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ON THE OFF-MASS-SHELL CONTINUATION OF THE VENEZIANO MODEL

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The problem of incorporation of currents into the Veneziano model is studied. The case of axial vector current is treated in detail on the example of the off-shell $\pi\pi \rightarrow \pi\pi$, $A_\mu\pi \rightarrow \pi\pi$ an d $A_\mu\pi \rightarrow A_\mu\pi$ processes.

Introduction

Recently some interest has been devoted to the incorporation of currents into the Veneziano model. The case of vector currents has been extensively studied by BROWER and WEIS and others [1] in the framework of the generalized Veneziano model. At the present stage of development of this theory it is not yet possible to study axial currents in this framework. Therefore, as we want to study axial currents, we shall confine ourselves to four-point functions only. Thus our treatment will be more physical in the sense that the particles (external and leading trajectory particles) will have their physical masses.

As a particularly simple example we shall study the scattering amplitudes of the $A_\mu\pi \rightarrow A_\mu\pi$, $A_\mu\pi \rightarrow \pi\pi$, $\pi\pi \rightarrow \pi\pi$ processes. The reasons for choosing these processes are:

a) The related on-shell processes have very simple Veneziano amplitudes as only a single trajectory contributes and the problems of external particle spin can also be solved (at least to leading trajectory level) in this case.

b) These processes form a relatively closed subsystem in the bootstrap scheme of all processes.

c) Using the soft pion method, one may extract information on the pion electromagnetic form factor.

We first list some fundamental properties of our model which we believe mean an improvement compared with numerous other attempts. In a Veneziano model with linear trajectories and daughters it is natural to assume that the axial vector current is saturated not only by the π and A_1 mesons, therefore we take into account the higher mass π and A_1 mesons (Fig. 1), too. Furthermore, we have carefully carried out the off-shell continuation in the momentum of one final π in the $A_\mu\pi \rightarrow \pi\pi$, $\pi\pi \rightarrow \pi\pi$ amplitudes and also in the

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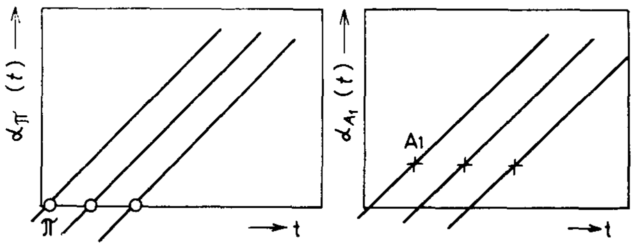

Fig. 1

momentum of one of the initial pions in the $\pi\pi \to \pi\pi$ amplitude. This is necessary, if one wants to compare the Veneziano model with current algebra and divergence algebra (e.g., it is essential in the determination of the pion form factor, too.)

Finally, we have to realize that at present we may hope to construct a model which is satisfactory on the leading trajectory level only. One reason for this is that it is the case for the $A_1\pi \to A_1\pi$ amplitude (which is related to the $A_\mu\pi \to A_\mu\pi$ amplitude); another reason will be mentioned below.

Preliminaries

The definitions of our amplitudes are the following:

$$T_{\mu\nu}\begin{pmatrix} iljn \\ qpq'k \end{pmatrix} = \int d^4 x e^{-ipx} \langle \pi q' j | T(A^l_\mu(x)\, A^n_\nu(0)) | \pi q i\rangle ,$$

$$T_\mu\begin{pmatrix} iljn \\ qpq'k \end{pmatrix} = i\int d^4 x e^{-i'px} \langle \pi q' j | T(A^l_\mu(x)\, \partial \cdot A^n(0)) | \pi q i\rangle , \qquad (1)$$

$$T\begin{pmatrix} iljn \\ qpq'k \end{pmatrix} = \int d^4 x e^{-ipx} \langle \pi q' j | T(\partial \cdot A^l(x)\, \partial \cdot A^n(0)) | \pi q i\rangle ,$$

where A_μ is the axial vector current, i, l, j, n are isospin indices, q, p, q', k are momenta.

The amplitudes are connected by the PCAC equations:

$$T_{\mu\nu}k^\nu = T_\mu + i(\delta_{il}\delta_{jn} - \delta_{in}\delta_{lj})(q+q')_\mu \frac{F_\pi(t)}{2(2\pi)^3}, \qquad (2)$$

$$T_\mu p^\mu = T + i\,\Sigma^{ln}(t)\,, \qquad (3)$$

where $F_\pi(t)$ is the pion electromagnetic form factor and $\Sigma^{ln}(t)$ is the σ-term:

$$\Sigma^{ln}(t) = -i\int d^4 x \delta(x_0)\, e^{-ipx} \langle \pi q' j | [A^l_0(x), \partial \cdot A^n(0)] | \pi q i\rangle . \qquad (3')$$

The residues of $T_{\mu\nu}$ at the appropriate values of p^2 and k^2 are proportional to the $A_1^*(\pi^*)\pi \to A_1^{**}(\pi^{**})\pi$ amplitudes, where the stars denote the higher mass A_1 (or π) particles. Similarly T_μ is connected to the $A_1^*(\pi^*)\pi \to \pi^{**}\pi$ and T to the $\pi^*\pi \to \pi^{**}\pi$ amplitudes.

In constructing the model, we shall proceed in the order $T \to T_\mu \to T_{\mu\nu}$. The number of independent invariant functions (using all the crossing and isospin constraints) is 2 for the amplitude T, 6 for T_μ and 20 for $T_{\mu\nu}$. Thus, for the sake of brevity, we shall deal here only with the $I_s = 2$ amplitudes and shall not write down all the formulas. We refer the reader for details and complete formulas to the original papers.

The amplitude T

The first task is to construct the $\pi^*\pi \to \pi^{**}\pi$ amplitude. This is given by the same expression as the $\pi\pi \to \pi\pi$ amplitude. E.g., the $I_s = 2$ amplitude is, as it is well known:

$$f_\varrho f_\varrho' \, B(t, u) \left(1 - \alpha(t) - \alpha(u)\right), \tag{4}$$

where

$$B(t, u) = \frac{\Gamma\left(1-\alpha(t)\right)\Gamma\left(1-\alpha(u)\right)}{\Gamma(2-\alpha(t)-\alpha(u))} \; ; \; \alpha(t) = \frac{1}{2} + b(t - m_\pi^2)$$

and b is the universal trajectory slope. As LOVELACE [2] pointed out, this expression is a good candidate for the off-shell amplitude, as it satisfies the Adler condition. Later on SUURA [3], OSBORN [4] and CSIKOR [5] pointed out the necessity of including the higher mass pion poles. So the off-shell amplitude is obtained by the replacements:

$$f_\varrho \to f(p^2), \; f_\varrho' \to f(k^2)$$

where

$$f(p^2) = \Gamma\left(\frac{1}{2} - \alpha(p^2)\right) \bar{f}(p^2)$$

and $\bar{f}(p^2)$ has no poles at all. The argument of the Γ function is $-\alpha_\pi(p^2)$, which yields the expected pole structure.

As pointed out by CSIKOR [5] and ELLIS and OSBORN [6], the daughter structure of this amplitude is far from being realistic. In fact, an infinite number of satellite terms are needed to correct this. So we content ourselves by saying that the above amplitude is correct only to the leading trajectory level.

The above ideas can be extended to other processes, such as $K\pi$ and KK scattering [4, 7]. This off-shell Veneziano model is in reasonable agreement with the chiral symmetry breaking model of GELL-MANN, OAKES and RENNER.*

* In the literature there has been some debate about this statement. We want to emphasize that all the diseases can be cured by taking into account explicit momentum dependence of the type described above and satellites. We refer to the recent paper by YONEYA et al. [8], which illustrates this point.

The amplitude T_μ

A number of attempts have been made to construct the amplitude T_μ already in the early times of Veneziano theory [9]. However, all these attempts keep the final state pions on the mass-shell (and take into account only the lowest π and A_1 poles). An amplitude which is off the mass-shell in both p_μ and k_μ has been given in [10]. This amplitude is by no means satisfactory as it contains an unphysical pole at $p^2 = 0$. A model which avoids this difficulty has been given in [11] and we shall now explain the main ideas of this work.

Before starting with the construction, it is convenient to state two simple theorems.

a) If a concrete model for T, T_μ satisfies the PCAC Eq. (3) (for all momenta p_μ, k_μ), exhibits the correct pole structure and satisfies the crossing conditions, then the expression of T_μ taken at $k_\mu = 0$ will be automatically transversal.

b) If in addition to the conditions of theorem a), the amplitude T satisfies the Adler–Weissberger condition, then $F_\pi(t)$, as determined from T_μ in the soft pion limit, has the correct normalization.

So in the construction we have to concentrate only on the conditions of these theorems.* We assume that T_μ is a sum of three terms:

$$T_\mu = T_\mu^{(1\times0)} + T_\mu^{(0\times0)} + t_\mu, \tag{5}$$

where $T_\mu^{(1\times0)}$ contributes only to the on-shell amplitudes $A_1^*\pi \to \pi^{**}\pi$, $T_\mu^{(0\times0)}$ contributes only to the on-shell amplitudes $\pi^*\pi \to \pi^{**}\pi$, and t_μ does not contribute to the amplitudes on the mass-shell. t_μ ensures that the PCAC Eq. (3) is fulfilled.

The $A_1^*\pi \to \pi^{**}\pi$ scattering amplitude is given by the well known Goebel, Blackmon, Wali expression [13], e.g. the $I_s = 2$ amplitude is:

$$\begin{gathered} f_\varrho'[\mu(k_\mu(2\alpha(u) - \alpha(t) - 1) - (q + q')_\mu(1 - \alpha(t))) \\ -2\mu'\,(k_\mu + (q + q')_\mu)]\,B\,(t,\,u), \end{gathered} \tag{6}$$

μ', μ correspond to the s and d wave-coupling constants of $A_1^* \to \varrho\pi$ decay, μ', μ are free parameters.

$T_\mu^{(1\times0)}$ is obtained from the expression (6) by the following replacements:

$$\begin{aligned} & f_\varrho \to f(k^2), \\ & \mu \cdot l_\mu \to \left\{ g_{\mu\lambda}\,\mu(p^2) - p_\mu p_\lambda \frac{\mu(p^2) - \mu(0)}{p^2} \right\} \cdot l^\lambda, \\ & \mu' \cdot l_\mu \to \left\{ g_{\mu\lambda}\,\mu'(p^2) - p_\mu p_\lambda \frac{\mu'(p^2) - \mu'(0)}{p^2} \right\} \cdot l^\lambda, \end{aligned} \tag{7}$$

* These theorems underline the importance of a careful continuation in the momentum k_μ. In fact, in s. [3, 12] this continuation was not carried out carefully, which led to difficulties in the determination of the pion form factor, namely at the kinematics $k_\mu = 0$ T_μ turned out to have a longitudinal component, too.

where l_μ is an arbitrary momentum. The function $f(k^2)$ is the same as before, and

$$\mu(p^2) = \Gamma\left(\frac{3}{2} - \alpha(p^2)\right)\bar{\mu}(p^2)$$

with $\bar{\mu}(p^2)$ a nonsingular function $\left(\frac{3}{2} - \alpha(p^2) = - \alpha_{A_1}(p^2)\right)$.

Similarly, $T_\mu^{(0x0)}$ is obtained from the $\pi^*\pi \to \pi^{**}\pi$ amplitude given in Eq. (5) by the following replacements:

$$f_\varrho \to \frac{f(p^2) - f(0)}{p^2}\, p_\mu, \qquad f'_\varrho \to f(k^2). \tag{8}$$

The above construction ensures that on the mass-shell $T_\mu^{(1x0)}$ and $T_\mu^{(0x0)}$ contribute only to the relevant amplitudes. Moreover $p^\mu\,(T_\mu^{(1x0)} + T_\mu^{(0x0)}) - T - i\,\Sigma^{ln}(t)$ has no singularity in the variable p^2, so we have a chance that t_μ (which is determined from the PCAC Eq. (3)) is also nonsingular in p^2.

The determination of t_μ is a little tricky.* For details we refer to [11]; here we note only that non-Veneziano type terms necessarily occur. To see this let us write down the PCAC equation (T_i denote the invariant functions of T_μ):

$$T_1 \cdot p^2 + T_2\,\frac{s + u - 2m_\pi^2}{2} + T_3\,\frac{s - u}{2} = T(s, t, u, p^2, k^2) - T(m_\pi^2, t, m_\pi^2, 0, t). \tag{9}$$

The last term is the σ term, which has clearly non-Veneziano form, and generates a non-Veneziano type term in t_μ, too. These non-Veneziano type terms exhibit fixed power behaviour for large s.

Using the soft pion method one gets the following expression for the pion electromagnetic form factor:

$$\begin{aligned} F_\pi(t) = \text{const.}\Big\{&(\mu(t) - \mu(0))\,(1 - \alpha(t)) + 2(\mu'(t) - \mu'(0)) - \\ &- 4t(\eta(t) - \bar{\eta}(t, 0)) + b\bar{f}(0)\,\Gamma\left(\frac{1}{2} - \alpha(0)\right) + \\ &+ f(0)\,\Gamma\left(\frac{3}{2} - \alpha(0)\right) b\left[\psi\left(\frac{1}{2}\right) - \psi\left(\frac{3}{2} - \alpha(t)\right)\right]\Big\} \cdot B(m_\pi^2, t), \end{aligned} \tag{10}$$

* Our main guide is that we have to avoid unphysical poles.

where $\psi(x) = \Gamma'(x)/\Gamma(x)$ is the logarithmic derivative of the Γ function, $\bar{\eta}(t)$, $\eta(t, 0)$ are unknown, nonsingular functions (which come from the nonuniqueness of t_μ). The pole structure of this expression is as expeeted. The lesson to be drawn from this complicated expression is, that without additional assumptions one cannot make any definite prediction for $F_\pi(t)$.

The $T_{\mu\nu}$ amplitude

The $T_{\mu\nu}$ amplitude can be constructed from the relevant on-shell amplitudes using very similar ideas. Of course, the calculations are rather lengthy (we have 20 independent invariant functions). So we point out here only the special difficulties. For details we refer to [14].

The first difficult point is the on-shell $A_1\pi \to A_1\pi$ amplitude. The new features are of course connected with the non-zero spin of the external particles. Furthermore, we have now non-trivial factorization conditions, as illustrated in

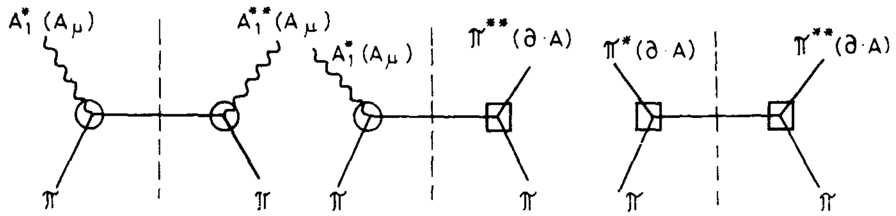

Fig. 2

Fig. 2. All these questions are treated in detail in [15] and [16]. We have essentially accepted the solution of LASLEY and CARRUTHERS with some modifications.* As a result we have got a *unique* three-parameter (+ free coupling constants) solution.

This solution can be extended with some effort to the $A_1^*\pi \to A_1^{**}\pi$ amplitudes. As one of the invariant functions vanishes if A_1^* and A_1^{**} is the same particle, the extension for $A_1^* \neq A_1^{**}$ yields nontrivial information.

As in the case of T_μ, $T_{\mu\nu}$ will be built up as a sum of several terms:

$$T_{\mu\nu} = T_{\mu\nu}^{(1\times1)} + T_{\mu\nu}^{(1\times0)} + T_{\mu\nu}^{(0\times1)} + T_{\mu\nu}^{(0\times0)} + t_{\mu\nu}.$$

The meaning of the various terms should be clear from the discussion of the T_μ amplitude. Only the construction of $T_{\mu\nu}^{(1\times1)}$ gives rise to new difficulties, as not all parameters appear in product form in the on-shell amplitudes (e.g.

* First the Adler condition is imposed incorrectly in [15]; secondly, we have removed one of the conditions of [15], which means a restriction on the signature of a daughter trajectory; third we have kept $m^2 \neq 0$ throughout the calculation. Details of this as well as comments on WHIPPMAN's paper are given in [17].

the coupling constants of the $A_1^* A_1^{**} \varrho$ vertex do not). Clearly some additional assumptions are needed here which, of course, increase the arbitrariness of the procedure.

We want to note that the factorization condition (illustrated in Fig. 2) is not satisfied automatically for our amplitude. So we have to impose this, which severely restricts the arbitrariness of $t_{\mu\nu}$.

A final remark concerns the FUBINI—DASHEN—GELL-MANN sum rule [18]. Our amplitude does not satisfy this sum rule automatically. If we impose it, we get severe restrictions for the pion electromagnetic form factor $F_\pi(t)$. Among others, this restriction fixes the ratio of the two coupling constants of $A_1 \to \varrho\pi$ decay, namely (using the notation of [19]) we get $/G_T/G_L/ = 0.90—1.1$. The experimental number is [20] 0.65 ± 0.25. While this is quite appealing, for large t the restricted $F_\pi(t)$ behaves only as $|t|^{-1/2} \log b|t|$. So we prefer to say that our amplitude does not satisfy the FDGM sum rule.

Concluding remarks

The procedure for the construction of current amplitudes outlined above is clearly applicable to other processes, too. The main difficulty is that until now we have no really reliable on-shell amplitudes for the physically most interesting cases [21], e.g. for the case of a proton target. An interesting attempt in this direction has been made by KONETSCHNY and MAJEROTTO [22], although the on-shell amplitudes and also the continuation are very crude yet. Clearly, a real progress in the construction of on-shell amplitudes would induce a progress in the construction of current amplitudes, too.*

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* A more refined continuation along the lines explained here is under investigation (private communication by W. KONETSCHNY).

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О ПРОДОЛЖЕНИИ МОДЕЛИ ВЕНЕЗИАНО ВНЕ МАССОВОЙ ПОВЕРХНОСТИ

Ф. ЧИКОР

Резюме

Изучена проблема включения токов в модель Венезиано. Детально рассматривается случай аксиального векторного тока на примере процессов вне массовой поверхности $\pi\pi - \pi\pi$, $A_\mu\pi \to \pi\pi$, и $A_\mu\pi \to A_\mu\pi$.