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**VALIDITY OF KOVÁCS AND NAGY'S EQUATION
AS A MEASURE OF THE TOTAL EQUIVALENT MEAN
SHEAR STRAIN IN TWISTED 99.7% Al WIRES**

By

M. R. SOLIMAN, F. H. HAMMAD* and G. A. HASSAN
METAL PHYSICS LABORATORY, NATIONAL RESEARCH CENTRE, CAIRO, EGYPT, UAR

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KOVÁCS and FELTHAM [1] and KOVÁCS and NAGY [2] derived an equation for the total equivalent mean shear strain $\bar{\gamma}$ of a wire plastically twisted about its axis at constant tensile load. In this equation $\bar{\gamma}$ consists of a torsional shear strain ND/L and the shear strain component equivalent to the associated tensile strain $\Delta L/L_0$, as follows:

$$\bar{\gamma} = \alpha\pi ND/L + \beta\Delta L/L_0, \quad (1)$$

where N is the number of turns of twist, D is the diameter of the wire, L_0 and L are the initial and instantaneous lengths of the wire, α and β are constants. KOVÁCS and FELTHAM [1] took the numerical values of α and β to be 1/3 and 2.24, respectively, but they indicated that these values were underestimated because the mean torsional strain was evaluated on the assumption that the wire was purely elastic: they pointed out that GAYDON'S [3] assumption of ideal plasticity would be more appropriate. Recent work [4, 5] has indicated that the values of α and β are $2\pi/3$ and 3, respectively.

Eq. (1) can be written in the form:

$$\Delta L/L_0 = -(\alpha/\beta)ND/L + \bar{\gamma}/\beta$$

or

$$\epsilon = -(\alpha/\beta)\Theta + \bar{\gamma}/\beta, \quad (2)$$

where $\epsilon = \Delta L/L_0$ and $\Theta = ND/L$.

By partial differentiation of ϵ with respect to θ at constant $\bar{\gamma}$ we get

* Atomic Energy Establishment, Egypt, U.A.R.

$$\begin{aligned}
 \left(\frac{\partial \epsilon}{\partial \theta} \right)_{\bar{\gamma}} &= - (\alpha/\beta) \\
 &= - 2\pi/9 \\
 &= - 0.698
 \end{aligned}
 \tag{3}$$

Eq. (3) shows that the rate of change of tensile strain per unit torsional strain at constant $\bar{\gamma}$ has a constant value of -0.698 determined by the constants α and β . Since the value of $\left(\frac{\partial \epsilon}{\partial \theta} \right)_{\bar{\gamma}}$ can be determined experimentally, this

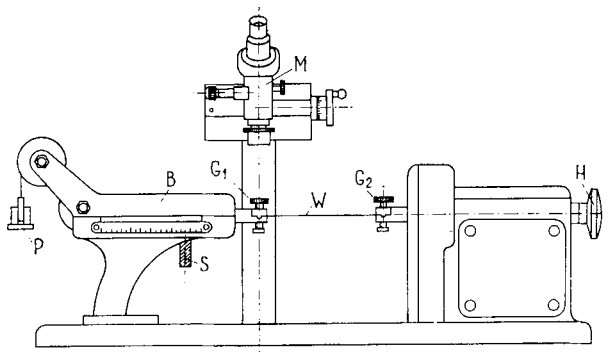


Fig. 1. Schematic diagram of twisting machine

gives a direct test of the validity of the Kovács—Nagy equation [Eq. (1)]. We have used this approach in an investigation on wires made of commercial purity Al (99.7% Al + 0.25% Fe).

Annealed Al wires (0.05 cm diameter, 15 cm long) were subjected to uniform twisting at room temperature using the machine shown in Fig. 1. The associated change in sample length was measured up to fracture with an accuracy of 10^{-3} cm. The axial stresses applied on the sample undergoing torsional deformation varied from 0.75 to 2.25 kg/mm²; such stresses did not exceed the yield stress.

Fig. 2 shows that for samples with the same thermal history, $\Delta L/L_0$ at certain ND/L increases as the magnitude of the applied stress is increased.

The values of $\left(\frac{\partial \epsilon}{\partial \theta} \right)_{\bar{\gamma}}$ were calculated from Fig. 2 using the least squares method. These values are given in Table I and are plotted against $\bar{\gamma}$ in Fig. 3. The average value of $\left(\frac{\partial \epsilon}{\partial \theta} \right)_{\bar{\gamma}}$ over the whole range of $\bar{\gamma}$ is approximately equal to the constant $\alpha/\beta = -0.7$. The resemblance of the experimentally determined values of α/β to that given by Eq. (3) indicates that the Kovács—Nagy equation is valid.

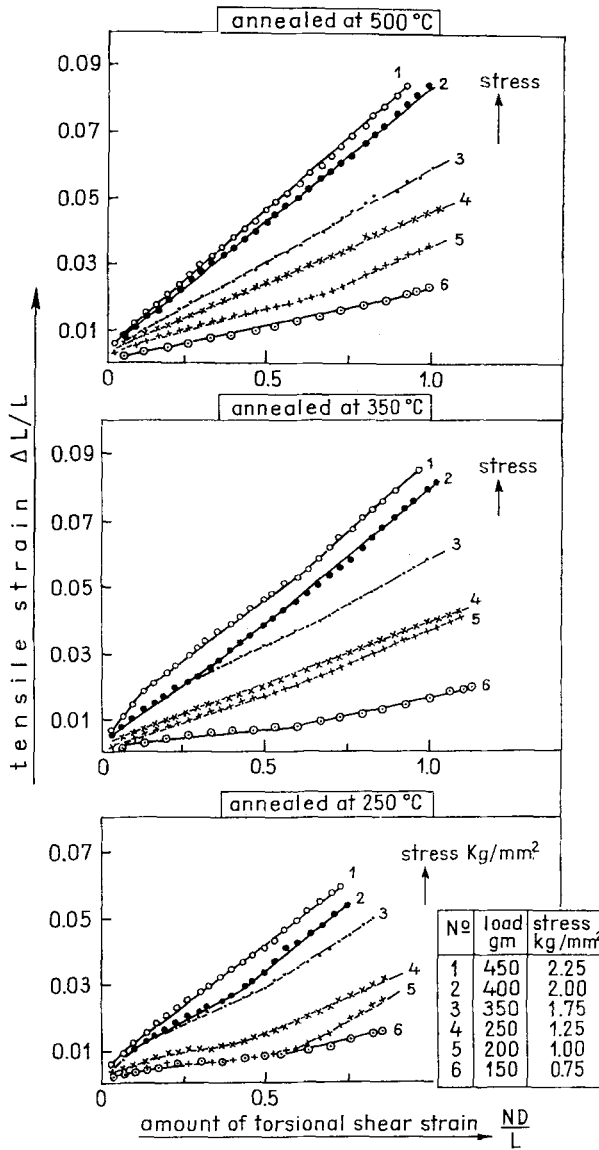


Fig. 2. Tensile strains $\Delta L/L_0$ associated with plastic twisting ND/L in pre-annealed aluminium wires under different constant loads

For specimens annealed at 500 °C before testing, scattered values of α/β are noted for $\bar{\gamma} < 100$. Under these conditions the values of α/β range between -0.6 and -0.4 . This may be attributed to the effect of annealing. Annealing at 250, 350 and 500 °C for 5 hours was found to influence the values of ND/L and $\Delta L/L_0$ as well as the angle of inclination between the torsional lines

Table I

$\bar{\gamma}$	$\left(\frac{\partial \epsilon}{\partial \theta}\right)_{\bar{\gamma}}$		
	250 °C	350 °C	500 °C
0.435		-0.65	
0.440			-0.52
0.450	-0.62		
0.480			-0.60
0.500	-0.80		
0.600			-0.60
0.620			-0.61
0.630		-0.70	
0.640	-0.82		
0.700	-0.65	-0.40	-0.40
0.800			-0.52
0.920		-0.50	
1.000	-0.70		-0.68
1.200	-0.63	-0.70	-0.70
1.400	-0.72	-0.65	
1.500			-0.70
1.600	-0.67	-0.63	
1.700	-0.82		
1.790		-0.76	
1.800	-0.80		
1.880			-0.62
2.000		-0.60	-0.70

Table II

Sample history	ϕ	ND/L	$\Delta L/L_0$
As received and twisted	47°	0.198	0.0069
Annealed at 250 °C	56°	0.924	0.0289
Annealed at 350 °C	75°	1.100	0.0420
Annealed at 500 °C	66°	1.020	0.0377

and wire axis, as shown in Table II. The appearance of torsional lines, which was used for determining Φ , is shown in Fig. 4. It is apparent that annealing at 250 and 350 °C causes the values of Φ , ND/L and $\Delta L/L_0$ to increase while annealing at 500 °C causes these values to decrease. Since Φ represents the

deformability of the specimen, annealing at 500 °C makes the material less deformable. This is also indicated by the decrease of ND/L and $\Delta L/L_0$.

The changes may be attributed to the presence of Al_3Fe , which has been observed to form under similar conditions [6, 7]. This is in agreement with the observations of PETTY [8] on the relation of deformability to the shape and size distribution of precipitate particles.

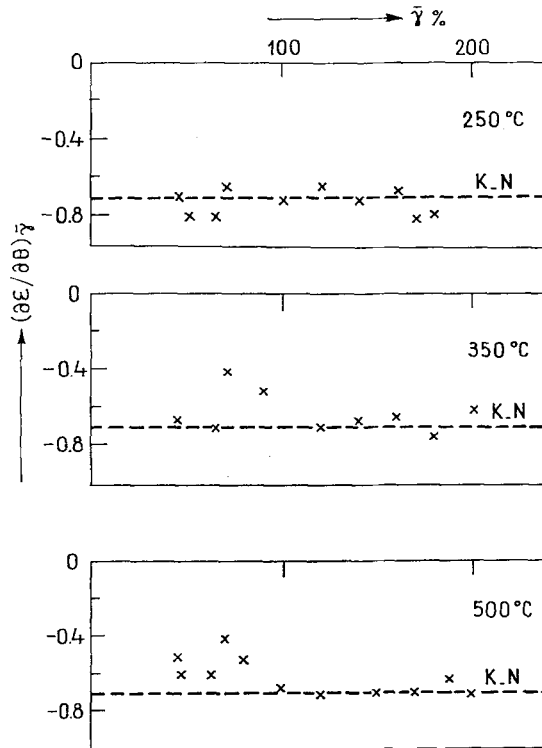


Fig. 3. Variation of the relative change of tensile strain with plastic torsion (W versus γ) from the KOVÁCS and NAGY value

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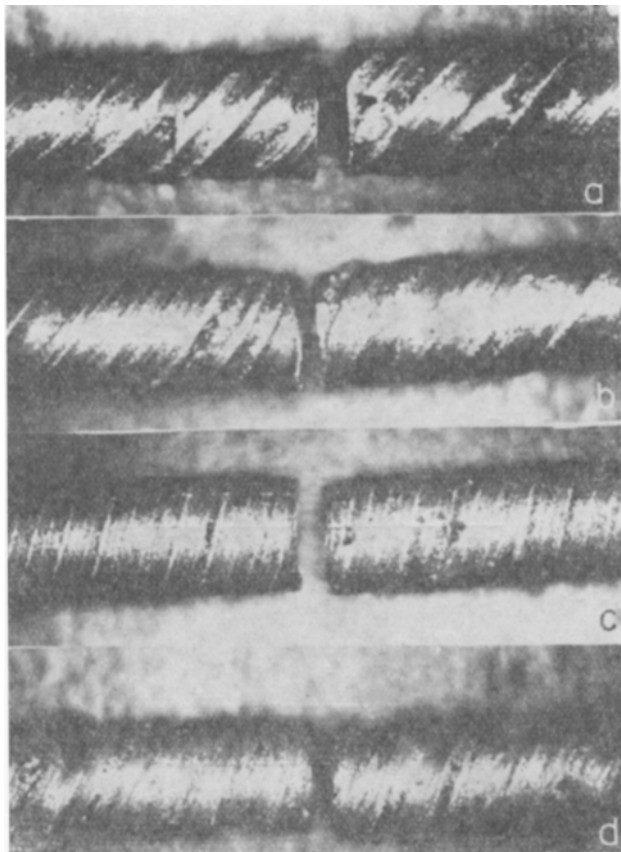


Fig. 4. Photographs of surfaces of Al samples twisted till fracture at room temperature. $50\times$.
Initial history:
a) As received; b) Annealed at 250 C; c) Annealed at 350 C; d) Annealed at 500 C

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