Acta Physica Academiae Scientiarum Hungaricae, Tomus 21 (1), pp. 1—15, (1966)

THE LORENTZ PRINCIPLE AND THE GENERAL THEORY OF RELATIVITY

PART I

HOMOGENEOUS PROPAGATION OF LIGHT

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(Received 23. XI. 1965)

This paper is the first of a series in which the formulation of the general theory of relativity in terms of the Lorentz principle is attempted. In this first part the Lorentz transformation is generalized so as to be applicable to parts of space where light is propagated homogeneously but possibly unisotropically. It is shown that the Lorentz principle in its ordinary form remains valid for such regions.

Introduction

§ 1. In the present article we show that the Lorentz principle which we have formulated in a number of papers [1]—[4] can be generalized so as to apply to the problems of general relativity. Just as in the case of the special theory, in the generalized form the principle leads to a mathematical formalism equivalent to that of the general theory of relativity. The approach through the generalized Lorentz principle gives, however, new physical aspect to the problems. In particular although the mathematical formalism of Riemann geometry is made use of the concept of curved space is left out of the considerations. We hope to come back to the philosophical aspects of the problem elsewhere, here we try to restrict ourselves to such an extent as possible to physical considerations only.

§ 2. Our considerations start from the fact that the mode of propagation of light is affected by gravitational fields. In a gravitational field the propagation of light can thus not any more be regarded to be isotropic and the velocity of propagation may vary both in time and with location.

So as to be able to generalize the Lorentz principle to the case of inhomogeneous mode of propagation of light we shall, as a first step, generalize the Lorentz transformation to regions with homogeneous mode of propagation. In the present Part I of this work we shall give the generalization of the Lorentz transformation to the case of unisotropic but what we shall call homogeneous mode of propagation of light. In Part II we shall further generalize the Lorentz transformation to the case of inhomogeneous propagation of light.

It will be seen that the Lorentz principle when interpreted in terms of the Lorentz transformation thus generalized yields essential parts of the formalism of general theory of relativity.

In Part III we shall re-interpret the considerations of Einstein giving the connection between gravitational field and its sources. We shall thus give the connection between the mode of propagation of light and the sources of gravitation.

With the considerations of Part III we shall arrive from our approach at the whole of the mathematical formalism of the general theory of relativity.

§ 3. Let us consider part of space in which light is propagated unisotropically so that the velocity of light in different directions should be different. Let us suppose, however, that the directional distribution is the same in different points and that it does not vary in time. Thus suppose that the velocity of propagation of light can be written as

$$\mathbf{c}(\varkappa) = \varkappa a(\varkappa) \qquad \begin{array}{l}\text{independent of } \mathbf{r}, t\\ \text{for any direction } \varkappa\end{array} \tag{1}$$

where $\varkappa$ is a unit vector.

Furthermore we shall restrict the function $a(\varkappa)$. Supposing that light is propagated unisotropically in the manner as we know light to be propagated in a homogeneous but unisotropic medium, we may suppose the following connection between the vector $\mathbf{r}$ pointing from a point P to Q and the measure of time t in which a signal starting from P reaches Q,

$$\mathbf{r}\mathbf{G}\mathbf{r} - c_0^2 t^2 = 0, \tag{2}$$

where $\mathbf{G}$ is a symmetric positive definite tensor with components $G_{ik} = G_{ki}$, $i, k = 1, 2, 3$ and c_0 is a velocity. The particular case $\mathbf{G} = 1$, $c_0 = c$, i.e. $G_{ik} = \delta_{ik}$ corresponds to the isotropic propagation of light.

§ 4. So as to generalize (2) a little further, we may suppose that the carrier of light moves with the constant velocity $\mathbf{v}$ relative to our system. If we describe the propagation of light with respect to a system K' in which the carrier of light is at rest, then we find for the coordinates of the two points P and Q which are at rest with respect to the system K:

$$\left.\begin{array}{l}\mathbf{r}'_P(t) = \mathbf{a} - \mathbf{v}t,\\ \mathbf{r}'_Q(t) = \mathbf{a} - \mathbf{v}t + \mathbf{r}\end{array}\right\} . \tag{3}$$

If a signal starts at $t = t_1$ from P and arrives at $t = t_2$ in Q we have to write

$$\left(\mathbf{r}'_P(t_1) - \mathbf{r}'_Q(t_2)\right) \mathbf{G} \left(\mathbf{r}'_P(t_1) - \mathbf{r}'_Q(t_2)\right) - c_0^2(t_1 - t_2)^2 = 0.$$

Rewriting the above relation and writing $t_2 - t_1 = t$ we find with the help of (3)

$$\mathbf{r}\mathbf{G}\mathbf{r} + 2\,\mathbf{r}\mathbf{V}\,t - c^2 t^2 = 0,$$

where we wrote $\mathbf{Gv} = \mathbf{V}$ and $c^2 = c_0^2 - \mathbf{vV}$. The above relation can also be written

$$\mathbf{xgx} = 0, \tag{4}$$

where we suppose $\mathbf{x}$ to be a four-vector with components

$$\mathbf{x} = \mathbf{r}, t$$

and $\mathbf{g}$ is a symmetric tensor of the fourth order with components

$$\mathbf{g} = \begin{pmatrix} \mathbf{G} & \mathbf{V} \\ \mathbf{V} & -c^2 \end{pmatrix}. \tag{5}$$

In the following we shall say that light is propagated homogeneously in a region if the propagation inside all parts of this region can be described by relation (4) and the tensor $\mathbf{g}$ has the form (5).

We suppose the components of $\mathbf{g}$ to be independent of $\mathbf{x}$ furthermore we suppose $-g_{44} = c^2 > 0$.

A particular case of (4) and (5) is the case considered in previous works, (see for instance [1]), i.e.

$$\mathbf{x\Gamma x} = 0 \text{ with } \mathbf{\Gamma} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -c^2 \end{pmatrix}. \tag{6}$$

§ 5. Suppose relation (4) (with given elements of the tensor $\mathbf{g}$) is valid in the measures of a system of reference K. We can form transforms of the coordinates, e.g.*

$$\mathbf{x}' = \mathbf{S}^{-1}\,\mathbf{x}, \tag{7}$$

and inserting (7) into (4) we find

$$\left.\begin{aligned} \mathbf{x}'\,\mathbf{g}'\,\mathbf{x}' &= 0 \\ \mathbf{g}' &= \widetilde{\mathbf{S}}\,\mathbf{g}\,\mathbf{S} \end{aligned}\right\}. \tag{8}$$

Thus taking the transformed coordinates to refer to a system K' we see that the propagation of light appears homogeneous also in the measures of K' but the tensor $\mathfrak{g}$ giving the detailed mode of propagation has in general different elements in the representation relative to K' than $\mathbf{g}$ representing

* The coordinate transformation itself may be an inhomogeneous transformation. $\mathbf{x}$ as used in relation (4) expresses a four-distance, thus it can be taken as the difference between two coordinate vectors e.g. $\mathbf{x} = \mathbf{x}_P - \mathbf{x}_Q$ and thus its transformation is homogeneous.

the propagation relative to K. (Here we denote by gothic symbols quantities irrespective of their representation).

§ 6. In particular if we prescribe the elements of the matrix $\mathbf{g}'$ we find a transformation $\mathbf{S}$ which leads from $\mathbf{g} \to \mathbf{g}'$ thus prescribed. Indeed, putting

$$\mathbf{S} = \mathbf{g}^{-1/2}\,\mathbf{g}'^{1/2} \tag{9}$$

(8) reduces to an identity. Thus the transformation with the matrix $\mathbf{S}$ as defined by (9) seems to give one suitable transformation, however, the transformation matrix $\mathbf{S}$ defined by (9) has in general complex elements. Writing $\mathbf{g}$ as a hypermatrix in the form (5) and using an analogous notation for the matrix $\mathbf{g}'$ we find that a matrix $\mathbf{S}$ giving the transformation (8) can be written

$$\mathbf{S} = \begin{pmatrix} \mathbf{G}^{-1/2}\,\mathbf{G}'^{1/2} & \mathbf{G}^{-1/2}\,\mathbf{G}'^{1/2}\,\mathbf{v}' - \mathbf{v}\,c'/c \\ 0 & c'/c \end{pmatrix}. \tag{9a}$$

Since $\mathbf{G}$ and $\mathbf{G}'$ are symmetric positive definite matrices, the matrix $\mathbf{S}$ as defined by (9a) has real elements only. For $\mathbf{v} = \mathbf{v}'$ or more generally if

$$\mathbf{G}'^{1/2}\,\mathbf{v}'/c' = \mathbf{G}^{1/2}\,\mathbf{v}/c$$

(9a) reduces to (9).*

Another transformation can be obtained as follows: denote by $\mathbf{O}$ and $\mathbf{O}'$ matrices with the help of which $\mathbf{g}$ respectively $\mathbf{g}'$ can be brought into diagonal form; thus suppose

$$\mathbf{O}^{-1}\,\mathbf{g}\,\mathbf{O} = \mathbf{D}, \qquad \mathbf{O}'^{-1}\,\mathbf{g}'\,\mathbf{O}' = \mathbf{D}'.$$

Thus remembering that $\mathbf{O}$ is an orthogonal matrix obeying $\tilde{\mathbf{O}} = \mathbf{O}^{-1}$ we can also put

$$\mathbf{S} = \mathbf{O}'\,\mathbf{D}'^{1/2}\mathbf{D}^{-1/2}\,\tilde{\mathbf{O}}, \tag{9b}$$

and we find that (9b) also satisfies (8).

Both transformations (9a) and (9b) have the following features:

1) If $\mathbf{g}' \to \mathbf{g}$ then $\mathbf{S} \to 1$.

2) The matrices thus defined are associative, i.e. if

$$\tilde{\mathbf{S}}\mathbf{g}\mathbf{S} = \mathbf{g}' \quad \text{and} \quad \tilde{\mathbf{S}}'\,\mathbf{g}'\,\mathbf{S}' = \mathbf{g}''$$

then we have also

$$\mathbf{g}'' = \tilde{\mathbf{S}}''\,\mathbf{g}\,\mathbf{S}'',$$

* I am indebted to P. KIRÁLY for drawing my attention to the fact that the definition (9) leads to matrices S with complex elements and also for pointing out that the alternative definition (9a) leads to transformation matrices with real elements only.

where S, S′ and S″ are all three given either by expressions of the type (9a), or all three are given by the expressions of the type (9b).

In particular we may put

$$\mathbf{g}' = \mathbf{\Gamma}$$

and thus we find that in the measures of K' the propagation appears isotropic. Conversely even if the real propagation of light is isotropic we can construct systems of references in which the propagation of light is characterized by an arbitrarily given tensor **g**.

Thus from the fact that the propagation of light appears homogeneous in one representation, it follows that it appears so in all other representations which are obtained from the former by linear transformation. There exist always among the possible representations such in which the propagation appears isotropic.

§ 7. One infers from the above that it is impossible to determine the elements of **g** from the result of measurement of the times of travels of signals of light. That this is indeed impossible we show presently by a consideration which is a generalization of considerations given earlier [4].

§ 8. We show presently that one can easily generalize the considerations which we have given elsewhere [2] for the case of isotropic propagation of light. Consider for this purpose a number of clocks near points $P_0, P_1, P_2, \ldots, P_n$. We show that taking, say, the clock P_0 as standard, we can synchronize the remaining clocks using light signals between the clocks and we can at the same time express the components of the coordinate vectors $r_0, r_1, r_2, \ldots, r_n$ of the position of the clocks in terms of the observed times of travels of light signals.

§ 9. So as to carry out the above synchronization, suppose P_0 to be the standard clock, we may synchronize the rates of the clocks P_k $k = 1, 2, \ldots$ by emitting signals with a period T from P_0 and adjust the rates of the clocks P_k $k > 0$ to the rythm of the signals thus received.

The procedure of synchronizing the rates of the clocks can be repeated by emitting in turn periodic signals from the points $P_1, P_2, \ldots$, etc. and it is to be expected that the signals thus emitted and received by the remaining clocks appear to be periodic when timed with the receiving clocks.

The latter procedure can be taken as a test of the assumption that the velocity of propagation of light is indeed constant in time and also a test of the assumption that the clocks P_k, $k = 0, 1, 2$ are in positions at constant distances from each other. Furthermore the possibility of synchronizing the rates of the clocks in a consistent manner supports the assumption that the rates of the clocks are constant indeed. We shall come back elsewhere to the analysis of this problem in greater detail.

We suppose thus that the rates of the clocks P_k, $k = 0, 1, 2, \ldots$ have been successfully synchronized. We show how the coordinate vectors $\mathbf{r}_k$ of the positions of the clocks can be determined in terms of the times of travels of signals between the clocks so synchronized.

§ 10. So as to obtain definite values for the coordinates we define a system of reference with the help of the positions of four of the clocks. Let us suppose that P_0, P_1, P_2, P_3 lie on the corners of a non degenerated tetrahedron. We can take P_0 to fix the origin of K while the points P_1, P_2, P_3 fix the direction of the axis of the system of reference K. In the system thus defined the coordinate vectors of the positions of the four clocks can be written

$$\mathbf{r}_0 = 0; \quad \mathbf{r}_1 = a_1, 0, 0; \quad \mathbf{r}_2 = 0, a_2, 0; \quad \mathbf{r}_3 = 0, 0, a_3 . \tag{10}$$

The numerical values of the components of the coordinate vector $\mathbf{r}_k$, $k = 1, 2, 3$ can be determined from the times of travels of light signals provided the components of $\mathbf{g}$ relative to K are known. This determination can be carried out in the following manner.

Denote the time of travel of a signal from P_n to P_m by $t_{n,m}$; denote the return time from P_n to P_m and back by

$$t_{n,m} + t_{m,n} = 2\, t_{nm} \tag{11}$$

further denote the difference

$$t_{n,m} - t_{m,n} = 2\, \Delta t_{nm} \tag{12}$$

(in the case of isotropic propagation of light we have of course $\Delta\, t_{n\,m} = 0$). Writing for the moment

$$\begin{gathered} t_{n,m} = t_1, \qquad t_{m,n} = t_2, \\ \mathbf{r}_m - \mathbf{r}_n = \mathbf{r}, \end{gathered} \tag{13}$$

then we have for the times of exchange of light signals between P_n and P_m

$$\left.\begin{aligned} \mathbf{rGr} + 2\,\mathbf{Vr}t_1 - c^2 t_1^2 &= 0, & \text{(a)} \\ \mathbf{rGr} - 2\,\mathbf{Vr}t_2 - c^2 t_2^2 &= 0. & \text{(b)} \end{aligned}\right\} \tag{14}$$

Solving the above equations into t_1 and t_2 we find using the notations (11), (12) and remembering (13)

$$\left.\begin{aligned} t_{nm}^2 &= (\mathbf{rGr})/c^2 + (\mathbf{Vr})^2/c^4, & \text{(a)} \\ \Delta t_{nm} &= \mathbf{Vr}/c^2, & \text{(b)} \\ \mathbf{r} &= \mathbf{r}_m - \mathbf{r}_n . & \text{(c)} \end{aligned}\right\} \tag{15}$$

In place of (15a) we may also write

$$\left.\begin{aligned} c^2 t_{nm}^2 &= (\mathbf{r}_m - \mathbf{r}_n)\overset{(3)}{\mathbf{g}}(\mathbf{r}_m - \mathbf{r}_n), && \text{(a)} \\ \text{where} \quad \overset{(3)}{\mathbf{g}} &= \mathbf{G} + (\mathbf{V} \circ \mathbf{V})/c^2. && \text{(b)} \end{aligned}\right\} \tag{16}$$

Observing the times t_{nm}, $nm = 0, 1, 2, \ldots$ we are in a position to determine the components of the coordinate vectors $\mathbf{r}_n$, $n = 1, 2, 3, \ldots$. In particular for $n = 0$, $m = k = 1, 2, 3$ we find from (16) and (10)

$$a_k = ct_{0k}/\sqrt{\overset{(3)}{g}_{kk}}, \quad k = 1, 2, 3. \tag{17}$$

Further introducing a vector $\mathbf{D}^{(n)}$ with components

$$D_k^{(n)} = c^2 \frac{t_{kn}^2 - t_{0n}^2 - t_{0k}^2}{2\, a_k} \qquad k = 1, 2, 3. \tag{18}$$

We find from (16a)

$$\mathbf{r}_n = \overset{(3)}{\mathbf{g}}{}^{-1}\, \mathbf{D}^{(n)} \qquad n = 0, 1, 2, \ldots \tag{18a}$$

However, (18a) gives only a necessary condition which the coordinates $\mathbf{r}_n$ have to satisfy. Whether the coordinates as given by (18a) in terms of the return times indeed satisfy the relations (16) has to be ascertained separately.

We consider the procedure in some more detail. Considering the four points P_k, $k = 0, 1, 2, 3$ and a fifth point P_l, $l > 3$, we may observe twenty return times between the various pairs of the five points. It follows from (16) that we must expect

$$t_{nm} = t_{mn}. \qquad n, m = 0, 1, 2, 3, l. \tag{19}$$

Equ. (19) gives thus ten conditions which have to be fulfilled by the observed return times if our assumptions about the mode of propagation of light is to be correct.

Supposing (19) to be fulfilled by the observed values, equ. (16a) provides us with further ten conditions. However, inserting (18) into (16a) remembering (17) nine out of the ten relations reduce to identities and we are left with one non trivial relation, i.e.

$$c^2 t_{0l}^2 = \mathbf{r}_l \overset{(3)}{\mathbf{g}} \mathbf{r}_l\,. \tag{20}$$

The above relation gives thus a further check of consistency of our assumptions.

One further check is obtained if we consider the time a signal takes to go round a triangle formed of three points. Writing thus

$$t_{klm} = t_{k,l} + t_{l,m} + t_{m,k}$$

we find with the help of (12)

$$t_{klm} - t_{kml} = 2(\Delta t_{kl} + \Delta t_{lm} + \Delta t_{mk})$$

and with the help of (15b) and (15c) we obtain

$$t_{klm} = t_{kml} \,. \tag{21}$$

The above relation can be checked directly by experiment.

If all the checks described above lead to satisfactory results then we may conclude: The fact that the rates of the clocks can be synchronized consistently, further the facts that (19), (20), and (21) are obeyed by the observed times of travels of signals support the hypothesis that in the region considered light is propagated homogeneously with a propagation tensor $\mathbf{g}$.

However, the checks do not really prove that the propagation tensor has indeed the value $\mathbf{g}$ used for the determination of the coordinates $\mathbf{r}_n$. Indeed, had we supposed that the propagation was not given by $\mathbf{g}$ but by a tensor

$$\mathbf{g}' = \tilde{\mathbf{S}}\mathbf{g}\mathbf{S} \,,$$

where $\mathbf{S}$ is a matrix with constant elements and det $\mathbf{S} \neq 0$, then the procedure described above would have led to coordinate vectors

$$\mathbf{r}'_n = \mathbf{S}^{-1}\, \mathbf{r}_n \,. \tag{22}$$

It is verified easily that provided the $\mathbf{r}_n$ obtained assuming the propagation to be given by $\mathbf{g}$ satisfy the checks described above, then automatically the coordinate vectors $\mathbf{r}'_n$ satisfy the corresponding relations involving the tensor $\mathbf{g}'$. We see therefore that *the analysis of times of travel of light signals can be used to ascertain whether or not light is propagated homogeneously, but no information can be obtained as to the components of the propagation tensor* $\mathbf{g}$ *from such measurements.*

It is interesting to note that it is usually strongly emphasized that observing the return times of light signals one cannot determine the velocity of the observer relative to the carrier of light.

We see from the above considerations that the latter statement contains only part of the real facts. The velocity of the observer relative to the carrier of light is contained in the components $V_k = g_{4k}$, $k = 1, 2, 3$ of $\mathbf{g}$.

As, however, *none* of the components of $\mathbf{g}$ can be determined by the exchange of light signals, it follows that *the observation of times of travels of signals are also unsuitable to determine whether or not light is propagated isotropically relative to its carrier*. Thus the ambiguity of the interpretation of the results is considerably larger than it is usually supposed to be.

§ 11. We may define as the measure r_{nm} of the distance between the points P_n and P_m as

$$r_{nm} = ct_{nm} = (\mathbf{r}_{nm} \overset{(3)}{\mathbf{g}} \mathbf{r}_{nm})^{1/2}. \tag{23}$$

The above relation in itself is a mere definition and has no physical contents. So as to obtain a physical statement we may take a solid rod AB, fix one of its ends A in the point P_n and turn it round into different directions. The end B of the rod thus will take up points upon a surface given by coordinate vectors

$$\mathbf{r}(\varkappa) = \mathbf{r}_n + \mathbf{l}(\varkappa)\,, \tag{24}$$

where $\varkappa$ is a two-component parameter defining the various orientations of the rod. Determining the $\mathbf{l}(\varkappa)$ by observing the behaviour of a real rod, we conclude from (23) and (24)

$$l(\varkappa) = \big(\mathbf{l}(\varkappa) \overset{(3)}{\mathbf{g}} \mathbf{l}(\varkappa)\big)^{1/2}$$

where $l(\varkappa)$ is the measure of the length the rod takes up when is pointed into the direction defined by $\varkappa$.

From experiments of the Michelson—Morley type it follows that the return time of a light signal travelling between the ends of a solid rod is not affected if the rod is turned round. This experimental result may be expressed by stating that for a solid rod turned round adiabatically we have

$$l(\varkappa) = l = \text{independent of } \varkappa \tag{25}$$

The latter relation implies that when it is turned round physical processes take place which make the solid rod to adapt itself to the measures obtained from light signals and defined by (23).

The observed relation (25) can be taken as the first step in formulating the Lorentz principle in its generalized form.

The Lorentz principle in the case of homogeneous propagation of light

§ 12. Let us consider a system of reference in which the propagation of light can be described by

$$\mathbf{xgx} = 0\,, \tag{26}$$

with a given tensor $\mathbf{g}$. We may change the system of reference and thus obtain new coordinates so that

$$\mathbf{x}' = \mathbf{M}\mathbf{x}\,. \tag{27}$$

(The transformation must be taken in the homogeneous form as $\mathbf{x}$ and $\mathbf{x}'$ represent four-distances.)

Introducing (27) into (26) we find

$$\mathbf{x}\widetilde{\mathbf{M}}\mathbf{g}\,\mathbf{M}\mathbf{x} = \mathbf{x}'\,\mathbf{g}\mathbf{x}'. \tag{28}$$

We see thus that if the matrix $\mathbf{M}$ is such as to obey relation

$$\widetilde{\mathbf{M}}\mathbf{g}\mathbf{M} = \Theta\mathbf{g} \qquad \Theta \neq 0, \tag{29}$$

then relation (26) written in terms of the coordinates $\mathbf{x}'$ reduces to

$$\mathbf{x}'\mathbf{g}\mathbf{x}' = 0\,. \tag{30}$$

We see therefore that there exists a set of systems of references K, K', . . . in all of which the propagation of light is expressed by the same algebraic expression of the form (26), i.e. by the same propagation tensor $\mathbf{g}$.

Relation (29) is the generalization of the definition of the Lorentz matrices [1], [4], i.e. of

$$\widetilde{\mathbf{\Lambda}}\mathbf{\Gamma}\mathbf{\Lambda} = \Theta\mathbf{\Gamma}\,. \tag{31}$$

In the following we shall restrict ourselves to consider transformations with $\Theta = +1$.

§ 13. The matrices $\mathbf{M}$ obeying (29) connect thus the systems of reference relative to which the propagation of light appears in the same form. The transformations $\mathbf{M}$ form (like the Lorentz transformations) a group. Indeed from

$$\widetilde{\mathbf{M}}\mathbf{g}\mathbf{M} = \mathbf{g} \tag{32}$$

follows, since $\det \mathbf{g} \neq 0$

$$\det \mathbf{M} = \pm 1\,. \tag{33}$$

Thus $\mathbf{M}$ possesses a reciprocal. We find thus from (32)

$$\mathbf{M}^{-1} = \mathbf{g}^{-1}\,\widetilde{\mathbf{M}}\mathbf{g}\,, \tag{34}$$

and therefore

$$\widetilde{\mathbf{M}}^{-1}\,\mathbf{g}\,\mathbf{M}^{-1} = \mathbf{g}\,. \tag{35}$$

Thus if $\mathbf{M}$ is a generalized Lorentz matrix, then $\mathbf{M}^{-1}$ is also such a matrix.

Furthermore we find that if two matrices **M** and **N** obey

$$\tilde{\mathbf{M}}\mathbf{g}\mathbf{M} = \mathbf{g} \quad \text{and} \quad \tilde{\mathbf{N}}\mathbf{g}\mathbf{N} = \mathbf{g}$$

then we have also

$$(\widetilde{\mathbf{M}\mathbf{N}})\,\mathbf{g}\,\mathbf{M}\mathbf{N} = \mathbf{g}\,.$$

Thus the matrices **M** obeying (32) form indeed a group.

§ 14. The generalized Lorentz transformation, which was introduced as giving transformation between the coordinates of different system of reference, can — just like the ordinary Lorentz transformation — be given a new meaning.

Considering the inhomogeneous transformation

$$\mathbf{X}^* = \mathbf{M}\mathbf{X} + \mathbf{m}\,, \tag{36}$$

where we write capital **X** for a coordinate four vector so as to distinguish it from the four-distances for which we wrote **x**.

We may consider **X** and **X*** as four-coordinates of two events, say $\mathfrak{E}$ and $\mathfrak{E}^*$ both coordinates relative to one system of reference K. Thus the transformation (36) can be taken as to refer to coordinates relative to one system of reference only and thus the transformation orders to an event $\mathfrak{E}$ represented by **X** another event $\mathfrak{E}^*$ represented by the coordinate **X***.

Considering instead of a single event $\mathfrak{E}$ some physical system $\mathfrak{Q}$ containing a number of points which may be also moving relative to each other, then transforming the coordinates of the points $\mathfrak{P}_1, \mathfrak{P}_2, \ldots, \mathfrak{P}_n$ of $\mathfrak{Q}$ we obtain new points $\mathfrak{P}_1^*, \mathfrak{P}_2^*, \ldots, \mathfrak{P}_n^*$ forming a new physical system $\mathfrak{Q}^*$. The system $\mathfrak{Q}^*$ is obtained from $\mathfrak{Q}$ by generalized Lorentz transformation. We may write symbolically

$$\mathscr{M}_{\mathfrak{p}}(\mathfrak{Q}) = \mathfrak{Q}^*\,, \tag{37}$$

where $\mathscr{M}_{\mathfrak{p}}$ is the operator describing the change from $\mathfrak{Q}$ into $\mathfrak{Q}^*$ and $\mathfrak{p}$ stands for the parameter characterizing the transformation. Relation (37) expressed in its representation relative to a system K of reference may be written

$$\mathscr{M}_{\mathbf{p}}(\mathbf{Q}) = \mathbf{Q}^* \tag{38}$$

where

$$\mathbf{p} = K(\mathfrak{p})\,, \qquad \mathbf{Q} = K(\mathfrak{Q})\,, \qquad \mathbf{Q}^* = K(\mathfrak{Q}^*)$$

are the representations of the various quantities relative to K.

Written more explicitly, if we denote the representation of the four-coordinate vectors of a point $\mathfrak{P}_\nu$ of $\mathfrak{Q}$ by $\mathbf{x}_\nu$

$$\mathbf{x}_\nu^* = \mathbf{M}_{\mathbf{p}}\,\mathbf{x}_\nu + \mathbf{m} \qquad \nu = 1, 2, \ldots, n\,. \tag{39}$$

where $\mathbf{M}_\mathbf{p}$ is a matrix obeying the relation (32), the components of $\mathbf{p}$ are the parameters specifying the transformation and $\mathbf{m}$ is a four-vector with constant components.

§ 15. Considering the transition $\mathfrak{Q} \to \mathfrak{Q}^*$, i.e. the Lorentz deformation with parameter $\mathfrak{g}$ relative to a new system of reference, then we find

$$\mathbf{x}_\nu'^* = \mathbf{M}_{\mathbf{p}'} \mathbf{x}_\nu' + \mathbf{m}' \tag{40}$$

where we have

$$\mathbf{M}_{\mathbf{p}'} = \mathbf{M}^{(\mathfrak{q})} \mathbf{M}_\mathbf{p} \mathbf{M}^{(\mathfrak{q})-1}; \tag{41}$$

here $\mathbf{M}^{(\mathfrak{q})}$ is the matrix of the coordinate transformation leading from $K \to K'$ Thus the latter coordinate transformation written explicitly

$$\mathbf{x}_\nu' = \mathbf{M}^{(\mathfrak{q})} \mathbf{x}_\nu + \mu \qquad \nu = 1, 2, \ldots, n \tag{42}$$

in place of the above relation we may also write

$$K' = \mathcal{M}^{(\mathfrak{q})}(K),$$

where we have denoted by $\mathcal{M}^{(\mathfrak{q})}$ the inhomogeneous operator containing the matrix $\mathbf{M}^{(\mathfrak{q})}$ and the vector μ.

§ 16. From relation (41) we see how the deformation $\mathfrak{Q} \to \mathfrak{Q}^*$ is represented relative to various systems of references in which the propagation tensor $\mathfrak{g}$ has the same representation.

More precisely, we may state that the propagation of light in a certain region of space is given by a tensor $\mathfrak{g}$. The representation of $\mathfrak{g}$ relative to a number of systems of references $K, K', \ldots, K''$ is the same, e.g.

$$K(\mathfrak{g}) = K'(\mathfrak{g}) = K''(\mathfrak{g}) = \ldots = \mathbf{g}.$$

Considering a coordinate transformation of the type

$$\bar{\mathbf{X}} = \mathbf{S}\mathbf{X} + \mathbf{s},$$

where

$$\tilde{\mathbf{S}}\mathbf{g}\mathbf{S} = \bar{\mathbf{g}} \neq \mathbf{g}.$$

We obtain from a system of reference K another system of reference $\bar{K}$ so that in the latter the propagation of light is described by a tensor $\bar{\mathbf{g}}$.

From the system of reference $\bar{K}$ we can form a group of systems of references $\bar{K}, \bar{K}', \bar{K}'', \ldots$ in each of which the tensor $\mathfrak{g}$ has the same representation $\bar{\mathbf{g}}$. The latter are connected by matrices $\bar{\mathbf{M}}$ obeying the relation

$$\tilde{\bar{\mathbf{M}}}\mathbf{g}\bar{\mathbf{M}} = \bar{\mathbf{g}}.$$

A Lorentz deformation $\mathfrak{Q} \to \mathfrak{Q}^*$ can be expressed by an operator of the set $\overline{\mathbf{M}}$ if we consider it in one of the representations $\overline{K}, \overline{K}', \overline{K}'', \ldots$. We find easily that the connection between the operators $\mathbf{M}$ and $\overline{\mathbf{M}}$ is given by

$$\overline{\mathbf{M}}_{\mathbf{q}} = \mathbf{S}^{-1} \mathbf{M}_{\mathbf{q}} \mathbf{S} \,. \tag{43}$$

Thus a Lorentz deformation $\mathfrak{Q} \neq \mathfrak{Q}^*$ can be represented by a set of operators

$$\mathbf{M}_{\mathbf{q}}, \mathbf{M}_{\mathbf{q}'}, \mathbf{M}_{\mathbf{q}''}, \ldots$$

relative to systems of references K, K', K'', ... in all of which $\mathfrak{g}$ is represented by a tensor $\mathbf{g}$.

The same Lorentz deformation can also be represented by operators

$$\overline{\mathbf{M}}_{\mathbf{q}}, \overline{\mathbf{M}}_{\mathbf{q}'}, \overline{\mathbf{M}}_{\mathbf{q}''},$$

relative to systems of references $\overline{K}, \overline{K}', \overline{K}'', \ldots$ in which $\mathfrak{g}$ appears to be represented by a tensor $\overline{\mathbf{g}}$ different from $\mathbf{g}$.

Considering relations (41) and (43) we find an important common feature of all the representations of a deformation $\mathfrak{Q} \to \mathfrak{Q}^*$. Indeed, *the matrices*

$$\mathbf{M}_{\mathbf{q}}, \mathbf{M}_{\mathbf{q}'}, \ldots, \overline{\mathbf{M}}_{\mathbf{q}}, \overline{\mathbf{M}}_{\mathbf{q}'}, \ldots,$$

have all the same eigenvalues.

The eigenvalues of a Lorentz matrix can be chosen to be of the form (see [3])

$$e^{i\varphi},\ e^{-i\varphi},\ \sqrt{\frac{c+v}{c-v}},\ \sqrt{\frac{c-v}{c+v}}$$

we see thus that *any representation of one Lorentz deformation has the same eigenvalues characterized by the parameters φ and v.* The latter result holds — as we see — also in the case of unisotropic propagation of light and it holds also if we consider systems of references in which the propagation tensor $\mathfrak{g}$ is represented by different matrices $\mathbf{g}, \overline{\mathbf{g}}, \ldots$, etc.

§ 17. We are now in a position to generalize the Lorentz principle to the case of homogeneous but possibly unisotropic propagation of light.

We state: *the laws of nature possess such symmetries that, provided $\mathfrak{Q}$ is a real physical system, then any Lorentz deformed form $\mathfrak{Q}^* = \mathscr{M}\mathfrak{p}(\mathfrak{Q})$ of $\mathfrak{Q}^*$ is also a possible system obeying the same laws of as $\mathfrak{Q}$.*

Furthermore, *if a system $\mathfrak{Q}$ is adiabatically accelerated then it changes its configuration into a Lorentz deformed form of its original configuration.*

The above formulation of the Lorentz principle regarding its form is identical with the former formulation (see [2]). We have extended its content by generalizing the Lorentz transformation to the case of unisotropic but homogeneous propagation of light.

§ 18. We make a concluding remark. The formulation of the Lorentz principle in its restricted form as was done in a previous work is based on the failure of a series of experiments to observe effects of translational motion. This failure is attributed to a peculiar symmetry of laws of nature which symmetry causes that to any effect which might arise from the translational motion relative to the aether, other effects appear which exactly compensate the former. This symmetry itself could be described adequately by the Lorentz principle.

The earlier considerations are based on the assumption that light is propagated isotropically relative to its carrier, the aether.

The generalized considerations show that supposing light was after all not carried isotropically in the aether but if the propagation be of the more general type which we denoted as homogeneous, even then, the symmetry discussed above might persist and this symmetry might prevent us not only to locate the distinguished system of reference K_0 which is at rest to the carrier of the light, but it equally prevents us to determine the propagation tensor $\mathbf{g}_0$ which describes the propagation of light relative to its carrier.

The extension of the symmetry properties of nature in this fashion is based on pure speculation. Experimentally the adequacy of this extension could be checked if we could carry out experiments, say with a Michelson interferometer in a region of space where we have good reason to believe the propagation of light to be unisotropic. If an experiment in such a region were to lead to a negative effect in spite of the unisotropy, then this result would directly justify the extension of the Lorentz principle.

At the moment such experiments do not exist. The generalization of the Lorentz principle we have given here can be in spite of the lack of direct evidence be justified.

Indeed, we shall show that the generalization of the Lorentz principle we have given here is a necessary intermediate step to its further generalization to the case of inhomogeneous propagation of light. In the case of the inhomogeneous propagation of light observable effects are found and the theory of these effects can be obtained by a straightforward further generalization which we discuss in the second part of this paper.

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ПРИНЦИП ЛОРЕНЦА И ОБЩАЯ ТЕОРИЯ ОТНОСИТЕЛЬНОСТИ
Часть I.

Л. ЯНОШИ

Резюме

Настоящая работа является первой из серии работ, в которых предлагается формулировка общей теории относительности на основе принципа Лоренца. В этой первой части преобразование Лоренца обобщается так, что оно становится применимым к областям пространства, где свет распространяется однородно, но быть может неизотропно. Показано, что для таких областей принцип Лоренца остается верным в своей обычной форме.