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IMPEDANCE, EQUIVALENT CIRCUIT AND STABILITY BEHAVIOUR OF MEDIUM-PRESSURE DISCHARGES

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An equivalent circuit of a medium pressure column is derived at low current and stability behaviour is considered on this basis.

Studies of the dynamic behaviour of discharges [1], [2] have shown that it is possible to derive quantitative information about the efficiency of various elementary processes (direct ionization, stepwise ionization) in the discharge mechanism. This possibility is of particular importance in instances when single elementary processes (e.g. triple collisions) have but a small effect on the static characteristics of a discharge, but a significant influence on the dynamic behaviour. From investigations of the dynamic behaviour it was found that tentative suggestions about the stability of the discharge [3], [4] could be made, although in order to get easily surveyable stability criteria it was necessary to derive an equivalent circuit of the discharge [5]. The positive low-pressure columns at weak currents have already been investigated in this direction. The object of this paper is first to derive an equivalent circuit of a positive medium-pressure column at low current and then to consider stability on this basis.

1. Equivalent circuit of a medium-pressure column at low current

The differential equation for the impedance behaviour of the positive column of a medium-pressure column, which is necessary to build up a suitable equivalent circuit, may be derived satisfactorily from the following system of initial equations [2]:

$$\begin{aligned} \partial Ne/\partial t + Ne/\tau_e - P_e &= 0, \\ \partial M/\partial t + M/\tau_m - P_m &= 0, \\ \partial R/\partial t + R/\tau_r - P_r &= 0, \\ J - \pi e_0 r_\sigma^2 N_e b_e E &= 0, \end{aligned} \tag{1}$$

where P_e/N^2, P_m/N^2 etc. are used as defined in [2].

Since plasma parameters E, Ne, M and R are to be subjected to variations whenever a disturbance δI_0 is impressed on the discharge current I_0 and only minor disturbances will be permitted

$$\left(\frac{\delta I_0}{I_0} \ll 1, \ \frac{\delta E}{E_0} \ll 1\right),$$

taking the above expressions with

$$\begin{aligned}
E &= E_0(1 + x_1) = E_0 + u\,,\\
N_e &= N_{e0}(1 + x_2)\,,\\
M &= M_0(1 + x_3)\,,\\
R &= R_0(1 + x_4)\,,\\
I &= I_0(1 + x_5) = I_0 + i\,,
\end{aligned} \tag{2}$$

we get the following system of equations:

$$\begin{aligned}
\frac{\partial x_2}{\partial t} &= b_{11}x_1 + b_{12}x_2 + b_{13}x_3 + b_{14}x_4\,,\\
\frac{\partial x_3}{\partial t} &= b_{21}x_1 + b_{22}x_2 + b_{23}x_3 + b_{24}x_4\,,\\
\frac{\partial x_4}{\partial t} &= b_{31}x_1 + b_{32}x_2 + b_{33}x_3 + b_{34}x_4\,,\\
x_5 &= b_{41}x_1 + x_4.
\end{aligned} \tag{3}$$

The b_{ik} and a_{ik} defined in [2] or b_i, respectively, show the correlations

$$\begin{aligned}
b_{11} &= -Na_{11}, & b_{31} &= -N\left(\frac{Ne/N}{R/N}\right)a_{41},\\
b_{12} &= -Nb_1, & &\\
b_{13} &= -Na_{13}, & b_{32} &= -N\left(\frac{Ne/N}{R/N}\right)a_{42},\\
b_{14} &= -Na_{14}, & &\\
b_{21} &= -N\left(\frac{Ne/N}{M/N}\right)a_{21}\,, & b_{33} &= -N\left(\frac{Ne/N}{R/N}\right)a_{43}\,,\\
b_{22} &= -N\left(\frac{Ne/N}{M/N}\right)a_{22}, & b_{34} &= -N\left(\frac{Ne/N}{R/N}\right)b_3,\\
b_{23} &= -N\left(\frac{Ne/N}{M/N}\right)b_2, & b_{41} &= a_{31},\\
b_{24} &= -N\left(\frac{Ne/N}{M/N}\right)a_{24}. & &
\end{aligned} \tag{4}$$

The wanted differential equation for the dynamic behaviour of a positive column which has been subjected to a minor disturbance is obtained by stepwise elimination of x_2, x_3 and x_4 through repeated differentiation of Eqs. (3):

$$\frac{c_0}{a_0}\frac{d^3 i}{dt^3}+\frac{c_1}{a_0}\frac{d^2 i}{dt^2}+\frac{c_2}{a_0}\frac{di}{dt}+\frac{c_3}{a_0}i = \\ =\frac{d^3 u}{dt^3}+\frac{a_1}{a_0}\frac{d^2 u}{dt^2}+\frac{a_2}{a_0}\frac{du}{dt}+\frac{a_3}{a_0}u. \tag{5}$$

The state of the plasma is characterized by the coefficients c_0, $c_1 \ldots a_0$, a_1, where combinations of the b_{ik} occur. With reference to the three inertial factors which show as a time derivation in the balance equation we get a linear homogeneous equation of third order.

In earlier experimental and theoretical investigations (Fig. 1) it was discovered that a marked resonance effect among others could be formed for the impedance condition, which suggests that a corresponding equivalent circuit (Fig. 2) is possible for weak-current discharges at medium pressures. With reference to the three inertial factors which are reflected as time derivations in the balance equations three reactances were taken into consideration in the equivalent circuit.

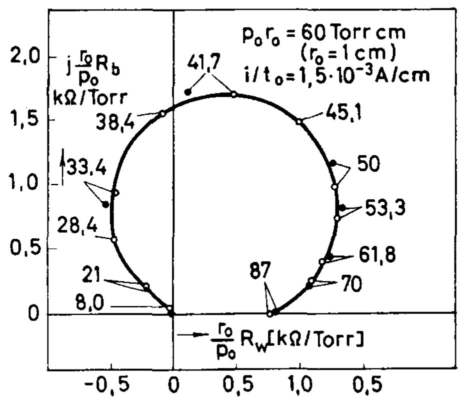

Fig. 1. Calculated and measured impedance characteristics of a neon discharge [2]. [$p_0 r_0 = 60$ torr cm; $r_0 = 1$ cm; $i/r_0 = 1.5 \cdot 10^{-3}$ A/cm]. ○ theory; ● experiment. Numerical values refer to the corresponding reduced frequencies in cps/torr

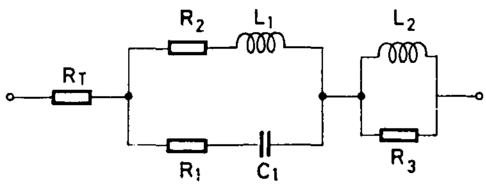

Fig. 2. Equivalent circuit of the positive column of a low-current medium-pressure glowdischarge (. . . taking the three intertial factors into consideration)

The differential equation for this at first empirically introduced equivalent circuit is:

$$\alpha \frac{d^3 i}{dt^3} + \beta \frac{d^2 i}{dt^2} + \gamma \frac{di}{dt} + \delta i = \frac{d^3 u}{dt^3} + A \frac{d^2 u}{dt^2} + B \frac{du}{dt} + Cu, \tag{6}$$

where coefficients α, β, ... are combinations of the quantities R_T, R_1, R_2, R_3, L_1, L_2 and C_1.

Eqs. (5) and (6) are isomorphic; so that with due regard to the plasma parameters reflected in the b_{ik} it is immediately possible to calculate the resistances, inductances and capacities by comparing coefficients. In the calculations presentation of the quantities of the equivalent circuit in a similarity scheme was abandoned from the first both because a higher pressure had to be expected with greater triple collision destruction in the medium-pressure discharge and because this process does not conform with the B-invariant similarity law [6].

From the comparison of coefficients a system of seven equations with seven unknown quantities is obtained. The solution of the equation system is here restricted to real — and hence physically suggestive — values for reactances: the solution was carried out graphically and numerically.

The quantities of the equivalent circuit were calculated by using the characteristic quantities of the homogeneous medium-pressure neon column for a given case of discharge [2] ($p_0 = 60$ torr; $I_0 = 1{,}5 \cdot 10^{-3}$ A; $r_0 = 1$ cm). The detailed calculation yielded the following results:

$L_1 = 1.55$ Henry	$R_2 = -1.74 \cdot 10^4$ Ohm
$L_2 = 1 \cdot 10^{-3}$ Henry	$R_T = +1.60 \cdot 10^4$ Ohm
$C_1 = 2.67 \cdot 10^{-9}$ Farad	$R_1 = +2.90 \cdot 10^4$ Ohm
$R_3 = 1.30 \cdot 10^2$ Ohm	

The calculated values were tested by substituting them into the following relation for the impedance of the equivalent circuit:

$$Z = R_T + \frac{R_1 R_2 + \dfrac{L_1}{C_1} + j\left(R_1 \omega L_1 - R_2 \dfrac{1}{\omega C_1}\right)}{R_1 + R_2 + j\left(\omega L_1 - \dfrac{1}{\omega C_1}\right)} + \frac{R_3 j \omega L_2}{R_3 + j\omega L_2}. \tag{7}$$

The check demonstrated that the last term on the r.h.s. of the relation (7) contributes only very little to Z, hence the impedance behaviour of the positive column of a medium-pressure discharge is essentially determined by the first two terms, i.e. by the equivalent circuit of Fig. 3.

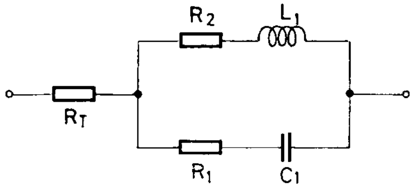

Fig. 3. "Reduced" equivalent circuit

2. Stability considerations

The following considerations refer only to discharges with a long positive column, the dynamic behaviour of which is essentially characterized by that of the positive column and are based on a procedure for establishing stability criteria outlined by GRANOWSKI [3], except that the differential characteristic of a discharge in an equivalent circuit representation is used as the starting point instead of a differential characteristic derived immediately from the balance equation.

In the case investigated here the differential characteristic of the gaseous space in the equivalent circuit scheme is:

$$A^* \frac{d^2i}{dt^2} + B^* \frac{di}{dt} + D^* i = A'' \frac{d^2 u}{dt^2} + B'' \frac{du}{dt} + u \,, \tag{8}$$

where

$$\begin{aligned} A^* &= R_T C_1 L_1 + R_1 C_1 L_1, \\ B^* &= R_1 R_T C_1 + R_2 R_T C_1 + R_2 R_1 C_1 + L_1, \\ D^* &= R_T + R_2, \\ A'' &= C_1 L_1, \\ B'' &= R_1 C_1 + R_2 C_1. \end{aligned}$$

For the further derivation of stability criteria we need a differential characteristic of the outer circuit in a general form (Fig. 4) which is taken over from an earlier work [4] (cf. also [3]):

$$CRL \frac{d^2 i}{dt^2} + L \frac{di}{dt} + Ri = - CR \frac{du}{dt} - u \,. \tag{9}$$

The general approach in investigating stationarity problems is to permit a small disturbance of a given state and to find out the time changes. If the disturbance recedes after a certain lapse of time the discharge is a stationary one, if not, it is non-stationary. As the differential equations (8) and (9) are linear in i and u, the following form is used for the disturbances of current and voltage:

$$\begin{aligned} i &= i_0\, e^{\omega t}, \\ u &= u_0\, e^{\omega t}, \end{aligned} \tag{10}$$

where i_0 refers to the initial deviation of the current from the stationary value I_0, and u_0 refers to the initial voltage deviation (i_0 and ω are generally complex quantities).

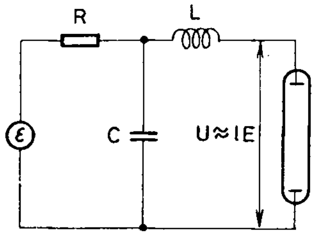

Fig. 4. Electrical circuit for the general case: U = voltage across the tube, E = voltage gradient, l = length of the positive column

With (10), by evaluating the differential characteristics of the gaseous space (8) and of the outer circuit (9), the following equation for calculating ω is obtained:

$$g_4\omega^4 + g_3\omega^3 + g_2\omega^2 + g_1\omega + g_0 = 0 \tag{11}$$

with

$$\begin{aligned} g_0 &= R + R_A\,, \\ g_1 &= L + L_1 + CRR_A + RR_1C_1 + RR_2C_1 + R_1R_TC_1 + R_2R_TC_1 + R_2R_1C_1\,, \\ g_2 &= LCR + LC_1\{R_1 + R_2\} + L_1C_1\{R_T + R_1 + R\} + \\ &\quad + CC_1R\{R_1R_T + R_2R_T + R_2R_1 + L_1C_1\}\,, \\ g_3 &= CC_1LR\{R_1 + R_2\} + C_1LL_1 + CC_1L_1R\{R_1 + R_T\}\,, \\ g_4 &= RCC_1LL_1\,. \end{aligned}$$

$R_A = R_T + R_2$ is the differential resistance at zero frequency. The behaviour is evidently stationary when the real parts of all roots of Eq. (11) are negative. According to the theorem of HURWITZ [7], the necessary and sufficient condition for this is:

$$D_1 = g_1 > 0\,; \qquad D_2 = \begin{vmatrix} g_1 & g_0 \\ g_3 & g_2 \end{vmatrix} > 0$$

$$D_3 = \begin{vmatrix} g_1 & g_0 & 0 \\ g_3 & g_2 & g_1 \\ 0 & g_4 & g_3 \end{vmatrix} > 0; \qquad D_4 = \begin{vmatrix} g_1 & g_0 & 0 & 0 \\ g_3 & g_2 & g_1 & g_0 \\ 0 & 0 & 0 & g_4 \end{vmatrix} > 0 \tag{12}$$

where $g_0 > 0$. This stationarity condition requires that all coefficients g_μ ($\nu = 0 \ldots 4$) of the characteristic equation should be positive in sign. In the following we shall restrict ourselves to a discussion of the necessary conditions g_ν ($\nu = 0 \ldots 4$) > 0, because it is rather difficult to deal in a complex discussion with stationarity conditions in the general form (12).

From the above necessary conditions one immediately obtains the well-known Kaufmann criteria

$$R + R_A > 0, \tag{13}$$

$$L + CRR_A > 0, \tag{14}$$

when there is a restriction to the static characteristics, i.e. when L_1, C_1 and R_1 are made identical with zero. In the general case, however, one has to make full use of all the criteria for stability tests.

As presumed by Kaufmann one cannot change the first stability criterion (13) just by taking into account the dynamic behaviour, while in the second criterion it is necessary to add the inductance L_1 of the discharge once to the inductance L of the outer space (cf. [4]); furthermore, additional terms, which are supplied by the self-capacity of the discharge C_1 with the resistances R_T, R_1, R_2 or R, respectively, will occur for the second condition. Apart from these "extended" Kaufmann conditions three more conditions will occur which are accounted for only by the consideration of the dynamic behaviour of the discharge. Consequently, these conditions become effective when the dynamic behaviour of the circuit is co-determined by the discharge.

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ИМПЕДАНС, ЭКВИВАЛЕНТНАЯ СХЕМА И СТАБИЛЬНОСТЬ РАЗРЯДА СРЕДНЕГО ДАВЛЕНИЯ

Г. ДОЙЧ

Резюме

Определена эквивалентная схема столба маломощного разряда среднего давления и рассмотрены условия стабильности.