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BROKEN SCALE INVARIANCE IN INELASTIC LEPTON–NUCLEON SCATTERING

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Broken scale invariance in inelastic lepton–nucleon scattering is discussed studying current commutators near to light-cone and their equal time limits. Models, based on WILSON's ideas, are proposed here to provide a more general frame than canonical models.

I. Introduction

I shall mainly review some work I have done in collaboration with CICCARIELLO, GATTO and TONIN (Sections IV, V and VI) [1, 2], and some previous related results by other authors (Sections I, II and III), concerning broken scale invariance and its applications to inelastic lepton–hadron scattering.

Finally (Section VII), I shall briefly comment on some relevant very recent experimental results from SLAC [40, 41].

Since the local algebra of currents has been proposed by GELL-MANN, a lot of sum rules have been derived which connect measurable quantities to ETC's between local operators. Particularly interesting are the ETC's involving e.m. or weak currents or their derivatives.

The model-independent parts of such commutators are easy to write down, at least if one believes in some symmetry scheme. More intriguing to compute are the model-dependent parts. To solve the problem, suggestions have been sought in canonical Lagrangian models such as the quark model, the gluon model, field algebra, or others.

Unfortunately, as JOFFE and VAINSTEIN [3], JACKIW and PREPARATA [4] and ADLER and TUNG [5] have shown, equal time commutators calculated by naive canonical manipulations of the field operators generally cannot be used in asymptotic sum rules. In fact, they do not always agree with those computed from Feynman diagrams via $q_0 \to i\infty$ limit.

The source of trouble has to be sought in the singular nature of products of local operators evaluated at the same space-time point. Such singularities can hardly be treated correctly unless one is able to solve exactly the model.

An economical way to bypass the problem, has been suggested by WILSON [6]. According to WILSON the commutator of two local operators, $A(x)$ and $B(0)$, can be expanded, when $x_\mu \to 0$, in the following asymptotic series:

$$[A(x),\ B(0)] \approx \sum O_n(0)\ C_n(x). \tag{1}$$

In Eq. (1) the equality holds in the weak sense, the $C_n(x)$'s are tempered distributions which contain the whole x_μ-dependence and for locality must vanish outside the light cone; the $O_n(0)$'s form a generally infinite set of independent local operators.

From Eq. (1), by taking the limit $x_0 \to 0$, one formally gets the following expansion for the equal time commutator between $A(x)$ and $B(0)$:

$$[A(\vec{x},0), B(0)] = \sum_{k=0} S^{\tau_1 \ldots \tau_k}(A,B;0)\, \partial_{\tau_1} \ldots \partial_{\tau_k} \cdot \delta^{(3)}(\vec{x}). \tag{2}$$

$S^{\tau_1 \ldots \tau_k}$, the k-order Schwinger term, may also be infinite (in this sense Eq. (2) is a formal development of an equal time commutator): when finite, it is a linear combination of local operators and has physical dimension (in units of length)

$$l_S(k) = l_A + l_B + k + 3, \tag{3}$$

$_A$ and l_B, which are negative numbers, are the physical dimensions of A and B.

The r.h.s. of Eq. (2) turns out to be certainly a finite sum in theories that do not contain dimensioned parameters, provided the number of operators with dimensions $\geqq l_A + l_B + 3$ is finite.

If, moreover, the set of such operators is known, the r.h.s. of Eq. (2) is determined apart from a few numerical constants. Theories that do not contain dimensioned parameters are insensible to a change of the unit of length, that is with respect to a scale transformation or, which is the same, with respect to a dilatation of space and time:

$$x_\mu \to e^\lambda x_\mu; \qquad \lambda \text{ real}$$

II. Scale and conformal invariance

The idea that scale invariance could be a useful concept in theoretical physics is rather old and dates to the works of GURSEY [7], WESS [8], FULTON, ROHRLICH and WITTEN [9], and KASTRUP [10].

KASTRUP [11] and MACK [12] in particular suggested that strong interactions become scale-invariant at short distances, that is at large energies, when all the masses and dimensioned coupling constants of renormalized interactions loose their relative weights.

Such asymptotic scale invariance, however, cannot be considered the consequence of an exact symmetry. In fact, it is easy to prove that in a scale-invariant theory, discrete states with non-vanishing masses are ruled out, cross-sections fall off too rapidly, the lack of dimensioned parameters prevents one from defining asymptotic states: to mention but a few of the difficulties one meets.

Therefore asymptotic scale invariance must be considered only as a broken symmetry. This amounts to saying that one believes in the existence of a limiting theory, called by WILSON skeleton theory, which obtains when all masses and dimensioned coupling constants vanish.

In such a situation, whenever the theory is renormalizable, one expects only operators of dimensions $\geqslant l_S(k)$, as given in Eq. (3), to contribute to the k-order Schwinger-term; those of dimensions $> l_S(k)$, when finite, occur suitably multiplied by symmetry breaking parameters.

In Lagrangian field theories scale invariance is often accompanied by a larger space-time symmetry, associated to the group of conformal transformations. The conformal group is a 15-parameter non-compact and non-semisimple Lie group, isomorfic to SO(2, 4). It contains as a subgroup the Poincaré group, and is defined as the group of the following non-linear transformations in the Minkowsky space:

$$x'_\mu = a_\mu + \Lambda_\mu{}^\nu x_\nu, \qquad \text{(inhomogeneous Lorentz transformations)}$$

$$x'_\mu = e^\lambda x_\mu; \ \lambda \text{ real}, \qquad \text{(dilatations)}$$

$$x'_\mu = \frac{x_\mu + c_\mu x^2}{1 + 2cx + c^2 x^2} . \qquad \text{(special conformal transformations)}$$

The Lie algebra of the conformal group is specified by the following commutators among the generators of infinitesimal transformations:

$$[M_{\mu\nu}, D] = 0 , \tag{4a}$$

$$[P_\mu, D] = iP_\mu , \tag{4b}$$

$$[M_{\mu\nu}, K_\varrho] = i(g_{\varrho\nu} K_\mu - g_{\varrho\mu} K_\nu) , \tag{4c}$$

$$[P_\mu, K_\nu] = 2\, i(g_{\mu\nu} D - M_{\mu\nu}), \tag{4d}$$

$$[K_\mu, K_\iota] = 0 , \tag{4e}$$

$$[D, K_\mu] = iK_\mu , \tag{4f}$$

where $M_{\mu\nu}$ and P_μ are the generators of the Poincaré group, D is the dilatation charge and K_μ are the generators of special conformal transformations.

The commutators among the generators of the inhomogeneous Lorentz group are well known and have not been written down.

The field-theoretically admissible representations of the conformal algebra have been discussed in a review paper by MACK and SALAM [13]. They are defined through the following relations:

$$[\Phi_a(x), P_\mu] = i\,\partial_\mu\,\Phi_a(x)\,,$$
$$[\Phi_a(x), M_{\mu\nu}] = i\,[(x_\mu\,\partial_\nu - x_\nu\,\partial_\mu)\,\delta_a^b (i\,\Sigma_{\mu\nu})_a{}^b\,]\,\Phi_b(x)\,,$$
$$[\Phi_a(x), D] = i\,(-l_a^b + x^\varrho\,\partial_\varrho\,\delta_a^b)\,\Phi_b(x)\,,$$
$$[\Phi_a(x), K_\mu] = i\,[-2l_a^b x_\mu + (2x_\mu x^\varrho\,\partial_\varrho - x^2\,\partial_\mu)\,\delta_a^b - 2\,i\,x^\varrho(\Sigma_{\mu\varrho})_a{}^b + \varkappa_a{}^b]\,\Phi_b(x)\,,$$

where $\Phi_a(x)$ is a finite or infinite set of local fields, $\Sigma_{\mu\nu}, l$ aud $\varkappa_\mu$ are finite or infinite matrices, and a sum over repeated indices is understood.

According to the type of the matrices $\Sigma_{\mu\nu}$, l and $\varkappa_\mu$, one gets the following classes of representations:

1. $\varkappa_\mu \equiv 0$. l is real and proportional to a unit matrix if $\Sigma_{\mu\nu}$ form an irreducible representation of the Lie algebra of the homogeneous Lorentz group.
2. $\Sigma_{\mu\nu}$, l and $\varkappa_\mu$ are finite dimensional; $\varkappa_\mu \neq 0$, but nilpotent.
3. $\Sigma_{\mu\nu}$, l and $\varkappa_\mu$ are infinite dimensional.

A local operator $\Phi(x)$ satisfying the above commutation relations will be said to be covariant with respect to conformal transformations. For the 1. class representations, l will be called the scale dimension of the field; it necessarily coincides with the physical dimension of $\Phi(x)$ only if the theory is scale-invariant.

III. Broken conformal invariance in Lagrangian field theories

It is also instructive to see how conformal symmetry comes about in canonical Lagrangian field theories.

This point has been analyzed for instance by WESS [8], MACK and SALAM [13] and GROSS and WESS [14].

In canonical Lagrangian theories of conformal covariant fields a slightly modified form of Nöether's theorem teaches how to construct the generators of the conformal transformations out of the canonical conjugate variables.

One can verify in this way that the divergence of the dilatation current vanishes, as expected, if and only if there are no dimensioned parameters in the theory.

It has also been proved [13, 14] that in a large class of Lagrangian field theories, including among others almost all renormalizable ones, the divergences of the special conformal currents are proportional to the divergence of the dilatation current, so that scale invariance implies invariance with respect to the entire conformal group. When such a situation is realized, one speaks of minimal breaking of conformal symmetry. A minimal breaking of conformal symmetry in canonical Lagrangian field theories allows for a redefinition, à la Belinfante–Möller, of the energy momentum tensor, as a symmetric tensor, which I shall call $\theta_{\mu\nu}$. In terms of $\theta_{\mu\nu}$ the conformal currents, their divergences and their associated charges assume the following simple form:

$$\theta_{\mu\nu} = \theta_{\nu\mu}\,, \tag{5a}$$

$$M_{\mu\nu\varrho} = x_\nu\,\theta_{\mu\varrho} - x_\varrho\,\theta_{\mu\nu}\,, \tag{5b}$$

$$D_\mu = x^\nu\,\theta_{\mu\nu}\,, \tag{5c}$$

$$K_{\mu\nu} = x_\nu\,D_\mu + x^\varrho\,M_{\mu\nu\varrho}\,, \tag{5d}$$

$$\partial^\mu\,\theta_{\mu\nu} = 0\,, \tag{6a}$$

$$\partial^\mu\,M_{\mu\nu\varrho} = 0\,, \tag{6b}$$

$$\partial^\mu\,D_\mu = \theta^\mu_\mu\,, \tag{6c}$$

$$\partial^\mu\,K_{\mu\nu} = 2x_\nu\,\theta^\mu_\mu\,, \tag{6d}$$

$$P_\mu = \int d^3x\theta_{0\mu}(x)\,, \tag{7a}$$

$$M_{\mu\nu} = \int d^3xM_{0\mu\nu}(x)\,, \tag{7b}$$

$$D = \int d^3xD_0(x)\,, \tag{7c}$$

$$K_\mu = \int d^3xK_{0\mu}(x)\,. \tag{7d}$$

If the symmetry is exact, $\theta_{\mu\nu}$ is traceless and carries pure spin 2.

This result is due to CALLAN, COLEMAN and JACKIW [15] who have also proved that the matrix elements of $\theta_{\mu\nu}$ are less singular than the homologous matrix elements of any other permissible energy-momentum tensor, in every order of perturbation theory.

IV. A non-Lagrangian model exhibiting smoothly broken conformal and U(3) ⊗ U(3) symmetries

After the short digression of the preceding sections let me now leave the limits of Lagrangian field theories. I want to retain, however, the following results which will be some of the defining hypotheses of the model worked out by CICCARIELLO, GATTO, TONIN and myself [2].

I shall assume:

i) the existence of a symmetric divergenceless energy momentum tensor, $\theta_{\mu\nu}$.

ii) The possibility of defining the charges

$$P_\mu,\ M_{\mu\nu},\ D \text{ and } K_\mu$$

in terms of $\theta_{\mu\nu}$ as specified in Eqs. (6a, b, c, d) and (7,a b, c, d).

iii) The existence of a symmetry limit in which the energy momentum tensor is traceless and transforms covariantly with respect to the entire conformal group as a first class tensor with scale dimension –4 and spin 2. In this limit the charges P_μ, $M_{\mu\nu}$, D and K_μ form a Lie algebra which is isomorfic to the Lie algebra of the conformal group.

To further specify the model, we have assumed – according to WILSON's philosophy – the existence of the following linearly independent local operators:

a) The traceless part of the energy-momentum tensor.

b) The 18 currents j^α_μ, where $\alpha = (a, A)$, $a = 0, \ldots, 8$ and A specifies the parity.

The associated charges

$$Q^\alpha = \int d^3x j^\alpha_0(x)$$

are the generators of a chiral U(3)⊗U(3).

c) The scalar and pseudoscalar fields w^A which are SU(3)⊗SU(3) singlets but not U(3)⊗U(3) singlets.

d) The scalar and pseudoscalar fields u^a which transform as tensors of a representation $(3,\bar{3})\otimes(\bar{3},3)$ of SU(3) ⊗ SU(3).

The transformation properties of these operators with respect to P, C, and PCT are the usual ones.

The breaking of the U(3) ⊗ U(3) symmetry is specified by assuming that the operator

$$\theta^{(s)}_{00} \equiv \theta_{00}(x) - \varepsilon^\alpha(\langle u^\alpha - \langle u^\alpha\rangle_0) - \varepsilon^A(w^A(x) - \langle w^A\rangle_0) \tag{8}$$

is a U(3) ⊗ U(3) singlet.

The symmetry breaking parameters ε^α and ε^A are dimensioned constants.

An immediate consequence of this assumption is $PCAC$ in the form:

$$\partial^\mu j^\alpha_\mu = S^{\alpha BC} \varepsilon^B w^C + F^{\alpha\beta\gamma} \varepsilon^\beta u^\gamma, \tag{9}$$

where $S^{\alpha BC}$ and $F^{\alpha\beta\gamma}$ are related to the transformation properties of w^A and u^α with respect to U(3) ⊗ U(3) transformations.

The skeleton theory is obtained in the limit ε^α, $\varepsilon^A \to 0$; it is conformal and U(3) ⊗ U(3)-invariant and the limit of the operators mentioned above transform in it covariantly as tensors of the first class with the following scale dimensions:

$$l_\theta = -4\,; \quad l_j = -3\,; \quad l_w = -\Delta'; \quad l_u = -\Delta\,.$$

Apart from eventual derivatives of such operators or c-numbers, there are no other operators with scale dimensions > -4.

In the true theory the breaking of the conformal symmetry is specified through Eq. (6c):

$$\partial^{\mu} D_{\mu} = \theta^{\mu}_{\mu}$$

and the assumption that θ^{μ}_{μ} is not an independent field. We have proved that in our hypotheses*

$$\theta^{\mu}_{\mu} = (4-\Delta)\, \varepsilon^{\alpha} (u^{\alpha} - \langle u^{\alpha} \rangle_0) + (4-\Delta')\, \varepsilon^{A} (w^{A} - \langle w^{A} \rangle_0). \tag{10}$$

This relation states the partial conservation of the dilatation current. In fact, it can also be derived following the standard procedure used to prove PCAC provided one adds the assumption that the singlet part of $\theta_{\mu\nu}$, $\theta^{(s)}_{00}(x)$ has scale dimension (-4).

In our model this has not been assumed but can be proved to be true.

Let me now justify some of our assumptions and make some additional remarks about their significance.

The use of the group U(3) ⊗ U(3) as a broken symmetry group is not new; it had already been considered by GELL-MANN, who also emphasized that the non-conservation of the axial baryon number is required by the high mass of the η' [16]. Δ and Δ' are not generally entire numbers. WILSON has pointed out that the renormalized fields have not necessarily the same dimensions as the unrenormalized ones; in fact, they do not generally satisfy the same canonical commutation relations. This fact, which has been explicitly checked in the Thirring model by WILSON [17] and by LOWENSTEIN [18], can be considered as a renormalization effect of the dilatation charges Δ and Δ'.

The assumptions Δ, $\Delta' < 4$ express the requirement that the breakings of the internal and of the conformal symmetries occur together and assure that they are due to a superrenormalizable piece of the Hamiltonian.

The condition $\Delta \geq 1$ is an immediate consequence of the semipositivity of the spectral function in the Lehmann representation for $\langle 0 \,|\, T\{u^{\alpha}(x) u^{\alpha}(0)\} \,|\, 0 \rangle$, which requires this object being at least as singular as $1/x^2$ when $x_{\mu} \to 0$.

An analogous reasoning gives $\Delta' \geq 1$.

The number of local fields with low dimensions, allowed in the model, is the minimum consistent with a symmetry scheme based on a broken U(3) ⊗ ⊗ U(3).

The existence of the fields w^{A} (first proposed by GLASHOW [19]) as vehicles of symmetry breaking is required for instance to justify the large mean mass of the ϱ-multiplet and the masses of nucleons, as has been noted by WILSON [6].

* See also [12].

V. Computing equal time commutators

Let me now come to the technical problem of computing the equal time commutators among the local operators of the model.

We have assumed that they are regular in the symmetry limit ε^A, $\varepsilon^\alpha \to 0$, i.e. that the symmetry is smoothly broken, at least as far as the equal time commutators are concerned, Particularly strong restrictions come from this assumption, if taken in conjunction with the hypothesis of the existence of a limited number of operators with low dimensions. All other conditions to be imposed on the equal time commutators of the model come from the assumed internal and space–time symmetries. Let me discuss this point in greater detail, starting from Eq. (2). The eventual tensor properties of A and B with respect to transformations of some group (whether it is a symmetry group or not) fix the tensor properties of the Schwinger terms with respect to transformations of the same group.

Such tensor properties can be analyzed conveniently in terms of infinitesimal transformations. This in turn amounts to requiring the validity of the Jacobi identity at different times:

$$[Q(t_2), [A(\vec{x}, t_1), B(0)]] = [[Q(t_2), A(\vec{x}, t_1)], B(0)] + [A(\vec{x}, t_1), [Q(t_2), B(0)]], \quad (11)$$

where $Q(t_2)$ is a generator of the group. If the symmetry is exact so that Q does not depend on time, it is sufficient to take the limit $t_1 \to 0$ in Eq. (11) in order to get an equal times Jacobi identity. But if the symmetry is broken and Q does depend on time, in order to get an equal times Jacobi identity it is not sufficient to take in Eq. (11) the two limits $t_2 \to 0$, $t_1 \to 0$, when such limits cannot be interchanged. Thus, in general, one must expect that the equal times Jacobi identity, among two covariant local operators and a non-conserved charge, is violated by terms which are proportional to the breaking parameters.

It may also occur that the equal times Jacobi identity is satisfied in the skeleton theory, where the symmetry is exact, but not in the true theory, because in the latter one or both of the operators A and B loose their exact covariance. In this case too, corrections proportional to the breaking parameters must be expected.

In any case, however, the violating terms are not completely arbitrary, but can be determined apart from a few parameters by a spurion analysis.

So, practically, in our model we must impose the following set of conditions:

A) Conditions which come from the discrete symmetries P, C and PCT are almost obvious and will not be discussed.

B) Poincaré covariance. Considerable technical advantages are obtained by introducing a positive time-like vector n_μ, which acts as a spurion of the Lorentz group, and by substituting to the equal time commutators, the commutators $\delta(nx)\,[A(x), B(0)]$ calculated on the space-like hyperplane of equation $n \cdot x = 0$. In this way one obtains formal covariance by sight. $S^{\tau_1 \cdots \tau_k}(A, B; 0)$ becomes a function of n_μ and its n_μ-dependence can be easily analyzed through differential methods. The validity of the equal times Jacobi identities involving one generator of the inhomogeneous Lorentz group allows to determine completely the Schwinger terms of the equal time commutator between $A(x)$ and $B(0)$.

These conditions must clearly be satisfied by the ETC's of any acceptable theory, and in our model they turn out to be essential in checking or imposing consistency between our commutators and conservation or partial conservation properties of the local operators involved.

C) D and K_μ covariance must be imposed only within the skeleton theory; in the true theory they are violated by terms proportional to the symmetry breaking parameters. The conditions which come from dilatation covariance have already been discussed.

K_μ covariance gives rise to complicated relations among the Schwinger terms of the equal time commutator between A and B. Their most striking effects can be roughly resumed in the following statement: the local operator $\partial_{\mu_1} \ldots \partial_{\mu_j} C(0)$ $(j = 1, 2, \ldots)$ contributes in the k-order Schwinger term of $[A(x), B(0)]_{\text{e.t.}}$ if $C(0)$ contributes in the $(k + j)$-order Schwinger term of the same commutator [2].

D) Further conditions must be obeyed by the equal time commutators involving the (0μ) components of the energy-momentum tensor. These conditions come from the covariance of the operators of the skeleton theory with respect to transformations of the conformal group and from the particular form that has been postulated for the generators in terms of $\theta_{\mu\nu}$.

Restrictions of this kind concern Schwinger terms up to the second order [2]. Those coming from dilatations and special conformal transformations may be violated by terms proportional to the symmetry breaking parameters.

E) Covariance with respect to U(3) $\otimes$ U(3) transformations implies obviously that all the Schwinger terms in the equal time commutator between A and B can get contributions only from operators which transform according to a representation contained in the Kronecker product of the representations according to which A and B transform. This statement must be taken in a strict sense only within the skeleton theory: in the true theory it must be substituted by the results of a spurion analysis.

In this way, by means of simple group theoretical considerations, we have been able to compute in our model, in terms of a few numerical parameters, all the equal time commutators among the local operators w^A, u^α, j_μ^α and $\theta_{\mu\nu}$ and the equal time commutators between j_μ^α and $\partial_\varrho j_\nu^\beta$. The only exceptions are the equal time commutators involving only space components of the energy-momentum tensor.

We have thus obtained a realization of GELL-MANN's program of extending current algebra to include the energy-momentum tensor.

Some of the commutators we have calculated are reported, in truncated form, in Tables I, II, III and IV. In the following section I shall discuss only some of their most striking aspects and important applications.

Table I

$$[\theta_{00}(\vec{x},0),\ \theta_{00}(0)]_T = -i\partial_0\,\theta_{00}\,\delta^{(3)}(x) + 2i\theta_{0j}\,\partial_j\,\delta^{(3)}(x)$$

$$[\theta_{00}(\vec{x},0),\ \theta_{0l}(0)]_T = -i\partial_0\,\theta_{0l}\,\delta^{(3)}(x) + i(\theta_{jl}\,\partial_j + \theta_{00}\,\partial_l)\,\delta^{(3)}(x)$$

$$[\theta_{00}(\vec{x},0),\ \theta_{kl}(0)]_T = -i\partial_0\,\theta_{kl}\,\delta^{(3)}(x) + i(\theta_{0l}\,\partial_k + \theta_{0k}\,\partial_l)\,\delta^{(3)}(x)$$

$$[\theta_{0k}(\vec{x},0),\ \theta_{0l}(0)]_T = -i\partial_k\,\theta_{0l}\,\delta^{(3)}(x) + i(\theta_{0l}\,\partial_k + \theta_{0k}\,\partial_l)\,\delta^{(3)}(x)$$

$$\begin{aligned}[\theta_{0k}(\vec{x},0),\ \theta_{lm}(0)]_T = {} & -i\partial_k\,\theta_{lm}\,\delta^{(3)}(x) + \\ & + i\left\{\frac{4}{3}\delta_{nk}\,\theta_{lm} + \frac{1}{3}\delta_{lm}\,\theta_{nk} + \frac{1}{2}[\delta_{nl}\,\theta_{km} + \delta_{nm}\,\theta_{kl} - \right.\\ & - \delta_{kl}\,\theta_{nm} - \delta_{km}\,\theta_{nl}] - \frac{1}{9}\delta_{nk}\,\delta_{lm}[\theta_{00} - (4-\Delta')^2\,\varepsilon^B\,w^B - \\ & - (4-\Delta)^2\,\varepsilon^\beta\,u^\beta] - p_{nk\,,lm}^{n'k',l'm'}\,[k_1^{(1)}\,\varepsilon_{n'l'}\,\theta_{k'm'} + \\ & \left. + k_2^{(1)}\,\delta_{n'l'}\,\delta_{k'm'}\,\theta_{jj} - \delta_{n'l'}\,\delta_{k'm'}(k_3^{(1)}\,\varepsilon^\beta\,u^\beta + k_4^{(1)}\,\varepsilon^B\,w^B)]\right\}\partial_n\,\delta^{(3)}(x)\end{aligned}$$

$$p_{nk\,,lm}^{n'k',l'm'} \equiv \frac{1}{2}(p_{nk}^{n'k'}\,p_{lm}^{l'm'} + p_{nk}^{l'm'}\,p_{lm}^{n'k'})$$

$$p_{nk}^{n'k'} \equiv \frac{1}{2}(\delta_{nn'}\,\delta_{kk'} + \delta_{nk'}\,\delta_{kn'} - \frac{2}{3}\delta_{nk}\,\delta_{n'k'})$$

$k_1^{(1)}, k_2^{(1)}, k_3^{(1)}$ and $k_4^{(1)}$ are real numbers.

In the r.h.s. 's all local operators are evaluated at $x = 0$.

Table II

$$[j_0^\alpha(\vec{x},0),\ \theta_{00}(0)]_T = i\partial^\mu\,j_\mu^\alpha\,\delta^{(3)}(x) + ij_r^\alpha\,\partial_r\,\delta^{(3)}(x)$$

$$[j_0^\alpha(\vec{x},0),\ \theta_{0m}(0)]_T = ij_0^\alpha\,\partial_m\,\delta^{(3)}(x)$$

$$\begin{aligned}[j_0^\alpha(\vec{x},0),\theta_{mn}(0)]_T = {} & -i\delta_{mn}\left[\frac{3-\Delta}{3}\,\partial^\mu\,j_\mu^\alpha + \frac{\Delta+\Delta'}{3}\,\varepsilon^B\,S^{\alpha BC}\,w^C\right]\delta^{(3)}(x) + \\ & + i\left[\frac{1}{2}j_m^\alpha\,\partial_n + j_n^\alpha\,\partial_m) - 2k_1\,p_{mn}^{m'n'}\,j_{m'}^\alpha\,\partial_{n'}\right]\delta^{(3)}(x)\end{aligned}$$

$$[j_k^\alpha(\vec{x},0),\ \theta_{00}(0)]_T = i[\partial_0\,j_k^\alpha - \partial_k\,j_0^\alpha]\,\delta^{(3)}(x) + ij_0^\alpha\,\partial_k\,\delta^{(3)}(x)$$

$$\begin{aligned}[j_k^\alpha(\vec{x},0),\ \theta_{0m}(0)]_T = {} & i\left[\left(k_1 - \frac{1}{2}\right)\partial_k\,j_m^\alpha - \frac{2}{3}\,k_1\,\partial_m\,j_k^\alpha + \left(k_1 + \frac{1}{2}\right)\delta_{km}\,\partial_r\,j_r^\alpha\right]\delta^{(3)}(x) + \\ & + i\left[\left(1 + \frac{2}{3}k_1\right)j_k^\alpha\,\partial_m + \left(\frac{1}{2} - k_1\right)j_m^\alpha\,\partial_k - \left(\frac{1}{2} + k_1\right)\delta_{mk}\,j_r^\alpha\,\partial_r\right]\delta^{(3)}(x)\end{aligned}$$

$$[j^{\alpha}(\vec{x},0),\theta_{mn}(0)]_T = i\left\{\left(k_1+\frac{1}{2}\right)p_{mn}{}^{ll}\delta_{kr}(\partial_s j_0^{\alpha} - \partial_0 j_s^{\alpha}) + \frac{1}{3}\delta_{mn})\pm(\partial_0 j_k^{\alpha}) + (\partial_k j_0^{\alpha})\right\}\delta^{(3)}(x) +$$

$$+ i\left\{j_0^{\alpha}\left[\frac{1}{2}(\delta_{km}\partial_n + \delta_{kn}\partial_m) + k_1(\delta_{km}\partial_n) + (\delta_{kn}\partial_m - \frac{2}{3}\delta_{mn}\partial_k)\right]\right\}\delta^{(3)}(x)$$

k_1 is a real number.

$$S^{\alpha BC} = 3s\sqrt{\frac{2}{3}}\,\delta^{a0}\,\delta^{A\mathcal{A}}\,(\delta^{B\mathcal{A}}\,\delta^{C\mathcal{V}} - \delta^{B\mathcal{V}}\delta^{C\mathcal{A}})$$

s is an integer $\neq 0$.

In the r.h.s. 's all local operators are evaluated at $x = 0$.
Only truncated commutators are reported.

Table III

$$[j_0^{\bar\alpha}(\vec{x},0), j_0^{\bar\beta}(0)]_T = iC^{\bar\alpha\bar\beta\bar\gamma}\,j_0^{\bar\gamma}\,\delta^{(3)}(x)$$

$$[j_0^{\bar\alpha}(\vec{x},0), j_k^{\bar\beta}(0)]_T = iC^{\bar\alpha\bar\beta\bar\gamma}\,j_k^{\bar\gamma}\,\delta^{(3)}(x)$$

$$[j_l^{\bar\alpha}(\vec{x},0), j_k^{\bar\beta}(0)]_T = -i\left\{b_1\,C^{\bar\alpha\bar\beta\bar\gamma}\,\delta_{kl}\,j_0^{\bar\gamma} + b_2\,D^{\bar\alpha\bar\beta\bar\gamma}\,\varepsilon_{klm}\,j_m^{\bar\gamma} + E^{\bar\alpha\bar\beta\bar C}\,\varepsilon_{klm}\,j_m^{(0,C)}\right\}\delta^{(3)}(x)$$

$$[j_0^{\bar\alpha}(\vec{x},0), u^{\beta}(0)]_T = i\,F^{\bar\alpha\beta\gamma}\,u^{\gamma}\,\delta^{(3)}(x)$$

$$[j_k^{\bar\alpha}(\vec{x},0), u^{\beta}(0)]_T = 0$$

Notations: $\alpha = (a, A)$: $a = 0, 1, 2, \ldots 8$

$A = \mathcal{V}$ (vector or scalar) or $\mathcal{A}$ (axial or pseudoscalar).

$\bar\alpha = (\bar a, A)$: $\bar a = 1, 2, \ldots, 8$

Summation over repeated indices is always understood.

$C^{\alpha\beta\gamma} \equiv f^{abc}\,\psi^{ABC}$; $C^{\bar\alpha\bar\beta\bar\gamma}\,f^{\bar a\bar b\bar c}\,\psi^{ABC}$

f^{abc} structure constants of U(3)

$f^{\bar a\bar b\bar c}$ structure constants of SU(3)

$\psi^{ABC} = \begin{cases} 0 \text{ if the number of "axial" indices } \mathcal{A} \text{ is odd} \\ 1 \text{ if the number of "axial" indices } \mathcal{A} \text{ is even} \end{cases}$

$D^{\alpha\beta\gamma} \equiv d^{abc}\,\bar\psi^{ABC}$; $D^{\bar\alpha\bar\beta\bar\gamma} = d^{\bar a\bar b\bar\gamma}\,\bar\psi^{ABC}$

d^{abc} are defined according to Gell-Mann's proposal:

$$\{\lambda^a ; \lambda^b\} = 2d^{abc}\,\lambda^c$$

$\bar\psi^{ABC} = \begin{cases} 0 \text{ if the number of "axial" indices } \mathcal{A} \text{ is even} \\ 1 \text{ if the number of "axial" indices } \mathcal{A} \text{ is odd} \end{cases}$

$$E^{\bar\alpha\bar\beta\bar c} \equiv \left(\sqrt{\frac{2}{3}}\,b_2 + b_3\right)\delta^{\bar\alpha\bar\beta}\,\delta^{CA} + \left(\sqrt{\frac{2}{3}}\,b_2 + b_4\right)\delta^{\bar a\bar b}\,(\delta^{A\mathcal{V}}\,\delta^{B\mathcal{A}} + \delta^{A\mathcal{A}}\,\delta^{B\mathcal{V}})\,\delta^{C\mathcal{V}}.$$

$$F^{\bar\alpha\beta\gamma} = f^{\bar abc}\,\psi^{ABC}\,\delta^{A\mathcal{V}} + d^{\bar abc}\,\delta^{A\mathcal{A}}(\delta^{B\mathcal{A}}\,\delta^{C\mathcal{V}} - \delta^{B\mathcal{V}}\,\delta^{C\mathcal{A}}).$$

b_1, b_2, b_3, b_4, are real numbers.

Only truncated commutators are reported.

Table IV

$$[j_k^{\alpha}(\vec{x}\ 0), \partial_0 j_l^{\bar{\beta}}(0)]_T = i \Big\{\delta^{\bar{\alpha}\bar{\beta}}\, c_1\, \delta_{kl}\, \theta_{jj} + c_2\, \theta_{kl}] +$$

$$+ G^{\{\bar{\alpha}\bar{\beta}\}\gamma\delta}\, \delta_{kl}\, \varepsilon^{\gamma} u^{\delta} + G^{\{\bar{\alpha}\bar{\beta}\}CD}\, \delta_{kl}\, \varepsilon^{C} w^{D} + \frac{1}{2} C^{\bar{\alpha}\bar{\beta}\bar{\gamma}}[(1 + b_1)\cdot$$

$$\cdot(\partial_k j_l^{\gamma} - \partial_l j_k^{\bar{\gamma}}) + \partial_k j_l^{\bar{\gamma}} + \partial_l j_k^{\bar{\gamma}} - b_1\, \delta_{kl}\, \partial^{\mu} j_{\mu}^{\bar{\gamma}}] -$$

$$- \frac{b_2}{2} D^{\bar{\alpha}\bar{\beta}\bar{\gamma}}\, \varepsilon_{klm}(\partial_0 j_m^{\bar{\gamma}} - \partial_m j_0^{\bar{\gamma}}) - \frac{E^{\bar{\alpha}\bar{\beta}C}}{2}\, \varepsilon_{klm}(\partial_0 j_m^{(0,C)} - \partial_m j^{(0,C)}\Big\}$$

$$\delta^{(3)}(x) - i\{C^{\bar{\alpha}\bar{\beta}\bar{\gamma}}\, [\delta_{lm} j_k^{\bar{\gamma}} + \delta_{km} j_l^{\bar{\gamma}} + b_1\, \delta_{kl}\, j_m^{\bar{\gamma}}] +$$

$$+ b_2\, D^{\bar{\alpha}\bar{\beta}\bar{\gamma}}\, \varepsilon_{klm}\, j_0^{\bar{\gamma}} + E^{\bar{\alpha}\bar{\beta}C}\, \varepsilon_{klm}\, j_0^{(0,C)}\}\, \partial_m\, \delta^{(3)}(x)$$

$$[j_0^{\bar{\alpha}}(\vec{x}, 0), \partial_0 j_k^{\bar{\beta}}(0)]_T = iC^{\bar{\alpha}\bar{\beta}\bar{\gamma}}\, \partial_0 j_k^{\bar{\gamma}}\, \delta^{(3)}(x) + i\,\{b_1\, C^{\bar{\alpha}\bar{\beta}\bar{\gamma}}\, \delta_{kl}\, j_0^{\bar{\gamma}} + b_2\, D^{\bar{\alpha}\bar{\beta}\bar{\gamma}}\, \varepsilon_{klm}\, j_m^{\bar{\gamma}} +$$

$$+ E^{\bar{\alpha}\bar{\beta}C}\, \varepsilon_{klm}\, j_m^{(0,C)}\}\partial_l\, \delta^{(3)}(x)$$

c_1 and c_2 are real numbers.

$G^{\{\bar{\alpha}\bar{\beta}\}\gamma\delta}$ and $G^{\{\bar{\alpha}\bar{\beta}\}CD}$ are sets of real constants.

In the r.h.s. 's all local operators are evaluated at $x = 0$.
Only truncated commutators are reported.

VI. Discussion of the equal time commutators

A) The equal time commutators $[\theta_{\mu\nu}, \theta_{\varrho\sigma}]$ (see Table I), at least for the moment, are interesting only from a theoretical point of view. In connection with them we have rederived as a particular case, SCHWINGER's [23] theorem, and checked previous model-independent results by BOULAWARE and DESER [24].

B) The equal time commutators between the components of a SU(3) current and those of the energy-momentum tensor are reported in Table II. From the first three rows of the Table, one realizes that the U(3) ⊗ U(3) singlet part of $\theta_{\mu\nu}$ is a non-covariant operator which differs from $\theta_{\mu\nu}$ only for the addition of spin 0 fields:

$$\theta_{\mu\nu}^{(s)} = \theta_{\mu\nu} - g_{\mu\nu}[\varepsilon^{\alpha}(u^{\alpha} - \langle u^{\alpha}\rangle_0) + \varepsilon^{A}(w^{A} - \langle w^{A}\rangle_0)] - (-g_{\mu\nu} + g_{\mu 0}\, g_{\nu 0})\cdot$$
$$\cdot\left[\frac{\Delta}{3}\, \varepsilon^{\alpha}(u^{\alpha} - \langle u^{\alpha}\rangle_0) + \frac{\Delta'}{3}\, \varepsilon^{A}(w^{A} - \langle w^{A}\rangle_0)\right]. \tag{12}$$

As already noted, the most interesting commutators, because of their connection to experimentally measurable quantities, are the commutators among currents and currents derivatives. They determine [25–32]:

a) The divergent part of e.m. self masses and the leading divergences of weak self masses;

b) The asymptotic behaviours of e.m. and weak amplitudes, and a number of electro- and neutrino-production sum rules, some of which I shall mention. To this end I have recalled in Appendix A some standard notations and results relevant to the problem of inelastic lepton scattering from unpolarized targets.

c) For the ETC's $[j^\alpha_\mu j^\beta_\nu]$ which are reported in Table III, we have found an expression which is more general than the one provided by the quark model. In the $P \to \infty$ limit the ETC's between two SU(3) $\otimes$ SU(3) currents determine many sum rules, of which I wish to recall the DASHEN–FUBINI–GELL-MANN sum rule [33–35], the backward BJORKEN asymptotic sum rule [36] and the GROSS and LLEWELLYN-SMITH sum rule [30].

Recently JACKIW et al. [31] and CORNWALL et al. [32] have proved that the integral

$$\int_0^2 \frac{F_l(\omega)}{\omega} d\omega ,$$

provided it converges, is proportional to the q-number first order Schwinger term in the ETC between $j_k^{\text{e.m.}}$ and $j_k^{\text{e.m.}}$. In our model we find no operator Schwinger terms in this ETC, so the integral diverges or vanishes. In the second case the semipositivity of $F_l(\omega)$ implies that the integrand is zero.

D) The equal time commutators $[j^\alpha_\mu, \partial_0 j^\beta_\nu]$ are reported in Table IV. Their forward matrix elements determine in the $P \to \infty$ limit the asymptotic sum rules related to electro- and neutrino-production, which are expressed in terms of integrals over the functions $F_i(\omega)$ and $\bar{F}_i(\omega)$ defined in Eqs (A.3) and (A.7). Only terms which do not contain derivatives contribute to the integral over d^3x of these forward matrix elements. Therefore, as is seen from Table IV, only the matrix elements of the energy momentum tensor contribute to the sum rules. The other non-derivative terms are in fact matrix elements of spin 0 fields, and vanish when $P_z \to \infty$. So the asymptotic sum rules for electro- and neutrino-production on nucleons can be expressed in terms of only the two parameters c_1 and c_2, which multiply the components of $\theta_{\mu\nu}$ appearing in the expression of the equal time commutator (see Table IV), and their r.h.s.'s turn out to be proportional to the masses of the targets. MACK [37] has pointed out that this result holds whichever is the target, owing to the universality of the forward matrix elements of the energy-momentum tensor.

The CALLAN–GROSS sum rules [28], when evaluated in our model, give [1, 2]:

$$I_t = \int_0^2 d\omega \omega F_t(\omega) = -2/3\, c_1 \left\{= 2/3\, c_2\right\}^*, \tag{13a}$$

$$I_l = \int_0^2 d\omega \omega F_l(\omega) = 2/3(c_1 + c_2) \left\{= 0\right\}^*. \tag{13b}$$

The results marked with a star hold only if the sum rule of JACKIW et al. [31] converges. The same convention will be used in the sequel too. Comparing with the experimental data obtained at SLAC [38] for $F_l(\omega)$ we find:

$$\{-c_1 = 0.54 \pm 0.06\}^* \tag{14}$$

The neutrino production sum rules are similarly determined in terms of the two parameters c_1 and c_2 [1, 2]:

$$\bar{I}_t = \int_0^2 d\omega\omega\{\bar{F}_t^\nu(\omega) + \bar{F}_t^{\bar{\nu}}(\omega)\} = -4c_1 \ \{= 2.16 \pm 0.24\},^* \tag{15a}$$

$$\bar{I}_l = \int_0^2 d\omega\omega\{\bar{F}_l^\nu(\omega) + \bar{F}_l^{\bar{\nu}}(\omega)\} = -4(c_1 + c_2) \ \{= 0\},^* \tag{15b}$$

$$\bar{I}_3 = \int_0^2 d\omega\omega\{\bar{F}_3^\nu(\omega) + \bar{F}_3^{\bar{\nu}}(\omega)\} = 0, \tag{15c}$$

where the label $\nu(\bar{\nu})$ refers to neutrino (anti-neutrino) processes.

Noting that when $P_z \to \infty$, the forward matrix elements of $\theta_{\mu\nu}$ coincide with those of its singlet part $\theta_{\mu\nu}^{(s)}$, defined in Eq. (12), it is possible to calculate the ratio of the total neutrino + antineutrino cross-sections into $S = 1$ and $S = 0$ states [1, 2]:

$$\frac{\sigma_{\text{tot}}(\Delta S = 1)}{\sigma_{\text{tot}}(\Delta S = 0)} = \tan^2\theta\,, \tag{16}$$

where θ is the Cabibbo angle.

BJORKEN has shown [36] that when E is much greater than M, but not so large as to make the cut offs due to unitarity, or intermediate boson exchange, operative

$$\sigma_{\text{tot}}^{\nu p} + \sigma_{\text{tot}}^{\bar{\nu} p} \to \frac{G^2 ME}{\pi}\frac{i}{4}\left(\dot{J}_{zz} - \frac{4}{3}\dot{J}_{xx} - \frac{2}{3}i\dot{J}_{xy}\right), \tag{17a}$$

$$\sigma_{\text{tot}}^{\nu n} + \sigma_{\text{tot}}^{\bar{\nu} n} \to \frac{G^2 ME}{\pi}\frac{i}{4}\left(\dot{J}_{zz} - \frac{4}{3}\dot{J}_{xx} + \frac{2}{3}i\dot{J}_{xy}\right), \tag{17b}$$

where the definition of Eq. (A.9) has been used. In our model the r.h.s.'s of Eqs. (17a and b) coincide and can be computed in terms of c_1 and c_2. The result is [2]

$$\frac{3c_2 - c_1}{3}\,\frac{G^2 ME}{\pi}\left\{=(0.7 \pm 0.1)\frac{G^2 ME}{\pi}\right\}^*. \tag{18}$$

If one neglects $S = 1$ transitions, $\sigma^{\nu n} = \sigma^{\bar{\nu} p}$ is obtained as the result of a simple isospin rotation and the experimental result [39]

$$\sigma_{tot}^{\nu p}+\sigma_{tot}^{\nu n}=\frac{G^2 ME}{\pi}(1.02\pm 0.3) \tag{19}$$

can be compared with the starred result of Eq. (18). The agreement is not fantastic, but within the experimental errors. I wish to recall, however, that the assumption of convergence of the integral

$$\int_0^2 d\omega \frac{F_l(\omega)}{\omega}$$

is essential to the prediction of Eq. (18).

VII. Additional remarks and conclusion

At the recent Conference of Kiev [40] and at the beginning of this Symposium, [41] some new data from SLAC have been reported which seem to be in disagreement with the universality of $J_{\mu\nu}$ (defined in Eq. (A.9)) predicted by our model. In fact, the function $\nu W_2^{\text{neutron}}(\omega, q^2)$ has been found sensibly different from $\nu W_2^{\text{proton}}(\omega, q^2)$. So, before concluding, I think some comments are in order about such a possible discrepancy. Various possibilities exist:

1. One may be so optimistic as to hope that the function $\nu W_2^{\text{neutron}}(\omega, \nu)$ which for large values $(1/16\leq \omega \leq 2)$ of ω has been found $< \nu W_2{}^{\text{proton}}(\omega, \nu)$, at small values of ω becomes so large that the integrals $I_{t(l)}^{\text{neutron}}$ and $I_{t(l)}^{\text{proton}}$ defined in Eqs. (13a, b) coincide.

2. One may prefer to modify the model, for instance, by admitting the existence of a set of 16 operators $\theta^{\alpha}_{\mu\nu}$, which carry spin 2 and dimensions —4, and transform as tensors of a representation (8, 1) + (1, 8) of SU(3) $\otimes$ SU(3). Such operators can contribute in the equal time commutator $[\dot{j}^{\alpha}_{\mu}, \partial_{\varrho}\dot{j}^{\beta}_{\nu}]$ and their contributions which depend on three new parameters, are competitive with those of $\theta_{\mu\nu}$.

If the sum rule of Jackiw et al. converges, one of the parameters can be determined; besides, the ratio F/D for the coupling of $\theta^{\alpha}_{\mu\nu}$ to the nucleons can be computed from independent data, so beside c_1 two new parameters enter the asymptotic sum rules for electro- and neutrino-production on protons and neutrons, and the available experimental data are sufficient only to determine them. No testable prediction can be made in this case.

It must also be noted that in this way one gives up one of the most appealing features of the model, consisting in a sort of bootstrap which realizes in the equal time commutators among the (minimum number of) operators which are required to generate and to break the symmetries.

3. In a recent paper [43] NAUENBERG has shown, through a phenomenological fit, that the experimental data for the proton are not yet asymptotic enough as to assure that $\nu W_2^{\text{proton}}(\omega, \nu)$ really scales.

If credit is given to such an interpretation of experiments, the data for the neutron too must be considered only sub-asymptotic, and one need not modify the model to justify them. The situation is illustrated by the following example. Suppose

$$\nu W_2(\omega, \nu) = F_2(\omega) + \left(\frac{M^2}{\nu}\right)^{\eta/2} F_2'(\omega) . \tag{20}$$

If $\eta > 0$ is sufficiently small and $F_2'(\omega)$ depends on isotopic spin, the contribution of the second term in the r.h.s. of Eq. (20) can account for the differences between the neutron and proton data, but cannot be distinguished from the contribution of an exactly scaling term.

In our model such contribution could be explained for instance by simply assuming that the operators $\theta^{\alpha}_{\mu\nu}$ exist, but with dimensions —$(4 + \eta)$, so they would contribute in WILSON's [44] expansion for $[j^{\alpha}_{\mu}(x), \partial_{\varrho} j^{\beta}_{\nu}(0)]$, but not at equal times. While waiting for more precise or more asymptotic experiments, this position is perhaps the most appealing one to adopt from our point of view.

In any case, whichever is the interpretation of the experimental data, the philosophy which is behind the model keeps its validity.

To conclude I would stress that models of the kind proposed here, based on WILSON's ideas, appear to provide a more general frame than canonical models, allowing inclusion of limiting cases of the canonical formalism. In this sense, such models are related for instance to the discussion of possible limiting cases of field algebra [45–48], where ambiguous products $j_{\mu}(x) j_{\nu}(x)$ of operators, taken at the same point, appear in the equal time commutators. Such ambiguities are here evaded essentially through the assumption of the existence of a small number of low-dimension operators which, after fixing a broken symmetry scheme, for want of something better, have been selected according to a principle of economy.

Appendix

In this Appendix some standard notations and results relevant to the problem of inelastic lepton scattering from unpolarized targets are recalled.

In the laboratory frame E and E' are the energies of the incident and scattered lepton, P_{μ} is the momentum of the target and M its mass, θ is the scattering angle of the lepton; $q^2 = -4EE' \cdot \sin^2 \theta/2$ and $\nu = q \cdot P = M(E - E')$ are the squared momentum transfer and the energy transfer to the leptons, respectively. The structure functions for electroproduction are defined from

$$\frac{1}{M^2}\left(P_\mu-\frac{(P\cdot q)\,q_\mu}{q^2}\right)\left(P_\nu-\frac{(P\cdot q)\,q_\nu}{q^2}\right)W_2(q^2,\nu)-\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)\times$$
$$\times\, W_1(q^2,\nu)=\frac{P_0}{M}\int\frac{d^4x}{2\pi}\,e^{iqx}\,\langle P|[j_\mu^{\text{e.m.}}(x),J_\nu^{\text{e.m.}}(0)]|P\rangle\,. \tag{A.1}$$

The inelastic differential cross section in terms of W_1 and W_2 is

$$\frac{d\sigma}{d\Omega dE'}=\frac{\alpha^2}{4E^2\sin^4\theta/2}\,[W_2(q^2,\nu)\cos^2\theta/2+2W_1(q^2;\nu)\sin^2\theta/2]; \tag{A.2}$$

Following BJORKEN [26] I shall also define:

$$\omega=-q^2/\nu$$
$$F_1(\omega)=\lim_{\substack{\nu\to\infty\\ \omega\text{ fixed}}} MW_1(q^2;\nu)\,, \qquad \text{A.3a)}$$

$$F_1(\omega)=\lim_{\substack{\nu\to\infty\\ \omega\text{ fixed}}}\ \nu/M\ \ W_2(q^2;\nu), \tag{A.3b}$$

$$F_t(\omega)=F_1(\omega), \tag{A.4a}$$

$$F_l(\omega)=\frac{F_2(w)}{\omega}-F_1(\omega)\,. \tag{A.4b}$$

The structure functions for neutrino production are defined from

$$\frac{P_0}{M}\int\frac{d^4x}{2\pi}\,e^{iq\cdot x}\langle P|[j_\mu(x),j_\nu^+(0)]|P\rangle=\frac{P_\mu P_\nu}{M^2}\,\overline{W}_2(q^2;\nu)-$$
$$-\,g_{\mu\nu}\,\overline{W}_1(q^2;\nu)-\frac{i}{2M}\,\varepsilon_{\mu\nu}{}^{\alpha\beta}P_\alpha q_\beta\overline{W}_3(q^2;\nu)+\ldots \tag{A.5}$$

and the cross section is:

$$\frac{\pi}{EE'}\frac{d\sigma}{d\Omega dE'}=\frac{E'}{E}\frac{G^2}{2\pi}\left[\overline{W}_2(q^2;\nu)\cos^2\theta/2+\right.$$
$$\left.+\,2\overline{W}_1(q^2;\nu)\sin^2\theta/2+\frac{E+E'}{M}\cdot\ {}_3(q^2;\nu)\sin^2\theta/2\right]. \tag{A.6}$$

For neutrino-production one defines:

$$\bar{F}_1(\omega) = \lim_{\substack{\nu\to\infty \\ \omega \text{ fixed}}} M\bar{W}_1(q^2;\nu)\,. \tag{A.7a}$$

$$\bar{F}_2(\omega) = \lim_{\substack{\nu\to\infty \\ \omega \text{ fixed}}} \nu/M\, \bar{W}_2(q^2;\nu)\,, \tag{A.7b}$$

$$\bar{F}_3(\omega) = \lim_{\substack{\nu\to\infty \\ \omega \text{ fixed}}} \nu/M\, \bar{W}_3(q^2;\nu)\,, \tag{A.7c}$$

$$J_{\mu\nu} \equiv \lim_{P_z\to\infty} \int d^3x \,\langle P_z|[J_\mu(\vec{x},0), J_\nu^+(0)]|P_z\rangle\,, \tag{A.8}$$

$$\dot{J}_{\mu\nu} = \lim_{P_z\to\infty} \int \frac{d^3x}{P_0} \left\langle P_z \left\| \left[\frac{d}{dt} J_\mu(\vec{x},0),\, J_\nu^+(0)\right] \right\| P_z \right\rangle. \tag{A.9}$$

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НАРУШЕННАЯ КАЛИБРОВОЧНАЯ ИНВАРИАНТНОСТЬ ПРИ НЕУПРУГОМ РАССЕЯНИИ ЛЕПТОНОВ НА НУКЛОНАХ

Г. САРТОРИ

Резюме

С помощью изучения коммутаторов тока вблизи светового конуса рассматривается нарушенная калибровочная инвариантность при неупругом рассеянии лептонов на нуклонах. Предложены модели, основанные на идеях Вильсона, и обеспечивающие более общее описание, чем канонические модели.