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SCATTERING OF LIGHT BY LIGHT USING ELECTRON-POSITRON COLLIDING BEAMS

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We discuss the process $e^- + e^+ \to \gamma +$ hadrons. Using automodelity, vector-meson dominance and the parton model, we give an estimate of the deep-inelastic cross section, showing that it might be comparable to the cross section of the deep-inelastic reaction $e^- + e^+ \to H +$ anything H being a singled-out hadron. Some of the one-meson contributions are calculated in the quark model.

I. Introduction

As electron–positron and electron–electron storage rings will be considerably developed in the next few years, further new interesting experiments will be feasible if the energies and luminosities anticipated are achieved [1]. First of all the reactions of higher order in electromagnetic interactions are interesting. By storage ring machines the following reactions may be studied:

$$e^- + e^+ \to \text{hadrons} \tag{1}$$

$$e^- + e^+ \to \text{hadrons} + \gamma \tag{2}$$

$$e^- + e^+ \to \text{hadrons} + \mu^- + \mu^+ \tag{3}$$

$$e^- + e^{\mp} \to \text{hadrons} + e^- + e^{\mp} \tag{4}$$

The corresponding Feynman diagrams and the expected order of the cross sections are given in Fig. 1. Process (2) is of third order in the electromagnetic coupling constant and its cross section most likely does not decrease faster than the cross section for process (1).

The significance of process (2), which can be considered as generalized Bhabha and Möller scattering, is based on the increase of the cross section near forward direction with increasing energy. As a consequence, the additional factors α and α^2 in the cross sections for process (2) and (1) will be compensated at higher energies as first shown by Low ten years ago [2]. Each process above gives information for the *C*-even hadronic states, and their cross sections are connected with the fundamental process of photon–photon scattering.

The present practical importance of Reaction (4) has been realized at Orsay [3], Novosibirsk [4] and SLAC [5]. It represents a powerful method especially for the study of photon–photon scattering. The cross section is given as follows [5]:

$$\sigma^H(s) \sim \alpha^2 \left(\ln \frac{E}{m_e}\right)^2 \int_{s_{th}}^{4E^2} \frac{ds}{s} f\left(\sqrt{\frac{s}{4E^2}}\right) \sigma^H_{\gamma\gamma}(s)\,, \tag{5}$$

where $\sigma^H_{\gamma\gamma}(s)$ is the cross section for the reaction $\gamma\gamma \rightarrow$ hadrons, s is the c.m. energy squared, and $f(x)$ is a known function

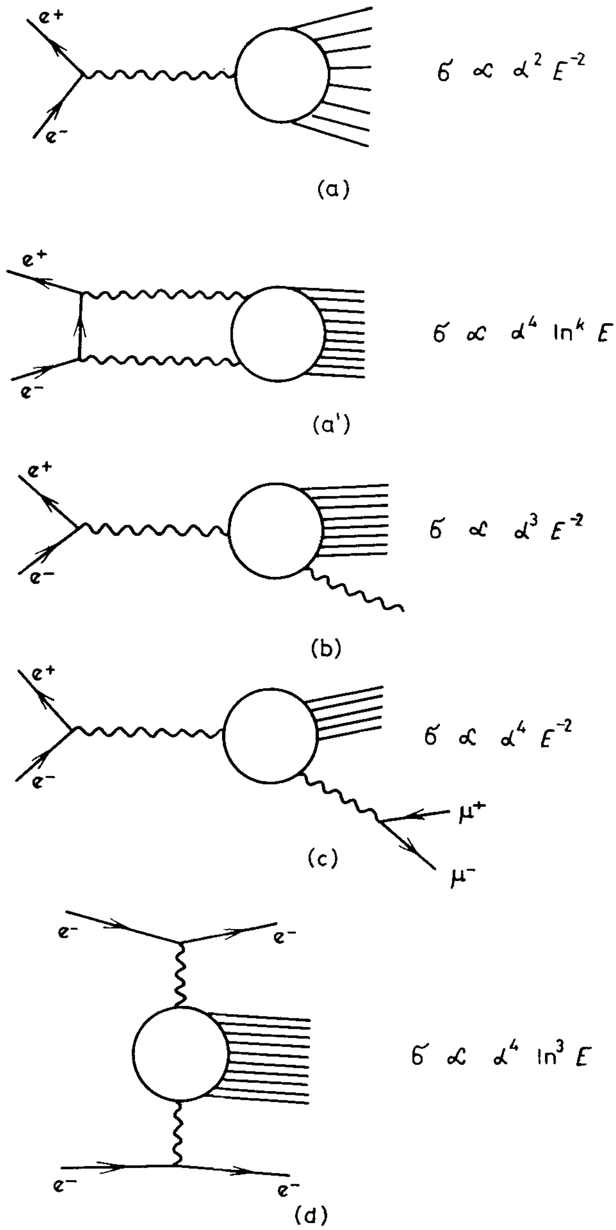

Fig. 1

$$f(x) = (2 + x^2)^2 \log \frac{1}{x} - (1 - x^2)\ (3 + x^2).$$

(As to the resonant scattering of light by light see [6]).

We discuss only Reaction (2), in the deep-inelastic region. Its importance in the investigation of hadron systems with even charge parity conjugation has been studied by CREUTZ and EINHORN [7]. In particular, they thoroughly investigated the $\gamma\pi^-\pi^+$ final state.

In Section II the kinematics is presented and we discuss how to minimize the background. In Section III, using vector-meson dominance (VDM) and the parton model, we give a rough estimate of the deep-inelastic cross section. Finally, in Section IV we briefly review the one-meson contributions.

II. Kinematics

In the one-photon exchange approximation there are two types of amplitudes, as shown in Fig. 2.

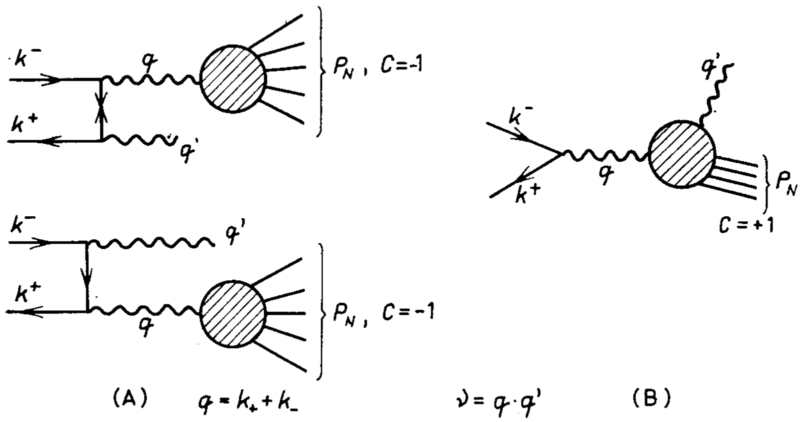

Fig. 2. Feynman diagrams which represent Reaction 2

For experiments which treat the charges symmetrically the interference term between these two types of diagrams vanishes by charge conjugation invariance

$$d\sigma = d\sigma^- + d\sigma^+ \propto |A|^2 + |B|^2. \tag{6}$$

The magnitude of A up to order e^3 can be exactly calculated using Q.E.D. and the knowledge of the cross section for Reaction (3). Therefore, we can measure the magnitude of B in a "charge symmetric" experiment.

The contributions of the C-odd amplitudes to the spin average cross sections with respect to the variable ν and Ω (solid angle of the final photon) are:

$$\frac{d\sigma^{(-)}}{d\Omega\, d\nu} = \frac{8\nu\alpha^3}{q^2} \frac{(1-a^2/\nu)^2+\cos^2\Theta}{\sin^2\Theta}\ \varrho(q^2-2\nu)\,, \tag{7}$$

where ϱ is defined as follows:

$$\varrho_{\mu\nu}(\tilde{q}) = \sum_N (2\pi)^4 \delta(q-q'-P_N) \langle 0|J_\mu(0)|P_N\rangle \langle P_N|J_\nu(0)|0\rangle = \\ = (-\tilde{q}^2 g_{\mu\nu} + \tilde{q}_\mu \tilde{q}_\nu)\, \varrho(\tilde{q}^2)\,, \tag{8}$$

where $\tilde{q} = q - q'$, $\tilde{q}^2 = q^2 - 2\nu$.

$J_\mu(x)$ is the electromagnetic current of hadrons. Using knowledge of $\varrho(\tilde{q}^2)$ from Reaction (3), this contribution can be removed. The C-even spin average cross section has the form

$$\frac{d\sigma^{(+)}}{d\nu\, d\Omega} = \frac{2\alpha^3 \nu}{q^6} \left[\overline{W}_1^\gamma(q^2, \nu) + \frac{1}{2} \sin^2 \Theta\, \overline{W}_2^\gamma(q^2, \nu) \right], \tag{9}$$

where the inelastic form-factors $\overline{W}_i^\gamma$ are defined as

$$\overline{W}_{\mu\nu}^\gamma = \sum_N (2\pi)^4 \delta(q - q^1 - p_N) \int d^4x\, d^4y\, e^{-iq(x-y)}\, \Gamma_{\mu\nu}^N(x, y)\,, \tag{10}$$

where

$$\Gamma_{\mu\nu}^N(x,y) = \langle N|\, T(J_\mu(x)\, J_\varrho(0)\, |0\rangle \langle N|\, T(J_\nu(y)\, J^\varrho(0)\, |0\rangle$$

and

$$\overline{W}_{\mu\nu}^\gamma = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) \overline{W}^\gamma{}_1(q^2, \nu) + \frac{1}{\nu} \left(q_\mu - \frac{q^2}{\nu} q'_\mu \right) \left(q_\nu - \frac{q^2}{\nu} q'_\nu \right) \overline{W}_2^\gamma(q^2, \nu). \tag{11}$$

A more detailed kinematical analysis including spin-dependent effects is presented in [8].

Performing such experiments one encounters the question of distinguishing the internal bremsstrahlung process, represented by diagram B in Fig. 2, from the large background of external bremsstrahlung and photons coming from π^0 decays. Because of the known features of the external bremsstrahlung photons, the photons radiated by the hadron blob have to be observed at large angles with respect to the direction of the beam and the momenta of the other charged particles involved. To reduce the background one has to know something about the production mechanism of hadrons. (As to the background, see the analysis of the inelastic Compton scattering given in [9].)

Bjorken and Brodsky have pointed out [10] that there are two possible extremes. On the one hand, the jet picture, where the distribution of the transverse momenta of secondaries relative to a particular axis is given by an exponential law. Therefore, energetic photons measured at large angles with respect to the jet axis should mainly be due to internal bremsstrahlung (see Fig. 3(a)). On the other hand, the statistical model predicts a distribution of secondaries which falls off with energy exponentially. According to this model, the mean value of the energy of pions should be $\langle E_\pi \rangle \sim 400$ MeV. If the production of hadrons exhibits such a "statistical" behaviour, very energetic photons transversal to the beam directions should be observed (see Fig. 3(b)).

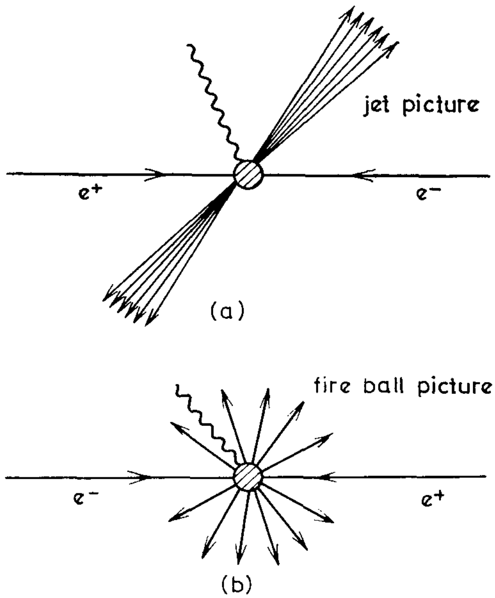

Fig. 3. Graphical schemes for the two possible extremes of the production mechanism of hadrons

Hopefully, measurements of Reaction (1) will provide us with the necessary information to set up an experiment where photons produced by internal bremsstrahlung can be distinguished from the large background.

III. Estimate for the deep-inelastic cross section

To give a rough estimate for the energy-dependence and the magnitude of the cross section, we use automodelity, VMD and the parton model.

The approximate automodelity or scale invariance principle was formulated for lepton—hadron processes at high energies and large momentum transfers [11].

This principle gives the asymptotic form of the structure functions $\overline{W}_i^\gamma(q^2, \nu)$ which are homogeneous functions of corresponding dimensions under the scale transformation

$$q \to \lambda q, \; q' \to \lambda q'. \tag{12}$$

From Eq. (9) it is clear that the functions $\overline{W}_i^\gamma$ are dimensionless. From the automodelity principle it follows that

$$\overline{W}_i^\gamma(\lambda^2 q^2, \lambda^2 \nu) = \overline{W}_i^\gamma r(q^2, \nu) \qquad i = 1, 2$$

These requirements can be satisfied by putting

$$\overline{W}_i^\gamma(q^2, \nu) = \bar{F}_i^\gamma\left(\frac{q^2}{2\nu}\right) \qquad i = 1, 2 \tag{13}$$

at high beam and photon energies.

The vector-meson dominance model can be used for the processes with real photons. It might, however, not be used for virtual photons in the far-off time-like region. We use VMD only for the real photon as shown in Fig. 4.

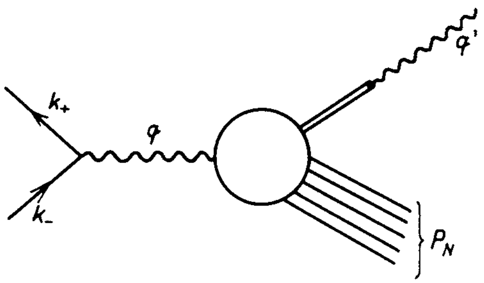

Fig. 4. Vector meson dominance for the real photon

The model connects the *C*-even part of the cross section for Reaction (2) with the cross section for the reaction

$$e^- + e^+ \to V + \text{hadrons}, \tag{14}$$

V being a singled-out vector meson (cf. [12] and [13]). The second rank tensor of Eq. (10) can be rewritten as

$$\overline{W}_{\mu\nu}^\gamma = \sum_N (2\pi)^4 \delta(q - q' - P_N)\left(\frac{1}{2\gamma\nu}\right)^2 \langle 0|J_\mu(0)|P_N, V(q')\rangle \times \\ \times \langle P_N, V(q')|J_\nu(0)|\rangle + \text{interference terms.} \tag{15}$$

Neglecting the interference term, we obtain

$$\overline{W}_1^\gamma = \sum \frac{3m_v\,\pi}{\gamma_v^2}\,\overline{W}_1^v, \tag{16a}$$

$$\overline{W}_2^\gamma = \sum \frac{3\pi}{\gamma_v^2}\,\frac{\nu}{m_v}\,\overline{W}_2^v\cdot\left(\frac{\nu}{q^2}\right), \tag{16b}$$

where $\overline{W}_1^v$ and $\overline{W}_2^v$ are the structure functions for the process of Eq. (14). They are connected with the structure functions of electroproduction on vector mesons by the substitution law:

$$\overline{W}^{v}_{\mu\nu}(q, q') = W^{v}_{\mu\nu}(q, -q'). \tag{17}$$

The cross section for electroproduction on vector meson has the usual form:

$$\frac{d\sigma}{dE'd\Omega} = \frac{\alpha^2}{4E^2 \sin^4 \Theta/2} \left(W^{v}_{2}(q^2, \nu) \cos^2 \Omega/2 + 2\, \overline{W}^{v}_{1} \sin^2 \Theta/2\right). \tag{18}$$

Integrating over the photon solid angle in Eq. (9), and introducing the new variable

$$\omega = \frac{2\nu}{q^2}, \tag{19}$$

we can rewrite the cross section given in Eq. (9) as follows:

$$\frac{d\sigma}{d\omega} = \frac{2\pi\alpha^3 \omega}{q^2} \left[\overline{W}^{\gamma}_{1} + \frac{1}{3} \overline{W}^{\gamma}_{2}\right].$$

Using the relations given in Eqs. (16), we obtain:

$$\frac{d\sigma}{d\omega} = \frac{2\pi\alpha^3 \omega}{q^2} \sum_{v} \left(\frac{3m_v \pi}{\gamma_v^2} \overline{W}^{v}_{1} + \frac{\pi}{2\gamma_v^2} \omega \frac{\nu}{m_v} \overline{W}^{v}_{2}\right).$$

In the limit $-q^2 \to \infty$, $\nu \to \infty$ and when ω is fixed, the BJORKEN scale invariance behaviour for $\overline{W}_i$ is assumed [14] to be:

$$\lim_{\substack{q^2, \nu \to \infty \\ \omega \text{ fixed}}} \frac{\nu}{m_v} \overline{W}^{v}_{2} = \overline{F}^{v}_{2}(\omega), \qquad \lim_{\substack{q^2, \nu \to \infty \\ \omega \text{ fixed}}} m_v \overline{W}^{v}_{1} = \overline{F}^{v}_{1}(\omega). \tag{20}$$

Obviously, the structure functions $\overline{W}^{\gamma}_{i}$ exhibit the same automodel behaviour:

$$\lim_{\substack{q^2, \nu \to \infty \\ \omega \text{ fixed}}} \begin{cases} \overline{W}^{\gamma}_{1} \approx \sum_{v} \frac{3\pi}{\gamma_v^2} \overline{F}^{v}_{v}(\omega) . \\ \overline{W}^{\gamma}_{2} \approx \sum \frac{3\pi}{2\gamma_v^2} \omega \overline{F}^{v}_{2}(\omega) . \end{cases} \tag{21}$$

Therefore, the deep-inelastic cross section for Reaction (2) reads as follows:

$$\frac{d\sigma^{(+)}}{d\omega} = \frac{6\pi^2 \alpha^3 \omega}{q^2} \sum \frac{1}{\gamma_v^2} \left(\overline{F}^{v}_{1}(\omega) + \frac{\omega}{6} \overline{F}^{v}_{2}(\omega)\right). \tag{22}$$

We assume that the asymptotic form factors $\bar{F}_1^v(\omega)$ and $\bar{F}_2^v(\omega)$ can be obtained by analytic continuation of the form factors of deep-inelastic electroproduction $F_1^v(\omega)$ and $F_2^v(\omega)$. This assumption has been proposed by several authors [12, 13]. It is worth noticing that such a property can be shown in a simple field theoretic model. In Veneziano-type models, however, the asymptotic form factor $\bar{F}_2(\omega)$ diverges [15].

Supposing that such an analytic continuation is possible, we find (see Eq. 17):

$$F_1^v(\omega) = \bar{F}_1^v(\omega), \tag{23a}$$

$$F_2^v(\omega) = -\bar{F}_2^v(\omega). \tag{23b}$$

For annihilation and electroproduction the physical region has a common boundary, therefore, we can estimate the magnitude of the cross section, Eq. (22), near $\omega \sim 1$ using the knowledge of the asymptotic form factors of electroproduction. The parton model of BJORKEN and PASCHOS [9] should be useful in estimating the order of magnitude of $F_{1,2}^v(\omega)$. For spin 1/2 partons we have:

$$F_1^v(\omega) = \frac{1}{2}\,\omega F_2^v(\omega). \tag{24}$$

Therefore, the cross section (Eq. (22)) can be written as

$$\frac{d\sigma^+}{d\omega} = \frac{2\pi^2\alpha^3}{q^2}\,\omega^2 \sum_v \frac{1}{\gamma_v^2}\,F_2^v(\omega)\,, \quad \text{near } \omega \sim 1. \tag{25}$$

Since the parton–antiparton cloud gives the main contributions, the magnitude of $F_2^v(\omega)$ is of the same order as for electroproduction on protons. The behaviour of $F_2^v(\omega)$ near $\omega \sim 1$, however, is different from $F_2^{\text{proton}}(\omega)$. In such parton models (the parton spin is 1/2) we have at $\omega = 1$ the following threshold theorems [12]:

for fermions

$$F_2(\omega) = C_n(\omega - 1)^{2n+1} + \ldots \qquad n = 0, 1, 2, \ldots, \tag{26a}$$

and for bosons:

$$F_2(\omega) = C_n'(\omega - 1)^{2n} + \ldots \qquad n = 0, 1, 2. \tag{26b}$$

Therefore, assuming smooth behaviour near $\omega \sim 1$ and that $C_0 \neq 0$, we find that $\bar{F}_2(\omega)$ is larger for bosons than for fermions (see Fig. 5).

The most important contribution comes from the ϱ meson, Using the experimental values of the factors γ_v [1], we obtain:

$$\frac{d\sigma^\gamma}{d\omega} = \frac{d\sigma^v}{d\omega} \geqslant 0.1\,. \tag{27}$$

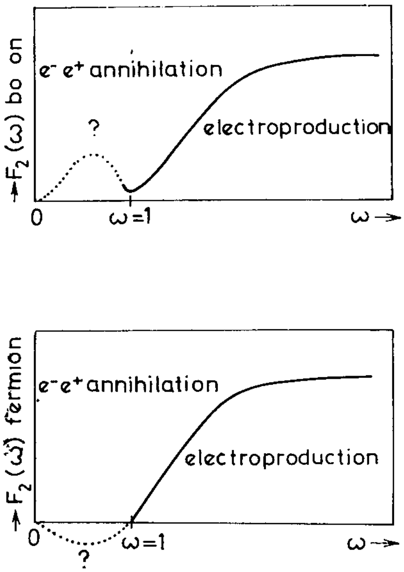

Fig. 5. Asymptotic structure functions for electroproductions and their possible continuations to the region of annihilation

Integrating the cross section (Eq. (25)) over the region $\omega = 0.77$–0.99, at the virtual photon mass square $q^2 \approx 50\ \mathrm{GeV}^2$ we find

$$\Delta\sigma \sim 10^{-35} - 10^{-36}\ \mathrm{cm}^2.$$

Therefore, despite the additional factor α in the cross section for Reaction (2), we expect that it will be comparable with the cross section of the deep-inelastic reaction $e^+ + e^- \rightarrow H +$ anything, H being a singled-out hadron [12].

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РАССЕЯНИЕ СВЕТА НА СВЕТЕ, ИСПОЛЬЗУЯ СТАЛКИВАЮЩИЕСЯ ЭЛЕКТРОННЫЙ И ПОЗИТРОННЫЙ ПУЧКИ

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Резюме

Рассмотрены процессы $e^- + e^+ \rightarrow \gamma +$ адроны. Пользуясь аппроксимацией с помощью векторных мезонов, партонной моделью, и автомодельностью, сделана оценка для глубоко неупругого сечения, которая показывает, что оно сравнимо с сечением глубоко неупругой реакции $e^- + e^+ \rightarrow H +$ что- то, где H-есть выбранный адрон. Некоторые из одномезонных вкладов вычислены по модели кварков.