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DEEP-INELASTIC SCATTERING OF POLARIZED ELECTRON BEAM FROM POLARIZED NUCLEON TARGET

By

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Spin-dependent effects in deep-inelastic electron—proton scattering are discussed.

Introduction

In this report we hope to convince you that spin-dependent effects in deep-inelastic electron–proton scattering are interesting and should yield new information on many questions arisen in the intensive analysis of the spin-averaged SLAC-MIT data [1]. These data shed light on the behaviour of two famous structure functions $W_1(q^2, \nu)$ and $W_2(q^2, \nu)$ vehemently discussed during the last two years [2].

Our group at the Eötvös University started a program last July to investigate the spin-dependent effects measurable by using polarized electron beam scattered on polarized nucleon target. The motivation for this program was that "theoretical explanations" of the SLAC-MIT data on $W_1(q^2, \nu)$ and $W_2(q^2, \nu)$ differed widely. One possibility to test these ideas is to turn to polarization effects.

It turns out from our analysis [3] that the spin-dependent structure functions, which we denote here by $d(q^2, \nu)$ and $g(q^2, \nu)$ should be more selective than the present spin-averaged data.

To our knowledge, there has been only little effort in the literature to shed more light on spin-dependent effects in deep-inelastic electron–proton scattering. Some time ago (1966) BJORKEN wrote down a sum rule for the corresponding cross sections [4] and dismissed it as "worthless". This negative conclusion has been reconsidered in a recent SLAC preprint [5]. Beyond this, we have found only a few attempts to clarify the polarizability contribution of the spin-dependent functions to the hyperfine splitting in the hydrogen atom [6]. Hyperfine splitting is interesting in itself, and we have investigated this problem in the light of the present theoretical situation [3].

I. Kinematics of the scattering process

The scattering amplitude is shown in Fig. 1. Here (p, α) denotes the four-momentum and polarization vector of the proton; (k_1, β) and (k_2, β') are similar notations for the electron beam before and after the emission of a virtual photon of four-momentum q. P_n stands for the hadrons produced in the collision.

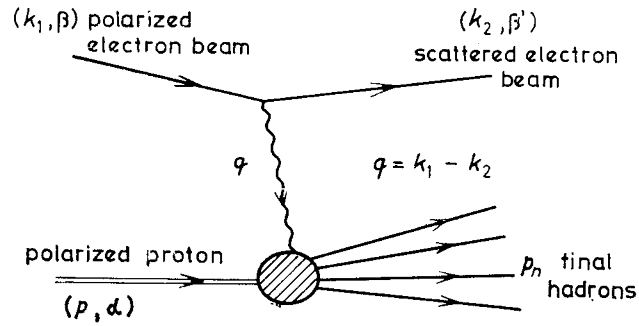

Fig. 1. Inelastic scattering of polarized electron beam from polarized proton target

We sum over final hadrons and the polarization of the scattered electron beam. The differential cross section with proton polarization α and electron polarization β is*:

$$d\sigma_{\alpha\beta} = \frac{1}{4} e^4 q^{-4} [(k_1 \cdot p)^2 - m^2 M^2]^{-1/2} \cdot L^{\beta}_{\mu\nu} \cdot W^{\nu\mu}_{\alpha} \cdot \frac{d^3 k_2}{(2\pi)^3 2E'} \, . \tag{1}$$

The leptonic part is given by

$$L^{\beta}_{\mu\nu} = \sum_{\beta'} \overline{W}_{\beta'}(k_2)\, \gamma_\mu W_\beta(k_1) \cdot \overline{W}_\beta(k_1)\, \gamma_\nu W_{\beta'}(k_2) \, . \tag{2}$$

The hadron amplitude is split into symmetric and antisymmetric parts in the (μ, ν) indices:

$$W^{\alpha}_{\mu\nu}(p, q) = \int d^4 x e^{iqx} \langle p \cdot \alpha | [J_\mu(x), J_\nu(0)] | p, \alpha \rangle = W^{s}_{\mu\nu}(p, q) + iW^{A}_{\mu\nu}(p, q) \, . \tag{3}$$

From (PT) invariance we read off

$$\begin{aligned} W^{s}_{\mu\nu}(p, q) &= \frac{1}{2} \sum_{\alpha} \int d^4 x e^{iqx} \langle p, \alpha | [J_\mu(x), J_\nu(0)] | p, \alpha \rangle , \\ iW^{A}_{\mu\nu}(p, q) &= \frac{1}{2} \int d^4 x e^{iqx} \left\{ \langle p, \alpha | [J_\mu(x), J_\nu(0)] | p, \alpha \rangle - (\alpha \rightarrow -\alpha) \right\} . \end{aligned} \tag{4}$$

* Normalization of states $< p', r \,|\, p, s > = 2p_0 (2\pi)^3 \delta(p' - p) \delta_{rs}$, spinor normalization: $\overline{W} \cdot W = 2m$, $p^2 = M^2$ and $\nu = p \cdot q$.

The definition of the four real structure functions is:

$$W^{s}_{\mu\nu}(p,q)=\left(\frac{q_\mu q_\nu}{q^2}-g_{\mu\nu}\right)4\pi M\cdot W_1(q^2,\nu)+\frac{1}{M^2}\left(p_\mu-\frac{pq}{q^2}q_\mu\right)\times$$
$$\times\left(p_\nu-\frac{pq}{q^2}q_\nu\right)4\pi M\cdot W_2(q^2,\nu)\,, \tag{5}$$

$$W^{A}_{\mu\nu}(p\cdot q)=\epsilon_{\mu\nu\varrho\sigma}\,q^{\varrho}\,\alpha^{\sigma}\,d(q^2,\nu)+(\alpha q)\,\epsilon_{\mu\nu\varrho\sigma}\,q^{\varrho}\,p^{\sigma}g(q^2,\nu)\,. \tag{9}$$

Similar splitting can be performed on the leptonic piece of the amplitude:

$$L^{\beta}_{\mu\nu}=L^{s}_{\mu\nu}+iL^{A}_{\mu\nu}\,,\quad L^{A}_{\mu\nu}=2m\,\epsilon_{\mu\nu\varrho\sigma}\cdot q^{\varrho}\,\beta^{\sigma}\,. \tag{7}$$

In order to analyze spin-dependent effects, we turn to the antisymmetric combination:

$$\frac{d^2\sigma^{\uparrow\uparrow}}{d\Omega dE'}-\frac{d^2\sigma^{\uparrow\downarrow}}{d\Omega dE'}=\alpha^2\frac{E'}{\pi\cdot M\cdot E}\,\frac{1}{q^2}\,\cdot$$
$$\cdot\left\{(E+E'\cos\theta)\,d(q^2,\nu)+(E-E'\cos\theta)\,(E+E')\,Mg(q^2,\nu)\right\}. \tag{8}$$

Here E and E' are, respectively, the initial and final electron energies, as viewed in the laboratory frame, and θ is the electron scattering angle. $d\sigma^{\uparrow\uparrow}$ is the cross section when the spins of electron and proton are parallel and along the direction of motion of the incident electron; $d\sigma^{\uparrow\downarrow}$ is the cross section for antiparallel spins. The electron mass is neglected in Eq. (8).

II. Light-cone behaviour and scaling laws

It is widely recognized [7] that deep-inelastic electron scattering measures the light-cone behaviour of the commutator functions of two electromagnetic currents sandwiched between identical proton states. We go to coordinate-space by Fourier transformation:

$$X^{A}_{\mu\nu}(p,x)=(2\pi)^{-4}\int d^4q e^{-iqx}\,W^{A}_{\mu\nu}(p,q)\,,$$
$$iX^{A}_{\mu\nu}(p,x)=\frac{1}{2}\left\{\langle p,\alpha|[J_\mu(x),J_\nu(0)]|p,\alpha\rangle-(\alpha\to-\alpha)\right\}. \tag{9}$$

Similarly, the Fourier transforms of the structure functions:

$$A_d(p,x)=(2\pi)^{-4}\int d^4q e^{-iqx}d(q^2,\nu),$$
$$A_g(p,x)=(2\pi)^{-4}\int d^4q e^{-iqx}\,g(q^2,\nu).$$

In terms of these amplitudes we write:

$$X^A_{\mu\nu}(p,x) = i\epsilon_{\mu\nu\varrho\sigma}\,\partial^\varrho \alpha^\sigma A_d(p,x) - (\alpha\cdot\partial)\,\epsilon_{\mu\nu\varrho\sigma}\,\partial^\varrho p^\sigma A_g(p,x)\,. \tag{10}$$

$A_d(p, x)$ and $A_g(p, x)$ are invariant functions of two independent Lorentz scalars; x^2 and $M^{-2}(px)^2$ or x^2 and $-x^2 + M^{-2}(px)^2$ can be choosen for convenience.

We have proved that $A_d(p, x)$ and $A_g(p, x)$ vanish outside the light-cone and they have correct support properties in momentum space to apply the Jost—Lehmann representation to them. In the proton rest frame:

$$\begin{aligned} A_d(p,x) &= i\int_0^\infty d\lambda^2 \Delta(x,\lambda^2)\,\varphi_d(x_0,\lambda^2)\,, \\ \Delta(x,\lambda^2) &= \frac{1}{2\pi}\,\epsilon(x_0)\left\{\delta(x^2) - \theta(x^2)\,\frac{\lambda^2}{2}\,\frac{J_1(\sqrt{\lambda^2 - x^2})}{\sqrt{\lambda^2\cdot x^2}}\right\}. \end{aligned} \tag{11}$$

Provided the integral

$$a_d\,(M^{-2}(px)^2) = \int_0^\infty d\lambda^2\cdot\varphi_d(M^{-2}(px)^2,\lambda^2)$$

converges, we can write $A_d(p, x)$ in the form

$$A_d\,(p,x) = \frac{i}{2\pi}\,\epsilon(x_0)\,\delta(x^2)\,a_d(x_0) + R_d(p,x)\,, \tag{12}$$

where $R_d\,(p, x)$ is less singular than $1/x^2$ on the light-cone. The Fourier transformed form of Eq. (12) yields the asymptotic behaviour in the deep-inelastic limit when $\omega = -2\nu/q^2$ is fixed and $\nu \to \infty$:

$$\begin{aligned} d(q^2,\nu) &\sim \frac{1}{\nu}\,\alpha(\xi), \quad \xi = \frac{M}{\omega}\,, \\ a_d\,(x_0) &= -\,\frac{1}{2\pi M}\int_{-M}^{M} d\xi\, e^{i\xi x_0}\,\alpha(\xi)\,. \end{aligned} \tag{13}$$

$\alpha(\xi)$ has to vanish for $|\,\xi\,| > M$.

$R_d(p, x)$ cancels the leading δ-singularity in the forbidden region in momentum space where $d(q^2, \nu)$ is forced to vanish because of support conditions.

It may happen that the leading δ-singularity of $A_d\,(p, x)$ is missing in special dynamical models. In that case $\int d\lambda^2\cdot\varphi_d(x_0, \lambda^2) = 0$ is satisfied identically. Apart from more singular cases, the integral

$$b_d(M^{-2}(px)^2) = -\frac{1}{4}\int_0^\infty d\lambda^2 \cdot \lambda^2 \cdot \varphi_d(M^{-2}(px)^2) \tag{14}$$

converges and we write

$$A_d(p,x) = \frac{i}{2\pi}\,\epsilon(x_0)\,\theta(x^2)\,b_d(M^{-2}(px)^2) + S_d(p,x)\,, \tag{15}$$

where $S_d(p, x)$ vanishes on the light-cone in the limit $x^2 \to 0$. Again, Fourier representation shows that the scaling limit is determined by the leading light-cone singularity in Eq. (15):

$$\begin{aligned} d(q^2,\nu) &\to \frac{\gamma(\xi)}{2\nu^2}\,, \\ b_d(x_0) &= -\frac{1}{M^2}\int_0^M d\xi\,\frac{\sin \xi x_0}{x_0}\,\gamma(\xi)\,, \end{aligned} \tag{16}$$

where $\gamma(\xi)$ is restricted to $|\xi| \leq M$.

The observed scaling behaviour of $W_1(q^2, \nu)$ and $W_2(q^2, \nu)$ puts stringent restrictions on the "scaling" of $d(q^2, \nu)$ and $g(q^2, \nu)$. The general constraints have been derived from $W^a_{\mu\nu}(p, q) \cdot a^\mu \cdot a^{\nu*} \geq 0$, which is valid for any complex four-vector a^μ. With properly choosen a_μ's we find four inequalities:

$$\begin{aligned} &V_1(q^2,\nu) + M^2 \cdot V_2(q^2,\nu) \geq 0, \\ &q^2 V_1(q^2,\nu) + \nu^2 \cdot V_2(q^2,\nu) \geq 0, \\ &d^2(q^2,\nu) \leq (V_1(q^2,\nu) + M^2 \cdot V_2(q^2,\nu))\,(q^2 \cdot V_1(q^2,\nu) + \nu^2 \cdot V_2(q^2,\nu)), \\ &g^2(q^2,\nu) \leq M^{-2}(q^2 \cdot V_1 + \nu^2 \cdot V_2) \cdot V_1^2\,(\nu/M\sqrt{V_1 + M^2 V_2} - \\ &\qquad - \sqrt{q^2 V_1 + \nu^2 V_2})^{-2}. \end{aligned} \tag{17}$$

Here we have introduced two local functions instead of $W_1(q^2, \nu)$ and $W_2(q^2, \nu)$:

$$\begin{aligned} W^s_{\mu\nu}(p,q) = {}& [q_\mu q_\nu - q^2 g_{\mu\nu}] \cdot V_1(q^2,\nu) + \\ &+ [(p_\mu q_\nu + p_\nu q_\mu)(pq) - p_\mu p_\nu \cdot q^2 - (pq)^2 \cdot g_{\mu\nu}]\, V_2(q^2,\nu)\,. \end{aligned} \tag{18}$$

We put into (17) the observed behaviour of $W_1(q^2, \nu)$ and $W_2(q^2, \nu)$. We find *in scaling limit* the very interesting upper bounds on $d(q^2, \nu)$ and $g(q^2, \nu)$:

$$d(q^2,\nu) \leq \frac{1}{\nu^{1/2}} \cdot \text{scaling function, if } F_l(\xi) \neq 0,$$

$$d(q^2, \nu) \leq \frac{1}{\nu} \cdot \text{(scaling function)} \qquad \text{if } F_l(\xi) = 0,$$

$$g(q^2, \nu) \leq \frac{1}{\nu^{3/2}} \cdot \text{(scaling function)} \qquad \text{if } F_l(\xi) \neq 0, \tag{19}$$

$$g(q^2, \nu) \leq \frac{1}{\nu^2} \cdot \text{(scaling function)} \qquad \text{if } F_l(\xi) = 0.$$

Here $V_1(q^2, \nu) \to 1/2\nu \; F_l(\xi) \cdot \xi^{-1}$ and $F_l(\xi) = 0$ is allowed by the present data. The restriction (19) on $d(q^2, \nu)$ is consistent with a leading $\epsilon(x_0)\delta(x^2)$ singularity in $A_d(p, x)$. However, derivatives of $\delta(x^2)$ are forbidden by the bound in *any local representation* for $A_d(p, x)$.

The $\delta(x^2)$-singularity in $A_g(p, x)$ is ruled out by (19). The corresponding "smooth" scaling behaviour follows from the form, analogous to (15):

$$A_g(p, x) = \frac{i}{2\pi} \epsilon(x_0) \cdot \theta(x^2) \cdot b_g(M^2(px)^2) + S_g(p, x), \tag{20}$$

$$g(q^2, \nu) \to \frac{\beta(\xi)}{2\nu^2},$$

$$b_g(x_0) = -\frac{1}{M^2} \int_0^M d\xi \, \frac{\sin \xi x_0}{x_0} \beta(\xi). \tag{21}$$

From this analysis, we expect the scaling law for $\nu \cdot d(q^2, \nu)$ or $\nu^2 \cdot d(q^2, \nu)$ depending on more detailed dynamics, and for $\nu^2 \cdot g(q^2, \nu)$. Fractional powers of ν could appear in the scaling laws, but this would imply less regular "theories" which we do not want to discuss here.

The missing $\delta(x^2)$-singularity in $g(q^2, \nu)$ is not a real surprise, because we have used the experimental input for $V_1(q^2, \nu)$ and $V_2(q^2, \nu)$. The $\delta(x^2)$-singularity is missing in $V_2(q^2, \nu)$ and the four structure functions are coupled through the inequalities in (19).

The next step is to calculate the equal-time commutators. It is easy to see from Eqs. (9), (12), (15) and (20) that

$$X_{ik}^A(p, x_0 = 0, \mathbf{x}) = a_d(0)\delta^{(3)}(\mathbf{x}) \cdot \epsilon_{ikl} \cdot \alpha^l \tag{22}$$

Only the $\delta(x^2)$-singularity of $Ad(p, x)$ contributes to the equal-time commutator of the space-space components. No gradient terms appear in (22). In quark algebra [4, 5] we have

$$[J_i(0, \mathbf{x}), J_k(0)] = -2i\,\epsilon_{ikl} \cdot J_5^l(0) \cdot \delta^{(3)}(\mathbf{x}) + \text{gradient terms}. \tag{23}$$

The commutator algebra in (23) sandwiched between identical proton states, is

$$X^{A}_{ik}(p, x_0 = 0, \mathbf{x}) = 4M \cdot Z\delta^{(3)}(\mathbf{x}) \cdot \epsilon_{ikl} \cdot \alpha^{l} + \text{gradient terms} \tag{24}$$
$$\langle p, \alpha \mid J^{\mu}_{5}(0) \mid p, \alpha \rangle = -2MZ \cdot \alpha^{\mu}.$$

In our analysis Z is given in terms of a measurable integral:

$$Z = -\frac{1}{4\pi M^2}\int_0^M d\xi \alpha(\xi) \ . \tag{25}$$

From isospin algebra:

$$Z = \begin{cases} \bar{Z} + \frac{1}{6}\left|\frac{G_A}{G_V}\right| & \text{quark algebra; proton target;} \\ \bar{Z} - \frac{1}{6}\left|\frac{G_A}{G_V}\right| & \text{quark algebra; neutron target.} \end{cases}$$

Here $\left|\frac{G_A}{G_V}\right| \approx 1.2$ is the ratio of β-decay coupling constants and $\bar{Z}$ is a model-dependent isoscalar contribution. Depending upon the sign of $\bar{Z}$, the magnitude of Z must be greater than 0.2 for either proton or neutron target [5]. In field algebra the antisymmetric piece of the equal-time commutator should vanish identically.

III. J-plane analysis

The forward virtual Compton amplitude is defined by

$$M^{\alpha}_{\mu\nu}(p, q) = i \int d^4 x e^{iqx} \langle p, \alpha | T(J_\mu(x), J_\nu(0)) | p, \alpha \rangle + \text{polynomial}.$$

The scattering process, described by the S-matrix element $\epsilon^{*}_{\mu}(q^2)\, M^{\mu\nu}_{\alpha}(p, q)\ \epsilon_{\nu}(q^2)$ is shown in Fig. 2. $\epsilon_{\mu}(q^2)$ is the polarization vector of the virtual photon. We define the symmetric and antisymmetric pieces:

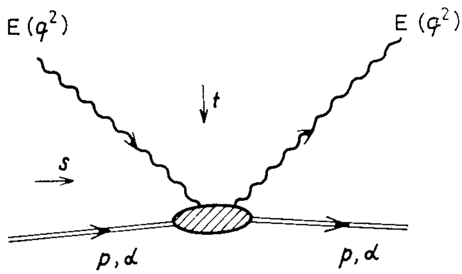

Fig. 2. Virtual Compton scattering on polarized proton

$$M^{s}_{\mu\nu}(p,q) = \frac{1}{2}[M^{\alpha}_{\mu\nu}(p,q) + M^{-\alpha}_{\mu\nu}(p,q)],$$

$$M^{A}_{\mu\nu}(p,q) = \frac{1}{2}[M^{\alpha}_{\mu\nu}(p,q) - M^{-\alpha}_{\mu\nu}(p,q)].$$

The imaginary parts are

$$\mathrm{Im}\, M^{s}_{\mu\nu}(p,q) = W^{s}_{\mu\nu}(p,q)\,,$$
$$\mathrm{Im}\,\big(-iM^{A}_{\mu\nu}(p,q)\big) = W^{A}_{\mu\nu}(p,q)\,.$$

The covariant expansion of the antisymmetric amplitude is

$$M^{A}_{\mu\nu}(p,q) = i\epsilon_{\mu\nu\varrho\sigma}\, q^{\varrho}\, \alpha^{\sigma}\cdot D(q^2,\nu) + i(\alpha q)\,\epsilon_{\mu\nu\varrho\sigma}\, q^{\varrho}\, p^{\sigma}\, G(q^2,\nu)\,. \qquad (26$$

The imaginary parts of the scalar amplitudes are the spin-dependent structure functions:

$$\mathrm{Im}\, D(q^2,\nu) = d(q^2,\nu),\ \ \mathrm{Im}\, G(q^2,\nu) = g(q^2,\nu).$$

$D(q^2,\nu)$ is even in ν, $G(q^2,\nu)$ is odd in ν. It is convenient to perform the Regge—Sommerfeld–Watson transformation on two linear combinations* given by

$$H_1(q^2,\nu) = \frac{M^2}{2}[\mathrm{D}(q^2,\nu) + (pq)\, G(q^2,\nu)]\,,$$

$$H_2(q^2,\nu) = -\frac{M^4}{2(pq)}\, G(q^2,\nu)\,.$$

There are twelve independent S-channel helicity amplitudes but eight of them vanish in the forward direction. Crossing gives us $H_1(q^2,\nu)$ and $H_2(q^2,\nu)$ in terms of the t-channel helicity amplitudes:

$$H_1(q^2,\nu) = \frac{M^4}{4(\nu^2 - q^2M^2)}\Big[\frac{\nu}{M}\big(F^{s}_{1-\frac12;1-\frac12}(q^2,\nu) - F^{s}_{1\frac12;1\frac12}(q^2,\nu)\big) + \sqrt{2q^2}\,F^{s}_{1\frac12;0-\frac12}(q^2,\nu)\Big]\,,$$

$$H_2(q^2,\nu) = -\frac{M^4}{4(\nu^2 - q^2M^2)}\Big[\frac{M}{\nu}\big(F^{s}_{1-\frac12;1-\frac12}(q^2,\nu) - F^{s}_{1\frac12;1\frac12}(q^2,\nu)\big) + \frac{\sqrt{2q^2}}{q^2}\,F_{1\frac12;0-\frac12}(q^2,\nu)\Big]\,. \qquad (27)$$

* In the first paper of [3] the imaginary parts of $H_1(q^2,\nu)$ and $H_2(q^2,\nu)$ are used as $W_3(q^2,\nu)$ and $W_4(q^2,\nu)$ (up to a constant factor).

Eight conspiracy equations are derived for the eight vanishing S-channel amplitudes expressed in terms of the t-channel ones. After R–S–W transformation on the t-channel helicity amplitudes we get poles + cuts +background:

$$H_1(q^2,\nu)=\sum_i \beta_i^-(q^2)\cdot \nu^{\alpha_i^-(0)-1}+\sum_k q^2\beta_k^+(q^2)\cdot\frac{\alpha_k^+(0)-1}{\alpha_k^+(0)}\cdot \nu^{\alpha_k^+(0)-2}+ \\ + \text{cuts} + \text{background} \tag{28}$$

The signs $\pm$ refer to signature. The leading Pomeranchuk trajectory is decoupled from $H_1(q^2, \nu)$ and $H_2(q^2, \nu)$. This can be demonstrated by turning to the conspiracy equations. The leading singularity, which satisfies the conspiracy equations and theorems on the spin-dependence of high-energy amplitudes [8], is the negative parity piece of the Pomeranchuk cut:

$$\begin{aligned} d(q^2,\nu) &= \beta_p(q^2)\frac{\nu^{\alpha_p(0)}}{\ln\nu}+\text{lower terms},\\ g(q^2,\nu) &= -\beta_p(q^2)\frac{\nu^{\alpha_p(0)-1}}{\ln\nu}+\text{lower terms}.\end{aligned} \tag{29}$$

There is an attempt to describe the leading scaling behaviour by the leading term in the R–S–W expansion [9]. This idea tries to identify the leading light-cone singularity with the leading J-plane object.

The scaling functions $\alpha(\xi)$ and $\beta(\xi)$ are singular at $\xi = 0$ in that case, and the residue function $\beta_p(q^2)$ has definite q^2-asymptotics to give rise to the desired scaling behaviour of $d(q^2, \nu)$ and $g(q^2, \nu)$. Integrals involving $\alpha(\xi)$ and $\beta(\xi)$ remain meaningful even with $\alpha(\xi)$ and $\beta(\xi)$ singular at $\xi = 0$, since $A_d(p, x)$ and $A_g(p, x)$ are tempered distributions, so that every operation has to be treated in distribution-theoretic sense. In this picture we find the scaling law:

$$\begin{aligned} d(q^2,\nu) &\sim \frac{1}{2\nu}\alpha(\xi),\\ g(q^2,\nu) &\sim -\frac{1}{2\nu^2}\alpha(\xi).\end{aligned} \tag{30}$$

First, one takes the scaling of $\nu^2 g(q^2, \nu)$ for granted, then the behaviour of $d(q^2, \nu)$ follows from (29). The common $\beta_p(q^2)$ in (29) gives rise to the very interesting point that $d(q^2, \nu)$ and $g(q^2, \nu)$ are *not independent* in scaling limit.

This result is independent of the location of the leading J-plane singularity in the deep-inelastic limit if such an object can be singled out at all. The character of the J-plane singularity appears in the behaviour of $\alpha(\xi)$ near to $\xi = 0$. (30) follows from the R–S–W representation and the identification mentioned above.

IV. Spin-dependent effects in the parton model

We have calculated the structure functions $d(q^2, \nu)$ and $g(q^2, \nu)$ in a simple field-theoretic model [10] which indicates point-like parton interpretation in the spin-dependent case, too. The calculation is tedious and lengthy. The results are transparent and provocative because they do not contain any free parameters.

We have found scale invariance for $\nu \cdot d(q^2, \nu)$ and $\nu^2 \cdot g(q^2, \nu)$ with explicit scaling functions in the deep inelastic region at large values of ω [3].

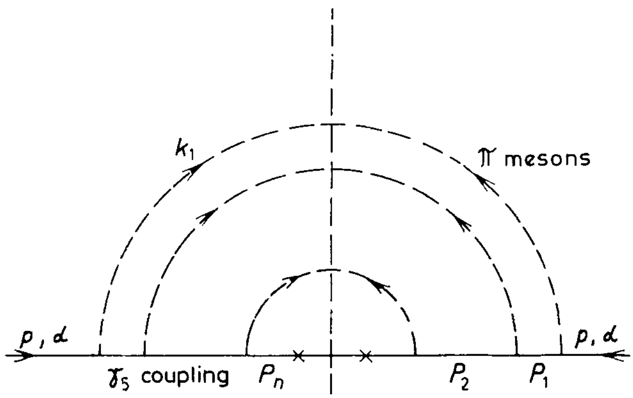

Fig. 3. The current scatters on the proton. Scattering on pions does not give rise to spin-dependent effects

We summarize here only the essential points in the calculation. The technique is the same as applied by Drell et al. to the spin-averaged amplitudes. We "undress" the current operator and go into the interaction picture with the U-matrix

$$U(t) = \left(e^{-i\int_{-\infty}^{t} dt' H_I(t')}\right)_+ .$$

The free or "undressed" current is related to the fully interacting current by $J_\mu(x) = U^{-1}(t)_{j\mu}(x)U(t)$ where $j_\mu(x)$ has the same form in terms of in-fields as does $J_\mu(x)$ in terms of interacting Heisenberg fields.

One proves for the spin-dependent amplitude in scaling limit:

$$\lim_{\substack{q^2 \cdot \nu \to \infty \\ \omega > 1}} W^{\alpha}_{\mu\nu}(p, q) = \int d^4 x e^{iqx} [\langle UP_{\alpha}| j_{\mu}(x) j_{\nu}(0) | UP_{\alpha}\rangle]_{p\to\infty} \qquad |UP_{\alpha}> = U(0)|\, P, \alpha >. \tag{31}$$

Equation (31) suggests parton-interpretation [10, 11] which may be more general than simple models, manifested in $H_1(t)$. Detailed analysis in the pseudoscalar theory shows that one gets the main contribution to the spin-dependent structure functions from the ladder diagrams (Fig. 3).

We give some explicit formulae to indicate the main points of the calculation. The contribution of π^0 mesons with n rungs is

$$|Up_{\alpha}> = \text{const.} \int \frac{d^3 p_n}{\sqrt{2E_n}} \cdot \prod_{i=1}^{n} \frac{d^3 k_i}{\sqrt{2\omega_i}} \delta^{(3)}(\vec{p} - \vec{p}_n - \vec{k}_1 - \ldots - \vec{k}_n) \times$$
$$\sum_{\beta} \frac{\bar{u}(p_n, \beta)\, \gamma_5 (M + \hat{p}_{n-1\, n-1})\, \gamma_5 \ldots \gamma_5 (M + \hat{p}_1)\, \gamma_5\, u(p, \alpha)}{(2E_1)\ldots(2E_{n-1})(E_p - E_1 - \omega_1)\ldots(E_p - E_n - \omega_1 - \ldots - \omega_n} \tag{32}$$
$$|p_n \beta; k_1 k_2 \ldots k_n>,$$

$$W^{\alpha}_{\mu\nu}(p, q) = \text{const.} \int \prod_{i=1}^{n} \frac{d^3 k_i}{2\omega_i} \delta(q^2 + 2p_n q) \times$$
$$\times \frac{T_r \left\{ (M + \hat{p}) \frac{1 + \gamma_5 \hat{\alpha}}{2} \gamma_5 \ldots (M + \hat{p}_n) \gamma_{\mu} (M + \hat{p}_n + \hat{q}) \gamma_{\nu} (M + \hat{p}_n) \gamma_5 \ldots \gamma_5 (M + \hat{p}) \gamma_5 \right\}}{(2E_1)^2 \ldots (2E_n)^2 (E_p - E_1 - \omega_1)^2 \ldots (E_p - E_n - \omega_1 - \ldots - \omega_n)^2}. \tag{33}$$

To include the contribution of charged pions is only simple algebra. The antisymmetric piece of Eq. (33) permits the calculation of $d(q^2, \nu)$ and $g(q^2, \nu)$ in the pseudoscalar theory:

$$iW^{A}_{\mu\nu}(p, q) = \text{const.} \int \prod_{i=1}^{n} \frac{d^3 k_i}{2\omega_i} \delta(q^2 + 2q_n q)$$
$$\frac{Tr \{(M + \hat{p})\, \hat{\alpha} (M + \hat{p}_1)\, \gamma_5 \ldots \gamma_{\mu} (M + \hat{p}_n + \hat{q}) \gamma_{\nu} \ldots}{(2E_1)^2 \ldots (2E_n)^2 (E_p - E_1 - \omega_1)^2 \ldots} \tag{34}$$
$$\frac{\gamma_5 (M + \hat{p}_1)\, \gamma_5 \}}{(E_p - E_1 - \omega_1 - \ldots - \omega_n)^2}.$$

One starts from Eq. (34) to work out the practical details. For the calculations we refer to [3]. It turns out that the scaling laws (for proton or neutron):

$$d(q^2, \nu) \sim \frac{1}{2\nu} \alpha(\xi),$$
$$g(q^2, \nu) \sim \frac{1}{2\nu^2} \beta(\xi) \tag{35}$$

are general consequences of the special transverse momentum cut-off [10] in a large class of models. We have tried this in the pseudoscalar model and in other models where the pions were replaced by vector mesons [3]. At large ω we have found the scaling function, both for proton and neutron, without free parameters. This makes it possible to estimate the polarization effects in different parton models.

The physical interpretation is simple and transparent. The current is scattered on point-like constituents, $d(q^2, \nu)$ and $g(q^2, \nu)$ measure the spin-distribution of the partons inside the physical nucleon:

$$W^{\alpha}_{\mu\nu}(p, q) = \sum_N P(N) \int_0^1 dx\, f_N(x)\, W^{\beta(p_n,\alpha,p)}_{\mu\nu}(p_n, q)\,, \tag{36}$$

where $p_n = xp$. $P(N)$ is the probability that we find N partons inside the proton, $f_N(x)$ is the probability that the "proton—parton" has a four-momentum xp. The polarization $\beta(P_n, \alpha, p)$ of this parton depends on the polarization of the physical proton.

The current scatters on the spin one-half charged constituents described by $W^{\beta}_{\mu\nu}(p_n, q)$. The results one deduces from these models can be generalized to parton models without concrete field-theoretic background [12]. In our Letter [3] we have chosen a simple quark model to study spin-dependent effects. In the light of the field-theoretic analysis we have now more general results.

We emphasize again that the scaling law in (35) seems to be rather common property of different dynamical models based on point-like constituents. The scaling functions can be calculated explicitly and we find sizeable polarization effects in the deep-inelastic region.

V. Conclusion

Spin-dependent effects in the deep-inelastic region should be analyzed by measuring the asymmetry

$$A = \frac{d\sigma_{\alpha\beta} - d\sigma_{-\alpha\beta}}{d\sigma_{\alpha\beta} + d\sigma_{-\alpha\beta}}\,. \tag{37}$$

Incident polarized electron or muon beams should be focused onto a polarized target. Scattered electrons or muons at fixed angles are momentum-analyzed and identified using magnetic spectrometers. The theoretical estimations [3, 5] predict raw asymmetries which may well be within the range of electron scattering experiments in the future.

Table I

Predictions and correlations of the different theoretical descriptions

Scaling behaviour	Constitution of the proton	Light-cone singularity Constitution of the electric current	Leading light-cone singularity leading J-plane object
$d(q^2,\nu) \sim \frac{1}{2\nu}\alpha(\xi)$ $g(q^2,\nu) \sim \frac{1}{2\nu^2}\beta(\xi)$	parton-like point constituent with nonzero spin $\alpha(\xi) \neq -\alpha(\xi)$ $Z = -\frac{1}{4\pi M}\int_0^M d\xi\alpha(\xi)$	$A_d(p,x) \sim \epsilon(x_0)\,\delta(x^2)$ $A_g(p,x) \sim \epsilon(x_0)\,\theta(x^2)$ non-vanishing equaltime space-space commutator is possible constituents with non-zero spin are expected	condition of identification: $\alpha(\xi) = -\beta(\xi)$
$d(q^2,\nu) \sim \frac{1}{2\nu^2}\gamma(\xi)$ $g(q^2,\nu) \sim \frac{1}{2\nu^2}\beta(\xi)$	not parton-like	$A_d(p,x) \sim \epsilon(x_0)\,\theta(x^2)$ $A_g(p,x) \sim \epsilon(x_0)\,\theta(x^2)$ equal-time commutator vanishes no restriction on the spin-constitution	identification impossible
$g(q^2,\nu) \sim \frac{1}{2\nu^\epsilon}\beta(\xi)$ $\frac{3}{2} \leq \epsilon < 2$	not parton-like	$\sigma_s(q^2,\nu) = 0 \rightarrow$ half-integer spin constituents are present $A_g(p,x) \sim \epsilon(x_0)\,\theta(x^2)\,b(x_0)$ $b(x_0)$ singular	condition of identification: $d(q^2,\nu) \sim \frac{1}{2\nu^{1+\epsilon}}\alpha(\xi)$ $\alpha(\xi) = -\beta(\xi)$

Longitudinally polarized muon beams have been formed from the decay of pions in flight at the major accelerator sites. The Serpukhov accelerator and the Batavia accelerator will give rise to more intense and higher-energy pion beams; this should make muon experiments of this kind feasible. It may also be possible to produce a high-energy polarized electron beam at SLAC.

Another attempt in thinking about the feasibility of the spin-dependent measurement is to use the unpolarized SLAC beam and to try to measure the polarization of the *scattered* electron beam.

We are aware of the experimental difficulties. The motivation for our analyzis is that experimental data on $d(q^2, \nu)$ and $g(q^2, \nu)$ could help a lot in testing different ideas stimulated by the SLAC–MIT experiment.

We have investigated the consistency of Finite-Energy Sum Rules with ideas presented here and the contribution of $d(q^2, \nu)$ and $g(q^2, \nu)$ to the hyperfine splitting of the hydrogen ground-state in different theoretical models [3].

In conclusion, the predictions of the different theoretical considerations are summarized in Table I.

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ГЛУБОКО НЕУПРУГОЕ РАССЕЯНИЕ ПОЛЯРИЗОВАННОГО ПУЧКА ЭЛЕКТРОНОВ НА ПОЛЯРИЗОВАННОЙ НУКЛОННОЙ МИШЕНИ

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Резюме

Рассмотрены эффекты, зависящие от спина при рассеянии электронов на протонах.