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ENTROPIC CHANGES IN A MEASURED QUANTUM-MECHANICAL OBJECT

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The processes connected with the entropy of measured quantum mechanical objects are investigated. The entropic characteristics of the measurement process are determined by means of the so-called entropy balance. It is shown that the Shannon entropy of quantal objects described by non-commutative operators is always positive in the post-measurement state. From the entropy balance of quantum mechanical measurement also follows that the measurement of the characteristics of quantal objects provides some information on quantities not measured.

Introduction

In the last decades the conceptual and mathematical apparatus of information theory has been successfully applied in various fields of physical science [1, 2, 3]. These applications have been possible because the physical phenomena in question are to a great extent describable by means of probability or statistical formalism. The probabilistic interpretation of quantum mechanics permits a direct application of the terms of information theory also in the description of an individual particle, as to each quantal object can be attached a value of its probability uncertainty, the measure of which is called the (information-theoretical) entropy [4].

Although the entropy of a quantal object may generally change in any physical process, the change that takes place during a measurement is especially important. Before measurement (the pré-measurement state) the measured observable generally has a non-zero entropy. After the measurement (the post-measurement state) this entropy is decreased, and in the optimal case it completely vanishes. As the measured quantal object is described by a set of observables which are mutually stochastically related, the removal of the entropy of a measured observable generally affects the entropy of the non-measured observable of the object. The determination of these various entropic changes forms the subject of this paper.

The entropic changes in a measured object play an important role in the physical description of the link between the measured micro-object and measuring macrophysical instrument, since according to the entropy law the total entropy of the whole measuring complex — measuring instrument and measured object — should increase (or at least be constant) during the measurement, so that the negative entropic change of the measured object must be compensated for by a corresponding positive entropic change of the measuring instrument. By considering the measuring instrument as a physical statistical system, it is possible to determine this necessary positive change of its physical entropy using the well known relation between the physical and information-theoretical entropy [5].

From the point of view of information theory, the measuring process may be simulated by a mathematical model, called the entropic model of quantum measurement [6]. In this model the measured object and measuring instrument are represented by two probability systems. The measured observable τ_0, a random variable, is determined on the set of the physical states of the measured object. The measuring instrument may occur in one of its pointer positions representing the basic set on which the random variable τ_m is defined through the scale values attached to the pointer positions. During the measurement a statistical linkage is established between the random variables τ_0 and τ_m, by means of which the measuring instrument obtains information about the measured object. The measure of the statistical linkage between the random variables τ_0 and τ_m is in information theory given by a quantity called the information [7].

We shall first recall some terms of information theory which will be used in the further physical considerations. The measure of the probability uncertainty of a quantal observable represents the information-theoretical entropy, which is defined as follows [8]:

A random variable (observable) $\tilde{x}$ defined on a complete set of physical states S with the probability distribution given by the scheme

S	A_1	A_2	$\dots$	A_n
P	p_1	p_2	$\dots$	p_n
$\tilde{x}$	x_1	x_2	$\dots$	x_n

$$\sum_{i=1}^{n} p_i = 1,$$

has an information-theoretical entropy (the magnitude of its probability uncertainty) of

$$H = -\sum_{i=1}^{n} p_i \log_a p_i . \tag{1}$$

If not otherwise stated, we shall take $a = e$. The entropy as a function of the variables $p_1, p_2, \dots, p_n$ (i.e. the elements of the probability distribution of the random variable $\tilde{x}$) fulfils a system of important mathematical axioms [8].

In order to quantitatively characterize the statistical linkage between the random variables $\tilde{x}$ and $\tilde{y}$ we need the data given by the transfer matrix [9]:

$$R = \begin{pmatrix} r_1(1) & \dots & r_1(n) \\ \vdots & & \vdots \\ r_n(1) & & r_n(n) \end{pmatrix},$$

where $r_i(j)$ is the conditional probability for assuming the j-th value of the random variable $\tilde{y}$ when the random variable $\tilde{x}$ has its i-th value. The information contained in the random variable $\tilde{y}$ about the random variable $\tilde{x}$ is given [10] by the formula

$$I(\tilde{x};\tilde{y}) = \sum_{i,j} p_i r_i(j) \log \frac{r_i(j)}{\sum_k p_k r_k(j)}, \tag{2}$$

which can be rearranged in the form

$$I = -\sum_i q_i \log q_i + \sum_i p_i \sum_j r_i(j) \log r_i(j), \tag{3a}$$

where

$$q_i = \sum_k p_k r_k(i).$$

If the random variables $\tilde{x}$ and $\tilde{y}$ are continuous with the density functions $p(x)$ and $q(y)$, respectively, formula (2) takes the form

$$I = \int_Y \int_X p(x) r_x(y) \log \frac{r_x(y)}{q(y)} \, dx \, dy, \tag{3}$$

where $r_x(y)$ represents the transfer function between the random variables $\tilde{x}$ and $\tilde{y}$.

The entropy of a measured quantal object changes by various ways, depending on the measuring conditions. When only one observable is measured, not only does its probability uncertainty change but also the probability uncertainties of those observables of the measured object with which it is statistically linked. Depending on the physical situation of the measurement, one may use various entropic characteristics of the measured quantal object in its description. The most important of these are the following: The total probability uncertainties of the measured object in its pre-or post-measurement state (H_b or H_a, respectively) and their difference ($\Delta H = H_a - H_b$), the pre- and post-measurement probability uncertainties of the measured observable, the entropic change of the non-measured observables during the measurement, etc. Which of them is used depends on the character of the problem.

1. The entropic changes in a quantal measurement

The basic assumption of the theory of quantum measurements is that in measuring a quantum-mechanical object one marks a system of eigenfunctions of the measured observable [11] and the result of a measurement represents one of these eigenvalues. The general wave function of the quantal object is thus reduced to an eigenfunction. This change of wave function is accompanied by a corresponding change in the entropy of the measured object, here considered as an information source. We shall now determine this entropic change for the measured quantal object.

Consider the general quantal object Σ described by an assembly of observables $G_1, G_2, \ldots, G_s$ to which the operators $\hat{O}_1, \hat{O}_2, \ldots, \hat{O}_s$ are associated. We shall denote by symbols $\{\varphi_{i_1}\}, \{\varphi_{i_2}\}, \ldots, \{\varphi_{i_s}\}$ the sets of eigenfunctions of the operators $\hat{O}_1, \hat{O}_2, \ldots, \hat{O}_s$ satisfying the quantal eigenvalue equations

$$\hat{O}_j \varphi_{i_j} = g_{i_j} \varphi_{i_j}, \qquad j = 1, 2, \ldots, s$$

where g_{i_j} denotes the i-th eigenvalue of the operator $\hat{O}_j$. We shall further denote by $U_{j_k l_m}$ the elements of the operator of the unitary transformation between the systems of eigenfunctions $\{\varphi_{j_k}\}$ and $\{\varphi_{l_m}\}$, whence

$$\varphi_{j_k} = \sum_{l_m} U_{j_k l_m} \varphi_{l_m} .$$

Let the quantal object Σ under study be described by the wave function

$$\Psi(x) = \sum_{i_1} \mu_{i_1} \varphi_{i_1} = \sum_{i_2} \mu_{i_2} \varphi_{i_2} = \ldots = \sum_{i_s} \mu_{i_s} \varphi_{i_s} .$$

The elements of the probability distributions $P_1, P_2, \ldots, P_s$, which are given on the sets of physical states of the quantal object Σ for the observable $G_1, G_2, \ldots, G_s$ are determined by means of the well-known equation

$$p_{i_j} = \mu_{i_j} \mu_{i_j}^{+} = |\mu_{i_j}|^2 . \qquad i = 1, 2, \ldots, n. \tag{4}$$

Between the observables $G_1, G_2, \ldots, G_s$ certain statistical dependences may exist which are described by means of the assembly of transfer matrices $R(p, t)$, $p, t = 1, 2, \ldots, s$, whose elements are determined by the elements $r_{i_p}(j_t)$ of the operator of the unitary transformation $U_{i_p j_t}$ between the p-th and t-th sets of eigenfunctions:

$$r_{i_p}(j_t) = U_{i_p j_t} \cdot U_{i_p j_t}^{+} = |U_{i_p j_t}|^2 \cdot \qquad p, t = 1, 2, \ldots, s. \tag{4a}$$

The total probability uncertainty $H(P^{(z)})$ of the quantal object Σ is given by the entropy of the joint probability distribution $P^{(z)}$ [12], which is defined on the product set $Z = S_1 \oplus S_2 \oplus \ldots \oplus S_s$, where $S_1 = \{s_1^{(1)}, s_2^{(1)}, \ldots, s_n^{(1)}\}$, $S_2 = \{s_1^{(2)}, \ldots, s_n^{(2)}\}$, ..., $S_s = \{s_1^{(s)}, \ldots, s_n^{(s)}\}$ are the sets of quantum states of observables $G_1, G_2, \ldots, G_s$. The elements of the product set Z represent all ordered n-groups of quantum states $(s_i^{(1)}, s_j^{(2)}, \ldots, s_m^{(s)})$, on which a vector random variable $\tilde{z} = \{g_{i_1}, g_{i_2}, \ldots, g_{i_s}\}$ is determined, where g_{i_k} represents k-th eigenvalue of the observable G_i. Denoting by $P_{i_1, i_2 \ldots, i_s}$ the elements of the joint probability distribution $P^{(z)}$, the total entropy of the measured quantal object is, according to Eq. (1), given by the relation

$$H(P^{(z)}) = - \sum_{i_1, i_2, i_3, \ldots, i_s} P_{i_1 i_2 \ldots i_s} \log P_{i_1 i_2 \ldots i_s} . \tag{5}$$

The elements of the joint probability distribution $P^{(z)}$ can also be written in the form

$$P_{i_1 i_2 \ldots i_s} = p_{i_1} \cdot r_{i_1}(i_2) \cdot r_{i_2}(i_3) \ldots r_{i_{s-1}}(i_s) , \tag{6}$$

where $r_{i_k}(i_{k+1})$, $k = 1, 2, \ldots, s-1$, represents the element of the transfer matrices $R(k, k+1)$, and p_{i_1} is an element of the probability distribution of the i-th observable.

Substituting Eq. (6) into (5) we find

$$\begin{aligned} H(P^{(z)}) = &- \sum_{i_1} p_{i_1} \log p_{i_1} - \sum_{i_1} p_{i_1}^{(1)} \cdot \sum_{i_2, i_3, \ldots, i_s} r_{i_1}(i_2) \cdot \\ &\cdot r_{i_2}(i_3) \ldots r_{i_{s-1}}(i_s) \cdot \log r_{i_1}(i_2)\, r_{i_2}(i_3) \ldots r_{i_{s-1}}(i_s) . \end{aligned} \tag{7}$$

Eq. (7) determines the total entropy of the measured quantal object with the joint probability $P^{(z)}$ in its pre-measurement state. Since there are generally statistical dependences between the observable describing this object, its entropy has a smaller value than the sum of the entropies of the individual observables. We have [13]

$$H(P^{(z)}) \leqq H(P_1) + H(P_2) + \ldots H(P_s) , \tag{7a}$$

where $P_1, P_2, \ldots, P_s$ are the probability distributions of the observables $G_1, G_2, \ldots, G_s$. In the relation (7a) the sign $=$ is to be taken only when all the observables are stochastically independent from each other, i.e. when

$$P^{(z)}_{i_1 i_2 \ldots i_s} = \prod_{k=1}^{s} p_{i_k} ,$$

where p_{i_k} represents the element of the probability distribution of the k-th observable.

The entropy change of the measured quantal object Σ during the measurement is given by the entropy balance of measurement, i.e. by the difference between the total entropy of the object in its pre-measurement state $H_b(P^{(z)})$ and in its post-measurement state $H_a(P^{(z)})$:

$$\Delta H_t = H_a(P^{(z)}) - H_b(P^{(z)}). \tag{8}$$

During the measurement of the observable G_1 the probability distribution of the measured object changes. Denoting by $w = \{ w_{i_1}, w_{i_1}, \ldots, w_{n_1} \}$ the post-measurement probability distribution of the measured observable, we can write according to Eq. (8) the total entropy change

$$\begin{aligned} \Delta H_t = -\sum_{i_1} w_{i_1} \log w_{i_1} + \sum_{i_1} p_{i_1} \log p_{i_1} - \sum_{i_1} (w_{i_1} - p_{i_1}) \cdot \sum_{i_2, i_3, \ldots, i_s} \\ r_{i_1}(i_2) \cdot r_{i_2}(i_3) \ldots r_{i_{s-1}}(i_s) \cdot \log r_{i_1}(i_2) \cdot r_{i_2}(i_j) \ldots r_{i_{s-1}}(i_s). \end{aligned} \tag{8a}$$

According to the entropy law a change ΔH_t in the entropy of the measured quantal object requires a minimal change $-k\Delta H_t$ in the physical entropy of the measuring instrument, k being Boltzmann's constant.

Let us now turn to the determination of the total post-measurement entropy $H_a(P^{(z)})$ of a measured quantal object. When only one observable is measured (for example G_1), the post-measurement entropy is

$$\begin{aligned} H_a = -\sum_{i_1} w_{i_1} \log w_{i_1} - \sum_{i_1} w_{i_1} \sum_{i_2, i_3, \ldots, i_s} r_{i_1}(i_2)\, r_{i_2}(i_3)\, r_{i_3}(i_4) \ldots \cdot \\ \cdot r_{i_{s-1}}(i_s) \cdot \log r_{i_1}(i_2)\, r_{i_2}(i_3)\, r_{i_3}(i_4) \ldots r_{i_{s-1}}(i_s). \end{aligned}$$

If the probability uncertainty of the measured observable is totally removed, we have

$$H_a(P^{(z)}) = H_b(P^{(z)}) - H(P_1).$$

Taking into account the relations (5) and (6), as well as the equation

$$H(P_1) = -\sum_{i_1=1}^{n} p_{i_1} \cdot \log p_{i_1},$$

we find

$$H_a(P^{(z)}) = -\sum_{i_1, i_2, \ldots, i_s} p_{i_1} r_{i_1}(i_2)\, r_{i_2}(i_3) \ldots r_{i_{s-1}}(i_s) \cdot \log r_{i_1}(i_2)\, r_{i_2}(i_3) \cdot r_{i_3}(i_4) \ldots r_{i_{s-1}}(i_s), \tag{9}$$

i.e. the total post-measurement entropy of the measured quantal object is equal to its general conditional entropy [13].

We shall next determine the foregoing entropic characteristics for the sake of simplicity, only for two observables G_1 and G_2. In this case relation (9) turns out to be

$$H_a(P^{(z)}) = -\sum_{i_1} p_{i_1} \sum_{i_2} r_{i_1}(i_2) \log r_{i_1}(i_2)\,. \tag{10}$$

Substituting Eqs. (4) and (4a) into Eq. (10), we have

$$H_a(P^{(z)}) = -\sum_{i_1} |\mu_{i_1}|^2 \sum_{i_2} |U_{i_1 i_2}|^2 \log |U_{i_1 i_2}|^2 . \tag{10a}$$

From Eq. (10) it is easy to see that the post-measurement entropy $H_a(P^{(z)})$ vanishes when

$$|U_{i_1 i_2}|^2 = \delta_{i_1 i_2} . \tag{11}$$

Consequently, for the corresponding operators $\hat{O}_1$ and $\hat{O}_2$ it holds that

$$\hat{O}_1 \hat{O}_2 - \hat{O}_2 \hat{O}_1 = 0, \tag{12}$$

i.e. the operators $\hat{O}_1$ and $\hat{O}_2$ commute. Thus we can state that only in the case when the operators associated with the observables G_1 and G_2 commute does the total entropy of the measured object vanish through the measurement of one of the observables.

When the observables $G_1, G_2, \ldots, G_m$ possess continuous spectra, then the eigenvalue problem can be written in the form

$$\hat{O}_i \, \varphi(l_i, x) = \mu(l_i) \, \varphi(l_i, x) . \tag{13}$$

Let the wave function $\Psi_c(x)$ of the considered quantum-mechanical system be

$$\Psi_c(x) = \int \mu(l_1) \cdot \varphi(l_1, x) \, dl_1 = \int \mu(l_2) \, \varphi(l_2, x) \, dl_2 \ldots = \int \mu(l_m) \cdot \varphi(l_m, x) \, dl_m . \quad (13a$$

The unitary transformation of the observables in this case turns out to be the integral one:

$$\varphi(l_i, x) = \int U(l_i, l_j) \, \varphi(l_j, x) \, dl_j . \tag{14}$$

The functions $p(l_1) = |\mu(l_1)|^2, p(l_2) = |\mu(l_2)|^2, \ldots, p(l_m) = |\mu(l_m)|^2$ and $r_{l_i}(l_j) = |U(l_i, l_j)|^2$ give the probability density functions of the observables $G_1, G_2, \ldots, G_m$ and the transfer functions $r_{l_i}(l_j)$ determining the statistical dependences between them, respectively. The change of the total entropy during the measurement and the post-measurement entropy of the measured object, while the observable G_1 is being completely measured, are found in a similar way:

$$\begin{aligned} \Delta H_t = H_a(P^{(z)}) - H_b(P^{(z)}) = &- \int u(l_1) \log u(l_1) \, dl_1 + \int p(l_1) \log p(l_1) \, dl_1 - \\ &- \int\int \ldots \int\int (u(l_1) - p(l_1) - p(l_1)) \, r_{l_1}(l_2) \, r_{l_2}(l_3) \ldots r_{l_{m-1}}(l_m) \\ &\log \left[r_{l_1}(l_2) \cdot r_{l_2}(l_3) \ldots r_{l_{m-1}}(l_m) \right] dl_1 \, dl_2 \ldots dl_m , \\ H_a(P_c^{(z)}) = - \int\int \ldots \int p(l_1) \cdot r_{l_1}(l_2) \cdot r_{l_2}(l_3) \ldots r_{l_{m-1}}(l_m) \cdot \log r_{l_1}(l_2) \cdot \\ \cdot \, r_{l_2}(l_3) \ldots r_{l_{m-1}}(l_m) \, dl_1 \, dl_2 \ldots dl_m . \end{aligned} \tag{15}$$

The foregoing quantities occurring in the entropy balance of measurement represent basic quantitative characteristics of the measuring process from the entropic point of view.

2. Information and the measurement

In the preceding Section we have dealt with the change of the probability uncertainty of a measured system when performing the measurement of one of its observables. We now turn to the problem, of how the probability uncertainty of an observable G_p changes when we measure the observable G_1, or in other words how much information about the observable G_p is contained in the observable G_1. Consider again the physical system described in the preceding Section. Determining the i-th value of the observable G_1, the entropy balance of the variable G_p is $H_p = -\sum_{i_p} p_{i_p} \log p_{i_p}$ in the pre-measurement state and $H_i' = \Sigma_{j_p} r_{i_1}(j_p) \log r_{i_1}(j_p)$ in the post-measurement state. Therefore, the mean change of the probability uncertainty in the observable G_p when a measurement of the observable G_1 is performed is

$$\Delta H = H_p - \sum_i p_i H_i' = I(G_1; G_p) = - \sum_{i_p} p_{i_p} \log p_{i_p} + \\ + \sum_{i_1} p_{i_1} \sum_{j_p} r_{i_1}(j_p) \ln r_{i_1}(j_p) \,. \tag{16}$$

We can see that the expression for the mean entropy change ΔH is identical with that for the information (see Eq. (2)).

The change of the total entropy of the joint probability distribution of an assembly of observables $G_2, \ldots, G_s$ during the measurement of the observable G_1 is

$$\Delta H = I(G_1; G_2, \ldots, G_s) = - \sum_{i_2, i_3, \ldots, i_s} P_{i_2 i_3 \ldots i_s} \log P_{i_2 i_3 \ldots i_s} + \\ + \sum_{i_1, i_2, \ldots, i_s} p_{i_1} \cdot r_{i_1}(i_2) \cdot r_{i_2}(i_3) \ldots r_{i_{s-1}}(i_s) \log r_{i_1}(i_2)\, r_{i_2}(i_3) \ldots r_{i_{s-1}}(i_s). \tag{17}$$

Since the elements $P_{i_2 i_3} \ldots {}_{i_s}$ and p_{i_1} of the joint probability distributions of the observables $G_2, G_3, \ldots, G_s$ and the measured observable G_1, respectively, as well as the elements of the transfer matrices $R(1, 2)$, $R(2,3), \ldots$, are linked with the physical parameters of the measured quantum-mechanical object according to the relations (4) and (4a), when we substitute these relations into Eqs. (16) and (17) we get expressions in which only the quantum-mechanical terms occur.

Where the observables have a continuous probability distribution it is possible to give the mean magnitude of the entropic change of observables G_2, $G_3, \ldots, G_s$ by measurement of the continuous observable G_1. In accordance with Eqs. (17), (13) and (13a), we find

$$I = -\iint \ldots \int |\mu(l_2)|^2 \, |U(l_2, l_3)|^2 \cdot |U(l_3, l_4)|^2 \ldots |U(l_{s-1}, l_s)|^2 \cdot \\ \cdot \log [|\mu(l_2)|^2 \, |U(l_2, l_3)|^2 \cdot |U(l_3, l_4)|^2 \ldots |U(l_{s-1}, l_s)|^2] \, dl_2 \cdot \\ \cdot dl_3 \ldots dl_s + \iint \ldots \int |\mu(l_1)|^2 \, |U(l_1, l_2)|^2 \cdot |U(l_2, l_3)|^2 \ldots \cdot \\ \cdot |U(l_{s-1}, l_s)|^2 \cdot \log [|U(l_1, l_2)|^2 \cdot |U(l_2, l_3)|^2 \ldots |U(l_{s-1}, l_s)|^2] \cdot \\ \cdot dl_1 \, dl_2 \ldots dl_s . \tag{18}$$

It can be shown that the maximum entropy change during the measurement will be obtained when $|U_{i_1 j_p}|^2 = r_{i_1}(j_p) = \delta_{i_1 j_p}$ or when $|U(l_i, l_j)|^2 = \delta(l_i - l_j)$, i.e. when the observables G_1 and G_p are compatible. This shows that the criterion of simultaneous measurability of the physical observables can be expressed by means of their entropic characteristics. This is of importance in the mathematical analysis of the quantum-mechanical formalism [14].

Since the general aim of a measurement is to reduce the probability uncertainty of the measured system as much as possible, one may, using the relations (8a), (15) and (18), find parameters of the measured object and measuring instrument (e.g. elements of transfer matrices, etc.) for which the entropy balance of measurement becomes optimal.

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ИЗМЕНЕНИЕ ЭНТРОПИИ КВАНТОВО-МЕХАНИЧЕСКИХ ОБЪЕКТОВ ПРИ ИЗМЕРЕНИИ

В. МАЕРНИК

Резюме

Обсуждаются процессы, связанные с энтропией системы, при измерении квантово-механических объектов. Определены энтропийные характеристики процесса измерения, при измерениях проведенных в так называемом режиме баланса энтропии. Показано, что энтропия Шеннона квантованных объектов, которая описывается с помощью некоммутирующих операторов, после измерения всегда положительна. Далее, из баланса энтропии квантово-механических измерений следует, что измерение наблюдаемых характеристик квантовых объектов дает некоторые информации и о неизмеренных величинах.