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LEE MODEL AS A Z LIMIT*

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Limits are investigated in a generalized Lee model without cut off which reproduce the elementary + composite particle case found in the original model.

1. One of the most interesting problems in particle physics is to find some answer to the question as to which particles among the vast number of stable or instable particles are elementary and which are composed somehow from others. Different lines of attack offer certain types of criteria which might be especially useful in bootstrap calculations. In a Lagrangian field theory e.g. $Z \to 0$ limits are assumed to turn an elementary particle into a composite one. Detailed references concerning these problems can be found in [1].

Former investigations of the Lee model without cut-off (i.e. when a ghost is present) have shown that there the typical elementary + composite particle case arises [2], with the consequence that one of the particles is elementary and the other composite, but one cannot say which belongs to either type.

Here we deal with the question of the kind of limit which produces this situation. The model which we have chosen is the generalized model with two V-particles [3], which with a cut-off, has been investigated already from the point of view of a Z limit [4], where, of course, it gives a different result.

In the Lee model all information arises [5] from a function $h(z)$ having the form

$$h(z) = a + bz + z^2 G(z)$$

a, b finite

$$G(z) = \int \frac{k^2\, dk}{2\omega^3(\omega - z)} \, .$$

Thus, reproducing the results of the Lee model means just reproducing the above form for the corresponding function of the generalized theory.

* Dedicated to Prof. P. GOMBÁS on his 60th birthday.

2. The generalized model we are discussing is described by the Hamiltonian

$$
\begin{aligned}
H &= H_0 + H_1 \\
H_0 &= \sum_{i=1}^{2} m_i\, \delta_i\, \psi_{v_i}^{*}\, \psi_{v_i} + \int \omega(k)\, a^{*}(k)\, a(k)\, \mathrm{d}k\,, \\
H_1 &= -\sum_{i=1}^{2} \frac{g_i}{\sqrt{4\pi}} \int \frac{1}{\sqrt{2\omega}} \left[\psi_N^{*}\, \psi_{v_i}\, a^{*}(k) + \delta_i\, \psi_{v_i}^{*}\, \psi_N\, a(k)\right] \mathrm{d}k\,, \\
\omega &= \sqrt{m_0^2 + k^2}.
\end{aligned}
\tag{1}
$$

We have taken two V-particles, and fixed the energy scale by setting $m_N = 0$. The commutation relations are the conventional ones except for the V-particles, for which

$$\{\psi_{v_i}, \psi_{v_i}^{*}\} = \delta_i \tag{2}$$

is prescribed. $\delta_i = +1$ for a normal particle, $\delta_i = -1$ for a particle quantized with indefinite metric. m_i and g_i are the bare masses and coupling constants. We have taken the convenient form (1) in order to retain the maximal possible symmetries in the following equations [5]. H is Hermitian if g_i is real with $\delta_i = +1$, and pure imaginary for $\delta_i = -1$. Now we wish to solve the problem of the N—Θ sector. Here a general state possesses the form

$$
\begin{aligned}
|E> &= \left(\Sigma \alpha_i\, \delta_i\, \psi_{v_i}^{*} + \psi_N^{*} \int \varphi(k)\, a^{*}(k)\, \mathrm{d}k\right)|0>, \\
\psi_{v_i}|0> &= \psi_N|0> = a(k)\,|0> = 0\,, \qquad <0|0> = 1\,,
\end{aligned}
\tag{3}
$$

with the norm

$$<E|E> = \Sigma\, \delta_i\, \alpha_i^{*}\, \alpha_i + \int \varphi^{*}(k)\, \varphi(k)\, \mathrm{d}k. \tag{4}$$

The eigenvalue equation

$$H|E> = E|E>$$

gives

$$
\begin{aligned}
(m_i - E)\, \alpha_i &= \frac{g_i}{\sqrt{4\pi}} \int \frac{\mathrm{d}k}{2\omega}\, \varphi(k)\,, \\
(\omega - E)\, \varphi(k) &= \frac{\Sigma\, \alpha_i\, g_i}{\sqrt{4\pi}\sqrt{2\omega}}\,.
\end{aligned}
\tag{5}
$$

Thus, for real particle states

$$\varphi(k) = -\frac{\Sigma\, \alpha_i\, g_i}{\sqrt{4\pi}\sqrt{2\omega}\,(E - \omega + i\varepsilon)}\,, \tag{6}$$

in the continuous spectrum (S wave)

$$\varphi(k) = \frac{1}{k_0}\delta(k - k_0) - \frac{\Sigma\,\alpha_i g_i}{\sqrt{4\pi}\sqrt{2\omega}\,(E-\omega+i\varepsilon)} \tag{7}$$

(outgoing waves). Substituting (6) and (7) into (5) one gets

$$\begin{aligned} A_1\alpha_1 + B\alpha_2 &= 0\,, \\ B\alpha_1 + A_2\alpha_2 &= 0\,, \end{aligned} \tag{8}$$

or

$$\begin{aligned} A_1\alpha_1 + B\alpha_2 &= E_1\,, \\ B\alpha_1 + A_2\alpha_2 &= E_2\,. \end{aligned} \tag{9}$$

Here

$$A_i = m_i - E + g_{i(i)}^2\int\frac{k^2\,dk}{2\omega(E-\omega+i\varepsilon)}\,,$$

$$B = g_1 g_2\int\frac{k^2\,dk}{2\omega(E-\omega+i\varepsilon)}\,,$$

$$E_i = g_i\frac{k_0}{\sqrt{2E}}\,, \qquad E = \sqrt{m_0^2 + k_0^2}.$$

From (8) the necessary condition for finding real particle states is

$$A_1 A_2 - B^2 = 0\,. \tag{10}$$

For continuum states from (9)

$$\Sigma\,\alpha_i g_i = \frac{k_0}{\sqrt{2E}}\left[g_1^2(m_2 - E) + g_2^2(m_1 - E)\right](A_1 A_2 - B^2)^{-1}\,. \tag{11}$$

Next we distinguish four special cases:

i.) $m_1 = m_2 = m.$	ii.) $m_1 \neq m_2.$
a) $g_1^2 = -g_2^2,$	a) $g_1^2 = -g_2^2,$
b) $g_1^2 \neq -g_2^2,$	b) $g_1^2 \neq -g_2^2.$

In cases a) one of the particles is normal; the other is abnormal by prescription.

For the case i. a) (10) gives

$$(m - E)^2 = 0\,.$$

From (6) and (8) $\varphi = 0$, namely

$$\Sigma \alpha_i g_i = \alpha_1 g_1 - \frac{A_1}{B}\bigg|_{E=m} \cdot \alpha_2 g_2 = 0\,.$$

Actually, the eigenstate with (for the sake of definiteness)—$\delta_1 = \delta_2 = 1$

$$|E> = -\alpha \left(\psi^*_{v_1} + \frac{g_1}{g_2}\psi^*_{v_2}\right)|0> \tag{12}$$

possesses zero norm

$$<E|E> = 0\,.$$

Then from general principles it follows that a dipole ghost satisfying

$$(H - E)|D> = E|E>$$

must exist. Indeed, replacing

$$|D> = \left(\Sigma \beta_i \delta_i \psi^*_{v_i} + \psi^*_N \int \Phi(k)\, a^*(k)\, \mathrm{d}k\right)|0>$$

and (12), by $E = m$, one gets

$$-\frac{g_i}{\sqrt{4\pi}}\int \frac{\mathrm{d}k}{\sqrt{2\,\omega}}\,\Phi(k) = m\alpha_i\,,$$

$$(\omega - m)\,\Phi(k) = \frac{\Sigma \beta_i \alpha_i}{\sqrt{4\pi}\sqrt{2\omega}}\,.$$

From these

$$\Phi(k) = -\frac{\alpha m}{g_1 \displaystyle\int \frac{k^2\,\mathrm{d}k}{2\omega\,(m-\omega)}}\;\frac{1}{\sqrt{4\pi}\sqrt{2\omega}\,(m-\omega)}$$

and the state

$$|D> = \left[-\beta\psi^*_{v_1} + \left(\frac{\alpha m}{g_1 g_2 \displaystyle\int \frac{k^2\,\mathrm{d}k}{2\,\omega(m-\omega)}} - \frac{g_1}{g_2}\beta\right)\psi^*_{v_2} + \psi^*_N \int \Phi(k) a^*(k)\,\mathrm{d}k\right]|0>$$

Obviously $\langle E|D\rangle \neq 0$. Choosing β and

$$\gamma = \frac{\alpha}{g_1 \displaystyle\int \frac{k^2\,\mathrm{d}k}{2\,\omega(m-\omega)}}$$

one can appropriately prescribe

$$<D|D> = 0, \quad <E|D> = 1.$$

Similarly from (11) one can see that there is no scattering $S = 1$. These two interactions of equal strength distort the two bare eigenvectors into a complete dipole ghost situation. It is interesting that only the dipole ghost is surrounded by a meson cloud, otherwise a compensation has occurred which is, of course, trivial.

For the other cases it is most convenient to introduce the function of the complex variable

$$h(z) = -\frac{(m_1 - z)(m_2 - z)}{g_1^2(m_2 - z) + g_2^2(m_1 - z)} + \int \frac{k^2\,\mathrm{d}k}{2\,\omega(\omega - z)}\,. \tag{13}$$

From (8), (9), (10) and (11) by standard methods one can see that the dressed particle eigenvalues are given by

$$h(E) = 0\,, \tag{14}$$

and the N—Θ scattering S matrix is

$$S = 1 - \frac{i\pi k_0}{h(E + i\varepsilon)} = \frac{h(E - i\varepsilon)}{h(E + i\varepsilon)}\,. \tag{15}$$

Furthermore, the norms of the dressed particle eigenvectors are (for E real)

$$<E|E> = \frac{\delta_i|\alpha_i|^2\,[g_1^2(m_2 - E) + g_2^2(m_1 - E)]}{m_j - E + g_j^2\displaystyle\int \frac{k^2\,\mathrm{d}k}{2\omega(E - \omega)}} \cdot h'(z)|_{z=E}\,, \tag{16}$$

$$i \neq j\,, \quad \text{i.e.} \quad i = 1, j = 2 \quad \text{or} \quad i = 2, j = 1.$$

Now, without cut off, $h(z)$ with

$$G(z) = \int \frac{k^2\,\mathrm{d}k}{2\,\omega^3(\omega - z)}$$

i.b)

$$\begin{aligned} h(z) &= a + bz + z^2 G(z)\,, \\ a &= -\frac{m}{g_1^2 + g_2^2} + \int \frac{k^2\,\mathrm{d}k}{2\,\omega^2}\,, \\ b &= \frac{1}{g_1^2 + g_2^2} + \int \frac{k^2\,\mathrm{d}k}{2\,\omega^3}\,, \end{aligned} \tag{17}$$

ii.a)
$$h(z) = a + bz + cz^2 + z^2 G(z)\,,$$
$$a = \frac{m_1 m_2}{g_1^2(m_1 - m_2)} + \int \frac{k^2\,\mathrm{d}k}{2\,\omega^2}\,,$$
$$b = \frac{m_1 + m_2}{g_1^2(m_2 - m_1)} + \int \frac{k^2\,\mathrm{d}k}{2\,\omega^3}\,, \tag{18}$$
$$c = \frac{1}{g_1^2(m_1 - m_2)}\,.$$

ii.b)
$$h(z) = \frac{A}{E_0 - z} + a + bz + z^2 G(z)\,,$$
$$a = -\frac{g_1^2 m_1 + g_2^2 m_2}{(g_1^2 + g_2^2)^2} + \int \frac{k^2\,\mathrm{d}k}{2\,\omega^2}\,, \quad b = \frac{1}{g_1^2 + g_2^2} + \int \frac{k^2\,\mathrm{d}k}{2\,\omega^3}\,,$$
$$A = \frac{\big(g_1 g_2 (m_1 - m_2)\big)^2}{(g_1^2 + g_2^2)^3}\,, \quad E_0 = \frac{g_1^2 m_2 + g_2^2 m_1}{g_1^2 + g_2^2}\,. \tag{19}$$

All (real) constants a, b, c, A, E_0 are kept finite. After fixing them a model theory is defined. From the forms of b we see also that in cases i, ii, b) one or both of the V particles must be quantized with indefinite metric. $h(z)$ in the case of i.b) corresponds completely to that of the Lee model implying the same results, except, of course, that the two real states are produced now by two bare fields. The real particle state structure of ii.a) is the same as that of the Lee model; the structure of ii.b) corresponds to the model described in [6] implying three real V particle states.

3. Let us now turn to the problem of what type of limits reproduce the results of the original Lee model. From case i.a.) there is no transition, of course. (17) corresponds to the Lee model as it stands, (18) requires $c = 0$, (19) $A = 0$. On the other hand, the elementary composite particle case can be attributed to $|\alpha_i(\mathrm{E})|^2 = 0$, $i = 1$ or 2 with $h(E) = 0$. The connection and difference between this condition and $\det Z = 0$ can be found in [1,4]. Therefore let us study the behaviour of α_i. Since $\langle E|E\rangle$, $h'(E)$ are finite $|\alpha_i|^2$ is essentially determined from (16) by the factor

$$B_j = \frac{m_j - E + g_j^2 \displaystyle\int \frac{k^2\,\mathrm{d}k}{2\,\omega(E - \omega)}}{g_1^2(m_2 - E) + g_2^2(m_1 - E)}\,, \qquad \text{with} \qquad h(E) = 0\,.$$

Since in all cases we are discussing here a field theory with indefinite metric, $|\alpha_i|^2$ is not restricted to $0 \leqslant |\alpha_i|^2 \leqslant 1$. Actually, in general, it diverges. Anyway $|\alpha_i|^2$ vanishes if and only if $B_j = 0$. Using (13)

$$B_j = \frac{g_i^2(m_j - E)^2}{[g_1^2(m_2 - E) + g_2^2(m_1 - E)]^2} \, . \tag{20}$$

Here we are dealing with divergent quantities. Therefore, double care is necessary. These are the same as in the original model; the constants in equations (17), (18), (19) are kept finite, also the divergent integrals by cut off at ω; finally $\omega \to +\infty$. But then one has to treat the cases separately.

In case i.b)

$$B_j = \frac{g_i^2}{(g_1^2 + g_2^2)^2} = g_i^2 \left(b - \int \frac{k^2\,\mathrm{d}k}{2\,\omega^3} \right) .$$

Therefore, $|\alpha_i|^2$ vanishes if $g_i^2 = 0$; i.e. from the beginning there is no interaction at all with one of the particles. For case ii.a)

$$B_j = \frac{g_i^2(m_j - E)^2}{g_1^4(m_2 - m_1)^2} = c^2 g_i^2 (m_j - E)^2 .$$

From (18)

$$m_i = \frac{1}{2c} \left(- b + \frac{1}{g_i^2} + \int \frac{k^2\,\mathrm{d}k}{2\,\omega^3} \right),$$

$$\frac{1}{g_1^4} = \frac{1}{g_2^4} = \left(- b + \int \frac{k^2\,\mathrm{d}k}{2\,\omega^3} \right) - 4c \left(a - \int \frac{k^2\,\mathrm{d}k}{2\,\omega^2} \right),$$

therefore from $c = 0$, $|\alpha_i|^2$ vanishes for the particle with $g_i^2 \geq 0$. That is, the remaining one has to be quantized with indefinite metric.

For the last case, from (19)

$$g_1^2 = \frac{1 \pm \sqrt{1 - AD}}{2G}, \quad g_2^2 = \frac{1 \mp \sqrt{1 - AD}}{2G},$$

$$m_1 - m_2 = \frac{1}{G} \left(a - \int \frac{k^2\,\mathrm{d}k}{2\,\omega^2} \right) - E_0 ,$$

$$D = \frac{4}{(m_1 - m_2)^2\, G}, \quad G = b - \int \frac{k^2\,\mathrm{d}k}{2\,\omega^3} .$$

Then $A = 0$ leads to a particular g_i, let us say g_2, equal to zero, $|\alpha_2|^2 = 0$ as in the first case.

4. We have seen that the original Lee model without cut off can be generalized to the case of two V particles. The theory requires quantization with indefinite metric. Case i.a) is quite different from the Lee model; $m_1 = m_2$,

$g_1^2 \neq -g_2^2$ reproduces the results of the original model without requiring the condition

$$|\alpha_i(E_j)|^2 = |<0|\psi_{v_i}|E_j>|^2 = 0, \qquad j = 1, 2. \tag{21}$$

When (21) is required g_i must be zero. $m_1 \neq m_2$, $g_1^2 = -g_2^2$, $c = 0$ and $m_1 \neq m_2$, $g_1^2 \neq -g_2^2$, $A = 0$ correspond to the Lee model. In the first case $g_i^2 \neq 0$ at the beginning, in the second case one of the g_i is equal to zero giving $|\alpha_i(E_j)|^2 = 0$ for one i and all j in both cases.

It is interesting to notice the various behaviours of the different variants of the extended model. It can also be seen that the most general case (19) with its three real particle states can be produced from an appropriate model with three free V fields.

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МОДЕЛЬ ЛИ КАК Z-ПРЕДЕЛ

К. Л. НАДЬ

Резюме

Исследуются пределы в общей модели Ли без отреза, которые воспроизведут случай, элементарный + составной частицы, найденный в оригинальной модели.