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SPIN AND UNITARY SPIN IN THE DUAL RESONANCE MODEL

By

I. Montvay

INSTITUTE FOR THEORETICAL PHYSICS, ROLAND EÖTVÖS UNIVERSITY, BUDAPEST

The recent developments in constructing dual amplitudes with spins and unitary symmetry are briefly reviewed.

I. Introduction

A considerable amount of work in the Veneziano theory has been done in the model with neutral, scalar Reggeons only. In this model there is a single family of trajectories, the ground state being a neutral, scalar particle. As a consequence, the intercept of the leading trajectory is negative. One may ask how much this simple model reflects the real world. The mere fact that physicists consider such models expresses their strong belief that the physical world is not the only one possible mathematically. One can fulfil the axioms *including unitarity* without ϱ-mesons. Nevertheless ϱ-mesons do exist, thus one must once abandon the spinless world. In fact, a great deal of work has been done in the last two years in the direction of physical Reggeons with positive intercepts.

It is interesting to note that the two classical examples (the $\pi\pi\pi\omega$ and $\pi\pi\pi\pi$ amplitudes) both involve spin and unitary spin. In spite of this, the systematic inclusion of positive intercept trajectories and especially fermion trajectories represents a lot of nontrivial problems. After the success of the 4π amplitude it is tempting to try the many-pion amplitudes. The isospin structure of them was given quickly by Paton and Chan [1]. The integral representation of the invariant amplitude in the case of 4π is [2]:

$$\int_0^1 dv\, v^{-1-\alpha(s)}(1-v)^{-1-\alpha(t)}\left\{-1-\alpha(s)-\alpha(t)\right\},$$

where

$$s=(p_1+p_2)^2; \qquad t=(p_2+p_3)^2;$$

$$p_1+p_2+p_3+p_4=0; \qquad \alpha(s)=\alpha_\varrho(s)-1=a_\varrho-1+\alpha s.$$

Therefore, the invariant for the N-pion amplitude can be of the general form

$$\int d\,\varphi_N(v,p)\,P(v_{ij};s_{ij}).$$

Here p is a polynomial in the channel invariants

$$s_{i,j}=(p_{i+1}+p_{i+2}+\ldots+p_j)^2\,;\quad 0\le i<j\le N-1\,;\quad p_1+p_2+\ldots+p_N=0$$

and in the corresponding channel integration variables

$$v_{0,i}=v_i\,(i=1,\ldots,N-1)\,;\quad v_{i,j}=\frac{(1-v_{i+1}\,v_{i+2}\ldots v_{j-1})\,(1-v_i\ldots v_j)}{(1-v_{i+1}\ldots v_j)\,(1-v_i\ldots v_{j-1})}$$

$$v_1=v_{N-1}=0\,;\qquad (2\le i+1<j\le N-1)\,.$$

The integral without the polynomial P is the spinless N-point amplitude:

$$\int d\varphi_N(v,p)=\int\frac{dv_2\,dv_3\ldots dv_{N-2}}{(1-v_2v_3)\,(1-v_3v_4)\ldots(1-v_{N-3}\,v_{N-2})}\prod_{(i,j)}v_{i,j}^{-1-\alpha(s_{i,j})}=$$

$$=\int\prod_{i=2}^{N-2}dv_i\,v_i^{-1-\alpha(s_{0,i})}\,(1-v_i)^{a-1}\prod_{2\le i\le j\le N-2}(1-v_i\,v_{i+1}\ldots v_j)^{-2\alpha p_i p_{j+1}}\,.$$

Explicit expressions for the many-pion amplitude were given by OLIVE and ZAKRZEWSKY [3] and by RITTENBERG and RUBINSTEIN [4]. These amplitudes are, however, not completely satisfactory as they do not factorize along the daughter trajectories.

The many-pion amplitudes are still simple in the sense that external spins are not present in them. In the general case the amplitude has spin indices. Instead of an invariant one has to construct functions of the particles' momenta showing the appropriate covariance property. In principle it would be possible to deal with helicity amplitudes familiar in the Regge theory, or with several invariant amplitudes multiplying the independent covariants. A more direct and simple way is, however, to consider the M-functions ("spinor amplitudes") which are the S-matrix elements between spinor states. The M-functions are free from kinetical singularities and have simple covariance, crossing and factorization properties.

The factorization is a severe restriction and at the same time a very powerful tool in every zero-width resonance model. It says that the residuum of a pole corresponding to a given intermediate state must factorize into the product of the appropriate lower point functions. If we have, for instance, the 6π amplitude, then by factorization we can get $A_1\pi\pi\pi$, $A_2\pi\pi\pi$, $\varrho\varrho\pi\pi$, etc. By subsequent factorizations the whole amplitude can be traced back to the vertices. Therefore, the only physical content of a dual resonance model is the spectrum of particles and the trilinear couplings among them. A constructive way of building up the amplitudes is thus to specify the spectrum of particles (which can be external as well as intermediate states) and to fix their couplings. Then by calculating the residua and summing up the poles, one has the amplitude of an arbitrary process. It can be seen that the procedure is very similar to the van Hove model of Reggeization. The difference is that a simultaneous description of the crossed channels is also aimed both at low and high energies. There are several papers which start from this standpoint or more precisely from its advanced variant, the Reggeized supermultiplet [5]. The spectrum of these models is, however, too much unphysical as the internal states span SU(6, 6) representations.

The SU(6,6) symmetry which is a relativistic version of SU(6) points toward quarks. In fact, already the PATON—CHAN unitary symmetry factors for the N-point functions are easily interpreted in terms of the Rosner–Harari duality diagrams strongly reminiscent of quarks. Therefore, it is perfectly reasonable to try to extend the PATON—CHAN SU(3) factors to SU(6) in order to get a description of the external spins on the same footing as external unitary spin. Indeed, this was done by MANDELSTAM, BARDAKCI and HALPERN [6] (MBH). In the MBH model the M-functions for the hadron are projected out by factorization from the "scattering amplitudes" UB of quarks. Here B is the scalar Veneziano amplitude and U a spin and unitary spin factor, being a product of quark wave functions. The main failure of this model is also the unphysical degeneracy of intermediate states: there are 144 ground state mesons and 1728 ground state baryons (instead of 36 and 56, respectively). In spite of its unphysical features the MBH model is one of the greatest achievments of the last years, as a consequence of its fascinating simplicity and its factorization at all the daughter levels.

The unphysical degeneracy of the leading trajectory in SU(6, 6) theories is on the same footing as the long standing problem of the parity doubling of fermion Regge trajectories. It was demonstrated recently on an explicit example in the van Hove model by CARLITZ and KISLINGER [7] that there is a theoretical possibility to avoid the parity doubling by means of standing j-plane cuts. Such standing cuts were introduced in the Veneziano model by several authors to avoid the degeneracy of internal states [8–10]. In the SU(6,6) models (or the Sl(6, C) model [11]) the CARLITZ—KISLINGER cuts appear both in fermion and meson channels. This makes the high-energy predictions at high momentum transfer problematic.

An essentially different method of constructing Veneziano type amplitudes for general processes was proposed in a work of DOMOKOS, KÖVESI-DOMOKOS and SCHÖNBERG [12]. It is based on a general Fourier—Mellin integral representation which gives in pole approximation the Veneziano amplitude. In this proposal the internal Sl(2, C) group of Möbius transformations of the KOBA—NIELSEN integrand [13] for the spinless amplitude is combined in a nontrivial way with the Lorentz group transforming the external spins. The interesting possibility in this approach is that it gives a framework for going beyond the pole approximation.

The main drawback of all the above models is very easy to summarize: it is the lack of factorization. Even the factorization of the spinless amplitude holds only in a limited sense. Strictly speaking, factorization with ghost states cannot be regarded as factorization. Another uncertainty is due to the possible inclusion of satellite terms which do not contribute in one or more channels to the leading trajectory. An interesting question is whether it would be possible to make out of two bad things a good one, that is, to use the satellite ambiguity for ghost killing.

II. Ghost killing with satellites

The possibility of adding satellite terms to the generalized Veneziano amplitudes seems to leave a considerable freedom in the construction of dual resonance models. This arbitrariness is presumably weakened by factorization. Nevertheless, at the first sight it seems that every level density and every coupling strength can be fitted with a suitable choice of satellite terms.

Let us illustrate this in the simple case of scalar $N = 4$ point functions. The integral representation of a typical satellite term is

$$V(m, M; n, N) = \int_0^1 dv\, v^{-1-\alpha(s)} (1-v)^{-1-\alpha(t)} v^m (1-v)^n (\alpha s)^N (\alpha t)^M \qquad (1)$$
$$\alpha(s) = a + \alpha s .$$

This term gives contributions in the points $s_i = (i - a/\alpha)$; $\alpha(s) = M + i - m$, $M + i - m - 1, \ldots, 0$ ($i = m$, $m + 1$, $m + 2, \ldots$) on the s-channel Chew—Frautschi plot, and in $t_j = (j - a/\alpha)$; $\alpha(t) = N + j - n$, $N + j - n - 1, \ldots, 0$ ($j \geq n$) on the t-channel Chew—Frautschi plot. Let us suppose for the moment that we know the "good" residua, corresponding to the physical level density and physical couplings. Then in the representation

$$B(s,t) = \sum_{n=0}^{\infty} \sum_{\nu=0}^{n} \frac{S_{n\nu} t^\nu}{n - \alpha(s)} = \sum_{m=0}^{\infty} \sum_{\mu=0}^{m} \frac{T_{m\mu} S^\mu}{m - \alpha(t)} \qquad (2)$$

the coefficients $S_{n\nu}$ and $T_{m\mu}$ are known. The good residua can be built in an amplitude of Veneziano type if one adds satellite terms in the following order:

$$\begin{aligned} B(s,t) = {} & \sigma_{00}\, V(0, 0; 1, 0) + \tau_{00}\, V(1, 0; 0, 0) + \sigma_{11}\, V(1, 1; 2, 0) + \\ & + \sigma_{10}\, V(1, 0; 2, 0) + \tau_{11}\, V(2, 0; 1, 1) + \tau_{10}\, V(2, 0; 1, 0) + \qquad (3) \\ & + \sigma_{22}\, V(2, 2; 3, 0) + \sigma_{21}\, V(2, 1; 3, 0) + \sigma_{20}\, V(2, 0; 3, 0) + \\ & + \tau_{22}\, V(3, 0; 2, 2) + \tau_{21}\, V(3, 0; 2, 1) + \tau_{20}\, V(3, 0, 2, 0) + \ldots \end{aligned}$$

The coefficients σ_{00}, τ_{00}, σ_{11} . . . are chosen successively to fit S_{00}, T_{00}, S_{11}, . . . (For a similar construction see [14].) It is clear that the physical residua (without ghosts) can be fitted by this procedure on any finite number (k) of the lowest levels. The convergence as $k \to \infty$ depends, however, on the explicit form of the prescribed residua.

A different procedure for (partial) ghost killing was proposed in [15], where the ghosts were "killed" everywhere on a finite number ($k + 1$) of highest daughter trajectories. The amplitude is given there in the form

$$\begin{gathered} A(s,t) = A_s(s,t) + A_t(s,t) \\ A_s(s,t) = \int_0^1 dv\, v^{-1-\alpha(s)} (1-v)^{+k-\alpha(t)} (K_0(x) + vK_1(x) + \ldots + v^k K_k(x)) \\ x = \alpha(t)\, ; \quad A_t : (s \leftrightarrow t)\,. \end{gathered} \tag{4}$$

For definiteness let us choose the "good" residua to be

$$\bar{R}_n(\alpha(t)) = (-1)^n \frac{[\alpha(t) + a]^n}{n!}\,. \tag{5}$$

Then the functions $K_l(x)$ are given explicitly [15] by

$$K_l(x) = e^{-2x} \sum_{i_1 + 2i_2 + \ldots + li_l = i} \frac{1^{-i_1} \ldots l^{-i_l}}{i_1! \ldots i_l!} \left(k - a - \frac{x}{2}\right)^{i_1} \left(k - \frac{2x}{3}\right)^{i_2} \ldots \left(k - \frac{lx}{l+1}\right)^{i}. \tag{6}$$

Writing the amplitude $A(s, t)$ as a sum of s-channel poles

$$A(s,t) = \sum_{n=0}^{\infty} \frac{R_n(\alpha(t))}{n - \alpha(s)}\,, \tag{7}$$

it follows from the construction that the residua $R_n(\alpha(t))$ conicide with $_n(\alpha(t))$ for $n = 0, 1, \ldots, k$.

In $A(s, t)$ the leading trajectories of the two channels are separated, the piece A_s, for instance, contributes in the t-channel only to the ($k + 1$)-th and lower daughters, but it gives entirely the highest ($k + 1$) trajectories in the s-channel. Therefore, questions concerning duality may arise. Duality can be formulated requiring the fulfilment of the finite energy sum rule

$$\sum_{n=0}^{N} R_n(\alpha(t)) = N^{\alpha(t)+1} \frac{r(\alpha(t))}{\Gamma(a(t)+2)} + 0(N^{\alpha(t)})\, ; \, (N \to \infty) \tag{8}$$

($r(\alpha(t))$ being the residuum of the leading Regge-pole exchanged in the t-channel). This fixes the asymptotic behaviour of the residua in the following way:

$$R_N(\alpha(t)) = N^{\alpha(t)} \frac{r(\alpha(t))}{\Gamma(\alpha(t)+1)} + 0(N^{\alpha(t)-1})\,. \tag{9}$$

The amplitude $A(s, t)$ satisfies Eqs. (8) and (9) as it is meromorphic and Regge-behaved in s. But looking at Eq. (5) one sees at once that Eq. (9) can be valid only for $N \geqslant k$. This implies that a distinction can be made between the resonances occurring in $A(s, t)$: the resonances on the highest $k + 1$ trajectories are *physical* (they have physical couplings), but those on the lower daughters are *supplementary*, they are partly ghosts and they are responsible for the Regge behaviour in the direct channel. (One has to remark that a similar situation is to be expected in every dual model, as the asymptotic behaviour (Eq. (9)) can only be produced by the far-out daughters.)

III. Higher symmetries in dual models

In this Section we give a short and simple derivation of dual models with relativistic SU(6) symmetry in the case of meson four-point functions. The starting point is the PATON—CHAN unitary symmetry factor

$$\delta_{A_1}^{B_2} \delta_{A_2}^{B_3} \delta_{A_3}^{B_4} \delta_{A_4}^{B_1} . \tag{10}$$

Here A_i, B_i denote the SU(3) nonet indices of the i-th meson. An immediate generalization to relativistic SU(6) would be

$$\delta_{\alpha_1}^{\beta_2} \delta_{\alpha_2}^{\beta_3} \delta_{\alpha_3}^{\beta_4} \delta_{\alpha_4}^{\beta_1} , \tag{11}$$

where $\alpha_i = A_i a_i$, $\beta_i = B_i b_i$ and a_i, b_i denote the Lorentz spinor indices of the i-th meson. Eq. (11) is, however, incorrect as parity conservation requires

$$\begin{aligned} M(p_1 p_2 p_3 p_4)_{a_1 a_2 a_3 a_4}^{b_1 b_2 b_3 b_4} = \eta\, M(p_{1s} p_{2s} p_{3s} p_{4s})_{\dot{a}_1 \dot{a}_2 \dot{a}_3 \dot{a}_4}^{\dot{b}_1 \dot{b}_2 \dot{b}_3 \dot{b}_4} = \\ = \eta \left(\frac{\hat{p}_1}{m}\right)_{b_1 b_1'} \left(\frac{\hat{p}_2}{m}\right)_{b_2 b_2'} \left(\frac{\hat{p}_3}{m}\right)_{b_3 b_3'} \left(\frac{\hat{p}_4}{m}\right)_{b_4 b_4'} M(p_{1s} p_{2s} p_{3s} p_{4s})_{a_j'}^{b_i'} \left(\frac{\check{p}_1}{m}\right)_{a_1' a_1} \left(\frac{\check{p}_2}{m}\right)_{a_2' a_2} \times \\ \times \left(\frac{\check{p}_3}{m}\right)_{a_3' a_3} \left(\frac{\check{p}_4}{m}\right)_{a_4' a_4} ;\, p_s = (p_0, -\underset{\sim}{p})\,,\, \hat{p} = p^\mu \sigma_\mu \,,\, \check{p} = p_s^\mu \sigma_\mu . \end{aligned} \tag{12}$$

Here M is the four-meson M-function, $\eta = + 1$ is the product of the parities and m is the meson mass. Otherwise we used the notations of [9]. The most simple change of Eq. (11) consistent with parity conservation is the following:

$$\delta_{\alpha_1}^{\beta_2} = \delta_{A_1}^{B_2} \delta_{a_1}^{b_2} \rightarrow [\delta - (p_2 p_1)]_{\alpha_1}^{\beta_2} = \delta_{A_1}^{B_2} \left[\delta_{b_2 a_1} - \left(\frac{\hat{p}_2 \check{p}_1}{m^2}\right)_{b_2 a_1} \right]. \tag{13}$$

Therefore, a possible four-meson amplitude is:

$$\begin{aligned} M(p_1 p_2 p_3 p_4)_{\alpha_1 \alpha_2 \alpha_3 \alpha_4}^{\beta_1 \beta_2 \beta_3 \beta_4} = \int dv\, v^{-1-\alpha(s)} (1-v)^{-1-\alpha(t)} [\delta - (p_2 p_1)]_{\alpha_1}^{\beta_2} \cdot \\ \cdot [\delta - (p_3 p_2)]_{\alpha_2}^{\beta_3} [\delta - (p_4 p_3)]_{\alpha_3}^{\beta_4} [\delta - (p_1 p_4)]_{\alpha_4}^{\beta_1} . \end{aligned} \tag{14}$$

Here $\alpha(s)$ denotes the (negative intercept) trajectory of the 0^- particles in the 36-plet. This amplitude is identical to the MBH amplitude for 4 mesons. Applying the same method to meson—baryon scattering one arrives at the Sl(6, C) symmetric amplitude of [9] which coincides with the SU(6, 6) amplitude if the 4 coupling constants are properly choosen. Let us now investigate the factorization properties of the spin-dependent part of the above amplitude, say in the s-channel $[s = (p_1 + p_2)^2]$. The first and third factors belong entirely to the initial and final states, respectively. The other two factors can be written as follows:

$$[\delta - (p_3 p_2]^{\beta_3}_{\alpha_2} = \frac{1}{2} [\delta - (p_3 p_{12})]^{\beta_3}_{\varrho_{12}} [\delta + (p_{12} p_2)]^{\varrho_{12}}_{\alpha_2} + \\ + \frac{1}{2} [\delta + (p_3 p_{12})]^{\beta_3}_{\varrho_{12}} [\delta - (p_{12} p_2)]^{\varrho_{12}}_{\alpha_2} ; p_{ik} = p_i + p_k , p_{ik}^2 = m^2 . \tag{15}$$

The two terms on the right-hand side are already factorized, thus multiplying by the other two terms coming from the last factor in Eq. (14), one has four 36-plets as intermediate states. It can be shown that there are two 36^--plets and two 36^+-plets, one of each having imaginary couplings ("ghosts").

The superfluous intermediate states can be easily eliminated at the ground state pole by the method of [16], which consists of multiplying the unwanted pieces of the covariants by the integration variable of the channel:

$$M = \int_0^1 dv\, v^{-1-\alpha(s)} (1-v)^{-1-\alpha(t)} \left\{ \frac{1}{2} [\delta - (p_4 p_{23})]^{\beta_4}_{\varrho_{23}} [\delta + (p_{23} p_3)]^{\varrho_{23}}_{\alpha_3} + \right. \\ \left. + \frac{1-v}{2} [\delta + (p_4 p_{23})]^{\beta_4}_{\varrho_{23}} [\delta - (p_{23} p_3)]^{\varrho_{23}}_{\alpha_3} \right\} \left\{ \frac{1}{2} [\delta - (p_3 p_{12})]^{\beta_3}_{\varrho_{12}} \right. \\ \left. [\delta + (p_{12}\, p_2)]^{\varrho_{12}}_{\alpha_2} + \frac{v}{2} [\delta + (p_3 p_{12})]^{\beta_3}_{\varrho_{12}} [\delta - (p_{12} p_2)]^{\varrho_{12}}_{\alpha_2} \right\} \left\{ \frac{1}{2} \right. \\ [\delta - (p_2 p_{41})]^{\beta_2}_{\varrho_{41}} [\delta + (p_{41} p_1)]^{\varrho_{41}}_{\alpha_1} + \frac{1-v}{2} [\delta + (p_2 p_{41})]^{\beta_2}_{\varrho_{12}} \\ \left. [\delta - (p_{41} p_1)]^{\varrho_{12}}_{\alpha_1} \right\} \left\{ \frac{1}{2} [\delta - (p_1 p_{34}))^{\beta_1}_{\varrho_{34}} [\delta + (p_{34} p_4)]^{\varrho_{34}}_{\alpha_4} + \right. \\ \left. + \frac{v}{2} [\delta + (p_1 p_{34})]^{\beta_1}_{\varrho_{34}} [\delta - (p_{34} p_4)]^{\varrho_{34}}_{\alpha_4} \right\} . \tag{16}$$

In this amplitude the ground state is a pure (and physical) 36^--plet. The excited states, however, are still degenerate. The degeneracy everywhere can be removed by multiplying by some functions $f(v)$ and $f(1-v)$ instead of v and $(1-v)$, respectively if

$$\begin{array}{l} f(0) = 0 , \\ f(1) = 1 , \end{array} \quad \frac{d^n f(0,1)}{dv^n} = 0 \quad (n = 1, 2, \ldots) . \tag{17}$$

Such an amplitude was constructed in [17], where it was shown that in doing so the amplitude gets pieces with exponential behaviour. This shows that the scalar Veneziano spectrum multiplied by pure SU(6) couplings is no more dual in the sense of satisfying finite energy sum rules.

Another possibility of modifying Eq. (16) is to remove the degeneracy on the first $(k+1)$ trajectories by the method of [15]. The resulting amplitude is the following [9]:

$$\begin{aligned} M_s = &\int_0^1 dv\, v^{-1-\alpha(s)} (1-v)^{-1-\alpha(t)} [\delta-(p_2 p_1)]^{\beta_2}_{\alpha_1} [\delta-(p_4 p_3)]^{\beta_4}_{\alpha_3} \times \\ &\times (1-v)^{k+1} (-a)^{B_0} \int_0^1 d\tau\, \tau^{-a-1} (-\log\tau)^{B_0-1}\, e^{\alpha(t) v(\tau-1)} \left[R_0^{(k)}(v\,\alpha(t), \tau) + \right. \\ &\left. + \ldots + v^k R_k^{(k)}(v\alpha(t), \tau)\right] \left\{ [\delta-(p_3 p_2)]^{\beta_3}_{\alpha_2} [\delta-(p_1 p_4)^{\beta_1}_{\alpha_4} - \frac{\sqrt{a \log\tau}}{\Gamma\left(B_0+\frac{1}{2}\right)} \times \right. \\ &\times [(p_3 p_{12})-(p_{12} p_2)]^{\beta_3}_{\alpha_2} [\delta-(p_1 p_4)]^{\beta_1}_{\alpha_4} + [\delta-(p_3 p_2)]^{\beta_3}_{\alpha_2} [(p_1 p_{34})-(p_{31} p_4)]^{\beta_1}_{\alpha_4}] + \\ &\left. + \frac{a \log\tau}{\Gamma(B_0+1)} [(p_3 p_{12})-(p_{12} p_2)]^{\beta_3}_{\alpha_2} [(p_1 p_{34})-(p_{34} p_4)]^{\beta_1}_{\alpha_4} \right\}; \quad M = M_s + M_t, \end{aligned} \tag{18}$$

where

$B_0 =$ free parameter, $a = \alpha(0)$; $p_1 + p_2 + p_3 + p_4 = 0$;

$$\begin{aligned} R_i^{(k)}(x, \tau) = &\sum_{i_1+\ldots+l i_l = i} \frac{1^{-i_1} \ldots l^{-i_l}}{i_1! \ldots i_l!} \left[\tau\left(1+\frac{x\tau}{2}\right) + k - \frac{x}{2}\right]^{i_1} \\ &\ldots \left[\tau^l \left(1+\frac{l x\tau}{l+1}\right) + k - \frac{l x}{l+1}\right]^{i_l}. \end{aligned} \tag{19}$$

The piece M_t can be obtained from M_s interchanging the roles of the s- and t-channels.

In summary, the factorization properties of the dual amplitudes of spinning particles offer a lot of questions to be studied in the future. The application of the dual model to most of the physical processes becomes possible presumably only after knowing the answers to these questions.

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СПИН И УНИТАРНЫЙ СПИН В ДУАЛЬНОЙ РЕЗОНАНСНОЙ МОДЕЛИ

И. МОНТВАИ

Резюме

Приведён краткий обзор последних достижений в области построения дуальных амплитуд с унитарной симметрией и спинами.