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SCALE INVARIANCE, GOLDSTONE BOSONS AND THE f′ TRAJECTORY*

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We propose that the f′ trajectory at $\alpha_{f'} = 0$ can serve the role of a Goldstone boson for cale invariance and discuss experimental consequences that follow from such an association.

Introduction

One of the most important questions on scale invariance in elementary particle physics is how good this invariance is in the real world. In this connection GELL-MANN [1] emphasized that there are two ways in which scale invariance manifests itself. In one way *(i)* the masses of all the particles will be massless in the limit of scale invariance. The invariance is violated to the extent that the nucleon mass, for example, has a value ∼1 GeV. In another way *(ii)*, the particles may be massive even in the limit of scale invariance provided there is a Nambu—Goldstone boson, i.e. a massless scalar meson. A measure of violation of scale invariance would be given by how much massive this Goldstone boson is in the actual world. The arguments on these points will be reviewed briefly in the following.

As an illustration consider the matrix element of the stress-energy-momentum tensor $\theta_{\mu\nu}$ between the states of a spinless particle. Using Lorentz covariance, parity conservation, time-reversal invariance, one can write the most general form given by [1]

$$\langle p'|\theta_{\mu\nu}|p\rangle = \frac{1}{2E}\left[2P_\mu P_\nu F(k^2) + (k^2\delta_{\mu\nu} - k_\mu k_\nu)G(k^2)\right], \tag{1}$$

where p_μ, p'_μ are the momenta of the particle in the initial and final states, and

$$P_\mu = \frac{1}{2}(p_\mu + p'_\mu), \quad k_\mu = p_\mu - p'_\mu .$$

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One can check that the form (1) is consistent with the conservation law $\partial_\mu \theta_{\mu\nu} = 0$. The first form factor $F(k^2)$ is normalized as

$$F(0) = 1. \tag{2}$$

One now uses the fact that $\theta_{\mu\nu}$ is traceless in the limit of scale invariance;

$$\theta_{\mu\mu} = 0. \tag{3}$$

This is derived from the geometrical consideration. The only question is how to modify the usual definition of the "canonical" stress–energy–momentum tensor derived from the field theoretic Lagrangian. See the papers by STRATHDEE and TAKAHASHI [2], and by GELL-MANN [1].

Imposing the condition (3) onto (1) one obtains

$$0 = \langle p'|\theta_{\mu\mu}|p\rangle = \frac{1}{2E}\,[2P^2\,F(k^2) + 3k^2G(k^2)], \tag{4}$$

which gives a constraint on two form factors $F(k^2)$ and $G(k^2)$. Particularly interesting is their behavior for $k^2 \approx 0$. Noting $P^2 \approx p^2 \approx -m^2$ in this limit one finds two cases: If $G(k^2)$ is finite at $k^2 = 0$, Eq. (4) gives

$$-2m^2F(0) = 0,$$

which combined with (2) gives

$$m^2 = 0.$$

This corresponds to the case *(i)* mentioned above. On the other hand, one may assume that $G(k^2)$ has a pole at $k^2 = 0$. Eq. (4) is then satisfied by

$$G(k^2) = \frac{2}{3}\,\frac{m^2}{k^2}\,. \tag{5}$$

This pole can be interpreted as the emergence of a massless scalar boson (the Nambu–Goldstone boson) which will be hereafter called the θ_λ meson.

The matrix element (1) is the "form factor" of a coupling of the particle to the graviton, since $\theta_{\mu\nu}$ is a source of the graviton. Eq. (5) will correspond to the fact that the diagram (a) of Fig. 1 contains a part which is dominated by the exchange of the θ_λ as illustrated by the diagram (b). One realizes a close analogy to the pion dominance in the weak coupling of a hadron to the leptons.

In the diagram (b) the constant for the graviton — θ_λ junction is denoted by f_θ (times the gravitational constant) while the coupling of a hadron labeled by i to θ_λ is denoted by $f_{\theta ii}$. The amplitude corresponding to the diagram (b) is then given by

$$G(t) = \frac{f_\theta f_{\theta ii}}{-t}, \tag{6}$$

where $t = -k^2$. Comparing this with (5) one obtains the relation

$$m_i^2 = \frac{3}{2} f_\theta f_{\theta ii}, \tag{7}$$

which is the exact analogue of the Goldberger–Treiman relation.

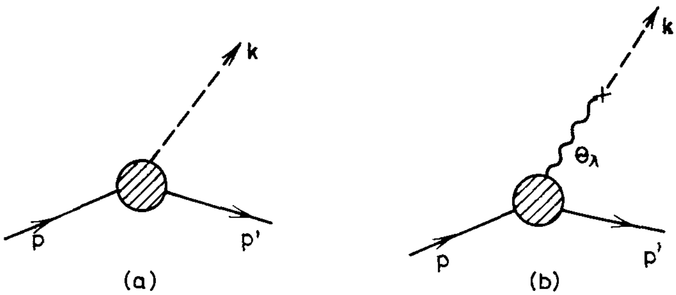

Fig. 1. Diagrams for $\langle p'|\theta_{\mu\nu}|p\rangle$

Now the question is whether there is a scalar meson which is light enough to be considered as an approximate Goldstone boson. The meson should be isoscalar as far as the electromagnetic mass differences are neglected. There are some candidates like ε (750 $\sim$ 900 MeV) or S* (1070 MeV). They are, however, almost as heavy as the nucleon or the ϱ meson, etc. This would mean that there is no preference of the second mechanism (the emergence of the Goldstone boson) over the first one in which finite masses of the ordinary particles are the manifestation of the violation of scale invariance.

It would be here worthwhile to recall that our concept of the particles had been changed drastically in the last decade. We have now the Regge pole theory. What we are going to do in this paper is to try to apply this new concept to the Goldstone boson so that one may consider the real world to be very close to the limit of scale invariance.

Being motivated in this way one can look at the Chew–Frautschi plot to find immediately that there is a trajectory which goes very close to the origin. It is the (exchange degenerate) φ — f′ trajectory. The value of t_θ for which this trajectory passes through the abscissa, is the order of $\pm m_\pi^2$. One might say that there is some spinless, isoscalar object having the squared mass t_θ. Of course, no such particle has been observed experimentally. One usually appeals to the so-called ghost killing mechanism. The simplest of such is the choosing nonsense mechanism by which the scattering amplitude corresponding to the exchange of this trajectory contains a numerator which behaves like $\sim \alpha(t)$ so that the pole behavior at $\alpha(t) = 0$ is cancelled. We are going to see if this "nonsense point" of the f′ trajectory is identified as the Goldstone boson we are looking for.

Consider scattering of the particle i and the particle j in the t channel. The scattering amplitude will then be given by

$$T(t) \sim \frac{\alpha(t)}{\sin \pi\alpha(t)} . \tag{8}$$

The Feynman amplitude, on the other hand, corresponding to the exchange of a meson θ_λ whose mass is assumed to be exactly zero just for simplicity is given by

$$T(t) \sim \frac{f_{\theta ii} f_{\theta jj}}{t} . \tag{9}$$

Comparing (8) and (9), and accepting the linear trajectory, one realizes that the coupling strength $f_{\theta ii}$ cannot be a constant, but should be proportional to $\sqrt{t}$;

$$f_{\theta ii}(t) = \sqrt{t}\, g_{\theta ii} , \tag{10}$$

where $g_{\theta ii}$ is a constant, or at least finite at $t = 0$. This is, no matter how strange in may appear, an inevitable conclusion from the choosing nonsense mechaitsm and the factorizability of the residue functions. Note also that the Veneziano—Lovelace amplitude gives the same result. Substituting (10) into (6) one obtains

$$G(t) = \frac{f_\theta g_{\theta ii} \sqrt{t}}{-t} .$$

The second term of (4) then vanishes for $t \to 0$ if f_θ is finite at $t = 0$. The only way in which the second term survives so that the nonsense point of the f′ trajectory serves as a Goldstone boson is that f_θ is inversely proportional to $\sqrt{t}$;

$$f_\theta(t) = \frac{1}{\sqrt{t}} F_\theta , \tag{11}$$

where F_θ is a constant, or at least finite at $t = 0$. One may argue at this point if a singular form (11) may cause any serious difficulty. Before answering this question, we discuss what the particle picture of our θ_λ will look like.

The behavior as given by Eq. (10) is supposed to be true for every hadron, so that the pole t^{-1} is always cancelled in any amplitude of the hadron interactions. This means that the θ_λ can never be observed as a particle as long as one is looking at the strong interactions of hadrons. The same would be true also for the interactions of hadrons and photons if the photons are absorbed or emitted always through vector mesons. The situation is not clear for the weak interaction. It is rather likely that the pole t^{-1} appears in the amplitudes involving gravitons. The θ_λ is then a real particle which may decay into a number of gravitons or be produced through the graviton-hadron collision, for example. In the following we discuss other questions which will arise from our approach.

I. Graviton—θ_λ mixing

Practically one does not have to worry about the singular behavior (11), since in the lowest order terms in the extremely small gravitational coupling constant the factor $1/\sqrt{t}$ is so designed as to be cancelled by another factor $\sqrt{t}$ in (10). In principle, however, one can investigate the effect of the higher order terms. We are particularly interested in the infinite sum of the diagrams like those in Fig. 2. This mixing problem between the θ_λ and graviton can be

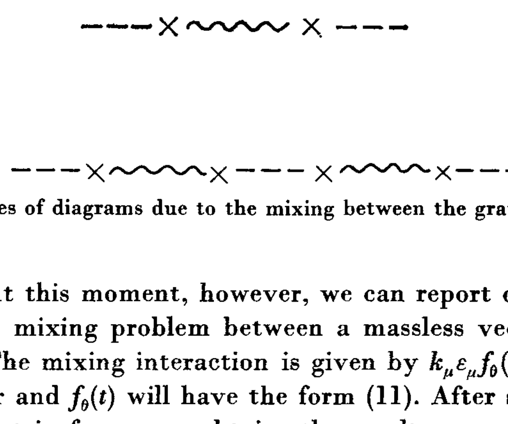

Fig. 2. Examples of diagrams due to the mixing between the graviton and the θ_λ

solved basically. At this moment, however, we can report only the results of an exercise — the mixing problem between a massless vector meson and a scalar meson, θ_λ. The mixing interaction is given by $k_\mu \varepsilon_\mu f_\theta(t)$, where ε_μ is the polarization vector and $f_\theta(t)$ will have the form (11). After solving the Dyson equation in the matrix form, one obtains the results summarized as follows:

(i) The vector meson remains massless after the mixing is turned on.

(ii) The squared mass t_θ is changed to $\tilde{t}_\theta$ by the amount which is second order in the gravitational coupling constant.

(iii) The fields V_μ and φ are related to the diagonalized fields $\tilde{V}_\mu$ and $\tilde{\varphi}$ as follows:

$$V_\mu = b_{11}\tilde{V}_\mu + b_{12}\partial_\mu\tilde{\varphi},$$
$$= b_{21}\partial_\mu\tilde{V}_\mu + b_{22}\tilde{\varphi}.$$

The constants are determined as

$$b_{11} = 1, \quad b_{12} \propto f_\theta(\tilde{t}_\theta),$$
$$b_{21} = 0, \quad b_{22} = 1.$$

The result of b_{12} shows that an infinity can be avoided if

$$\tilde{t}_\theta \neq 0.$$

These qualitative results are expected to remain the same if one replaces the vector meson by the graviton. The analogue of the first point *(i)* will say that main features of Einstein's gravitation theory remain unaltered. It is clear, however, that the second order tensor field which describes the graviton is no longer traceless. If the spinless part corresponding to this trace turns out to be massless, the resulting modification of Einstein's theory may be somewhat similar to DICKE's theory [3].

II. SU_3 transformation property of θ_λ

It is well-known that φ and f′ are very close to the "ideal" mixing of SU_3 so that they transform like $\bar{\lambda}\lambda$ in terms of quarks. It is natural to expect that our θ_λ also transforms in this way. If one appeals to the simple-minded quark counting model, one immediately obtains

$$g_{\theta\pi\pi} = 0\,, \qquad \frac{g_{\theta\eta_8\eta_8}}{g_{\theta K\bar{K}}} = \frac{4}{3}\,, \tag{12}$$

where η_8 represents the purely octet η meson. Combining (12) with (7), one gets the mass relations

$$m_\pi^2 = 0\,, \quad \frac{m_{\eta 8}^2}{m_K^2} = \frac{4}{3}\,,$$

which are consistent with GMO mass formula. If one applies the same procedure to the nonet vector and tensor mesons, one obtains the familiar equal spacing law, but with

$$m_\varrho^2 = 0, \quad m_{A_2}^2 = 0\,,$$

since the ϱ as well as the A_2 do not contain the strange quarks. One realizes that we need something else to give the masses of these lowest levels. It is not yet clear which of the mechanisms *(i)* and *(ii)* is responsible for these masses. (See, however, the following Section IV). Focusing our attention to the SU_3 breaking part at present, we modify the previous formula (7) to the form

$$\delta m_i^2 = \frac{3}{2} F_\theta g_{\theta ii}, \tag{13}$$

where we have replaced f_θ and $f_{\theta ii}$ by F_θ and $g_{\theta ii}$, respectively, by using (10) and (11).

Similar calculations can also be carried out for the fermions. Corresponding to (13) one obtains the result,

$$\delta m_i = 3F_\theta g_{\theta ii}. \tag{14}$$

Again the θ_λ gives no contribution to the nonstrange baryons like N or Δ. Contrary to the squared mass formula (13) for the mesons, one obtains the linear mass formula (14) for the baryons. Again using the simple quark counting for the estimate of the coupling constants, one obtains the equal spacing law for the baryon masses (neglecting the mass difference between Λ and Σ). For the later use, we quote one of the results

$$\frac{g_{\theta K\bar{K}}}{g_{\theta\Sigma\bar{\Sigma}}} = \frac{2m_K^2}{m_\Sigma - m_N}, \tag{15}$$

which is easily obtained from (13) and (14).

III. Broken chiral symmetry

According to our scheme, we find that the Hamiltonian ($= -\theta_{44}$) contains a part which is dominated by the θ_λ. This part clearly transforms like $\bar{\lambda}\lambda$. On the other hand, GMOR Hamiltonian

$$H' = u_0 + cu_8,$$

which violates chiral $SU_3 \times SU_3$ has the similar SU_3 transformation property. In fact, the constant c was found to be very close to $-\sqrt{2}$. In the approximation in which the pion mass is neglected, c is exactly equal to $-\sqrt{2}$ so that the whole H' transforms like $\bar{\lambda}\lambda$. One may speculate that the part of the Hamiltonian which is dominated by θ_λ is identical with the GMOR Hamiltonian. If this is really the case, the $SU_3 \times SU_3$ violating Hamiltonian is almost invariant under scale transformation, contrary to GELL-MANN's conjecture that GMOR Hamiltonian also violates scale invariance [1].

IV. Exotic nature of the θ_S

As was emphasized in Section II, we need something which lifts up the SU_3 multiplets other than pseudoscalar octet. The violation of scale invariance associated with this might be of the type *(i)* without the Goldstone boson. It seems still worthwhile to explore the possibility that there is another Goldstone boson, which will be called θ_S. We expect that there is another unitary singlet trajectory which is almost degenerate with the f′ trajectory. This trajectory will couple to the hadrons other than pairs of octet pseudoscalar mesons. This suggests that the new trajectory is exotic in a sense that it does not allow simple quark counting.

We also noticed that the GMOR Hamiltonian is the part dominated by the θ_λ. On the other hand, we know that there must be the part of the Hamiltonian which violates chiral $U_3 \times U_3$ but conserves $SU_3 \times SU_3$ [4]. The term having this property must also be exotic in a sense that it cannot be represented by the form $\bar{q}q$. It seems therefore natural to speculate that this is the part which is dominated by the θ_S.

In this connection it is also interesting to note that ARNOLD [5] proposed recently that there is an exotic, and Pomeron-like trajectory which has the unit intercept and the "normal" slope. Our θ_S may belong to a daughter of ARNOLD's trajectory.

V. An experimental test

If we make a full use of the simple-minded quark counting, we can predict results which can be tested in ordinary high energy physics. Noting that the f (1250) contains only non-strange quarks, one obtains, for example,

$$\frac{g_{fK\bar{K}}}{g_{f\Sigma\bar{\Sigma}}} = \frac{1}{2}\frac{g_{f'K\bar{K}}}{g_{f'\Sigma\bar{\Sigma}}},$$

where the factor 2 in the denominator of the right-hand side comes from the fact that the Σ^+, for example, contains two proton quarks, while it contains only one lamda quark. By assuming some smoothness condition along the trajectory, the right-hand side can be replaced by (1/2) $(g_{\theta K\bar{K}}/g_{\theta\Sigma\bar{\Sigma}})$, for which one can use Eq. (15). Further using the SU_3 argument, one arrives at the relation

$$\frac{g_{f\pi^+\pi^-}}{g_{fp\bar{p}}} \mathrel{\hat{=}} \frac{4}{3}\frac{m_K^2}{m_\Sigma - m_N}.$$

In order to test this new relation, we consider the ratio

$$R_\pi = \frac{\sigma_T(\pi N)_f}{\sigma_T(NN)_f}, \tag{16}$$

where $\sigma_T(\pi N)$, for example, is the part of the πN total cross section dominated by the exchange of the f. One may be able to extract such a part from the high energy πN total cross section by subtracting the asymptotic cross section.

With the aid of the optical theorem, the ratio (16) is put into the form

$$R_\pi = \frac{1}{2m_N} \frac{Im F(\pi N)_f}{Im F(NN)_f} ,$$

where $F(\pi N)_f$, which is the πN forward scattering amplitude dominated by the exchanged f, is proportional to $g_{f\pi\pi} g_{fNN}$, while $F(NN)_f$ is proportional to $(g_{fNN})^2$. One finally obtains

$$R_\pi = \frac{1}{2m_N} \frac{g_{f\pi^+\pi^-}}{g_{fp\bar{p}}} = \frac{2}{3} \frac{m_K^2}{m_N(m_\Sigma - m_N)} \cong 0.69 . \tag{17}$$

In the same way one obtains

$$R_K = \frac{\sigma_T(KN)_f}{\sigma_T(NN)_f} = \frac{1}{2} R_\pi \cong 0.35 . \tag{18}$$

The experimental data are shown in Table I. Since the asymptotic values for πN and NN cross sections have large uncertainties, we have used two sets of numbers [6–8]. Although it seems premature to draw any definite conclusion, the agreement is at least encouraging.

Table I

Experimental data (from 6 to 20 GeV/c) for R_π and R_K

	Case 1	Case 2
R_π	0.39–0.45	0.84–1.10
R_K	0.19–0.22	0.28–0.40

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8. The asymptotic cross sections, σ_∞, used are (See [6] and [7])
$\sigma(\mathrm{KN})_\infty = 17.0$ mb,
$\sigma(\pi\mathrm{N})_\infty = 20.5$ mb and $\sigma(\mathrm{NN})_\infty = 34.8$ mb for Case 1,
$\sigma(\pi\mathrm{N})_\infty = 18.1$ mb and $\sigma(\mathrm{NN})_\infty = 38.9$ mb for Case 2.

КАЛИБРОВОЧНАЯ ИНВАРИАНТНОСТЬ, БОЗОНЫ ГОЛЬДСТОУНА, И ТРАЕКТОРИЯ f′

Я. ФУДЗИЙ

Резюме

Предложена идея, согласно которой для сохранения калибровочной инвариантности траектория f′ при $\alpha_{f'} = 0$ может играть роль бозона Гольдстоуна. Дискутируются экспериментальные следствия, вытекающие из такой ассоциации.