\partial \eta^j_\mu} + \sum^{n^3} \frac{\partial}{\partial \zeta^k_\mu} \right\} \langle x; \xi^1, \ldots \xi^{n^+}; \eta^1, \ldots \eta^{n^-}; \zeta^1, \ldots \zeta^{n^3} | \sigma \rangle -$$

$$- \frac{i g}{2 \varkappa c} \int_\sigma d\sigma_\mu(x) \sum \left(\frac{i}{\hbar c} \right)^{n^+ + n^- + n^3} \int \cdots \int \langle \sigma | x; \xi^1, \ldots \xi^{n^+}; \eta^1, \ldots \eta^{n^-}; \zeta^1, \ldots \zeta^{n^3} \rangle$$

$$\prod^{n^+} d_{\nu i} \, d\sigma_{\nu i}(\xi^i) \prod^{n^-} d_{\nu_j} \, d\sigma_{\nu j}(\eta^j) \prod^{n^3} d_{\nu_k} \, d\sigma_{\nu k}(\zeta^k) \, \gamma_\varrho \, \gamma_5 \, \frac{\partial^{(!)}}{\partial x_\varrho} \{ \sqrt{2} \, \tau_- (n^+ - 1)^{1/2} \cdot$$

$$\cdot \langle x; x, \xi^1, \ldots \xi^{n^+}; \eta^1, \ldots \eta^{n^-}; \zeta^1, \ldots \zeta^{n^3} | \sigma \rangle + \sqrt{2}\, \tau_+ (n^- - 1)^{1/2} \cdot$$
$$\cdot \langle x; \xi^1, \ldots \xi^{n^+}; x, \eta^1, \ldots \eta^{n^-}; \zeta^1, \ldots \zeta^{n^3} | \sigma \rangle + \tau_3 (n^3 - 1)^{1/2} \cdot$$
$$\cdot \langle x; \xi^1, \ldots \xi^{n^+}; \eta^1, \ldots \eta^{n^-}; x, \zeta^1, \ldots \zeta^{n^3} | \sigma \rangle \} + c.c. -$$
$$- \lambda \frac{g^2}{2 M c^2} 2 \int_\sigma d\sigma_\mu (x) \sum \left(\frac{i}{\hbar c} \right)^{n^+ + n^- + n^3 - 1} \int_\sigma \ldots \int_\sigma \{ n^+$$
$$\langle \sigma | x; x, \xi^2, \ldots \xi^{n^+}; \eta^1, \ldots \eta^{n^-}; \zeta^1, \ldots \zeta^{n^3} \rangle$$
$$\prod_{i=2}^{n^+} d_{v_i}\, d\sigma_{vi} (\xi^i) \prod^{n^-} d\, v_j\, d\sigma_{vj} (\eta^j) \prod^{n^3} d_{vk}\, d\sigma_{vk} (\zeta^k) \cdot$$
$$\cdot \langle x; x, \xi^2, \ldots \xi^{n^+}; \eta^1, \ldots \eta^{n^-}; \zeta^1, \ldots \zeta^{n^3} | \sigma \rangle + \tag{58}$$

+ analoguous terms for n^- and n^3 + 2 mesons more +2 mesons less. Here the exclamation mark over $\frac{\partial}{\partial x_\varrho}$ indicates, that the succeeding amplitude should be differentiated with respect of the meson coordinate only. Further on the last two terms of (58) which are there not given explicitly are everywhere omitted, because they contribute nothing to the approximation to be dealt with below. In (58) x, ξ, η, ζ etc. are again four-dimensional vectors.

Before making an assumption concerning the trial functions, it is worth while to consider the following. A bare nucleon can be characterized by its parity, isotope spin and spin and these values remain the same for the real nucleon.

Earlier for a scalar field these requirements were fulfilled automatically, here in the choice of the trial function special attention has to be paid to these considerations. The only possibility to avoid the difficulties encountered in the calculation owing to this fact is not to allow around the real nucleon an arbitrary number of mesons.

Calculations were also carried out by taking into account several mesons [17]. To illustrate the configurational method we go only as far as the one-meson states, however, the recoil as well as the term of the form $\bar{\psi}\psi\Phi^2$ of the interaction energy are taken into account. Accordingly, only the following amplitudes can be chosen as differing from zero (again in the case of a t = const plane and by changing notation):

$$\langle x | t \rangle = C_0\, \varphi_{1/2,\,1/2} (x, 1/2, 1/2)$$
$$\langle x; \xi | t \rangle = C_1 \sqrt{\frac{2}{3}} \left\{ \sqrt{\frac{1}{3}} f_{1,1} (\xi - x, 1, 0)\, \varphi_{1/2,\,-1/2} (x, 1/2, 1/2) + \right.$$
$$\left. + \sqrt{\frac{2}{3}} f_{1,1} (\xi - x, 1, 1)\, \varphi_{1/2,\,-1/2} (x, 1/2, -1/2) \right\}$$

$$\langle x;\zeta\,|\,t\rangle = C_1\sqrt{\frac{1}{3}}\left\{\sqrt{\frac{1}{3}}\,f_{1,0}(\zeta-x,1,0)\,\varphi_{½,\,½}(x,{}^1/_2,{}^1/_2)+\right.$$
$$\left.+\sqrt{\frac{2}{3}}\,f_{1,0}(\zeta-x,1,1)\,\varphi_{½,\,½}(x,{}^1/_2,-{}^1/_2)\right\} \qquad (59)$$

for a proton and

$$\langle x\,|\,t\rangle = C_0\,\varphi_{½,\,-½}(x,{}^1/_2,{}^1/_2)$$

$$\langle x;\eta\,|\,t\rangle = C_1\sqrt{\frac{2}{3}}\left\{\sqrt{\frac{1}{3}}\,f_{1,-1}(\eta-x,1,0)\,\varphi_{½,\,½}(x,{}^1/_2,{}^1/_2)+\right.$$
$$\left.+\sqrt{\frac{2}{3}}\,f_{1,-1}(\eta-x,1,1)\,\varphi_{½,\,½}(x,{}^1/_2,-{}^1/_2)\right\}$$

$$\langle x;\zeta\,|\,t\rangle = C_1\sqrt{\frac{1}{3}}\left\{\sqrt{\frac{1}{3}}\,f_{1,0}(\zeta-x,1,0)\,\varphi_{½,\,-½}(x,{}^1/_2,{}^1/_2)+\right.$$
$$\left.+\sqrt{\frac{2}{3}}\,f_{1,0}(\zeta-x,1,1)\,\varphi_{½,\,-½}(x,{}^1/_2,-{}^1/_2)\right\} \qquad (60)$$

for a neutron. Here the outher indices of φ and f mean the isotope spin and its third component for the nucleon resp. the meson. By our choice the problems related to the isotope spin have been solved. The inner indices of φ refer to the angular momentum and its projection, and have in the usual representation of the γ-s the following form

$$\varphi_{½,\,½}(x,{}^1/_2,{}^1/_2) = \begin{pmatrix}\varphi_1\\0\\\varphi_2\\0\\0\\0\\0\\0\end{pmatrix}, \qquad \varphi_{½,\,½}(x,{}^1/_2,-{}^1/_2) = \begin{pmatrix}0\\-\varphi_1\\0\\-\varphi_2\\0\\0\\0\\0\end{pmatrix}$$

$\varphi_{½,\,-½}(½,½)$ and $\varphi_{½,\,-½}(½,-½)$ are the same, but their elements differ from zero at the lower 4 places. φ_1 and φ_2 are arbitrary spherical symmetric functions. Each φ is normalized to 1, and they are orthogonal to each other. For given φ_i, the values $\bar\varphi\varphi$, $\bar\varphi\gamma_4\varphi$ and $\int\bar\varphi\,(\hbar c\gamma_i\,\partial_i+Mc^2)\,\varphi\,dx$ are independent of the indices of the φ-s. This will be made use of later on. It is to be expected that the inner indices of the functions f will refer also to the angular momentum and its projection. For the moment let us consider them simply as distinguishing indices. Be the f-s normalized and the functions with different inner indices orthogonal to each other:

$$\frac{2}{\hbar c}\int f^*(-\Delta+\mu^2)^{½} f\,dx = 1\,,$$

$$\int f_{1,0}^{*}(x,1,0)\,(-\Delta+\mu^2)^{1/2} f_{1,0}(x,1,1)\,d\,x=0 \quad \text{etc.} \tag{61}$$

In this case the normalization condition $\langle t \mid t \rangle = 1$ is the following:

$$C_0^* C_0 + C_1^* C_1 = 1\,. \tag{62}$$

Substituting (59) into (58) and making use of what has been said about the φ-s and f-s we obtain

$$\frac{c}{i}\langle P_4 \rangle = C_0^* C_a E + C_1^* C_1 (E+\varepsilon) + C_0^* C_1 a + C_1^* C_0 a^* + C_1^* C_1 \gamma\,, \tag{63}$$

where

$$E = \int \bar{\varphi}\,(\hbar c \gamma_i \partial_i + M c^2)\,\varphi\, d\,x\,.$$

$$\varepsilon = 2\int\Big\{\frac{1}{3}\frac{1}{3} f_{1,0}^{*}(1,0)\,(-\Delta+\mu^2) f_{1,0}(1,0) + \frac{1}{3}\frac{2}{3} f_{1,0}^{*}(1,1)\,(-\Delta+\mu^2) f_{1,0}(1,1) + $$
$$+\frac{2}{3}\frac{1}{3} f_{1,1}^{*}(-\Delta+\mu^2) f_{1,1}(1,0) + \frac{2}{3}\frac{2}{3} f_{1,1}^{*}(1,1)\,(-\Delta+\mu^2) f_{1,1}(1,1)\Big\}\,d\,x\,,$$

$$a = \frac{g}{2\varkappa}\Big\{\sqrt{\frac{2}{3}}\sqrt{\frac{1}{3}}\,a_{1,1}(1,0)_i \int f_{1,1}(x,1,0)\,\partial_i\,\delta(x)\,d\,x +$$
$$+\sqrt{\frac{2}{3}}\sqrt{\frac{2}{3}}\,a_{1,1}(1,1)_i \int f_{1,1}(x,1,1)\,\partial_i\,\delta(x)\,d\,x +$$
$$+\sqrt{\frac{1}{3}}\sqrt{\frac{1}{3}}\,a_{1,0}(1,0)_i \int f_{1,0}(x,1,0)\,\partial_i\,\delta(x)\,d\,x +$$
$$+\sqrt{\frac{1}{3}}\sqrt{\frac{2}{3}}\,a_{1,0}(1,1)_i \int f_{1,0}(x,1,1)\,\partial_i\,\delta(x)\,d\,x\Big\}\,,$$

$$a_{1,1}(1,0)_i = \{0, 0, \sqrt{2}\}\,,$$
$$a_{1,1}(1,1)_i = \{\sqrt{2}\,, -i\sqrt{2}\,, 0\}\,,$$
$$a_{1,0}(1,0)_i = \{0, 0, 1\}\,,$$
$$a_{1,0}(1,1)_i = \{1, -i, 0\}\,,$$

$$\gamma = \lambda \frac{g^2}{2 M c^2}\, 2\, b\Big[\frac{2}{3}\frac{2}{3}\int f_{1,1}^{*}(x,1,1) f_{1,1}(x,1,1)\,\delta(x)\,d\,x +$$
$$+\frac{2}{3}\frac{1}{3}\int f_{1,1}^{*}(x,1,0) f_{1,1}(x,1,0)\,\delta(x)\,d\,x +$$

$$+\frac{1}{3}\frac{1}{3}\int f_{1,0}^{*}(x,1,0) f_{1,0}(x,1,0)\,\delta(x)\,dx +$$

$$+\frac{1}{3}\frac{2}{3}\int f_{1,0}^{*}(x,1,1) f_{1,0}(x,1,1)\,\delta(x)\,dx\Big],$$

$$b=\int \bar{\varphi}\varphi\,dx\,. \tag{64}$$

Here it has been used that $\int \bar{\varphi}\gamma_4\gamma_5\varphi\,dx \approx 0$, and also the term corresponding to the β of equation (37) has been omitted.

Varying (63) with respect to C^*, and considering (62) we obtain

$$\begin{aligned} &C_0(E-W)+C_1 a=0\,,\\ &C_0 a^* + C_1(E+\varepsilon+\gamma-W)=0\,. \end{aligned} \tag{65}$$

This system of equations has a nontrivial solution if *Det* $\neq 0$. From this the lower energy value is

$$W=E+\frac{\varepsilon+\gamma-(\varepsilon+\gamma)\sqrt{1+\dfrac{4\,a\,a^*}{(\varepsilon+\gamma)^2}}}{2}\approx E-\frac{aa^*}{\varepsilon+\gamma} \tag{66}$$

and the amplitudes belonging to this state

$$C_0=\frac{1}{\sqrt{1+\dfrac{a\,a^*}{(\gamma+\varepsilon)^2}}}\,,\qquad C_1=-\frac{a^*}{\varepsilon+\gamma}C_0\,. \tag{67}$$

From here it may be seen, that the approximation used in (66) for the extraction of the root does not make use of the small value of g, but of the fact, that the probability of single-meson states is small, compared with the bare-nucleon state. On the basis of the conclusions to be drawn from the preceding paragraph we may, however, hope that by permitting arbitrarily many mesons we would obtain essentially the same energy.

The determination of the f-s remains to be carried out. We determine the f-s also here from (66) by variation. The auxiliary conditions (61) should also be taken into account for the variation. However in our present approximation they are disregarded, although the f-s obtained as solution are to satisfy the conditions.

The solution of the set of equations obtained by variation is

$$f_{1,1}(x,1,0)=\frac{a}{\sqrt{2}}\left(-\Delta+\mu^2+\lambda\frac{g^2}{2\,M\,c^2}\,b\,\delta(x)\right)^{-1} a_{1,1}^{*}(1,0)_i\,\partial_i\,\delta(x)\,,$$

$$f_{1,1}(x,1,1) = \frac{a}{2}\left(-\Delta+\mu^2+\lambda\frac{g^2}{2Mc^2}b\,\delta(x)\right)^{-1} a^*_{1,1}(1,1)_i\,\partial_i\,\delta(x)\,,$$

$$f_{1,0}(x,1,0) = a\left(-\Delta+\mu^2+\lambda\frac{g^2}{2Mc^2}b\,\delta(x)\right)^{-1} a^*_{1,0}(1,0)_i\,\partial_i\,\delta(x)\,,$$

$$f_{1,0}(x,1,1) = \frac{a}{\sqrt{2}}\left(-\Delta+\mu^2+\lambda\frac{g^2}{2Mc^2}b\,\delta(x)\right)^{-1} a^*_{1,0}(1,1)_i\,\partial_i\,\delta(x)\,. \tag{68}$$

The normalization factor a can be determined from any f and we obtain always the same value. Similarly we may satisfy ourselves about the fact that the f-s of different inner indices are orthogonal.

Using (68) finally the energy of the field is in case of one proton

$$W = E - \left(\frac{g}{2\varkappa}\right)^2 \frac{3}{2}\int \partial_i\,\delta(x)\left(-\Delta+\mu^2+\lambda\frac{g^2}{2Mc^2}b\,\delta(x)\right)^{-1}\partial_i\,\delta(x)\,dx\,. \tag{69}$$

For a neutron the calculations can be carried out in the same way. Finally we receive back the functions (68) ($f_{1,1}\rightarrow f_{1,-1}$) and the energy (69).

Formulating the state vector of the total system from (6), (59), (60) and (68) it may be seen, that the determined state is the eigenstate of the total angular momentum and its projection, further on because the mesons are created in the p state also of the parity with correct eigenvalues.

In the present approximation the state vector characterizing the real nucleon has already been determined, so that now the value of an arbitrary operator characterizing the field can be determined. Below the magnetic momentum of the nucleon is calculated. The operator of the magnetic momentum is

$$\mathfrak{M} = :\frac{e\hbar}{2Mc}\int\bar{\psi}\,\gamma_4\frac{1+\tau_3}{2}\,\sigma\,\psi\,dx + \frac{ie}{2\hbar c}\int\left(\Phi\,[x,\nabla]\,\Phi^* - \Phi^*\,[x,\nabla]\,\Phi\right)dx: \tag{70}$$

From the obtained state functions we obtain the relation found by SACHS [45]

$$\langle\mathfrak{M}\rangle_P + \langle\mathfrak{M}\rangle_N = \frac{e\hbar}{2Mc}\left(1-\frac{4}{3}C_1^*C_1\right). \tag{71}$$

The numerical values of the magnetic momentum with a cutting off at $\delta\mu$ become in case of $\dfrac{g^2}{4\pi\hbar c} = 15$:

$$\langle \mathfrak{M} \rangle_P = 0{,}98 \qquad \langle \mathfrak{M} \rangle_N = -0{,}40 \qquad C_1^* C_1 = 0{,}32 \quad \text{if} \quad \delta = 4$$
$$= 1{,}04 \qquad = -0{,}29 \qquad = 0{,}19 \qquad = 3$$
$$= 1{,}04 \qquad = -0{,}14 \qquad = 0{,}08 \qquad = 2$$

In case of $\lambda = 0{,}2$, :

$$\langle \mathfrak{M} \rangle_P = 1{,}02 \qquad \langle \mathfrak{M} \rangle_N = -0{,}20 \qquad C_1^* C_1 = 0{,}14 \, , \qquad \delta = 3 \, .$$

These are in accordance with the earlier statements of SACHS: permitting only single meson states we obtain for the anomalous magnetic momentum of the nucleon wrong results. Taking into account the term of the interaction energy proportional to λ does not alter this fact either.

Let us determine now the electron charge distribution of the nucleon. Let us form with the determined state vector the mean value of the charge density-operator

$$\varrho (x) = : e \, \overline{\psi} \gamma_4 \frac{1 + \tau_3}{2} \psi + \frac{e}{\hbar c} \Phi^* d_4 \Phi :$$

Similarly to our other methods we obtain by considering what has been said about φ

$$\langle \varrho (x) \rangle_P = C_0^* C_0 e \, \overline{\varphi} (x) \gamma_4 \varphi (x) + \frac{1}{3} C_1^* C_1 \overline{\varphi} (x) \gamma_4 \varphi (x) + \frac{2}{3} C_1^* C_1 \frac{e}{\hbar c} \int \Big\{ \frac{1}{3} f_{1,1}^* (x - x^1, 1, 0) \, 2 \, (-\Delta + \mu^2)^{1/2} f_{1,1} (x - x^1, 1, 0) + \frac{2}{3} f_{1,1}^* (x - x^1, 1, 1) \cdot \cdot \, 2 \, (-\Delta + \mu^2)^{1/2} f_{1,1} (x - x^1, 1, 1) \Big\} \varphi (x^1) \gamma_4 \varphi (x^1) \, d x^1 ,$$

$$\langle \varrho (x) \rangle_N = \frac{2}{3} C_1^* C_1 e \, \overline{\varphi} (x) \gamma_4 \varphi (x) - \frac{2}{3} C_1^* C_1 \frac{e}{\hbar c} \int \Big\{ \frac{1}{3} f_{1,-1}^* (x - x^1, 1, 0) \cdot \cdot \, 2 \, (-\Delta + \mu^2)^{1/2} f_{1,-1} (x - x^1, 1, 0) + \frac{2}{3} f_{1,-1}^* (x - x^1, 1, 1) \cdot 2 \, (-\Delta + \mu^2)^{1/2} \cdot \cdot f_{1,-1} (x - x^1, 1, 1) \Big\} \overline{\varphi} (x^1) \gamma_4 \varphi (x^1) \, d x^1 . \tag{72}$$

From here making use of the fact that the φ-s with the same inner indices are identical functions, it can be read that the mesonic charge cloud of the real proton and neutron, — disregarding the sign — are the same.

$$\langle \varrho (x) \rangle_P + \langle \varrho (x) \rangle_N = e \, \overline{\varphi} (x) \gamma_4 \varphi (x) \, .$$

Finally from (72) in case of $\lambda = 0$ with the determined quantities the mesonic charge cloud becomes

$$e\, C_0^* C_0 \frac{g^2}{4\pi\hbar c} \frac{\mu}{8\pi^2 \varkappa^2} \int \left\{ \frac{\partial}{\partial x_i} \frac{e^{-\mu|x-x^1|}}{|x-\varkappa^1|} \frac{\partial}{\partial x_i} \frac{K_1(\mu|x-x^1|)}{|x-x^3|} \right\} \bar{\varphi}(x^1)\gamma_4 \varphi(x^1)\, dx^1 .$$

Let us finally calculate the mean value of the energy density of the field. With the aid of the determined state vector and in the approximation used in equation (66) we obtain that the energy density of the meson field is in the environment of the real nucleon

$$\langle \varrho^M(x) \rangle = \left(\frac{g}{2\varkappa}\right)^2 \frac{3}{2}\frac{1}{2} \int \bar{\varphi}(x^1)\gamma_4 \varphi(x^1) \{ \partial_i\, \partial_j (-\Delta + \mu^2)^{-1} \delta(x-x^1)$$
$$\partial_i\, \partial_j (-\Delta + \mu^2)^{-1} \delta(x-x^1) + \sqrt{-\Delta+\mu^2}\;\; \partial_j (-\Delta+\mu^2)^{-1}$$
$$\delta(x-x^1) \sqrt{-\Delta+\mu^2}\, \partial_j (-\Delta+\mu^2)^{-1} \delta(x-x^1) +$$
$$+ \mu^2 \partial_j (-\Delta+\mu^2)^{-1} \delta(x-x^1)\; \partial_j (-\Delta+\mu^2)^{-1} \delta(x-x^1) \} dx^1 . \qquad (73)$$

Indeed its integral over the whole volume agrees apart from the sign to the self-energy term of expression (69). In analogy with (47) it may however be assumed that taking into account the many-meson states the energy density of the field can be better approximated by the expression

$$\langle \varrho^M(x) \rangle = \left(\frac{g}{2x}\right)^2 \frac{3}{2} \int \bar{\varphi}(x^1)\gamma_4 \varphi(x^1) \{ \partial_i\, \partial_j (-\Delta+\mu^2)^{-1} \delta(x-x^1)\;\; \partial_i\, \partial_j \cdot$$
$$\cdot (-\Delta+\mu^2)^{-1} \delta(x-x^1) + \mu^2 \partial_j (-\Delta+\mu^2)^{-1} \delta(x-x^1)\, \partial_j (-\Delta+\mu^2)^{-1} \delta(x-x^1) \} dx^1 , \qquad (74)$$

the integral of which taken over the total volume agrees also with the second term of (69). From (74) in case of a point-like nucleon with $g^2/4\pi\hbar c = 15$ we obtain

$$\int\limits_{x^2 \geq \left(\frac{\hbar}{Mc}\right)^2} \langle \varrho(x) \rangle d x = \frac{g^2}{4\pi\hbar c} 0{,}99\, M c^2 = 14{,}8\, M c^2 ,$$

namely only the energy of the meson field extends to the considered part of the space. Since the total energy of the field is (neglecting the kinetic energy of the Dirac field) Mc^2, thus in such cases the energy present in the internal region is $-$ 13,8 Mc^2. It might be of interest to repeat the calculations of BHABHA by considering our present results.

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Appendix

Earlier the recoil of the nucleon was taken into consideration. Here the calculations taking into account the recoil are carried out in the momentum space. The present discussion shows clearly why the choice of trial function (34) means just the consideration of the recoil of the nucleon. Let us write (34) in the following form

$$\langle x^1; y^1, \dots y^n | \rangle =$$

$$= C_n \frac{1}{(2\pi\hbar)^{3/2}} \frac{1}{(2\pi\hbar)^{3n/2}} \int \varphi_n(p) e^{\frac{i}{\hbar}px} dp \prod_{i=1}^{n} \sqrt{\frac{\hbar^2 c^2}{2\omega_i}} f(k^i) e^{\frac{i}{\hbar}k^i y^i} dk^i , \quad (1)$$

where the recoil has to be taken into consideration by

$$\varphi_n(p) = \sum_s a_s \left(p + \sum_{i=1}^{n} k^i\right) u_s(p) . \quad (2)$$

$u_s(p)$ is owing to (7) an unit spinor characterizing a nucleon with momentum p, polarisation s (spin, isotope spin) and positive frequency, its explicit form is in the usual representation of the γ-s

$$\left(1 + \frac{c^2 p^2}{(E + Mc^2)^2}\right)^{-1/2} \begin{pmatrix} \delta_{1s} \\ \delta_{1s} \\ \frac{c(\sigma p)_{1s}}{E + Mc^2} \\ \frac{c(\sigma p)_{2s}}{E + Mc^2} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad \begin{array}{l} s = 1, 2 \\ \text{in case of } s = 3,4, \text{ elements} \neq 0 \text{ are} \\ \text{at the lower 4 places.} \end{array} \quad (3)$$

$$\sum_s \int a_s^*(p)\, a_s(p)\, dp = 1 ,$$

$$\omega = \sqrt{c^2 k^2 + m^2 c^4}$$

$$\int f^*(k) f(k)\, dk = 1 . \quad (4)$$

From (2) and (3) it can be seen, that neglecting the small components of the u-s (1) just agrees with (34). From the normalization condition $\langle t | t \rangle = 1$ it again follows that

$$\sum C_n^* C_n = 1 . \quad (5)$$

Let us first calculate the mean value of the momentum of the field. From (10) on the basis of the above we obtain

$$\langle P_i \rangle = \sum_s{}' \int p_i\, a_s^*(p)\, a_s(p)\, dp\,. \tag{6}$$

Similarly the mean value of the energy of the field is

$$\frac{c}{i}\langle P_4\rangle = \sum_n{}' C_n^* C_n \sum_s \int\ldots\int a_s^*(p) \prod^n f^*(k^j) \left(\sqrt{c^2 (p - \sum' k^j)^2 + M^2 c^4} + \right.$$

$$\left. + \sum^n{}' \sqrt{c^2 k^{j2} + m^2 c^4}\,\right) \cdot a_s(p) \prod^n f(k^j)\, dp \prod^n dk^j +$$

$$+ g \sum_n{}' (n+1)^{1/2} C_n^* C_{n+1} \sum_{s,s'}{}' \int\ldots\int a_s^*(p) \prod^n f^*(k^j) f(k^j)\, dk^j \,\cdot$$

$$\cdot\, \overline{u}_s(p - \sum^n{}' k^j)\, u_{s'}(p - q - \sum^n{}' k^j)\, a_{s'}(p)\, \frac{1}{(2\pi\hbar)^{3/2}} \sqrt{\frac{\hbar^2 c^2}{2\,\omega_q}}\, f(q)\, dq\, dp + C \cdot C. \tag{7}$$

Here the terms under the integral are still depending on n, thus GLAUBER and LUTTINGER's method cannot be applied to the solution of the equation obtained after the variation, therefore further approximations are used. By expansion we obtain

$$\sqrt{c^2 (p - \sum' k^j)^2 + M^2 c^4} \approx \sqrt{c^2 p^2 + M^2 c^4} + \frac{1}{2M} (\sum' k^j)^2 - \frac{1}{M} p \sum' k^j\,.$$

Assuming further that $a(p)$ and f are spherical symmetric, then the first term is

$$\sum_n{}' C_n^* C_n \left[E + n\left(\varepsilon + \frac{\overline{k^2}}{2M}\right)\right],$$

of course here

$$E = \sum_s{}' \int a_s^*(p) \sqrt{c^2 p^2 + M^2 c^4}\, a_s(p)\, dp\,,$$

$$\varepsilon = \int f^*(k) \sqrt{c^2 k^2 + m^2 c^4}\, f(k)\, dk\,,$$

$$\overline{k^2} = \int f^*(k)\, k^2 f(k)\, dk\,.$$

In the second term using the approximation (compare [5])

$$\overline{u}_s(p - \sum' k^j)\, u_{s'}(p - q - \sum' k^j) \to \overline{u}_s(p)\, u_{s'}(p) =$$

$$= \frac{1 - \dfrac{c^2 p^2}{(E + M c^2)^2}}{1 + \dfrac{c^2 p^2}{(E + M c^2)^2}}\, \delta_{s,s'} = g(p)\, \delta_{s,s'}$$

we obtain from (7) in such an approximation

$$\frac{c}{i}\langle P_4\rangle = \sum' C_n^* C_n \left| E + n\left(\varepsilon + \frac{\overline{k^2}}{2M}\right)\right] + \sum_n (n+1)^{1/2} C_n^* C_{n+1}\, a + \\ + \sum' n^{1/2} C_n^*\, C_{n-1}\, a^*$$

$$a = g\,\frac{1}{(2\pi\hbar)^{3/2}}\,\overline{g(p)}\int\sqrt{\frac{\hbar^2 c^2}{2\,\omega_q}}\,f(q)\,dq\,, \tag{8}$$

Our method is from here already the usual one, thus in the ground state we obtain

$$W^{(0)} = E - \frac{a\,a^*}{\varepsilon + \frac{\overline{k^2}}{2M}}\,. \tag{9}$$

This also minimizing with respect to f we obtain the solution for f

$$f(k) = a\,\frac{1}{\sqrt{\omega_k}}\cdot\frac{1}{\omega_k + \frac{k^2}{2M}}\,. \tag{10}$$

From here the energy of the field is

$$W^{(0)} = E - \frac{g^2}{2}\,\overline{g(p)}^2\,\frac{\hbar^2 c^2}{(2\pi\hbar)^3}\int\frac{1}{\omega_k\left(\omega_k + \frac{k^2}{2M}\right)}\,dk\,. \tag{11}$$

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МЕТОД СРЕДНЕЙ СВЯЗИ ТОМОНАГА, ПРИ ИСПОЛЬЗОВАНИИ КОНФИГУРАЦИОННО-ПРОСТРАНСТВЕННЫХ МЕТОДОВ

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Резюме

Определяются векторы состояния, характеризующие реальные нуклоны, методом средней связи, используя методы конфигурационного пространства в квантовой теории полей. Расматривается взаимодействие между нуклонным полем, описиваемым уравнением Дирака, и скалярным или псевдоскалярным мезонним полем. Образование пар пренебрегается. В рамках конфигурационного метода учитывается и отдача нуклонов. С помощью вектора состояния определяются средние значения некоторых физических величин в состоянии реального нуклона. Из-за конфигурационного метода, — особенно при расчете локальных физических величин — получается очень наглядная картина реального нуклона.