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POSSIBLE BOOTSTRAP ORIGIN OF MATHEMATICAL QUARKS*

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A set of self-consistency conditions based on very simple assumptions is derived. The conditions imply that hadrons have a surprising number of quark-model properties.

I. Introduction

My talk concerns the question, "Why does the quark model work?" But before I attempt to answer this question, I shall ask another, namely, "Why don't physicists worry about this question more than they do?"After all, the quark model is surprisingly successful. Five or six years ago, when this success was first becoming apparent, most physicists who were impressed with the quark model believed that the reason behind it was simply that quarks existed, and would be discovered. This was a reasonable hope. However, after years of unsuccessful quark searches, most of us do not believe in physical quarks any more. So why do we not worry more about the model's success? My own opinion is that we have become too familiar with the model by now, and some of us are so infatuated with learning the various rules and diagrams involving quarks that we forget that the reason why these rules work is still a mystery. In short, some physicists today think that the quark model works because it has always worked. As this is not really a satisfactory reason, I will propose another; that self-consistency conditions of the bootstrap type force hadrons to behave as quark composites.

Before discussing some self-consistency conditions, I will list and comment briefly on what I consider the three main successes of the quark model.

1) A simple rule for exotic states.

2) $SU(6)_W$ symmetry.

3) A simple matrix rule for meson–hadron–hadron coupling ratios.

You all know what exotic means: internal quantum numbers for which no known strongly coupled particles or resonances exist. The fact that there are exotic quantum numbers is not surprising. In any conceivable universe in which some particles possess nonzero values of an additive internal quantum number, the number of internal quantum states corresponding to particles is

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either finite or infinite. It is certainly not surprising if it is finite, and if it is, the nonresonating quantum states can be called exotic. Furthermore, some states made of two nonexotic particles will be exotic in such a universe. Thus, the quark model is successful not because exotics exist, but because the prescription for exoticity is simple in the model. The prescription is that only quark-antiquark and three-quark states are not exotic.

In order to discuss the second and third successes of the above list, I will first consider what the meson—quark—quark interaction should be in the quark model. Actually, if hadrons were composites of heavy quarks, bound very relativistically, it would not be easy to explain hadrons with realistic physical interactions. However, most physicists today do not try to think of physical quarks when they discuss quark models; rather they think of rules that can be expressed simply in terms of quarks. The term "mathematical quark" is used sometimes to mean that the quarks are not supposed to be real physical particles. My main aim here is to find why the mathematical quark is such a useful concept.

The following is a simple rule for the interaction of meson with mathematical quarks. The quantum numbers a of a meson M_a may be expressed in terms of a square matrix A_{ij}, where this matrix is the coefficient in an expansion of the quantum numbers in quark–antiquark states. Symbolically,

$$a \sim \Sigma_{ij} A_{ij} \bar{Q}_i Q_j. \tag{1}$$

The bar denotes an antiquark. The interaction constant for the process $b \rightarrow M_a + d$, where b and d are quarks, is simply λA_{db}, where λ is a constant of proportionality. One can represent this rule with the diagram of Fig. 1(a);

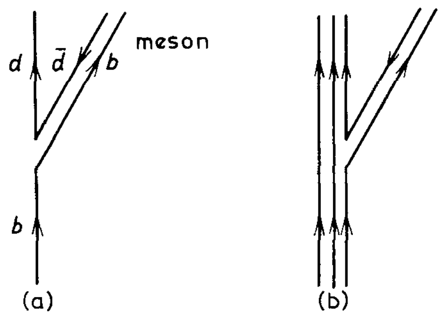

Fig.

this diagram means that the interaction constant is proportional to the coefficient of the $Q_d Q_b$ term in the expansion of the meson wave function.

If the hadrons are considered to be composed of quarks, a simple rule for meson–hadron–hadron interactions is to assume that this interaction is the sum of the interaction of the meson with all the constituent quarks. This is represented in Fig. 1(b) for the interaction of mesons with baryons (assumed composed of three quarks).

This rule automatically implies SU(n) symmetry, where n is the number of the quark states. If we include the quark spin-component quantum numbers, SU(6) is the group. Because of the charge-conjugation of the meson, one can show that the group must be applied in the $SU(6)_W$ manner; I do not have time to explain this here [1]. The point is that the quark model leads to $SU(6)_W$ naturally, so the success of this symmetry is a success of the quark model.

We next consider the question: does the simple interaction rule given above imply more than $SU(6)_W$ symmetry? Since the mesons correspond both to the singlet and regular representations of the group [i.e. to the representation $\mathbf{1} + \mathbf{35}$ of SU(6)], the above construction relates the interactions of the **1** and **35** meson states, and thus does imply more than the symmetry. This extra interaction prediction works fairly well, so I have listed it above as the third success of the quark model.

We must be a little careful about applying this interaction rule. Taken at face value it says that the interactions of the **35** and **1** mesons are proportional to matrix elements of the group generators and identity matrix. Such a rule would say that the meson interactions do not connect different representations, such as the 56 and 70-fold representations of SU(6). Since the observed even- and odd-parity baryon Regge trajectories correspond to these two representations, this rule would imply that such odd-parity states as the Λ^* (1520) and Σ^* (1660) would not decay strongly into meson-baryon states, in contradiction to experiment. Quark model enthusiasts are not troubled with this problem, because they point out that the odd-parity baryons correspond to three-quark states with some orbital angular momentum, and it is easy to write things down so that the meson-56–70- interaction exists. Thus, the physical existence of this interaction is neither a success nor failure of the quark model; I mention it here only because it is something any correct theory must predict.

I will give a short derivation of a set of consistency equations in Section II of this paper, and study the implications for MMM (meson – meson – meson) interactions and MBB (meson – baryon – baryon) interactions in Sections III(A) and III(B), respectively.

II. The consistency conditions

Some of the consistency equations that I will write are algebraically the same as equations derived from two to seven years ago. Among the people who have contributed to the development of the bootstrap equations are

CUTKOSKY [2], POLKINGHORNE [3], CHIU and FINKELSTEIN [4], and myself [5, 6]. The starting points for the derivations have been different; for example, a simple potential model was used in [2] and [3], and N/D dispersion relations were used in [5]. Somewhat later, in [6], superconvergence relations were used to derive similar equations. The recent development of the duality principle has led to great progress in this field, by allowing derivations of consistency conditions that are more plausible than the early derivations, and also lead to a more complete set of conditions. I will give a duality argument here.

I will consider both MM and MB scattering in the s-channel, at an intermediate energy, near the backward (small u) direction. It is assumed that t-channel Regge exchange may be neglected in this region. The duality principle may be written [7]:

$$\langle \mathrm{Im}\, T_{ui}^{\mathrm{Regge}} \rangle = \langle \mathrm{Im}\, T_{si}^{\mathrm{res}} \rangle . \tag{2}$$

Here i denotes the internal quantum numbers of the amplitude; i.e., i might represent $\pi^+ p$ scattering in the s channel, in which case it would correspond to $\pi^- p$ scattering in the u channel. The symbol T_{ui}^{Regge} denotes the amplitude for the exchange of u-channel Regge trajectories, T_{si}^{res} is the contribution of resonances to the s-channel amplitude, and $< >$ denotes some sort of semi-local average over energy. I assume that the external particles belong to a set of degenerate mesons and a set of degenerate baryons. If the amplitude i is exotic in the s channel, this condition implies that the trajectories of opposite signatures must be exchange degenerate, and that the residues are numerically equal. From now on I will simply assume the exchange degeneracy of the trajectories, and concentrate on the implications of the condition for the residues.

I now want to digress a little to show how a simple type of diagram, based on the duality principle of Eq. (2), makes plausible the fact that the quark model might lead to solutions of the consistency conditions. These are the "duality diagrams" of ROSNER and HARARI, and they have increased the popularity of consistency conditions based on duality [7, 8]. I will illustrate this with the baryon-exchange contributions to MB scattering. If the MBB interactions follow the rules of Fig. 1, then the left-hand side of Eq. (2) corresponds to the baryon-exchange diagram of Fig. 2(a), while the right-hand side of Eq. (2) corresponds to the baryon-resonance diagram of Fig. 2(b). However, it is clear that if one twists the lines a little, Figs 2(a) and 2(b) are identical, so it is not surprising that the quark model leads to a solution of the conditions. This argument is quite useful as a pedagogical device, but it does not serve the purpose of my talk, for two reasons. First, the argument is unclear in several aspects, among them the role played by the parities of the particles. Second, the quark construction is assumed at the beginning. I am interested in showing not just that a quark model satisfies the conditions, but also that models not

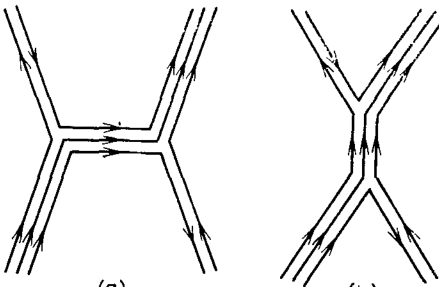

Fig. 2

interpretable in terms of quarks do not satisfy them. Therefore, I must write the consistency equations for the residues in an algebraic form.

The residue condition that follows from Eq. (2) is,

$$\alpha_i(X_{ui}^{(+)} - X_{ui}^{(-)}) = Z_{si}^{(+)} - Z_{si}^{(-)}, \tag{3}$$

where α_i is a kinematic factor, $X_{ui}^{(\pm)}$ is the sum of the residues at small u of the trajectories of signature $(\pm)$, and the $Z^{(\pm)}$ are the contributions of resonances of parities $(\pm)$ to the right-hand side of Eq. (2). The minus sign in front of $Z^{(-)}$ is appropriate, because the contributions to backward elastic scattering of resonances of opposite parities are opposite. The resonances lie on s-channel Regge trajectories, so that the Z_{si} are proportional to the residues of these trajectories. We define constants $\beta_i^{(\pm)}$ by the equation:

$$Z_{si}^{(\pm)} = \alpha_i \, \beta_i^{(\pm)} X_{si}^{(\pm)}, \tag{4}$$

where $X_{si}^{(\pm)}$ are the residues at small s of the s-channel trajectories. The constants $\alpha_i \, \beta_i^{(\pm)}$ involve phase space factors and also the variation of the residues between the Regge and resonances regions. Substitution of Eq. (4) into Eq. (3) yields

$$X_{ui}^{(+)} - X_{ui}^{(-)} = \beta_i^{(+)} X_{si}^{(+)} - \beta_i^{(-)} X_{si}^{(-)}. \tag{5}$$

It is important to realize that no assumption has been made between Eqs. (3) and (5), as the constants $\beta_i^{(\pm)}$ are defined by the requirement that Eq. (4) is true.

I now make the assumption that the $\beta_i^{(\pm)}$ are independent of the internal index i. This implies that the residues of the various trajectories are proportional as functions of energy. Basically, this assumption means that the degenerate mesons are dynamically similar, and the degenerate baryons are dynamically similar, i.e., these degeneracies are not accidental. A fuller discussion of the assumption is given in a recent paper [9].

It is now easy to find restrictions on the values of $\beta_i^{(\pm)}$. Application of Eq. (5) to a process exotic in the s channel implies $X_{ui}^{(+)} = X_{ui}^{(-)}$. Application to the crossed process (obtained by reversing the roles of the s and u channels) then implies that $\beta^{(+)} = \beta^{(-)}$. We then consider two possibilities; $X_{ui}^{(+)} - X_{ui}^{(-)}$ either is zero for all i, or is non-zero for at least one i. In the first case, the following consistency condition is automatically valid:

$$X_{ui}^{(+)} - X_{ui}^{(-)} = \pm (X_{si}^{(+)} - X_{si}^{(-)}), \tag{6}$$

although the $\pm$ sign on the right is superfluous in such a case. If $X_{ui}^{(+)} - X_{ui}^{(-)} \neq 0$ for some i, application of Eq. (5) to the amplitude for this i and to the crossed amplitude, together with the condition $\beta^{(+)} = \beta^{(-)}$, implies $\beta^{(+)2} = 1$, or Eq. (6). This equation is the condition that the $X_{si}^{(+)} - X_{si}^{(-)}$ are components of an eigenvector of the s–u crossing matrix, with eigenvalue ± 1. Thus, the condition is a generalization of that of Chew's reciprocal bootstrap model [10]. In our case, the sign of the eigenvalue depends on whether or not the residue changes sign between the resonance and Regge regions.

Our basic bootstrap equations are Eq. (6) and the corresponding equations that may be obtained for the s–t and u–t pairs of channels, together with the requirements that the set of lowest states on the mesonic and baryonic Regge trajectories should correspond to the sets of external mesons and baryons.

III. Implications of the conditions

(A) MMM interactions

I label the external particles in the s and u channels according to the scheme,

$$\begin{aligned} s: &\quad a + b \to c + d, \\ u: &\quad c + b \to a + d. \end{aligned} \tag{7}$$

We consider first the cases for which a and c are mesons of the same parity, and b and d are either both baryons of the same parity or both mesons of the same parity. We use the labels to refer to both internal quantum numbers and the z-components of the particle spins. We consider only backward scattering, for which the spin-components behave as internal quantum numbers under crossing. The set of mesons must include all the antiparticles, and so must be a self-conjugate set. A complete set of self-conjugate states exists in a self-conjugate set, for convenience, we take all meson states to be self-conjugate.

We consider first the case of MM scattering, where all external mesons are of the same parity. We define the coupling constant of the u-channel trajectory r of even signature (whose physical particles are of even parity) with the final state $a + d$ to be d_{adr} and the corresponding coupling of the trajectory s of odd signature to be f_{ads}. The quantities $X_{ui}^{(\pm)}$ are then given by

$$X_{ui}^{(+)} = \Sigma_r d_{adr} d_{cbr}^*, \quad X_{ui}^{(-)} = \Sigma_s f_{ads} f_{cbs}^*. \tag{8}$$

I have defined the X to be the residues at the small u appropriate for backward s-channel scattering. However, it is more convenient to evaluate the residues at the u of the lowest states on the trajectories, so that the d and f are ordinary meson–meson–meson coupling constants. Thus, the trajectories are labeled by their lowest states. This continuation in u actually introduces a proportionality constant into each of the equations of Eq. (8). However, my assumption concerning the proportionality of the residues of amplitudes of different i allows me to divide out one of the constants, and I choose the relative f/d normalization so that the other cancels also.

The self-conjugate property of the mesons a and c implies that the d_{ijk} and f_{ijk} are Hermitean in the final two indices. From this and Eq. (8), one may write the consistency condition of Eq. (6) in the form,

$$\Sigma_r d_{adr} d_{crb} - \Sigma_s f_{ads} f_{csb} \mp a \rightleftharpoons c = 0\,, \tag{9}$$

where $a \rightleftharpoons c$ denotes the preceding terms with a and c reversed. The physical states on one of the sets of trajectories have the same parity as the external particles. Our bootstrap hypothesis requires that the set of lowest states on this set of trajectories corresponds to the set of external mesons, and that we later must consider amplitudes with all possible external parity combinations. For MM scattering, we need not consider s–t and u–t channel consistency conditions, as we can permute indices so that every meson exchange is an s–u channel exchange.

I will describe the solution to this type of MMM consistency condition only briefly, as it has been worked out a few years ago. More details are given in [6].

We have taken all the meson states to be self-conjugate. Because of the Bose principle, the d's and f's refer to interactions of even and odd orbital parity, respectively. Since reversing the first two indices corresponds to reversing the directions of the two particles coupled to a trajectory particle, the d and f are symmetric and antisymmetric, respectively, in the first two indices. This, together with the Hermiticity in the last two indices, implies that the d are real and completely symmetric, and the f are imaginary and completely antisymmetric.

One can show that if the lower signs in Eqs. (6) and (9) are taken, there is no nontrivial solution. So we take the upper signs from here on out. This means that the eingenvalue of $X^{(+)}$—$X^{(-)}$ under $s \rightleftharpoons u$ crossing is $(+1)$. A rather trivial solution exists, with only one state interacting with itself with one symmetric interaction constant; we consider this too trivial. If all f's vanish, it can be shown that one can always choose an appropriate basis so that any solution becomes a direct sum of these disconnected one-state solutions. Hence, a non-triviality requirement implies that the f's do not all vanish.

If one sums Eq. (9) over all permutations of the external particles, a, b, c and d, including a minus sign for odd permutations, the d terms all are cancelled because of the symmetry property $d_{ijk} = d_{jik}$. The resulting equation can be written in the form

$$\Sigma_s \left(f_{ads} f_{csb} + f_{acs} f_{bsd} + f_{abs} f_{dsc}\right) = 0 \,. \tag{10}$$

This is the Jacobi identity for the f's. Together with the antisymmetry property, it implies that the f's must be proportional to the structure constants of a Lie group. A set of particles, all of whose interactions are of the f type, is the set of odd-parity mesons, so our bootstrap equation implies that the odd-parity mesons which interact with each other must correspond to the regular representation of a Lie group. The first use of this type of argument was by CUTKOSKY, who used a potential model involving only vector mesons [2].

We have not yet extracted all the information from Eq. (9), so we continue the argument of [6]. If the d's and f's are regarded as matrices in the space of the last two indices, and the upper sign in Eq. (9) is taken, this equation may be written in terms of matrix commutators, i.e.,

$$[d_a,\ d_c] - [f_a,\ f_c] = 0. \tag{11}$$

Since the f commutator exists, so must the d commutator, and so particles of both parities must exist. We may then consider the scattering amplitudes for which three of the external particles are of odd parity and one is of even parity. Our conclusions concerning the symmetry of the d's and f's remain valid if we use them to apply to all MMM interactions, the d and f applying when the product of the three intrinsic parities is even and odd, respectively. Our three odd-one even process leads then to a consistency equation in which each term is bilinear in the d and f. If we follow the procedure used in deriving Eqs. (9) and (11), this new equation in matrix form is,

$$[d_a,\ f_c] - [f_a,\ d_c] = 0. \tag{12}$$

This last equation is symmetric in the interchange $a \rightleftharpoons c$, corresponding to the choice of the lower sign in the crossing equation, Eq. (6). It is appropriate to choose this different sign when dealing with an amplitude for which the product of the intrinsic parities changes from the initial to the final state.

These amplitudes are odd in the momentum k_s, which is odd under crossing for collinear amplitudes (i.e., $k_s = -k_u$). Since the coupling constants are defined with this kinematic factor removed, an extra minus sign is introduced into the crossing equations for the coupling constants.

By taking a rather complicated permutation sum of the states a, b, c, and d, one can derive from this equation the matrix equation

$$[d_a, f_b] = - \Sigma_r f_{abr}(d_r) . \tag{13}$$

Since the f_{abr} are proportional to structure constants, this equation implies that d_a transforms either as a singlet under group transformations (if all f_{aij} vanish), or as a regular representation (if some f_{aij} do not vanish). Furthermore, it can be shown that the ratio between these two types of interactions is fixed. This requirement cannot be met with all simple Lie groups. For example, the only simple second rank group that gives a solution is SU(3); the other two (C_2 and G_2) do not, because they do not possess a completely symmetric interaction involving the regular representation only. It may be (though I have not proved it) that only SU(n) gives solutions, in which case the consistency conditions have implied another of the properties of the quark model.

The meson states of both parities must correspond to the singlet and regular representations (though the odd-parity singlet does not interact with pairs of odd-parity mesons). The quantum numbers of each parity set may be written in terms of the quark–antiquark construction of Eq. (1). It has been shown in [6] that the solution to the consistency equations is written simply in terms of these matrices, i.e.;

$$d_{abr} = C \text{ Tr } [(AB + BA) \ R], \tag{14a}$$

$$f_{abr} = C \text{ Tr } [(AB - BA) \ R], \tag{14b}$$

where C is a proportionality constant and Tr denotes the trace. In these equations, the indices are those of the quark—antiquark construction; there is an even-parity and odd-parity meson state for each pair of values of the indices. The d and f apply when the products of intrinsic parities are even and odd, respectively. This solution is a solution to the consistency equations obtained with all possible parity combinations for the external particles. The matrices for the singlet mesons are multiples of the identity matrix. The f constants involving singlets are zero, while Eq. (14a) defines the d constants for both singlet and regular-representation mesons. We conclude that our consistency equations do imply that mesons possess some properties of quark-antiquark composites.

I want to comment further on one feature of these solutions. Recall that by taking a simple permutation sum of Eq. (9), with the upper sign, I obtained a simple equation involving only the f constants. One cannot obtain a simple equation involving the d constants alone. Therefore, it appears that odd-parity mesons are the more fundamental, and it is not surprising that these are the lightest observed states on the exchange-degenerate meson trajectories. Physically, the lowest states on the even-parity trajectories are one unit of angular momentum higher than the lowest states on the corresponding odd-parity trajectories. I must emphasize again that the quantum numbers involved in the equations do not include total angular momentum, but are the internal quantum numbers and spin component along the direction of the collinear interactions.

(*B*) *MBB interactions*

I will simply assume that states of baryonic number greater than one should not exist. (I do not know whether or not simple solutions to the self-consistency equations exist that involve states of greater baryon numbers.) With this assumption, baryon—baryon scattering is simple, because all states are exotic. This implies that for every $BB \to BB$ amplitude the contribution- of exchange-degenerate mesonic trajectories of opposite signatures must cancel. We already found that the quantum numbers of the meson sets of opposite parity must be the same; now we find that the couplings to each baryon pair are the same for a pair of exchange-degenerate trajectories.

This simplifies the treatment of meson—baryon scattering. We need consider only amplitudes for which the parities of the external mesons are the same; the only important parities in the MB problem are those of the baryons. I will use the capital letters D and F to denote the MBB interactions; D applying when the two baryons have the same parity, F applying when the two baryons have opposite parity.

Again, I use the labels of Eq. (7) to describe the scattering in the s and u channels, with a and c self-conjugate mesons, and b and d baryons. The s–u channel consistency equations then follow from the same arguments as before. If b and d have the same parity, the condition may be obtained by replacing the f and d by F and D in Eq. (11). If b and d have opposite parity, one can make the same replacements in Eq. (12). The two resulting equations may be combined into one equation, if we regard the D_i and F_i as matrices corresponding to the meson i in a space which is the direct sum of the baryon states of even and odd parities. This combination equation is

$$[D_c - F_c,\ D_a + F_a] = 0. \tag{15}$$

We recall again that in the space of even and odd-parity baryon states, the D are non-zero only in the diagonal corners that connect states of the same parity, and the F are non-zero only in the off-diagonal corners.

In the case of meson—baryon scattering, the s–t and u–t channel consistency conditions are different in nature from the s–u conditions, and so must be considered also. In these conditions, the t-channel residues involve mesonic trajectories, and are bilinear in an MBB coupling constant and an MMM constant. Because of the symmetry of the MMM constants [the d's and f's of Sec. III(A)] it is convenient to take the sum and difference of the s–t and u–t equations. One does this by writing the s–t equation, reversing the a and c labels, and adding and subtracting. The resulting two equations are,

$(s-t)+(u-t)$

$$\{D_c+F_c, D_a+F_a\}_+ = K\,\Sigma_m\, d_{acm}(D_m+F_m), \tag{16}$$

$(s-t)-(u-t)$

$$[D_c+F_c, D_a+F_a] = K'\Sigma_m f_{acm}(D_m+F_m), \tag{17}$$

where K and K' are constants and the d and f are the MMM interaction constants of Eqs. (14b).

These three equations, Eqs. (15), (16) and (17), are the consistency equations for the MBB interactions. The derivation and discussion of these equations that I give here are contained in a paper soon to be published [11]. The MMM interaction constants f_{acm} of Eq. (17) are proportional to the structure constants of the group, if all indices refer to regular representation states, and $f_{acm}=0$ if one or more indices refer to singlet states. Thus Eq. (17) implies that the $D+F$ are a representation of the Lie algebra. If the meson index i refers to a regular-representation state, $(D+F)_i$ represents the generator associated with i; if i is a singlet state, the matrix $(D+F)_i$ commutes with all the generator matrices, and thus is diagonal in every irreducible subspace of the baryon space.

The anticommutator relation, Eq. (16), is new, and to the best of my knowledge, was first discussed in [11]. It is well-known that the anticommutators of matrices representing the generators can be written as a linear combination of the generator matrices and identity matrix *only* in the fundamental representation; for this representation these matrices form a complete set. Thus Eq. (16) implies that the $D+F$ behave like operators in *quark space*, i.e., in the space of the fundamental representation. It is easy to establish that Eq. (16) is satisfied in the quark representation. In fact, for SU(3), Eq. (16) is a standard equation for anticommutators given, for example, by GASIOROWICZ [12]. In GASIOROWICZ, the right-hand side contains a d_{acm} term and a $\delta_{ac}\, I$ term, where I is the identity matrix [12]. Our d is defined for singlet as well as regular representation states, and our d_{acm} term contains the $\delta_{ac}\, I$ term of [12] implicitly.

On the other hand, the baryons cannot correspond simply to quarks of one parity. If baryons of only one parity existed, the F would vanish, and Eqs. (15) and (17) would be contradictory. Thus the conditions imply that the baryons cannot be simple quarks, but must have many quark-model properties.

I have found two types of solutions to the three equations [11]. In the first, the quantum numbers and interactions of the sets of baryons of both parities are the same. The simplest example is the case where the baryons of each parity correspond to the fundamental representation. In this type of solution, each of the two terms of the commutator of Eq. (15) vanishes for every a and c. This implies that for MB scattering, each quantity $X_{ui}^{(+)} - X_{uj}^{(-)}$ vanishes, so there is no eigenvector of the s–u crossing matrix. In a bootstrap sense, the baryons are bootstrapped entirely by meson exchange, so we can say there is no static limit. This type of solution does not correspond to reality, as the quanta of the baryons of odd and even parities are not the same.

In the second type of solution, the two terms of the commutator of Eq. (15) do not each vanish for all a and c. In the simplest example that I have found, the baryons are all simple N-quark composites, where $N \geq 2$. Since the choice $N = 2$ would correspond to integral-spin baryons, I will consider the three-quark case, and call the quarks α, β and γ. The interaction $D + F$ is in the space of only one quark, which I will call the α quark. In symbols,

$$D_a + F_a = \varkappa A^\alpha, \tag{18}$$

where

$$\langle \alpha'\beta'\gamma' | A^\alpha | \alpha\beta\gamma \rangle = A_{\alpha'\alpha} \delta_{\beta'\beta} \delta_{\gamma'\gamma}.$$

Here $A_{\alpha'\alpha}$ is the matrix element associated with the meson state a by Eq. (1), and $\varkappa$ is a constant of proportionality. This interaction will satisfy Eqs. (16) and (17). The s–u condition, Eq. (15), will be satisfied also if $(D_c - F_c)$ is equal to $\varkappa C^\beta$, since the α and β quarks are independent.

These interactions will satisfy all the conditions provided that they are consistent with our definitions of D and F in the space of baryons of both parities. It must be possible to assign each irreducible representation in the baryon space a definite parity in such a way that the D connect only states of the same parity and the F connect only states of the opposite parity. This is easy to do with our choice of interactions, as the D and F matrices are given simply by

$$D_a = \frac{1}{2} \varkappa (A^\alpha + A^\beta)$$

$$F_a = \frac{1}{2} \varkappa (A^\alpha - A^\beta).$$

Clearly, if the baryon states of opposite symmetry under exchange of the α and β quarks are assigned opposite parity, everything will be all right.

If the group is (SU(6), the three-quark representations that are symmetric under the interchange of the first two quarks are the representations **56** and **70**, while those antisymmetric under this interchange are the **20** and another **70**. Thus, the corresponding solution involves the multiplets $\mathbf{56}^+$, $\mathbf{70}^+$, $\mathbf{70}^-$ and $\mathbf{20}^-$, where the superscript is the parity. In this solution, one can show that the $\mathbf{56}^+$ and $\mathbf{70}^-$ are coupled strongly together, and relatively weakly to the $\mathbf{70}^+$ and $\mathbf{20}^-$. Since the observed baryon spectrum seems to correspond to even parity trajectories of the **56** representation and odd-parity trajectories of the **70** representation, this solution may be valid approximately.

IV. Conclusion

At present we do not know how well the self-consistency conditions are met experimentally. The predicted results do not correspond exactly with experiment. On the other hand, we have made several assumptions that are unnecessary for the deriving of conditions, but were made only for simplicity, and we know these assumptions are not true exactly. An example is the assumption that meson states of the same parity (that are the lowest states on their Regge trajectories) are degenerate. It would be interesting to see if realistic modifications of some of our simple assumptions lead to predicted hadron spectra and hadron—hadron—hadron interaction ratios that agree even better with experiment. I hope that the next few years illuminate this question.

I can summarize my main result in two sentences. A set of self-consistency conditions based on very simple assumptions requires that the hadrons of any solution have a surprising number of quark-model properties. If these conditions are approximately valid physically, this might be the reason that the quark model works so well, even though quarks themselves do not exist.

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ВОЗМОЖНОСТЬ БУТСТРАПНОГО ПРОИСХОЖДЕНИЯ МАТЕМАТИЧЕСКИХ КВАРКОВ

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Резюме

Установлен ряд условий самосогласованности основанных на очень простых предположениях. Эти условия влекут за собой то, что адроны обладают удивительно большим числом свойств модели кварков.